# Package 'SPARRAfairness'

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Title Analysis of Differential Behaviour of SPARRA Score Across Demographic Groups

Maintainer James Liley <james.liley@durham.ac.uk></james.liley@durham.ac.uk>
Description The SPARRA risk score (Scottish Patients At Risk of admission and Re-Admission) estimates yearly risk of emergency hospital admission using electronic health records on a monthly basis for most of the Scottish population. This package imple ments a suite of functions used to analyse the behaviour and performance of the score, focusing particularly on differential performance over demographically-defined groups. It includes useful utility functions to plot receiver-operator-characteristic, precision-recall and calibration curves, draw stock human figures, estimate counterfactual quantities without the need to re-compute risk scores, to simulate a semi-realistic dataset.
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Encoding LITE-8

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LazyData true

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Imports cvAUC,ggplot2,ggrepel,mvtnorm

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Suggests knitr, rmarkdown

VignetteBuilder knitr

NeedsCompilation no

Author Ioanna Thoma [aut] (<a href="https://orcid.org/0000-0001-6928-2198">https://orcid.org/0000-0001-6928-2198</a>),

Catalina Vallejos [ctb] (<a href="https://orcid.org/0000-0003-2211-233X">https://orcid.org/0000-0003-2211-233X</a>),

Jill Ireland [ctb] (<a href="https://orcid.org/0000-0003-3578-4477">https://orcid.org/0000-0003-3578-4477</a>),

James Liley [cre, aut] (<a href="https://orcid.org/0000-0002-0049-8238">https://orcid.org/0000-0002-0049-8238</a>)

# **R** topics documented:

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ab() Shorthand to draw a red x-y line

# Description

ab

ab() Shorthand to draw a red x-y line

# Usage

ab(...)

# Arguments

.. passed to abline()

## Value

No return value, draws a figure

adjusted\_fdr 3

adjusted_fdr
--------------

#### **Description**

Estimates false discovery rate P(target=FALSE|score>cutoff,group=g) 'adjusted' for some category.

#### Usage

```
adjusted_fdr(
   scores,
   target,
   category,
   group1,
   group2,
   cutoffs = seq(min(scores, na.rm = TRUE), max(scores, na.rm = TRUE), length = 100),
   nboot = 100
)
```

### **Arguments**

```
scores vector of risk scores

target vector of values of target (which risk score aims to predict)

category vector of categories

group1 indices of group 1

group2 indices of group 2

cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)

nboot number of bootstrap samples for standard error
```

### Details

```
Namely, calculates sum\ (\ P(target=FALSE|score>cutoff, category=c, group=g)P(category=c|score<cutoff)\ ) where the sum is over categories c.
```

### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

```
# See vignette
```

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#### **Description**

Estimates false omission rate P(target=TRUE|score<=cutoff,group=g) 'adjusted' for some category.

#### Usage

```
adjusted_for(
   scores,
   target,
   category,
   group1,
   group2,
   cutoffs = seq(min(scores, na.rm = TRUE), max(scores, na.rm = TRUE), length = 100),
   nboot = 100
)
```

#### **Arguments**

```
scores vector of risk scores

target vector of values of target (which risk score aims to predict)

category vector of categories

group1 indices of group 1

group2 indices of group 2

cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)

nboot number of bootstrap samples for standard error
```

### Details

```
Namely, calculates sum\ (\ P(target=TRUE|score<=cutoff, category=c, group=g)P(category=c|score<cutoff)\ ) where the sum is over categories c.
```

### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

```
# See vignette
```

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all\_data

All data for fairness measures

### Description

This object contains all data from analysis of fairness measures in SPARRA v3 and v4.

#### Usage

```
all_data
```

#### **Format**

An object of class list of length 1260.

cal\_2panel

cal\_2panel Draws calibration curves (with legend) with a second panel underneath showing predicted differences.

#### **Description**

cal\_2panel Draws calibration curves (with legend) with a second panel underneath showing predicted differences.

### Usage

```
cal_2panel(
  cals,
  labels,
  col = 1:length(cals),
  lty = rep(1, length(col)),
  xy_lty = 2,
  xy_col = phs_colours("phs-magenta"),
  ci_col = rep(NA, length(col)),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

#### **Arguments**

cals	list of calibration objects, output from getcal().
labels	labels to use in legend
col	line colours
lty	line type, defaults to 1
xy_lty	line type for x-y line, defaults to 2 (dashed)
xy_col	line colour for x-y line, defaults to phs-magenta

6 counterfactual\_yhat

ci\_col colours to draw confidence intervals on lower panel; NA to not draw.

highlight if non-null, highlight a particular value

mar\_scale scale bottom and left margins by this amount. Also scales legend.

yrange\_lower y range for lower plot. If NULL, generates automatically

. . . other parameters passed to legend()

#### Value

No return value, draws a figure

#### **Examples**

# See vignette

 $counterfactual\_yhat \qquad \textit{counterfactual\_yhat}$ 

#### **Description**

Estimation of counterfactual quantities by resampling.

#### Usage

```
counterfactual_yhat(dat, X, x = NULL, G, g, gdash, excl = NULL, n = NULL)
```

## Arguments

dat	data frame containing variables in X, Yhat, G, excl. Variables in U are assumed to be colnames(dat)\(X U Yhat U G U excl)
X	set of variables and values on which to 'condition'; essentially we assume that any causal pathway from $G$ to Yhat is through $\boldsymbol{X}$
X	values of variables $\boldsymbol{X}$ on which to condition; can be null in which case we use marginal distribution of $\boldsymbol{X}$
G	grouping variable, usually a sensitive attribute
g	conditioned value of g
gdash	counterfactual value of g
excl	variable names to exclude from U
n	number of samples; if NULL return all

#### **Details**

Counterfactual fairness is with respect to the causal graph:

where

G=group (usually sensitive attribute); Yhat=outcome; X=set of variables through which G can act on Yhat, U=set of background variables;

We want the counterfactual Yhat\_g' <- G |X=x,G=g| (or alternatively Yhat\_g' <- G |G=g|).

counterfactual\_yhat 7

This can be interpreted as: the distribution of values of Yhat amongst individuals whose values of U are distributed as though they were in group G=g (and, optionally, had values X=x, but whose value of G is g'

Essentially, comparison of the counterfactual quantity above to the conditional YhatlG=g isolates the difference in Yhat due to the effect of G on Yhat through X, removing any effect due to different distributions of U due to different values of G.

To estimate Y'=Yhat\_g' <- G | G=g, we need to

- 1. Compute  $U'\sim(U|G=g)$
- 2. Compute the distribution X' as  $X' \sim (X|U \sim U', G=g')$
- 3. Sample Y'~(YhatlX~X',U~U')

To estimate Y'=Yhat\_g' <- G |X=x, G=g, we need to

- 1. Compute  $U'\sim(U|G=g,X=x)$
- 2. Compute the distribution X' as  $X' \sim (X|U \sim U', G=g')$
- 3. Sample Y'~(YhatlX~X',U~U')

This function approximates this samplying procedure as follows

- 1. Look at individuals with G=g (and optionally X=x)
- 2. Find the values of U for these individuals
- 3. Find a second set of individuals with the same values of U but for whom G=g'
- 4. Return the indices of these individuals

The values of Yhat for these individuals constitute a sample from the desired counterfactual.

#### Value

indices representing sample(s) from counterfactual Yhat\_g' <- G |X=x,G=g

```
set.seed(23173)
N=10000
# Background variables sampler
background_U=function(n) runif(n) \# U~U(0,1)
# Structural equations
struct_G=function(u,n) rbinom(n,1,prob=u) # G|U=u ~ Bern(u)
struct\_X=function(u,g,n) \ rbinom(n,1,prob=u*(0.5 + 0.5*g)) \ \# \ X \mid U=u,G=g \ ^{\circ} \ Bern(u(1+g)/2)
struct_Yhat=function(u,x,n) (runif(n,0,x) + runif(n,0,u))/2 # Yhat|X,N ~ (U(0,X) + U(0,U))/2
# To see that the counterfactual 'isolates' the difference in Yhat due to the
  causal pathway from G to Yhat through X, change the definition of struct_G to
  struct_G=function(u,n) rbinom(n,1,prob=1/2) # G|U=u ~ Bern(1/2)
#
\# so the posterior of U|G=g does not depend on g. Note that, with this definition, the
\# counterfactual Yhat_{G<01}|G=1 coincides with the conditional Yhat|G=0, since
  the counterfactual G<-1 is equivalent to just conditioning on G=1.
# By contrast, if we change struct_G back to its original definition, but
# change the definition of struct_Yhat to
```

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```
# struct_Yhat=function(u,x,n) (runif(n,0,1) + runif(n,0,u))/2 # Yhat|X,N \sim (U(0,1) + U(0,U))/2
# so Yhat depends on G only through the change in posterior of U from changing g,
# the counterfactual Yhat_{G<01}|_{G=1} coincides with the conditional Yhat_{G=1}.
# Sample from complete causal model
U=background_U(N)
G=struct_G(U,N)
X=struct_X(U,G,N)
Yhat=struct_Yhat(U,X,N)
dat=data.frame(U,G,X,Yhat)
# True counterfactual Yhat_{G <- 0}|G=1</pre>
w1=which(dat$G==1)
n1=length(w1)
UG1=dat$U[w1] # This is U|G=1
XG1=struct_X(UG1,rep(0,n1),n1)
YhatG1=struct_Yhat(UG1,XG1,n1)
# Estimated counterfactual Yhat_{G <- 0}|G=1</pre>
ind_G1=counterfactual_yhat(dat,X="X",G="G",g = 1, gdash = 0)
YhatG1_resample=dat$Yhat[ind_G1]
# True counterfactual Yhat_{G <- 0}|G=1,X=1</pre>
w11=which(dat$G==1 & dat$X==1)
n11=length(w11)
UG1X1=dat$U[w11] # This is U|G=1,X=1
XG1X1=struct_X(UG1X1,rep(0,n11),n11)
YhatG1X1=struct_Yhat(UG1X1,XG1X1,n11)
# Estimated counterfactual Yhat_{G <- 0}|G=1
ind_G1X1=counterfactual_yhat(dat,X="X",G="G",g = 1, gdash = 0,x=1)
YhatG1X1_resample=dat$Yhat[ind_G1X1]
# Compare CDFs
x=seq(0,1,length=1000)
oldpar = par(mfrow=c(1,2))
plot(0, type="n", xlim=c(0,1), ylim=c(0,1), xlab="Value",
     ylab=expression(paste("Prop. ",hat('Y')," < x")))</pre>
lines(x,ecdf(dat\$Yhat)(x),col="black") # Unconditional CDF of Yhat
lines(x,ecdf(dat$Yhat[which(dat$G==1)])(x),col="red") # Yhat|G=1
lines(x,ecdf(dat\$Yhat[which(dat\$G==\emptyset)])(x),col="blue") \ \# \ Yhat|G=\emptyset
# True counterfactual Yhat_{G <- 0}|G=1</pre>
lines(x,ecdf(YhatG1)(x),col="blue",lty=2)
# Estimated counterfactual Yhat_{G <- 0}|G=1</pre>
lines(x,ecdf(YhatG1_resample)(x),col="blue",lty=3)
```

dat2mat 9

```
legend("bottomright",
       c(expression(paste(hat('Y'))),
         expression(paste(hat('Y'),"|G=1")),
         expression(paste(hat('Y'),"|G=0")),
         expression(paste(hat(Y)[G %<-% 0],"|G=1 (true)")),</pre>
         expression(paste(hat(Y)[G %<-% 0],"|G=1 (est.)"))),</pre>
       col=c("black","red","blue","blue","blue"),
       lty=c(1,1,1,2,3),
       cex=0.5)
plot(0, type="n", xlim=c(0,1), ylim=c(0,1), xlab="Value",
     ylab=expression(paste("Prop. ",hat('Y')," < x")))</pre>
lines(x,ecdf(dat$Yhat[which(dat$X==1)])(x),col="black") # CDF of Yhat|X=1
lines(x,ecdf(dat\$Yhat[which(dat\$G==1 \& dat\$X==1)])(x),col="red") \ \# \ Yhat|G=1,X=1
lines(x,ecdf(dat\$Yhat[which(dat\$G==0 \& dat\$X==1)])(x),col="blue") \# Yhat|G=0,X=1
# True counterfactual Yhat_{G <- 0}|G=1,X=1</pre>
lines(x,ecdf(YhatG1X1)(x),col="blue",lty=2)
# Estimated counterfactual Yhat_{G <- 0}|G=1,X=1</pre>
lines(x,ecdf(YhatG1X1_resample)(x),col="blue",lty=3)
legend("bottomright",
       c(expression(paste(hat('Y|X=1'))),
         expression(paste(hat('Y'),"|G=1,X=1")),
         expression(paste(hat('Y'),"|G=0,X=1")),
         expression(paste(hat(Y)[G %<-% 0],"|G=1,X=1 (true)")),</pre>
         expression(paste(hat(Y)[G %<-% 0],"|G=1,X=1 (est.)"))),</pre>
       col=c("black","red","blue","blue","blue"),
       1ty=c(1,1,1,2,3),
       cex=0.5)
# In both plots, the estimated counterfactual CDF closely matches the CDF of the
# true counterfactual.
# Restore parameters
par(oldpar)
```

dat2mat

dat2mat

#### **Description**

Generates matrices for decomposition of admission type which can be used in plot\_decomp

```
dat2mat(dat, score, group1, group2, nquant = 20, cats = unique(dat$reason))
```

10 decomposition\_matrix

#### **Arguments**

data frame with population data, such as output from sim\_pop\_data. Must in-

clude a column reason

score risk scores corresponding to dat

group1 indices for group 1 group2 indices for group 2

nquant number of quantiles of code to use; default 20

cats vector of strings giving names of admission categories; default the unique values

in dat\$reason. Can include NAs.

#### **Details**

Generates two matrices with the following specifications: Each matrix corresponds to one group Columns are named with the admission types to be plotted. Any admission types including the string 'Died' are counted as deaths If the matrix has N rows, these are interpreted as corresponding to N score quantiles The (i,j)th entry of the matrix is the number of people admitted for reason i with a score greater than or equal to (j-1)/N and less than (j/N) who are in that group

#### Value

list with two objects matrix1 and matrix2 giving output matrices

#### **Examples**

# See vignette

#### **Description**

Matrix giving frequency of admission types for various groups at various score thresholds. Row names are of the form  $vX_Y_qZ$ , where X is version (3 or 4), Y is cohort (e.g., all, over 65, island postcode) and Z is quantile (1-20) of score. Column names are cause of admission or cause of death.

#### Usage

 $decomposition\_matrix$ 

### **Format**

An object of class data. frame with 520 rows and 41 columns.

demographic\_parity 11

 $demographic\_parity$   $demographic\_parity$ 

#### **Description**

Estimates demographic parity for a risk score (essentially cumulative distribution function)

#### Usage

```
demographic_parity(
   scores,
   group1,
   group2,
   cutoffs = seq(min(scores, na.rm = TRUE), max(scores, na.rm = TRUE), length = 100)
)
```

#### **Arguments**

scores vector of risk scores
group1 indices of group 1
group2 indices of group 2
cutoffs score cutoffs at which to estimate DP (default 100 evenly-spaced)

#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the proportion of scores in group g which are less than or equal to the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

### **Examples**

```
# See vignette
```

drawperson

drawperson

### Description

Draws a simple stock image of a person.

```
drawperson(
  xloc = 0,
  yloc = 0,
  scale = 1,
  headsize = 0.16,
  headangle = pi/8,
  headloc = 0.5,
```

12 drawperson

```
necklength = 0.1,
shoulderwidth = 0.1,
shouldersize = 0.05,
armlength = 0.4,
armangle = 7 * pi/8,
armwidth = 0.08,
leglength = 0.5,
legangle = 9 * pi/10,
legwidth = 0.15,
torsolength = 0.4,
...
)
```

#### **Arguments**

xloc x-axis offset from origin
yloc y-axis offset from origin
scale scale upwards from 1x1 box

headsize head size

headangle half angle of neck in terms of head

headloc location of centre of head relative to origin with scale 1

necklength neck length shoulder width

shouldersize size radius of arc for shoulder

armlength arm length

armangle angle of arm from horizontal

armwidth width of arm leglength leg length

legangle angle of leg from horizontal

legwidth width of leg torsolength length of torso

... other parameters passed to polygon()

#### **Details**

Draws a figure at a particular location. With defaults, has centre at origin and fits in 1x1 box. Dimensions customisable

#### Value

invisibly returns co-ordinates

```
\label{eq:plot_norm} \begin{split} & plot(\emptyset, xlim=c(-1,1), ylim=c(-1,1), type="n") \\ & drawperson(\emptyset,\emptyset,1,col="yellow",border="red",lwd=3,lty=2) \end{split}
```

drawprop 13

### **Description**

Illustrates a proportion as a set of people who are blue rather than red.

### Usage

```
drawprop(prop, ci, nxy = 10, col1 = "maroon", col2 = "lightblue", ...)
```

#### **Arguments**

prop	the proportion to illustrate
ci	half the 95% CI width of the proportion.
nxy	illustrate on an n x n grid
col1	colour to put the 'in' proportion
col2	the other colour
	passed to 'plot'

#### **Details**

Why anyone would want to think about a proportion this way is beyond the understanding of the authors, but the people have spoken.

### Value

No return value, draws a figure

#### **Examples**

```
# See vignette
```

|--|

## Description

For a given category (e.g., 'male', 'over 65') considers

- 1. all admissions for people in that category
- 2. all admissions for people in that category for which the SPARRA score was less than some threshold (e.g., false negatives

14 for\_breakdown

#### Usage

```
for_breakdown(
  decomp_table,
  group,
  threshold,
  inc_died = TRUE,
  ldiff = 0.005,
  ci = 0.95
)
```

#### **Arguments**

decomp\_table matrix for group; see specification in description

group name of group

threshold cutoff, rounded to nearest 0.05

inc\_died set to TRUE to include a second panel showing 'death' type admissions

ldiff specifically label points this far from xy line

ci set to a value <1 to draw confidence intervals at that value, or FALSE to not

draw confidence intervals.

#### Details

For each of these groups, we consider the breakdown of medical admission types. We then plot the frequency of admission types in group 1 against the difference in frequencies between group 1 and group 2 (group 2 minus group 1). An admission type which is relatively more common in group (1) indicates that, in the relevant category, the admission type tends to be associated with higher SPARRA scores (and is in a sense easier to predict). Such admission types will correspond to points below the line y=0. Admission types which are relatively more common in group 2 correspond to those which are relatively harder to predict. These correspond to points above the line y=0 Since points are close together, only those greater than a certain distance from 0 are marked.

Takes as an argument a matrix in which The matrix shows only data for the group in question Columns are named with the admission types to be plotted. Any admission types including the string 'Died' are counted as deaths If the matrix has N rows, these are interpreted as corresponding to N score quantiles in increasing order. The (i,j)th entry of the matrix is the number of people admitted for reason i with a score greater than or equal to (j-1)/N and less than (j/N) who are in that group

#### Value

ggplot figure (invisible)

```
# See vignette
```

getcal 15

getcal getcal()

### Description

Produces a set of points for a calibration plot.

### Usage

```
getcal(
   y,
   ypred,
   n = 10,
   kernel = FALSE,
   kernel_sd = 0.05,
   alpha = 0.05,
   c0 = 0,
   c2 = 0.1
)
```

## Arguments

У	class labels, 0/1 or logical
ypred	predictions Pr(Y=1), numeric vector
n	number of subintervals/points
kernel	set to TRUE to use kernel method
kernel_sd	kernel width for kernel method; see above
alpha	return a pointwise confidence envolope for conservative 1-alpha confidence interval
c0	for computing maximum bias; assume true covariance function is of the form $a0+a1x+a2x^2$ , with $ a0 < c0$ , $ a2 < c2$ (c1 does not matter)
c2	for computing maximum bias; assume true covariance function is of the form $a0+ a1x + a2x^2$ , with $ a0  < c0$ , $ a2  < c2$ (c1 does not matter)

#### **Details**

Uses either a binning method or a kernel method to determine height of points.

In both methods, considers n equally spaced subintervals of (0,1)

#### Value

a list with components x (expected calibration), y (observed calibration), n (number of samples in bins, if relevant), lower/upper (confidence interval on y)

```
# See vignette
```

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getprc	getprc()		

### **Description**

Comprehensive plotting function for precision-recall curve. Also calculates AUPRC and standard error.

### Usage

```
getprc(y, ypred, cv = NULL, res = 100)
```

#### **Arguments**

у	class labels, 0/1 or logical
ypred	predictions Pr(Y=1), numeric vector
CV	cross-validation fold assignments, if relevant. Changes estimate of standard error.
res	resolution. Returns this many equally-spaced points along the curve. Set res to null to return all points.

#### **Details**

Rather than returning points corresponding to every cutoff, only returns a representative sample of equally-spaced points along the curve.

Does not plot anything. Object can be plotted in a default way.

### Value

list containing: ppv, ppv for res points in every cv fold; sens, sensitivity for res points in every cv fold; auc, areas under the curve for each fold and average (note length is 1 greater than number of CV folds); se, standard error for AUC in each fold and standard error for average auc (note length is 1 greater than number of CV folds)

#### **Examples**

```
# See vignette
```

getroc	getroc() Comprehensive plotting function for receiver-operator characteristic curve. Also calculates AUROC and standard error.

### Description

Rather than returning points corresponding to every cutoff, only returns a representative sample of equally-spaced points along the curve.

```
getroc(y, ypred, cv = NULL, res = 100)
```

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### **Arguments**

У	class labels, 0/1 or logical
ypred	predictions $Pr(Y=1)$ , numeric vector
CV	cross-validation fold assignments, if relevant. Changes estimate of standard error.
res	resolution. Returns this many equally-spaced points along the curve. Set res to null to return all points.

#### **Details**

SE of AUROC with no CV structure is from Hanley and McNeil 1982. SE of AUROC with CV folds is from LeDell et al 2012

Does not plot anything. Object can be plotted in a default way.

#### Value

list containing: spec, specificity for res points in every cv fold; sens, sensitivity for res points in every cv fold; auc, areas under the curve for each fold and average (note length is 1 greater than number of CV folds); se, standard error for AUC in each fold and standard error for average auc (note length is 1 greater than number of CV folds)

#### **Examples**

```
# See vignette
```

groupmetric\_2panel

groupmetric\_2panel Draws plots of a group fairness metric with a second panel underneath

### Description

groupmetric\_2panel Draws plots of a group fairness metric with a second panel underneath

```
groupmetric_2panel(
  objs,
  labels,
  col = 1:length(objs),
  lty = rep(1, length(col)),
  yrange = NULL,
  ci_col = 1:length(objs),
  highlight = NULL,
  logscale = FALSE,
  mar_scale = 1,
  lpos = NULL,
  yrange_lower = NULL,
  ...
)
```

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#### **Arguments**

objs	list of fairness objects. Each should contain sub-objects 'x', 'y' and 'ci', which specify x and y values and half-widths of confidence intervals around y.
labels	labels to use in legend
col	line colours
lty	line type, defaults to 1
yrange	limit of y axis; defaults to 0,1
ci_col	confidence envelope colours. These will be transparent.
highlight	if non-null, draw a point at a particular cutoff
logscale	if TRUE, draw with log-scale.
mar_scale	scale bottom and left margins by this amount. Also scales legend.
lpos	legend position
yrange_lower	y range for lower plot. If NULL, generates automatically

#### Value

. . .

No return value, draws a figure

#### **Examples**

```
# See vignette
```

### Description

Estimates group fairness metric according to a specification vector of the form

other parameters passed to legend()

### Usage

```
group_fairness(
  specs,
  scores,
  target,
  group1,
  group2,
  cutoffs = seq(min(scores, na.rm = TRUE), max(scores, na.rm = TRUE), length = 100)
)
```

### Arguments

```
specs specification vector; see description
scores vector of risk scores
target vector of values of target (which risk score aims to predict)
group1 indices of group 1
group2 indices of group 2
cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)
```

integral 19

#### **Details**

```
c(A1,B1,C1,A2,B2,C2)
```

encoding a probability

P(A1,B1,C1|A2,B2,C2)

where

A1/A2 are events coded by 1:'score>= cutoff'; 0: 'score<cutoff' and NA: 1/TRUE B1/B2 are events coded by 1:'target=TRUE'; 0: 'target=FALSE' and NA: 1/TRUE C1/C2 are events coded by 1:'group=g'; and NA: 1/TRUE

For example, specs=c(NA,1,NA,0,NA,1) would encode false omission rate:

P(target=TRUE|score<cutoff,group=g)

#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

### **Examples**

# See vignette

integral

integral() Quick form for trapezoidal integration over range of x

### Description

integral() Quick form for trapezoidal integration over range of x

### Usage

```
integral(x, y = NULL)
```

#### **Arguments**

x x co-ordinates, or nx2 matrix of points

y y co-ordinates

### Value

trapezoidal estimate of integral of the xth value of y over range of x.

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logistic

Logistic

```
Description
```

```
Logistic function: 1/(1+exp(-x))
```

### Usage

```
logistic(x)
```

### Arguments

Х

argument

### Value

```
value of logistic(x)
```

## **Examples**

```
# Plot
x=seq(-5,5,length=1000)
plot(x,logistic(x),type="l")
```

logit

Logit

### **Description**

```
Logit function: -\log((1/x)-1)
```

# Usage

```
logit(x)
```

### **Arguments**

Х

argument

### Value

```
value of logit(x); na if x is outside (0,1)
```

```
# Plot
x=seq(0,1,length=100)
plot(x,logit(x),type="1")

# Logit and logistic are inverses
x=seq(-5,5,length=1000)
plot(x,logit(logistic(x)),type="1")
```

phs\_colours 21

phs\_colours phs\_colours

#### **Description**

Copied from github, "Public-Health-Scotland/phsstyles". Public Health Scotland colour scheme. Internal function.

## Usage

```
phs_colours(colourname = NULL, keep_names = FALSE)
```

#### **Arguments**

colourname name of colour; usually something like phs-blue. If NULL returns all colours. keep\_names keep names of colours in return list. Defaults to false.

#### Value

vector of colours, optionally with names.

plot.sparraCAL

Plot function for class sparraCAL

### Description

Plot function for class sparraCAL

#### Usage

```
## $3 method for class 'sparraCAL'
plot(
    x,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    add = FALSE,
    add_xy_line = TRUE,
    ...
)
```

### **Arguments**

```
x output from getcal()
cols colour to draw lines
add set to FALSE to add to existing plot
add_xy_line set to TRUE to draw an X-Y reference line.
... passed to lines()
```

22 plot.sparraPRC

#### Value

No return value, draws a figure

### **Examples**

```
# See vignette
```

plot.sparraPRC

Plot function for class above

## Description

Plot function for class above

### Usage

```
## $3 method for class 'sparraPRC'
plot(
    x,
    addauc = FALSE,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    ...
)
```

# Arguments

```
    x output from getprc()
    addauc set to TRUE to add text to the plot showing the (mean) AUC and SE.
    cols colour to draw lines
    passed to plot()
```

#### Value

No return value, draws a figure

```
# See vignette
```

plot.sparraROC 23

plot.sparraROC

Plot function for class sparraROC

### Description

Plot function for class sparraROC

#### Usage

```
## S3 method for class 'sparraROC'
plot(
    x,
    addauc = FALSE,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    ...
)
```

## Arguments

```
x output from getroc()
addauc set to TRUE to add text to the plot showing the (mean) AUC and SE.
cols colour to draw lines
... passed to plot()
```

### Value

No return value, draws a figure

### **Examples**

```
# See vignette
```

plot\_decomp

plot\_decomp

#### **Description**

Plots a bar graph of decomposition of FORP by cause of admission

```
plot_decomp(
  decomp1,
  decomp2,
  threshold,
  labels,
  inc_died = TRUE,
  mar_scale = 1
```

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#### **Arguments**

decomp1 matrix for first group; see specification in description
decomp2 matrix for second group; see specification in description
threshold score threshold to plot (between 0 and 1)
labels labels for group 1 and group 2

inc\_died set to TRUE to include a second panel showing 'death' type admissions

mar\_scale scale margins by this amount. Also scales legend.

#### **Details**

Takes two matrices as input with the following specifications: Each matrix corresponds to one group Columns are named with the admission types to be plotted. Any admission types including the string 'Died' are counted as deaths If the matrix has N rows, these are interpreted as corresponding to N score quantiles in increasing order. The (i,j)th entry of the matrix is the number of people admitted for reason i with a score greater than or equal to (j-1)/N and less than (j/N) who are in that group

#### Value

No return value, draws a figure

#### **Examples**

```
# See vignette
```

prc_2panel	prc_2panel Draws a PRC curve (with legend) with a second panel
	underneath showing precision difference.

### Description

prc\_2panel Draws a PRC curve (with legend) with a second panel underneath showing precision difference.

```
prc_2panel(
  prcs,
  labels,
  col = 1:length(prcs),
  lty = rep(1, length(col)),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

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### Arguments

list of sparraPRC objects. If of length 1, splits into folds
labels labels to use in legend

col line colours

lty line type, defaults to 1

highlight if non-null, draw a point at a particular cutoff

mar\_scale scale bottom and left margins by this amount. Also scales legend.

yrange\_lower y range for lower plot. If NULL, generates automatically

... other parameters passed to legend()

#### Value

No return value, draws a figure

#### **Examples**

```
# See vignette
```

### **Description**

roc\_2panel Draws a ROC curve (with legend) with a second panel underneath showing sensitivity difference.

#### Usage

```
roc_2panel(
  rocs,
  labels,
  col = 1:length(rocs),
  lty = rep(1, length(col)),
  xy_lty = 2,
  xy_col = phs_colours("phs-magenta"),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

### Arguments

rocs list of sparraROC objects (if one object, plots folds separately)
labels labels to use in legend
col line colours
lty line type, defaults to 1

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```
xy_lty line type for x-y line, defaults to 2 (dashed)
xy_col line colour for x-y line, defaults to red
highlight if non-null, add a point at this cutoff
mar_scale scale bottom and left margins by this amount. Also scales legend.
yrange_lower y range for lower plot. If NULL, generates automatically
other parameters passed to legend()
```

#### Value

No return value, draws a figure

### **Examples**

```
# See vignette
```

### **Description**

Simulates population data with a reasonably realistic joint distribution

## Usage

```
sim_pop_data(
  npop,
  coef_adjust = 4,
  offset = 1,
  vcor = NULL,
  coefs = c(2, 1, 0, 5, 3, 0, 0),
  seed = 12345,
  incl_id = TRUE,
  incl_reason = TRUE
)
```

# Arguments

npop	population size
coef_adjust	inverse scale for all (true) coefficients (default 4): lower means that hospital admissions are more predictable from covariates.
offset	offset for logistic model (default 1): higher means a lower overall prevalence of admission
vcor	a valid 5x5 correlation matrix (default NULL), giving correlation between variables. If 'NULL', values roughly represents realistic data.
coefs	coefficients of age, male sex, non-white ethnicity, number of previous admissions, and deprivation decile on hospital admissions, Default (2,1,0,5,3). Divided through by coef_adjust.
seed	random seed (default 12345)
incl_id	include an ID column (default TRUE)
incl_reason	include a column indicating reason for admission.

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#### **Details**

Simulates data for a range of people for the variables

- Age (age)
- Sex (sexM; 1 if male)
- Race/ethnicity (raceNW: 1 if non-white ethnicity)
- Number of previous hospital admissions (PrevAdm)
- Deprivation decile (SIMD: 1 most deprived, 10 least deprived. NOTE opposite to English IMD)
- Urban-rural residence status (urban\_rural: 1 for rural)
- Mainland-island residence status (mainland\_island: 1 for island)
- Hospital admission (target: 1/TRUE if admitted to hospital in year following prediction date)

Can optionally add an ID column.

Optionally includes an admission reason for samples with target=1. These admission reasons roughly correspond to the first letters of ICD10 categories, and can either correspond to an admission or death. Admission reasons are simulated with a non-constant multinomial distribution which varies across age/sex/ethnicity/urban-rural/mainland-island/PrevAdm values in a randomly- chosen way. The distributions of admission reasons are *not* however chosen to reflect real distributions, nor are systematic changes in commonality of admission types across categories intended to appear realistic.

#### Value

data frame with realistic values.

#### **Examples**

```
# Simulate data
dat=sim_pop_data(10000)
cor(dat[,1:7])
```

# See vignette

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