# Package 'SPARRAfairness'

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<b>Title</b> Analysis of Differential Performance of SPARRA Score Across Demographic Group
<b>Version</b> 0.0.0.1

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**Description** The SPARRA risk score (Scottish Patients At Risk of admission and Re-Admission) estimates yearly risk of emergency hospital admission using elec-

tronic health records on a monthly basis for most of the Scottish population. This package implements a suite of functions used to analyse performance of the score, focusing particularly on differential performance over demographically-defined groups. Includes useful utility functions to plot ROC, PRC and calibration curves, draw stock human figures, and estimate counterfactual quantities without the need to re-compute risk scores.

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Encoding UTF-8

LazyData true

Depends R (>= 3.5.0), stats, graphics, grDevices, matrixStats, mvtnorm, cvAUC, ranger

Roxygen list(markdown = TRUE)

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Suggests knitr, rmarkdown

VignetteBuilder knitr

NeedsCompilation no

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# Description

ab() Shorthand to draw a red x-y line

# Usage

ab(...)

## Arguments

... passed to abline()

 $adjusted\_fdr \qquad \qquad \textit{adjusted\_fdr}$ 

## Description

 $Estimates\ false\ discovery\ rate\ P(target=FALSE|score>cutoff,group=g)\ 'adjusted'\ for\ some\ category.$ 

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#### Usage

```
adjusted_fdr(
    scores,
    target,
    category,
    group1,
    group2,
    cutoffs = seq(min(scores, na.rm = T), max(scores, na.rm = T), length = 100),
    nboot = 100
)
```

#### **Arguments**

```
scores vector of risk scores

target vector of values of target (which risk score aims to predict)

category vector of categories

group1 indices of group 1

group2 indices of group 2

cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)

nboot number of bootstrap samples for standard error
```

#### **Details**

```
Namely, calculates sum\ (\ P(target=FALSE|score>cutoff, category=c, group=g)P(category=c|score<cutoff)\ ) where the sum is over categories c.
```

#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

#### **Examples**

```
# See vignette
```

## Description

Estimates false omission rate P(target=TRUE|score<=cutoff,group=g) 'adjusted' for some category.

4 all\_data

#### Usage

```
adjusted_for(
   scores,
   target,
   category,
   group1,
   group2,
   cutoffs = seq(min(scores, na.rm = T), max(scores, na.rm = T), length = 100),
   nboot = 100
)
```

#### **Arguments**

scores vector of risk scores

target vector of values of target (which risk score aims to predict)

category vector of categories

group1 indices of group 1

group2 indices of group 2

cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)

nboot number of bootstrap samples for standard error

#### **Details**

```
Namely, calculates sum\ (\ P(target=TRUE|score<=cutoff, category=c, group=g)P(category=c|score<cutoff)\ ) where the sum is over categories c.
```

#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

## **Examples**

```
# See vignette
```

all\_data

All data for fairness measures

#### **Description**

This object contains all data from analysis of fairness measures in SPARRA v3 and v4.

## Usage

all\_data

#### **Format**

An object of class list of length 1260.

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cal_2panel	cal_2panel Draws calibration curves (with legend) with a second
	panel underneath showing predicted differences.

## Description

cal\_2panel Draws calibration curves (with legend) with a second panel underneath showing predicted differences.

## Usage

```
cal_2panel(
  cals,
  labels,
  col = 1:length(cals),
  lty = rep(1, length(col)),
  xy_lty = 2,
  xy_col = phs_colours("phs-magenta"),
  ci_col = rep(NA, length(col)),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

## Arguments

cals	list of calibration objects, output from getcal().
labels	labels to use in legend
col	line colours
lty	line type, defaults to 1
xy_lty	line type for x-y line, defaults to 2 (dashed)
xy_col	line colour for x-y line, defaults to phs-magenta
ci_col	colours to draw confidence intervals on lower panel; NA to not draw.
highlight	if non-null, highlight a particular value
mar_scale	scale bottom and left margins by this amount. Also scales legend.
yrange_lower	y range for lower plot. If NULL, generates automatically
	other parameters passed to legend()

```
# See vignette
```

6 counterfactual\_yhat

## **Description**

Estimation of counterfactual quantities by resampling.

#### Usage

```
counterfactual_yhat(dat, X, x = NULL, G, g, gdash, excl = NULL, n = NULL)
```

#### **Arguments**

dat	data frame containing variables in X, Yhat, G, excl. Variables in U are assumed to be colnames(dat)\(X U Yhat U G U excl)
X	set of variables and values on which to 'condition'; essentially we assume that any causal pathway from $G$ to Yhat is through $X$
x	values of variables X on which to condition; can be null in which case we use marginal distribution of X
G	grouping variable, usually a sensitive attribute
g	conditioned value of g
gdash	counterfactual value of g
excl	variable names to exclude from U
n	number of samples; if NULL return all

#### Details

Counterfactual fairness is with respect to the causal graph:

where

G=group (usually sensitive attribute); Yhat=outcome; X=set of variables through which G can act on Yhat, U=set of background variables;

We want the counterfactual Yhat\_g' <- G |X=x,G=g (or alternatively Yhat\_g' <- G |G=g).

This can be interpreted as: the distribution of values of Yhat amongst individuals whose values of U are distributed as though they were in group G=g (and, optionally, had values X=x, but whose value of G is g'

Essentially, comparison of the counterfactual quantity above to the conditional YhatlG=g isolates the difference in Yhat due to the effect of G on Yhat through X, removing any effect due to different distributions of U due to different values of G.

To estimate Y'=Yhat\_g' <- G | G=g, we need to

- 1. Compute  $U' \sim (U|G=g)$
- 2. Compute the distribution X' as  $X' \sim (X|U \sim U', G=g')$
- 3. Sample Y'~(YhatlX~X',U~U')

To estimate Y'=Yhat\_g' <- G |X=x, G=g, we need to

1. Compute  $U'\sim(U|G=g,X=x)$ 

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- 2. Compute the distribution X' as  $X' \sim (X|U \sim U', G=g')$
- 3. Sample Y'~(YhatlX~X',U~U')

This function approximates this samplying procedure as follows

- 1. Look at individuals with G=g (and optionally X=x)
- 2. Find the values of U for these individuals
- 3. Find a second set of individuals with the same values of U but for whom G=g'
- 4. Return the indices of these individuals

The values of Yhat for these individuals constitute a sample from the desired counterfactual.

#### Value

indices representing sample(s) from counterfactual Yhat\_g' <- G |X=x,G=g

```
set.seed(23173)
N=10000
# Background variables sampler
background_U=function(n) runif(n) # U~U(0,1)
# Structural equations
struct\_G=function(u,n) \ rbinom(n,1,prob=u) \ \# \ G|U=u \ \sim \ Bern(u)
struct\_X=function(u,g,n) \ rbinom(n,1,prob=u*(0.5 + 0.5*g)) \ \# \ X|U=u,G=g \ ^{\sim} \ Bern(u(1+g)/2)
struct\_Yhat=function(u,x,n) \; (runif(n,0,x) + runif(n,0,u))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,X) + U(0,U))/2 \; \# \; Yhat \; |X,N \sim (U(0,X) + U(0,X) \; |X,N \sim (U(0,X) + U(0,X) \; |X,N \sim (U(0,X) + U(0,X) \; |X,N \sim (U(0,X) + U(0,X) + U(0,
# To see that the counterfactual 'isolates' the difference in Yhat due to the
# causal pathway from G to Yhat through X, change the definition of struct_G to
# struct_G=function(u,n) rbinom(n,1,prob=1/2) # G|U=u ~ Bern(1/2)
\# so the posterior of U|G=g does not depend on g. Note that, with this definition, the
\# counterfactual Yhat_{G<01}|G=1 coincides with the conditional Yhat|G=0, since
# the counterfactual G<-1 is equivalent to just conditioning on G=1.
# By contrast, if we change struct_G back to its original definition, but
# change the definition of struct_Yhat to
# struct_Yhat=function(u,x,n) (runif(n,0,1) + runif(n,0,u))/2 # Yhat|X,N ~ (U(0,1) + U(0,U))/2
# so Yhat depends on G only through the change in posterior of U from changing g,
\# the counterfactual Yhat_{G<01}|G=1 coincides with the conditional Yhat|G=1.
# Sample from complete causal model
U=background_U(N)
G=struct_G(U,N)
X=struct_X(U,G,N)
Yhat=struct_Yhat(U,X,N)
dat=data.frame(U,G,X,Yhat)
# True counterfactual Yhat_\{G \leftarrow \emptyset\} | G=1
```

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```
w1=which(dat$G==1)
n1=length(w1)
UG1=dat$U[w1] # This is U|G=1
XG1=struct_X(UG1,rep(0,n1),n1)
YhatG1=struct_Yhat(UG1,XG1,n1)
# Estimated counterfactual Yhat_{G <- 0}|G=1</pre>
ind_G1=counterfactual_yhat(dat,X="X",G="G",g = 1, gdash = 0)
YhatG1_resample=dat$Yhat[ind_G1]
# True counterfactual Yhat_{G <- 0}|G=1,X=1</pre>
w11=which(dat$G==1 & dat$X==1)
n11=length(w11)
UG1X1=dat$U[w11] # This is U|G=1,X=1
XG1X1=struct_X(UG1X1,rep(0,n11),n11)
YhatG1X1=struct_Yhat(UG1X1,XG1X1,n11)
# Estimated counterfactual Yhat_{G <- 0}|G=1</pre>
ind_G1X1=counterfactual_yhat(dat,X="X",G="G",g = 1, gdash = 0,x=1)
YhatG1X1_resample=dat$Yhat[ind_G1X1]
# Compare CDFs
x=seq(0,1,length=1000)
par(mfrow=c(1,2))
plot(0,type="n",xlim=c(0,1),ylim=c(0,1),xlab="Value",
     ylab=expression(paste("Prop. ",hat('Y')," < x")))</pre>
lines(x,ecdf(dat$Yhat)(x),col="black") # Unconditional CDF of Yhat
lines(x,ecdf(dat$Yhat[which(dat$G==1)])(x),col="red") # Yhat|G=1
lines(x,ecdf(dat$Yhat[which(dat$G==0)])(x),col="blue") # Yhat|G=0
# True counterfactual Yhat_{G <- 0}|G=1</pre>
lines(x,ecdf(YhatG1)(x),col="blue",lty=2)
# Estimated counterfactual Yhat_{G <- 0}|G=1</pre>
lines(x,ecdf(YhatG1_resample)(x),col="blue",lty=3)
legend("bottomright",
       c(expression(paste(hat('Y'))),
         expression(paste(hat('Y'),"|G=1")),
         expression(paste(hat('Y'),"|G=0")),
         expression(paste(hat(Y)[G %<-% 0],"|G=1 (true)")),</pre>
         expression(paste(hat(Y)[G %<-% 0],"|G=1 (est.)"))),</pre>
       col=c("black","red","blue","blue","blue"),
       1ty=c(1,1,1,2,3),
       cex=0.5)
plot(0,type="n",xlim=c(0,1),ylim=c(0,1),xlab="Value",
     ylab=expression(paste("Prop. ",hat('Y')," < x")))</pre>
lines(x,ecdf(dat\$Yhat[which(dat\$X==1)])(x),col="black") \ \# \ CDF \ of \ Yhat|X=1)
```

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```
lines(x,ecdf(dat$Yhat[which(dat$G==1 & dat$X==1)])(x),col="red") # Yhat|G=1,X=1
lines(x,ecdf(dat$Yhat[which(dat$G==0 & dat$X==1)])(x),col="blue") # Yhat|G=0,X=1
# True counterfactual Yhat_{G <- 0}|G=1,X=1</pre>
lines(x,ecdf(YhatG1X1)(x),col="blue",lty=2)
# Estimated counterfactual Yhat_{G <- 0}|G=1,X=1
lines(x,ecdf(YhatG1X1_resample)(x),col="blue",lty=3)
legend("bottomright",
       c(expression(paste(hat('Y|X=1'))),
         expression(paste(hat('Y'),"|G=1,X=1")),
         expression(paste(hat('Y'),"|G=0,X=1")),
         expression(paste(hat(Y)[G %<-% 0],"|G=1,X=1 (true)")),</pre>
         expression(paste(hat(Y)[G %<-% 0],"|G=1,X=1 (est.)"))),
       col=c("black","red","blue","blue","blue"),
       lty=c(1,1,1,2,3),
       cex=0.5)
# In both plots, the estimated counterfactual CDF closely matches the CDF of the
# true counterfactual.
```

dat2mat dat2mat

## Description

Generates matrices for decomposition of admission type which can be used in plot\_decomp

#### Usage

```
dat2mat(dat, score, group1, group2, nquant = 20, cats = unique(dat$reason))
```

#### Arguments

dat	data frame with population data, such as output from sim_pop_data. Must include a column reason
score	risk scores corresponding to dat
group1	indices for group 1
group2	indices for group 2
nquant	number of quantiles of code to use; default 20
cats	vector of strings giving names of admission categories; default the unique values in dat\$reason. Can include NAs.

#### **Details**

Generates two matrices with the following specifications: Each matrix corresponds to one group Columns are named with the admission types to be plotted. Any admission types including the string 'Died' are counted as deaths If the matrix has N rows, these are interpreted as corresponding to N score quantiles The (i,j)th entry of the matrix is the number of people admitted for reason i with a score greater than or equal to (j-1)/N and less than (j/N) who are in that group

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#### Value

list with two objects matrix1 and matrix2 giving output matrices

#### **Examples**

```
# See vignette
```

#### **Description**

Matrix giving frequency of admission types for various groups at various score thresholds. Row names are of the form  $vX\_Y\_qZ$ , where X is version (3 or 4), Y is cohort (e.g., all, over 65, island postcode) and Z is quantile (1-20) of score. Column names are cause of admission or cause of death.

## Usage

```
decomposition_matrix
```

#### **Format**

An object of class data. frame with 520 rows and 41 columns.

```
demographic\_parity demographic\_parity
```

## Description

Estimates demographic parity for a risk score (essentially cumulative distribution function)

## Usage

```
demographic_parity(
   scores,
   group1,
   group2,
   cutoffs = seq(min(scores, na.rm = T), max(scores, na.rm = T), length = 100)
)
```

#### **Arguments**

```
scores vector of risk scores
group1 indices of group 1
group2 indices of group 2
cutoffs score cutoffs at which to estimate DP (default 100 evenly-spaced)
```

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#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the proportion of scores in group g which are less than or equal to the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

#### **Examples**

```
# See vignette
```

drawperson

drawperson

#### **Description**

Draws a simple stock image of a person.

#### Usage

```
drawperson(
  xloc = 0,
  yloc = 0,
  scale = 1,
  headsize = 0.16,
  headangle = pi/8,
  headloc = 0.5,
  necklength = 0.1,
  shoulderwidth = 0.1,
  shouldersize = 0.05,
  armlength = 0.4,
  armangle = 7 * pi/8,
  armwidth = 0.08,
  leglength = 0.5,
  legangle = 9 * pi/10,
  legwidth = 0.15,
  torsolength = 0.4,
```

## **Arguments**

```
xloc x-axis offset from origin
yloc y-axis offset from origin
scale scale upwards from 1x1 box
```

headsize head size

headangle half angle of neck in terms of head

headloc location of centre of head relative to origin with scale 1

necklength necklength shoulderwidth shoulderwidth

12 drawprop

shouldersize size radius of arc for shoulder arm length armlength armangle angle of arm from horizontal armwidth width of arm leglength leg length legangle angle of leg from horizontal width of leg legwidth torsolength length of torso other parameters passed to polygon()

#### **Details**

Draws a figure at a particular location. With defaults, has centre at origin and fits in 1x1 box. Dimensions customisable

#### Value

invisibly returns co-ordinates

#### **Examples**

```
\label{eq:plot_norm} \begin{split} & plot(0,xlim=c(-1,1),ylim=c(-1,1),type="n") \\ & drawperson(0,0,1,col="yellow",border="red",lwd=3,lty=2) \end{split}
```

drawprop drawprop

#### **Description**

Illustrates a proportion as a set of people who are blue rather than red.

#### Usage

```
drawprop(prop, ci, nxy = 10, col1 = "maroon", col2 = "lightblue", ...)
```

## **Arguments**

prop	the proportion to illustrate
ci	half the 95% CI width of the proportion.
nxy	illustrate on an n x n grid
col1	colour to put the 'in' proportion
col2	the other colour
	passed to 'plot'

#### **Details**

Why anyone would want to think about a proportion this way is beyond the understanding of the authors, but the people have spoken.

getcal 13

## **Examples**

```
# See vignette
```

getcal getcal()

## Description

Produces a set of points for a calibration plot.

## Usage

```
getcal(
   y,
   ypred,
   n = 10,
   kernel = F,
   kernel_sd = 0.05,
   alpha = 0.05,
   c0 = 0,
   c2 = 0.1
)
```

## **Arguments**

у	class labels, 0/1 or logical
ypred	predictions Pr(Y=1), numeric vector
n	number of subintervals/points
kernel	set to TRUE to use kernel method
kernel_sd	kernel width for kernel method; see above
alpha	return a pointwise confidence envolope for conservative 1-alpha confidence interval
c0	for computing maximum bias; assume true covariance function is of the form $a0+a1x+a2x^2$ , with $ a0 ,  a2  (c1 does not matter)$
c2	for computing maximum bias; assume true covariance function is of the form $a0+a1x+a2x^2$ , with $a0<0$ , $a2<0$ (c1 does not matter)

#### **Details**

Uses either a binning method or a kernel method to determine height of points.

In both methods, considers n equally spaced subintervals of (0,1)

#### Value

a list with components x (expected calibration), y (observed calibration), n (number of samples in bins, if relevant), lower/upper (confidence interval on y)

```
# See vignette
```

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|--|

## Description

Comprehensive plotting function for precision-recall curve. Also calculates AUPRC and standard error.

## Usage

```
getprc(y, ypred, cv = NULL, res = 100)
```

#### **Arguments**

у	class labels, 0/1 or logical
ypred	predictions Pr(Y=1), numeric vector
CV	cross-validation fold assignments, if relevant. Changes estimate of standard error.
res	resolution. Returns this many equally-spaced points along the curve. Set res to null to return all points.

#### **Details**

Rather than returning points corresponding to every cutoff, only returns a representative sample of equally-spaced points along the curve.

Does not plot anything. Object can be plotted in a default way.

## Value

list containing: ppv, ppv for res points in every cv fold; sens, sensitivity for res points in every cv fold; auc, areas under the curve for each fold and average (note length is 1 greater than number of CV folds); se, standard error for AUC in each fold and standard error for average auc (note length is 1 greater than number of CV folds)

#### **Examples**

```
# See vignette
```

getroc	getroc() Comprehensive plotting function for receiver-operator characteristic curve. Also calculates AUROC and standard error.

## Description

Rather than returning points corresponding to every cutoff, only returns a representative sample of equally-spaced points along the curve.

#### Usage

```
getroc(y, ypred, cv = NULL, res = 100)
```

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## **Arguments**

У	class labels, 0/1 or logical
ypred	predictions $Pr(Y=1)$ , numeric vector
CV	cross-validation fold assignments, if relevant. Changes estimate of standard error.
res	resolution. Returns this many equally-spaced points along the curve. Set res to null to return all points.

#### **Details**

SE of AUROC with no CV structure is from Hanley and McNeil 1982. SE of AUROC with CV folds is from LeDell et al 2012

Does not plot anything. Object can be plotted in a default way.

#### Value

list containing: spec, specificity for res points in every cv fold; sens, sensitivity for res points in every cv fold; auc, areas under the curve for each fold and average (note length is 1 greater than number of CV folds); se, standard error for AUC in each fold and standard error for average auc (note length is 1 greater than number of CV folds)

#### **Examples**

```
# See vignette
```

groupmetric\_2panel

groupmetric\_2panel Draws plots of a group fairness metric with a second panel underneath

## Description

groupmetric\_2panel Draws plots of a group fairness metric with a second panel underneath

## Usage

```
groupmetric_2panel(
  objs,
  labels,
  col = 1:length(objs),
  lty = rep(1, length(col)),
  yrange = NULL,
  ci_col = 1:length(objs),
  highlight = NULL,
  logscale = FALSE,
  mar_scale = 1,
  lpos = NULL,
  yrange_lower = NULL,
  ...
)
```

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#### **Arguments**

list of fairness objects. Each should contain sub-objects 'x', 'y' and 'ci', which objs specify x and y values and half-widths of confidence intervals around y. labels labels to use in legend col line colours lty line type, defaults to 1 limit of y axis; defaults to 0,1 yrange ci\_col confidence envelope colours. These will be transparent. highlight if non-null, draw a point at a particular cutoff logscale if TRUE, draw with log-scale. mar\_scale scale bottom and left margins by this amount. Also scales legend. lpos legend position

y range for lower plot. If NULL, generates automatically

## Examples

# See vignette

yrange\_lower

group\_fairness group\_fairness

#### **Description**

Estimates group fairness metric according to a specification vector of the form

other parameters passed to legend()

#### Usage

```
group_fairness(
  specs,
  scores,
  target,
  group1,
  group2,
  cutoffs = seq(min(scores, na.rm = T), max(scores, na.rm = T), length = 100)
)
```

## **Arguments**

specs specification vector; see description
scores vector of risk scores
target vector of values of target (which risk score aims to predict)
group1 indices of group 1
group2 indices of group 2
cutoffs score cutoffs at which to estimate metric (default 100 evenly-spaced)

integral 17

#### **Details**

```
c(A1,B1,C1,A2,B2,C2)
```

encoding a probability

P(A1,B1,C1|A2,B2,C2)

where

A1/A2 are events coded by 1:'score>= cutoff'; 0: 'score<cutoff' and NA: 1/TRUE B1/B2 are events coded by 1:'target=TRUE'; 0: 'target=FALSE' and NA: 1/TRUE C1/C2 are events coded by 1:'group=g'; and NA: 1/TRUE

For example, specs=c(NA,1,NA,0,NA,1) would encode false omission rate:

P(target=TRUE|score<cutoff,group=g)

#### Value

matrix of dimension length(cutoffs)x4, with (i,2g-1)th entry the relevant fairness metric for group g at the ith cutoff value and (i,2g)th entry the approximate standard error of the (i,2g-1)th entry

## **Examples**

# See vignette

integral

integral() Quick form for trapezoidal integration over range of x

## Description

integral() Quick form for trapezoidal integration over range of x

## Usage

```
integral(x, y = NULL)
```

#### **Arguments**

x x co-ordinates, or nx2 matrix of points

y y co-ordinates

## Value

trapezoidal estimate of integral of the xth value of y over range of x.

18 logit

logistic

Logistic

# Description

```
Logistic function: 1/(1+exp(-x))
```

## Usage

```
logistic(x)
```

## Arguments

Χ

argument

## Value

```
value of logistic(x)
```

## **Examples**

```
# Plot
x=seq(-5,5,length=1000)
plot(x,logistic(x),type="l")
```

logit

Logit

## Description

```
Logit function: -\log((1/x)-1)
```

# Usage

```
logit(x)
```

## **Arguments**

Χ

argument

## Value

```
value of logit(x); na if x is outside (0,1)
```

```
# Plot
x=seq(0,1,length=100)
plot(x,logit(x),type="1")

# Logit and logistic are inverses
x=seq(-5,5,length=1000)
plot(x,logit(logistic(x)),type="1")
```

phs\_colours 19

#### **Description**

Copied from github, "Public-Health-Scotland/phsstyles". Public Health Scotland colour scheme. Internal function.

#### Usage

```
phs_colours(colourname = NULL, keep_names = FALSE)
```

#### **Arguments**

colourname name of colour; usually something like phs-blue. If NULL returns all colours. keep\_names keep\_names of colours in return list. Defaults to false.

#### Value

vector of colours, optionally with names.

plot.sparraCAL

Plot function for class sparraCAL

#### **Description**

Plot function for class sparraCAL

#### Usage

```
## $3 method for class 'sparraCAL'
plot(
    x,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    add = FALSE,
    add_xy_line = TRUE,
    ...
)
```

#### **Arguments**

```
x output from getcal()
cols colour to draw lines
add set to FALSE to add to existing plot
add_xy_line set to TRUE to draw an X-Y reference line.
... passed to lines()
```

```
# See vignette
```

20 plot.sparraROC

plot.sparraPRC

Plot function for class above

## Description

Plot function for class above

## Usage

```
## S3 method for class 'sparraPRC'
plot(
    x,
    addauc = FALSE,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    ...
)
```

## Arguments

```
    x output from getprc()
    addauc set to TRUE to add text to the plot showing the (mean) AUC and SE.
    cols colour to draw lines
    passed to plot()
```

#### **Examples**

```
# See vignette
```

plot.sparraROC

Plot function for class sparraROC

## Description

Plot function for class sparraROC

## Usage

```
## S3 method for class 'sparraROC'
plot(
    x,
    addauc = FALSE,
    cols = rep(phs_colours("phs-blue"), dim(x$sens)[1]),
    ...
)
```

plot\_decomp 21

#### **Arguments**

```
x output from getroc()
addauc set to TRUE to add text to the plot showing the (mean) AUC and SE.
cols colour to draw lines
... passed to plot()
```

#### **Examples**

```
# See vignette
```

plot\_decomp plot\_decomp

#### **Description**

Plots a bar graph of decomposition of FORP by cause of admission

## Usage

```
plot_decomp(
  decomp1,
  decomp2,
  threshold,
  labels,
  inc_died = TRUE,
  mar_scale = 1
)
```

#### **Arguments**

decomp1 matrix for first group; see specification in description
decomp2 matrix for second group; see specification in description

threshold score threshold to plot (between 0 and 1)

labels labels for group 1 and group 2

inc\_died set to TRUE to include a second panel showing 'death' type admissions

mar\_scale scale margins by this amount. Also scales legend.

#### **Details**

Takes two matrices as input with the following specifications: Each matrix corresponds to one group Columns are named with the admission types to be plotted. Any admission types including the string 'Died' are counted as deaths If the matrix has N rows, these are interpreted as corresponding to N score quantiles The (i,j)th entry of the matrix is the number of people admitted for reason i with a score greater than or equal to (j-1)/N and less than (j/N) who are in that group

```
# See vignette
```

22 roc\_2panel

prc_2panel	prc_2panel Draws a PRC curve (with legend) with a second panel
	underneath showing precision difference.

## Description

prc\_2panel Draws a PRC curve (with legend) with a second panel underneath showing precision difference.

## Usage

```
prc_2panel(
  prcs,
  labels,
  col = 1:length(prcs),
  lty = rep(1, length(col)),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

## Arguments

```
list of sparraPRC objects. If of length 1, splits into folds
prcs
                   labels to use in legend
labels
                   line colours
col
                   line type, defaults to 1
lty
highlight
                   if non-null, draw a point at a particular cutoff
mar_scale
                   scale bottom and left margins by this amount. Also scales legend.
yrange_lower
                   y range for lower plot. If NULL, generates automatically
                   other parameters passed to legend()
. . .
```

## **Examples**

```
# See vignette
```

roc_2panel	roc_2panel Draws a ROC curve (with legend) with a second panel
	underneath showing sensitivity difference.

#### **Description**

roc\_2panel Draws a ROC curve (with legend) with a second panel underneath showing sensitivity difference.

sim\_pop\_data 23

#### Usage

```
roc_2panel(
  rocs,
  labels,
  col = 1:length(rocs),
  lty = rep(1, length(col)),
  xy_lty = 2,
  xy_col = phs_colours("phs-magenta"),
  highlight = NULL,
  mar_scale = 1,
  yrange_lower = NULL,
  ...
)
```

#### **Arguments**

```
list of sparraROC objects (if one object, plots folds separately)
rocs
labels
                   labels to use in legend
col
                   line colours
lty
                   line type, defaults to 1
xy_lty
                   line type for x-y line, defaults to 2 (dashed)
xy_col
                   line colour for x-y line, defaults to red
highlight
                   if non-null, add a point at this cutoff
mar_scale
                   scale bottom and left margins by this amount. Also scales legend.
                   y range for lower plot. If NULL, generates automatically
yrange_lower
                   other parameters passed to legend()
```

## **Examples**

```
# See vignette
```

## **Description**

Simulates population data with a reasonably realistic joint distribution

#### Usage

```
sim_pop_data(
    npop,
    coef_adjust = 4,
    offset = 1,
    vcor = NULL,
    coefs = c(2, 1, 0, 5, 3, 0, 0),
    seed = 12345,
    incl_id = TRUE,
    incl_reason = TRUE
)
```

24 sim\_pop\_data

#### **Arguments**

population size npop inverse scale for all (true) coefficients (default 4): lower means that hospital coef\_adjust admissions are more predictable from covariates. offset for logistic model (default 1): higher means a lower overall prevalence of offset admission a valid 5x5 correlation matrix (default NULL), giving correlation between varivcor ables. If 'NULL', values roughly represents realistic data. coefs coefficients of age, male sex, non-white ethnicity, number of previous admissions, and deprivation decile on hospital admissions, Default (2,1,0,5,3). Divided through by coef\_adjust. random seed (default 12345) seed

include an ID column (default TRUE) incl\_id

incl\_reason include a column indicating reason for admission.

#### **Details**

Simulates data for a range of people for the variables

- Age (age)
- Sex (sexM; 1 if male)
- Race/ethnicity (raceNW: 1 if non-white ethnicity)
- Number of previous hospital admissions (PrevAdm)
- Deprivation decile (SIMD: 1 most deprived, 10 least deprived. NOTE opposite to English IMD)
- Urban-rural residence status (urban\_rural: 1 for rural)
- Mainland-island residence status (mainland\_island: 1 for island)
- Hospital admission (target: 1/TRUE if admitted to hospital in year following prediction date)

Can optionally add an ID column.

Optionally includes an admission reason for samples with target=1. These admission reasons roughly correspond to the first letters of ICD10 categories, and can either correspond to an admission or death. Admission reasons are simulated with a non-constant multinomial distribution which varies across age/sex/ethnicity/urban-rural/mainland-island/PrevAdm values in a randomly- chosen way. The distributions of admission reasons are not however chosen to reflect real distributions, nor are systematic changes in commonality of admission types across categories intended to appear realistic.

#### Value

data frame with realistic values.

```
# Simulate data
dat=sim_pop_data(10000)
pairs(dat[,1:7])
cor(dat[,1:7])
# See vignette
```

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