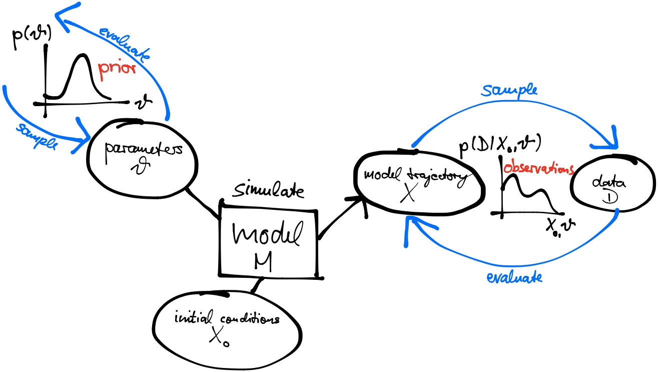
Outline

Approximate Bayesian Computation

1. Approximate Bayesian Computation

Review I

Deterministic models

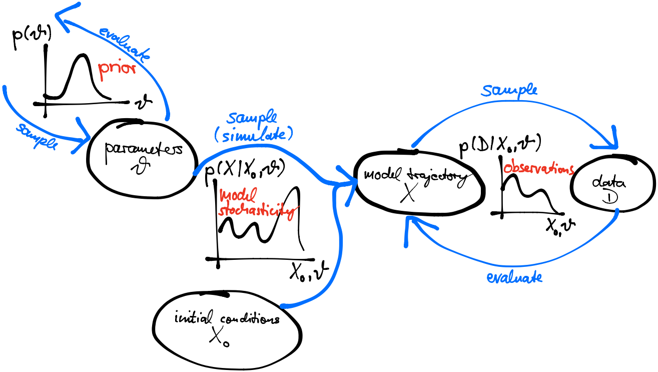


$$p(\theta|\mathrm{data}) \propto p(\mathrm{data}|\theta)p(\theta)$$

Use MCMC to get samples from it: $\theta_1, \theta_2, \theta_3, \ldots$

Review II

Stochastic models



- What is $p(\mathrm{data}|\theta)$
- need to marginalise: $p(\mathrm{data}|\theta) = \int_X p(\mathrm{data}|X,\theta)p(X|\theta)$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

propagate

$$p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

resample

$$p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

propagate

$$p(X_2|X_1,\theta)p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)
resample

$$p(\mathrm{data}_2|X_2,\theta)p(X_2|X_1,\theta)p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

. . .

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)
propagate

$$p(X_N|X_{1:N-1},\theta)p(\mathrm{data}_{N-1}|X_{N-1},\theta)\dots$$

$$\dots p(X_2|X_1,\theta)p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

evaluate

$$p(\mathrm{data}_N|X_N,\theta)p(X_N|X_{1:N-1},\theta)p(\mathrm{data}_{N-1}|X_{N-1},\theta)$$

$$\dots p(X_2|X_1,\theta)p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

evaluate

$$p(\mathrm{data}_N|X_N,\theta)p(X_N|X_{1:N-1},\theta)p(\mathrm{data}_{N-1}|X_{N-1},\theta)$$

$$\ldots p(X_2|X_1,\theta)p(\mathrm{data}_1|X_1,\theta)p(X_1|\theta)$$

average

$$\sum p(\mathrm{data}_N|X_N,\theta)p(\mathrm{data}_{N-1}|X_{N-1},\theta)\ldots p(\mathrm{data}_1|X_1,\theta)p(X|\theta)$$

Particle filter

Replaces the integral by sum over Monte Carlo samples (of trajectories)

evaluate

$$p(\mathrm{data}_N | X_N, \theta) p(X_N | X_{1:N-1}, \theta) p(\mathrm{data}_{N-1} | X_{N-1}, \theta)$$

$$\dots p(X_2 | X_1, \theta) p(\mathrm{data}_1 | X_1, \theta) p(X_1 | \theta)$$

average

$$\sum p(\mathrm{data}_N | X_N, \theta) p(\mathrm{data}_{N-1} | X_{N-1}, \theta) \dots p(\mathrm{data}_1 | X_1, \theta) p(X | \theta)$$

Result:

$$\sum p(\mathrm{data} | X) p(X | \theta)$$

Sampling from the posterior

Estimating the likelihood

	MCMC	SMC
SMC	PMCMC	SMC^2
ABC	ABC-MCMC	ABC-SMC

Approximate Bayesian Computation

Motivation

- ABC:
  - approximate the likelihood using (set of) summary statistics S
- Summary statistics
  - Something that is easy to calculate and approximates the likelihood
  - e.g., final size, number of peaks, height/timing of peaks
  - idea of sufficient summary statistics: to be exact, I need summary statistics that give the same result as the likelihood

Approximation:

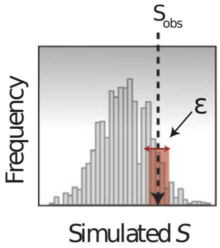
$$p(\mathrm{data}|\theta) \approx p(\,d(S_{\mathrm{data}}, S_{\mathrm{sim}(\theta)}) < \epsilon)$$

- S: summary statistics
- d: distance (e.g., absolute distance etc)
- ϵ: acceptance window

Approximation:

$$p(\text{data}|\theta) \approx p(d(S_{\text{data}}, S_{\text{sim}(\theta)}) < \epsilon)$$

- S: summary statistics
- d: distance (e.g., absolute distance etc)
- ϵ: acceptance window



Frequency that S_{sim} and S_{data} closer than ε

ABC-MCMC

1. Choose a summary statistic S, positive number ϵ, distance function $d(S_1, S_2)$ and initial θ
2. Calculate summary statistic S_{data} for the data
3. repeat:
   3.1 sample θ^* using transition kernel $q(\theta^*|\theta)$
   3.2 simulate trajectory + observation
   3.3 calculate summary statistic $S_{\text{sim}(\theta)}$
   3.4 calculate difference $d(S_{\text{sim}(\theta)}, S_{\text{data}})$
   3.5 if $d(S_{\text{sim}(\theta)}, S_{\text{data}}) < \epsilon$ accept with probability

   $$\min\left(1, \frac{p(\theta^*)}{p(\theta)}\frac{q(\ldots)}{q(\ldots)}\right)$$

   else reject
4. time spent at any θ approximates $p(\theta|y)$