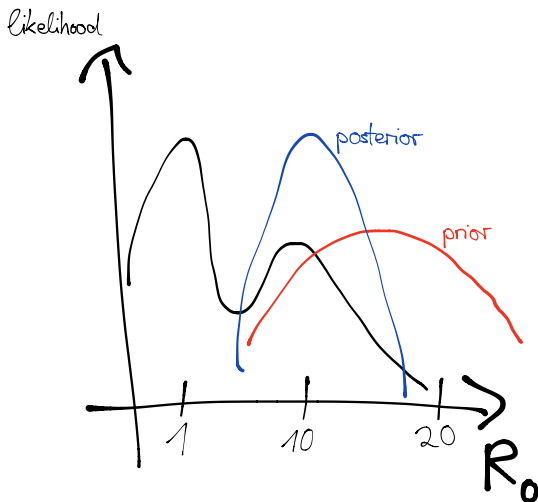


Revision

- posterior probabilities

$$p(\theta|\text{data}) = \frac{p(\text{data}|\theta)p(\theta)}{p(\text{data})}$$



Sampling from the posterior

We interpret $p(\theta|\text{data})$ as the probability distribution of a random variable θ , from which we **sample** (via MCMC)

Why sample?

1. explore parameter space
2. samples can be useful
 - ▶ explore interventions, forecasts

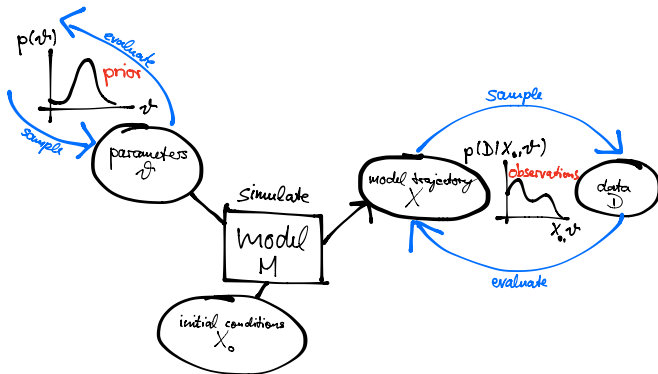
MCMC: Sampling from a distribution

- ▶ We can calculate (in a deterministic model) $p(\theta|\text{data})$ given any θ – how do we sample?

MCMC: Sampling from a distribution

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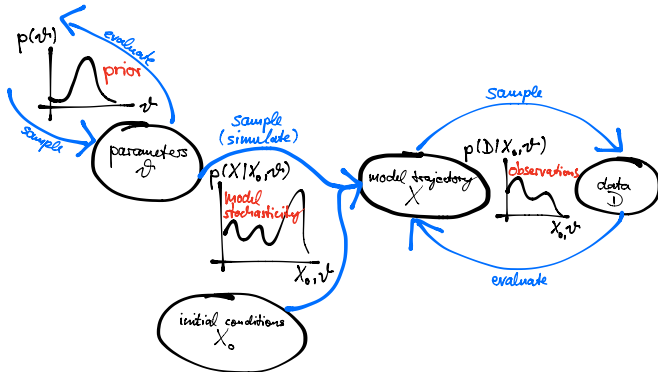
Deterministic process models



$$p(\theta|\text{data}) \propto p(\text{data}|\theta)p(\theta)$$

Use MCMC to get **samples** from it: $\theta_1, \theta_2, \theta_3, \dots$

Stochastic process models



Stochastic process models

- ▶ one θ can lead to many possible outcomes X
- ▶ we can
 1. sample from $p(X|\theta)$ (via simulation)
 2. evaluate the trajectory likelihood $p(\text{data}|X, \theta)$
- ▶ we can't directly evaluate the likelihood $p(\text{data}|\theta)$

$$p(\text{data}|\theta) = \sum_X p(\text{data}|X, \theta)p(X|\theta)$$

- ▶ The number of possible trajectories X for one value of θ is large (usually infinite)
- ▶ We replace the sum with a Monte Carlo (random) sample

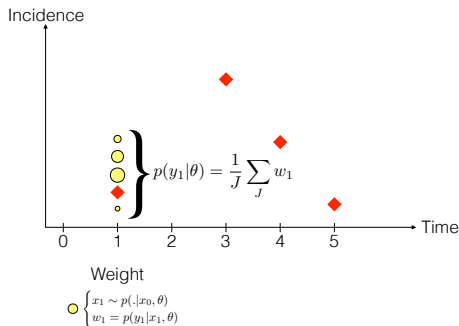
Sequential Monte Carlo (SMC) / Particle Filter I

We **sample** J trajectories $X_{J,1}$ from

$$p(X_{J,1}|\theta)$$

and average over

$$p(Y_1|X_{J,1}, \theta)$$



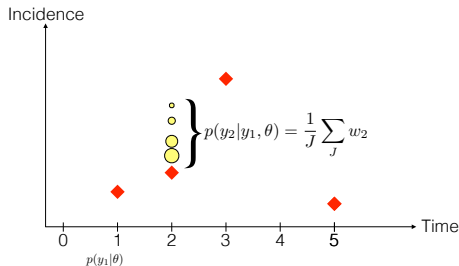
Sequential Monte Carlo (SMC) / Particle Filter II

We then **sample** J trajectories $X_{J,2}$ from

$$p(X_{J,2} | Y_1, \theta)$$

and average over

$$p(Y_2 | Y_1, X_{J,2}, \theta)$$

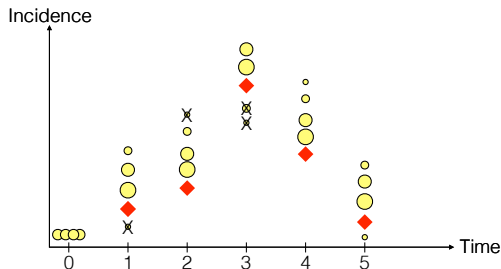


Sequential Monte Carlo (SMC) / Particle Filter III

The sum of all these (logged) values is

$$p(Y_{1:t}|\theta) \approx \prod_t p(Y_t|Y_{1:(t-1)}, \theta)$$

which is a **sample estimate** of the likelihood.

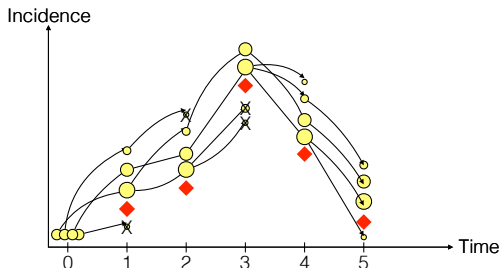


Sequential Monte Carlo (SMC) / Particle Filter IV

We can also retrieve **filtered trajectories**, that is samples from

$$p(X|\text{data})$$

by following the particles from the last point backwards.

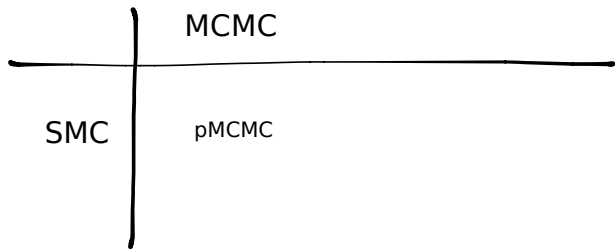


pMCMC

- ▶ Once we can estimate $p(\text{data}|\theta)$, we can combine this with the prior to evaluate the **posterior** $p(\theta|\text{data})$ for any θ .
- ▶ We can then use MCMC to sample from this: pMCMC

Sampling from the posterior

Estimating the
likelihood



Sampling from the posterior

Estimating the
likelihood

