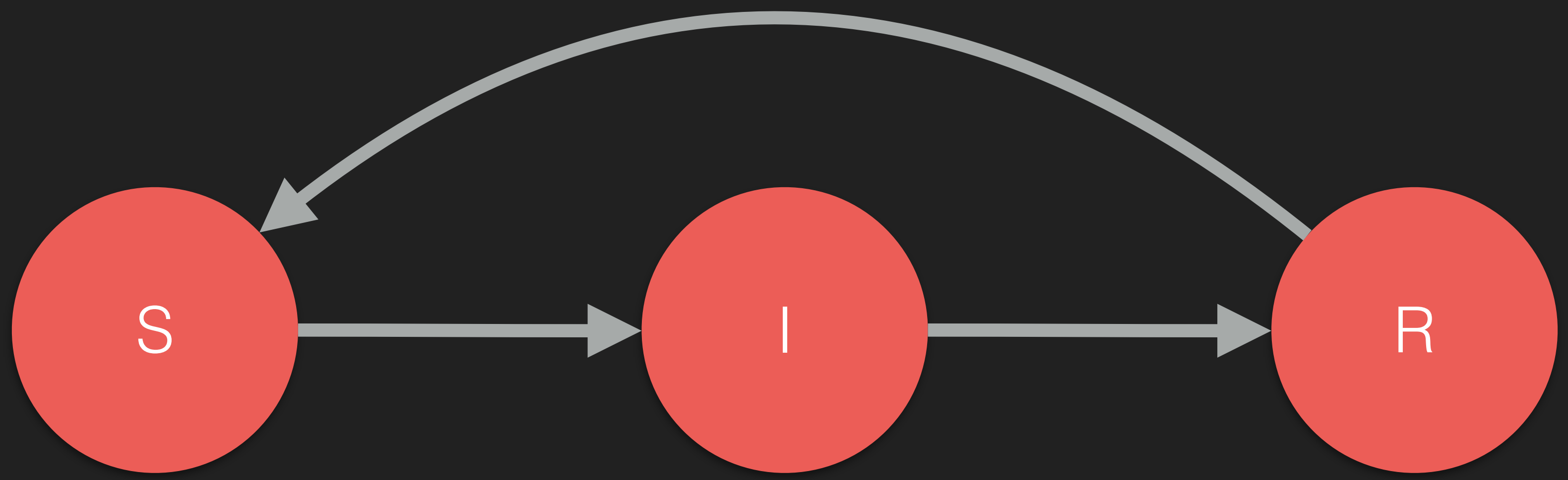
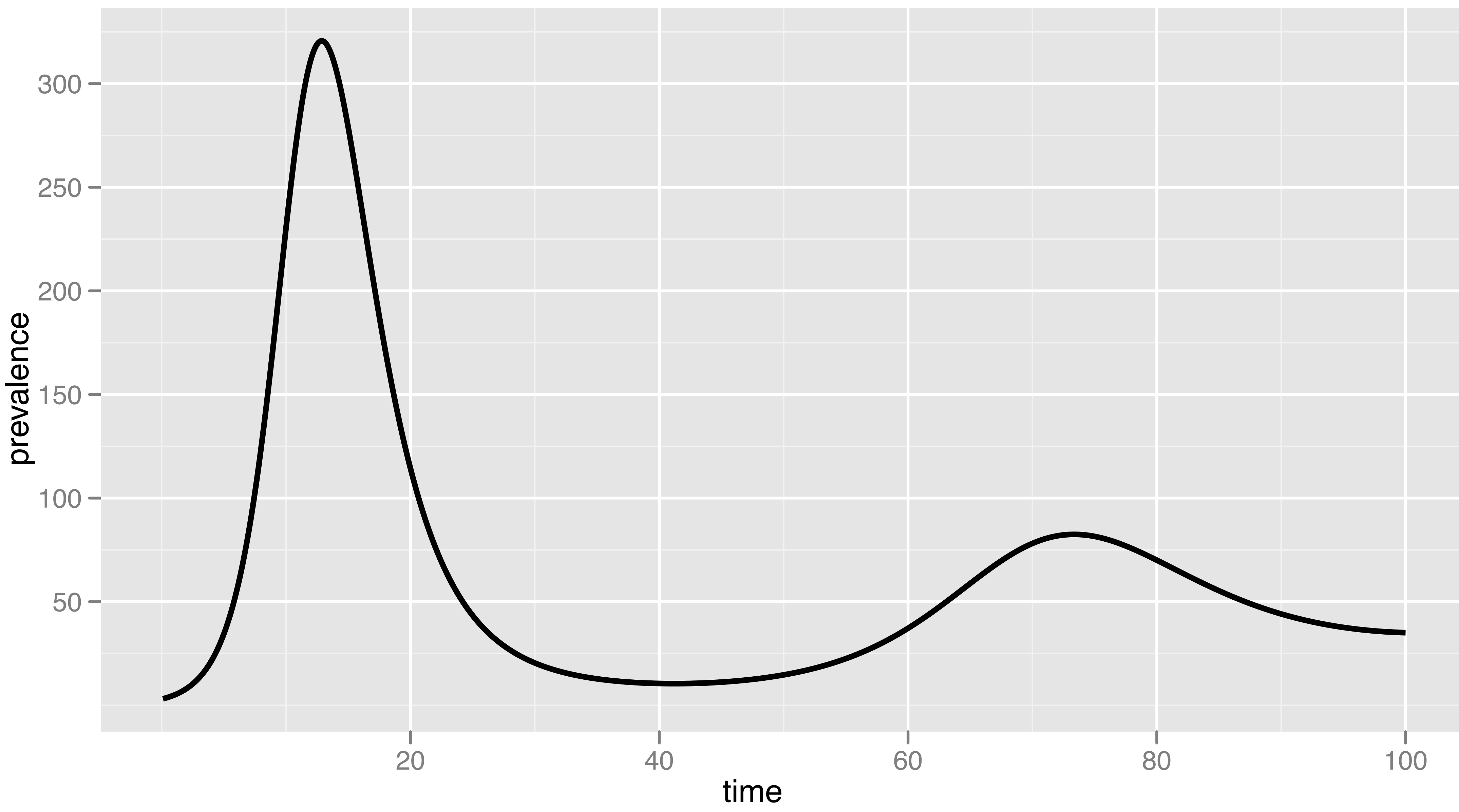
# Deterministic models

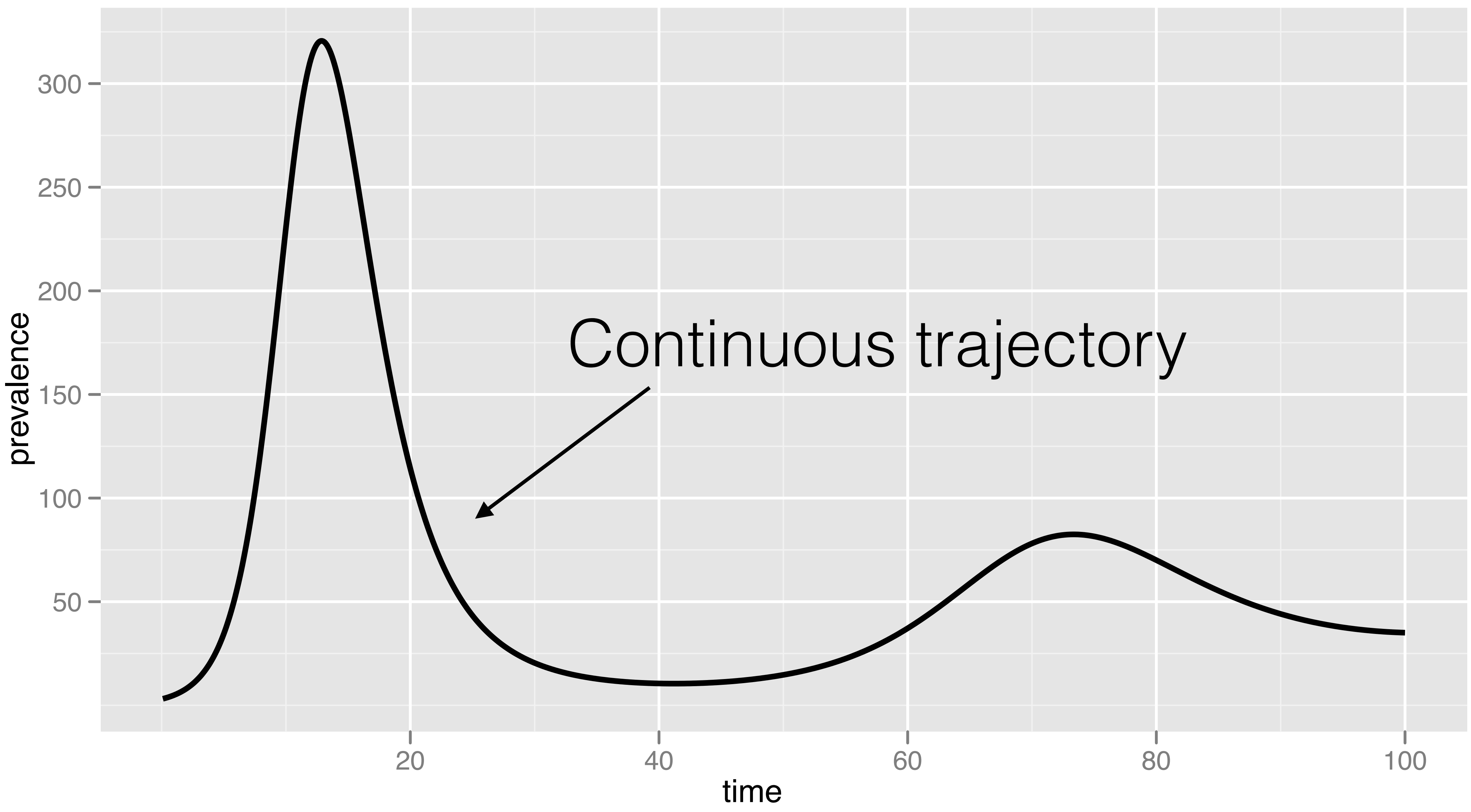



$$\frac{\mathrm{d}S}{\mathrm{d}t} = -\frac{\beta}{N}SI + \gamma(N - S - I)$$
$$\frac{\mathrm{d}I}{\mathrm{d}t} = \frac{\beta}{N}SI - \nu I$$

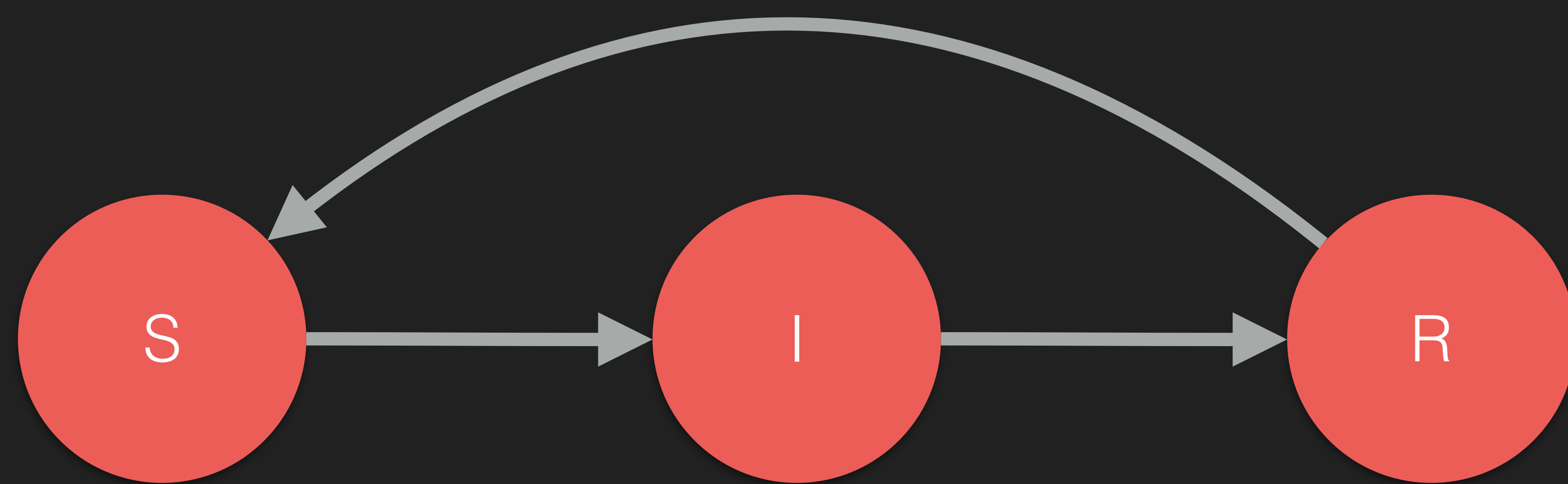
# One $\boldsymbol{\Theta}$ = One trajectory



# One $\boldsymbol{\Theta}$ = One trajectory

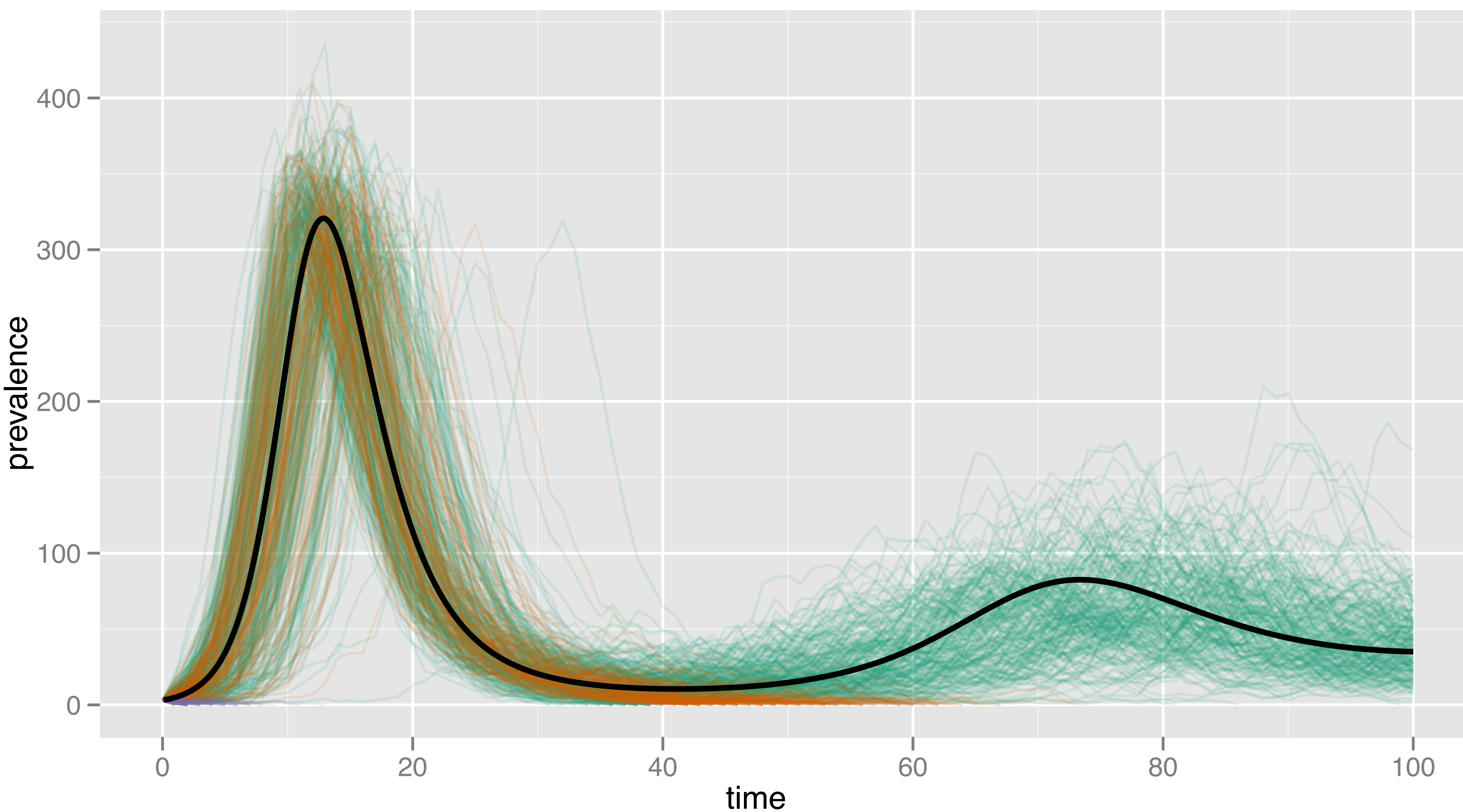


# Life is discrete & stochastic

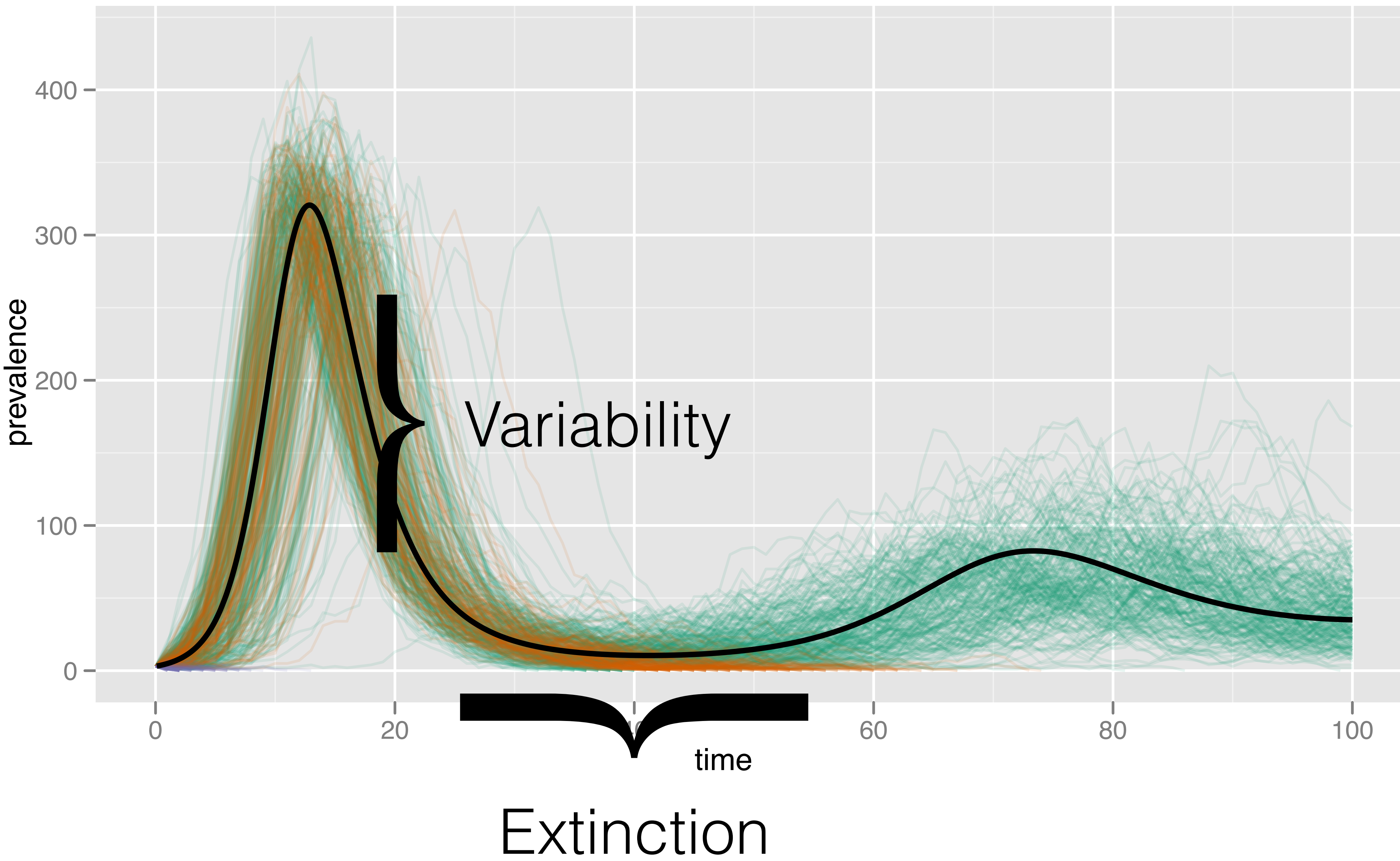



| Event | Transition | Jump intensity |
|---|---|---|
| Infection | $(s,i) \to (s-1, i+1)$ | $\beta si/N$ |
| Recovery | $(s,i) \to (s, i-1)$ | $\nu i$ |
| Loss of immunity | $(s,i) \to (s+1, i)$ | $\gamma(N-s-i)$ |

# One $\boldsymbol{\Theta}$ = Many trajectories



# One Θ = Many trajectories



# Inference

$$p(\theta|y) \propto p(y|\theta) \times p(\theta)$$

# Inference

Parameters

$$p(\theta|y) \propto p(y|\theta) \times p(\theta)$$

Data

# Inference

Posterior

Prior

$$p(\theta|y) \propto p(y|\theta) \times p(\theta)$$

Marginal likelihood

# Marginal likelihood

$$p(y|\theta) = \sum_{X} p(y|x, \theta) \times p(x|\theta)$$

All possible trajectories of the model

# Deterministic case

$$p(y|\theta) = \sum_X p(y|x,\theta) \times 1_{x=f(\theta)}$$

Perfectly known

# Deterministic case

$$p(y|\theta) = p(y|x = f(\theta), \theta) \times 1$$

ODE integration

That's what the function **`dTrajObs`** does.

# Marginal likelihood

$$p(y|\theta) = \sum_{X} p(y|x,\theta) \times p(x|\theta)$$

All possible trajectories of the model

# Stochastic case

$$p(y|\theta) = \sum_{X} p(y|x,\theta) \times p(x|\theta)$$

Can be billions!

No longer known

# Stochastic case

Trajectory of particle j

$$p(y|\theta) \approx \sum_{J} p(y|x_j, \theta) \times p(x_j|\theta)$$

J particles

# Stochastic case

Trajectory of particle j

$$p(y|\theta) \approx \sum_J p(y|x_j, \theta) \times p(x_j|\theta)$$

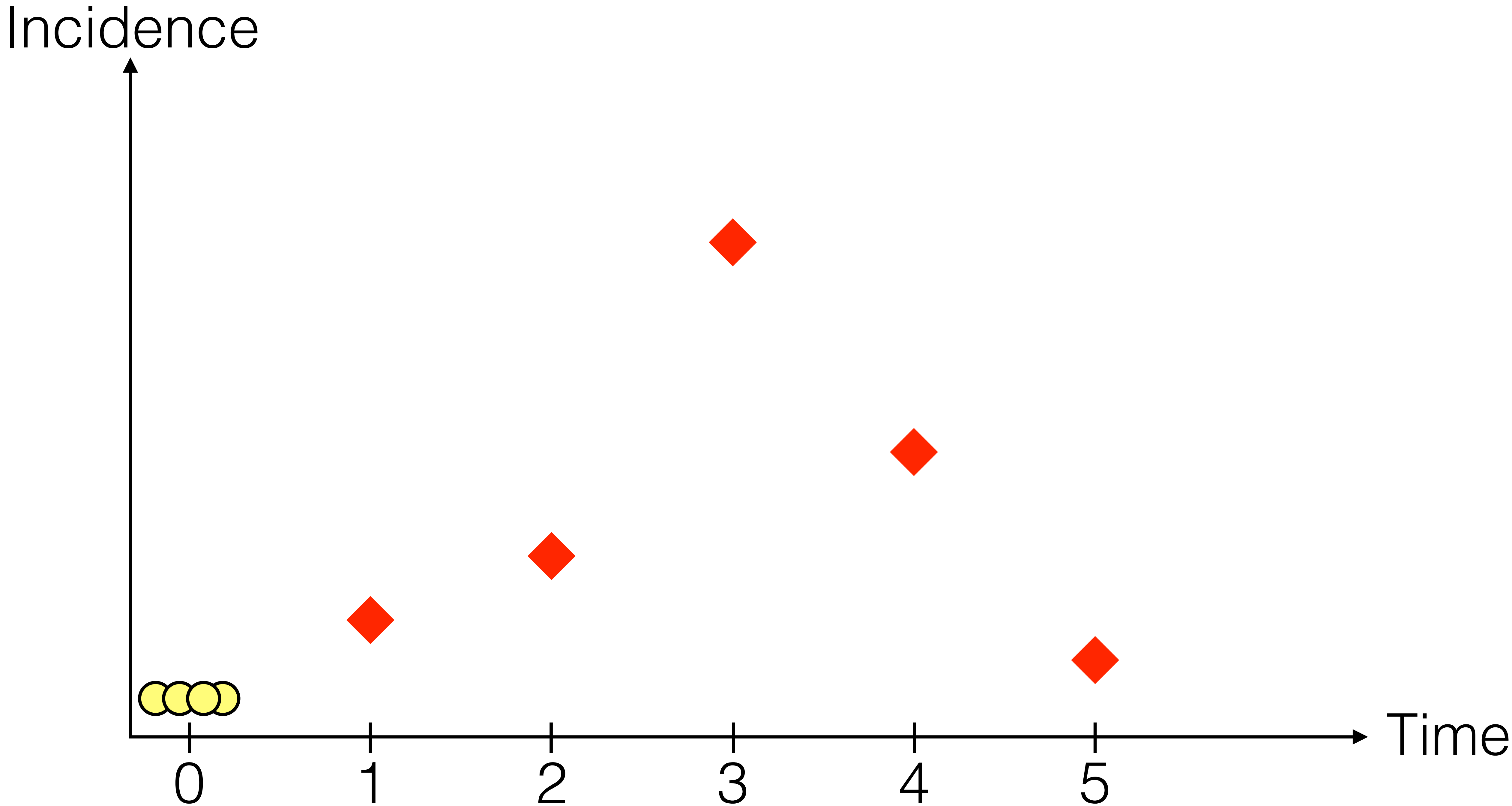
J particles

Monte-Carlo approximation

# Sequential Monte-Carlo

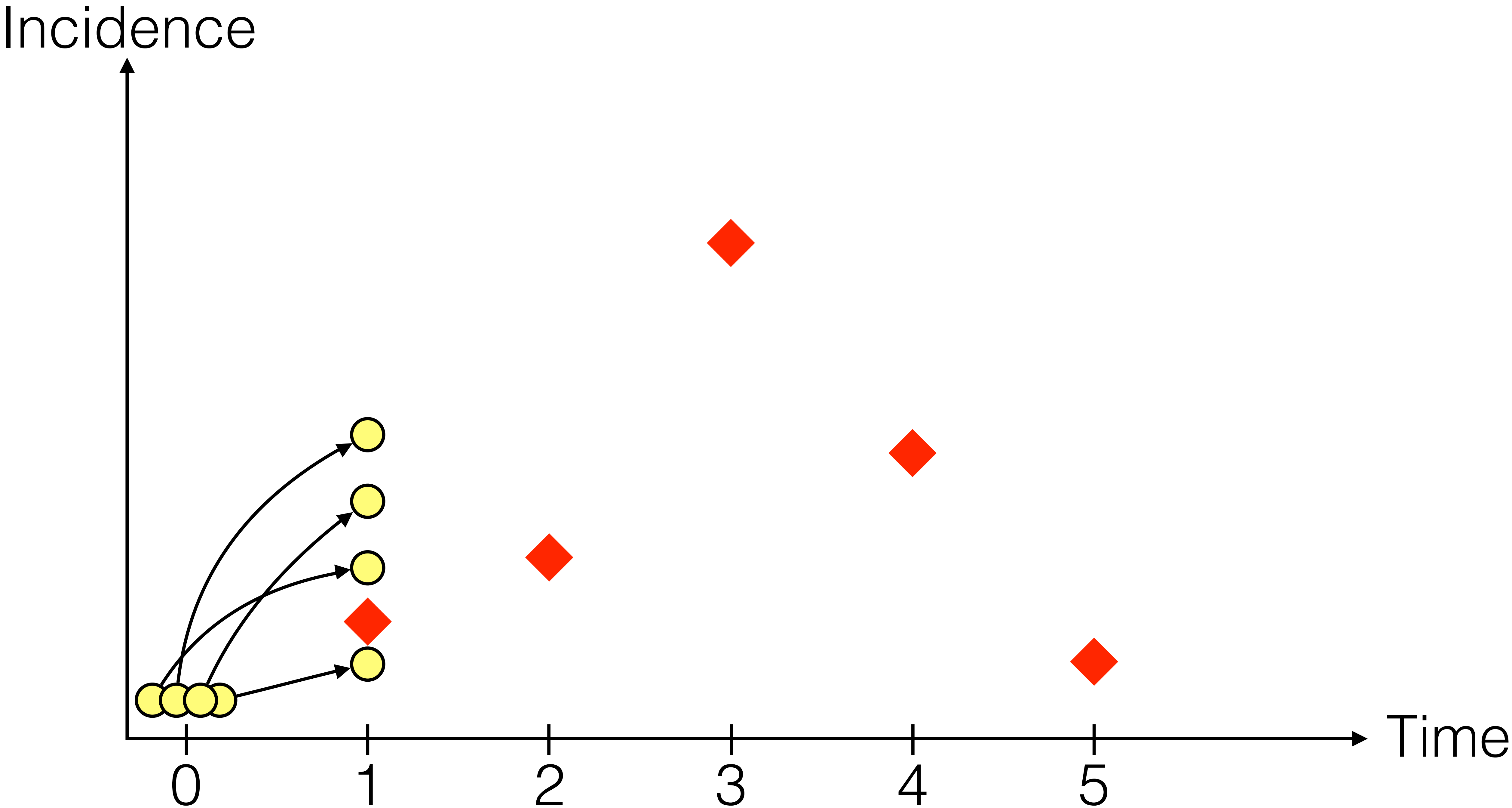
aka

# Particle Filtering



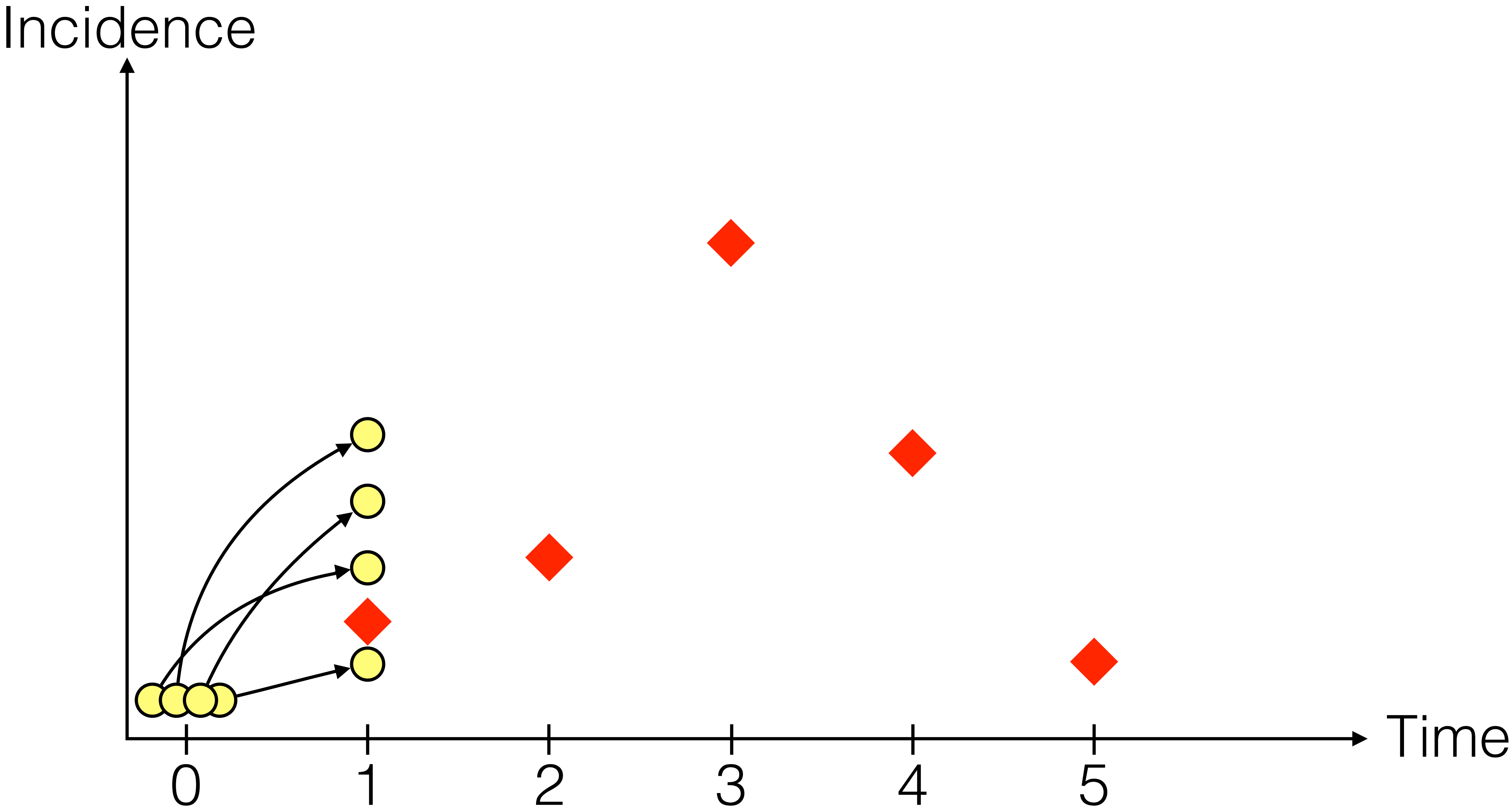

Initialise

$$\begin{cases} x_0 \sim p(.|\theta) \\ w_0 = 1/J \end{cases}$$




Propagate

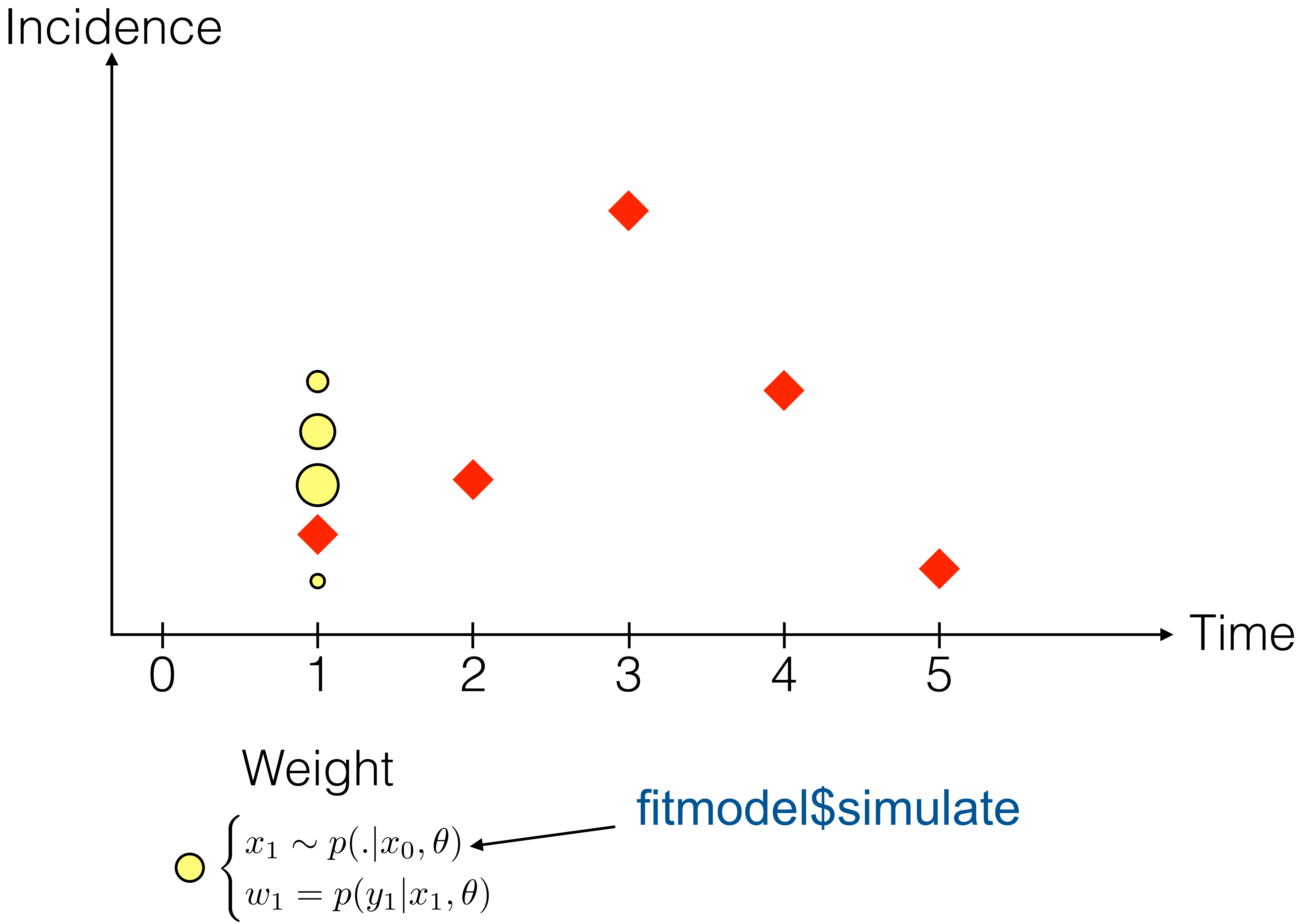
$$\begin{cases} x_1 \sim p(.|x_0, \theta) \\ \dots \end{cases}$$

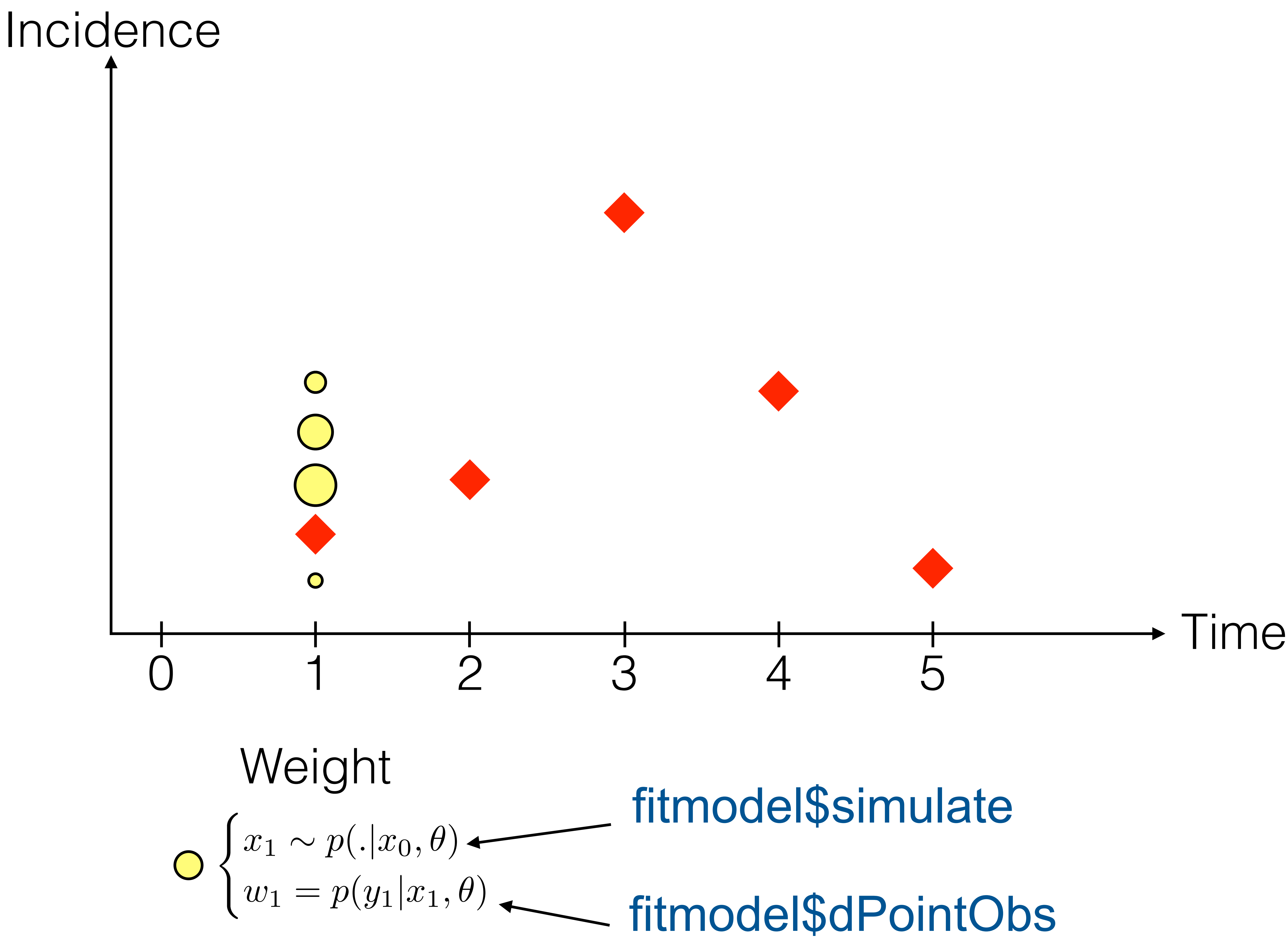


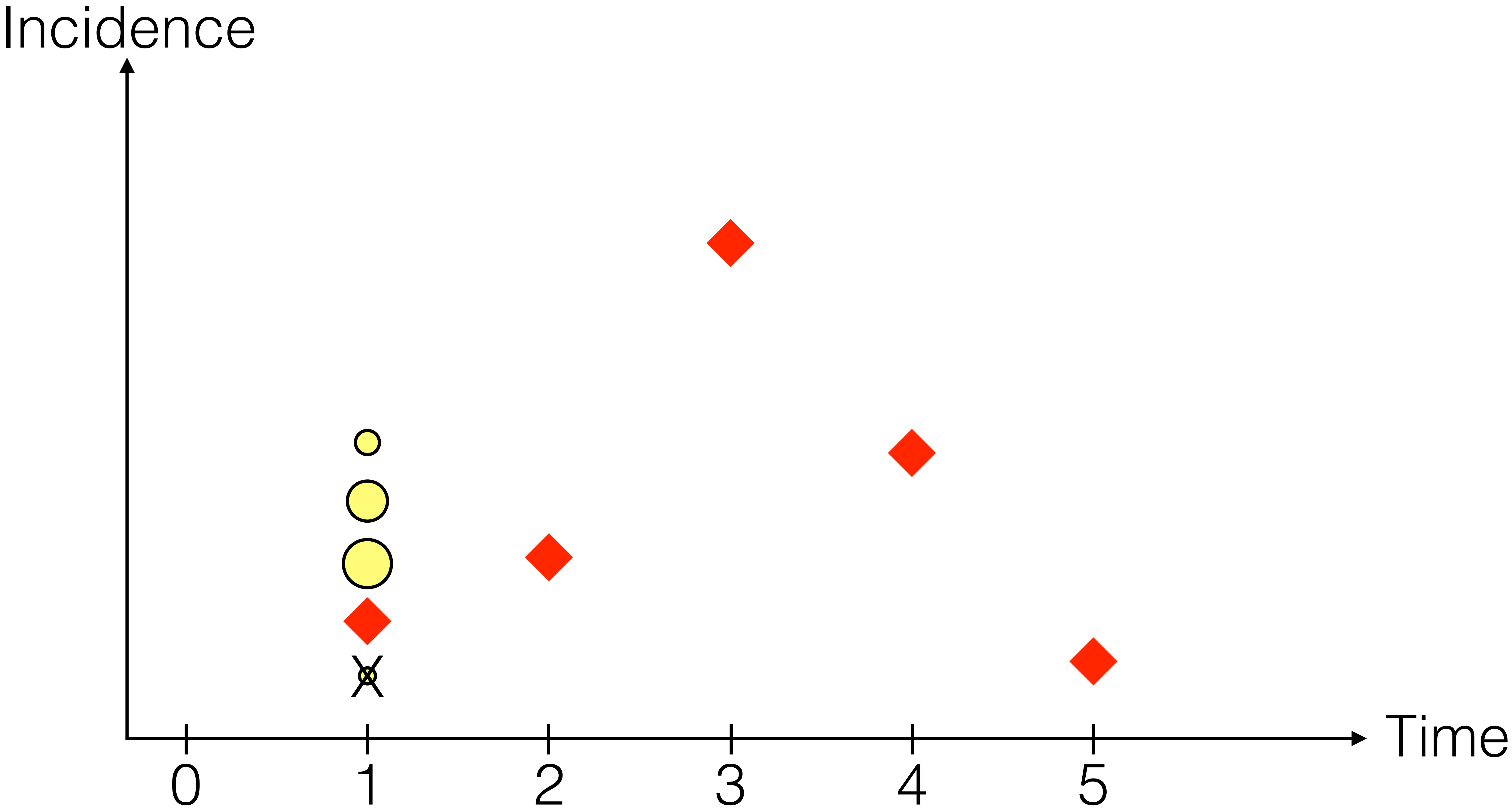

Propagate

$$\begin{cases} x_1 \sim p(.|x_0, \theta) \\ \dots \end{cases}$$

fitmodel$simulate

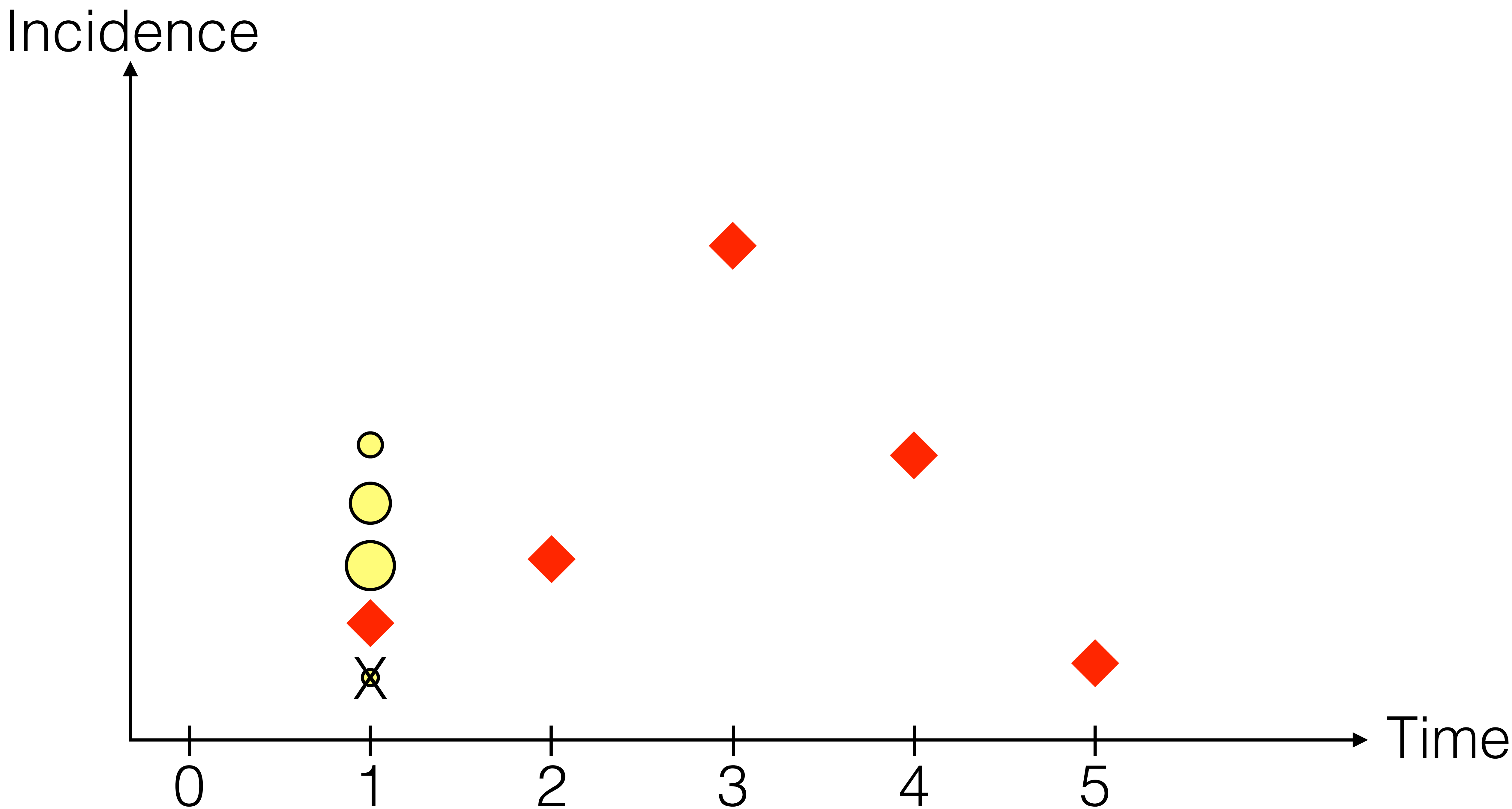

Incidence
Time
0
1
2
3
4
5
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate


Incidence
Time
0
1
2
3
4
5
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate
fitmodel$dPointObs




Resample

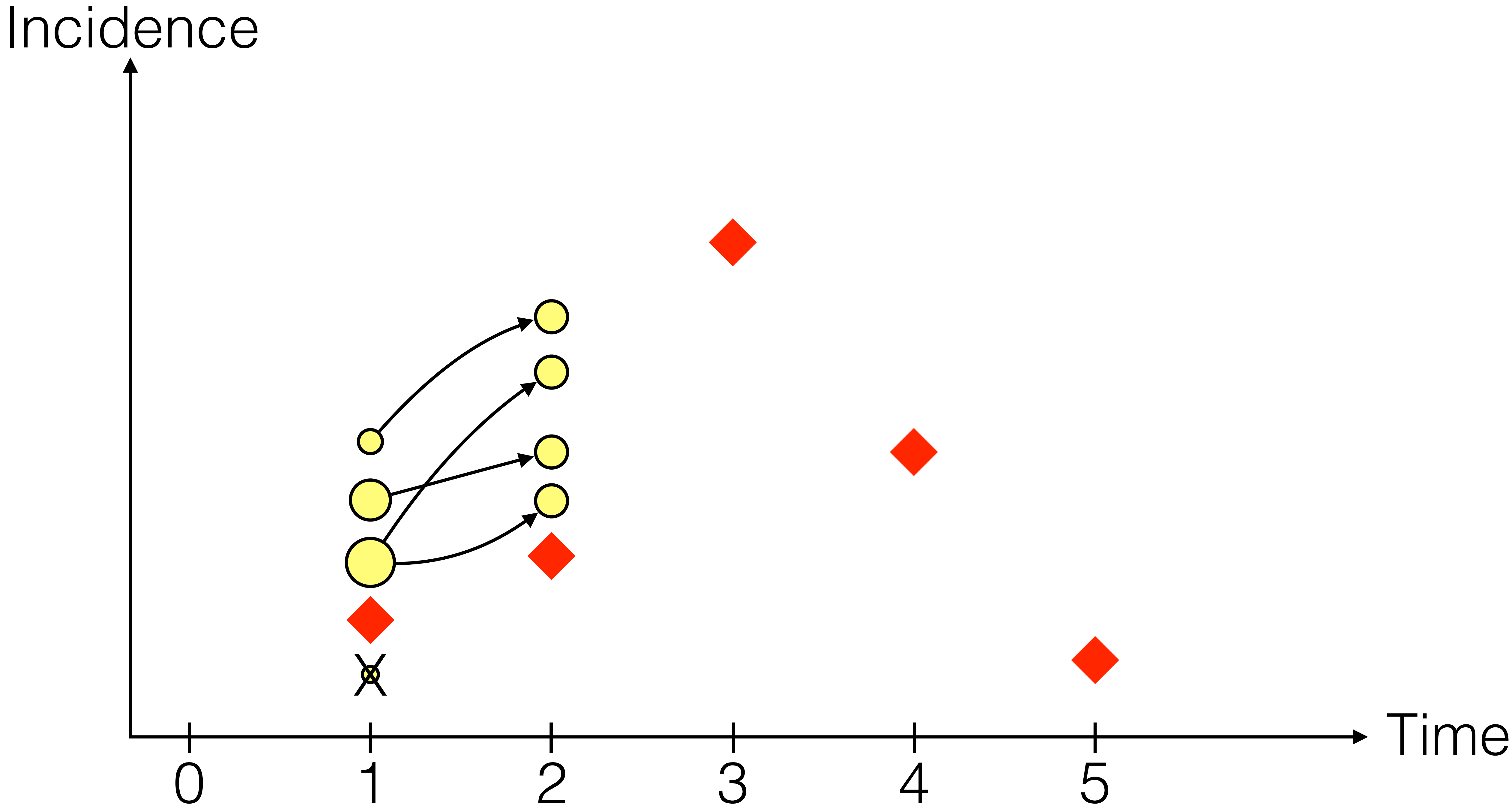
$\propto w_1$




Resample

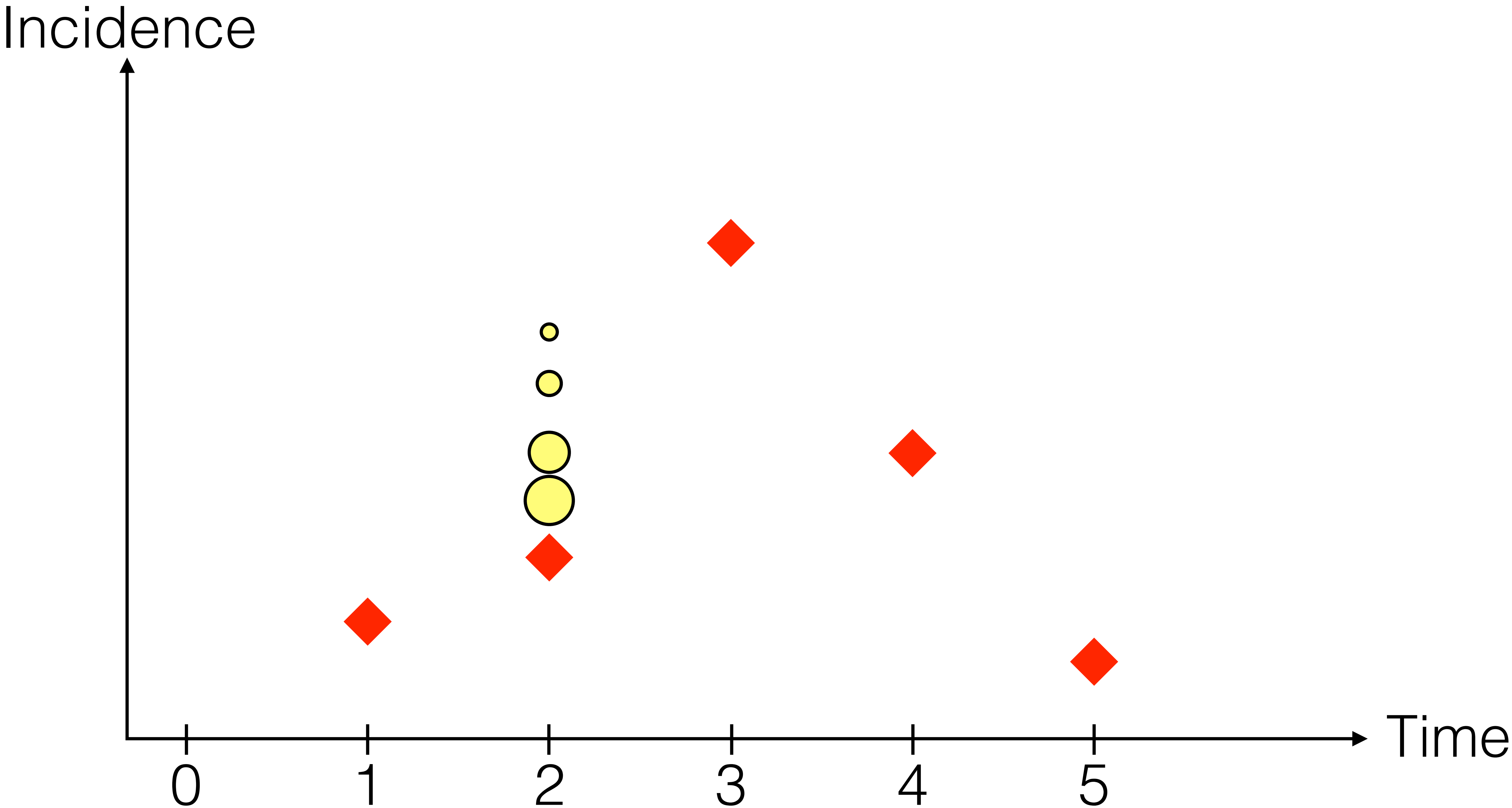
$\propto w_1$

**Use the R function sample(...)**



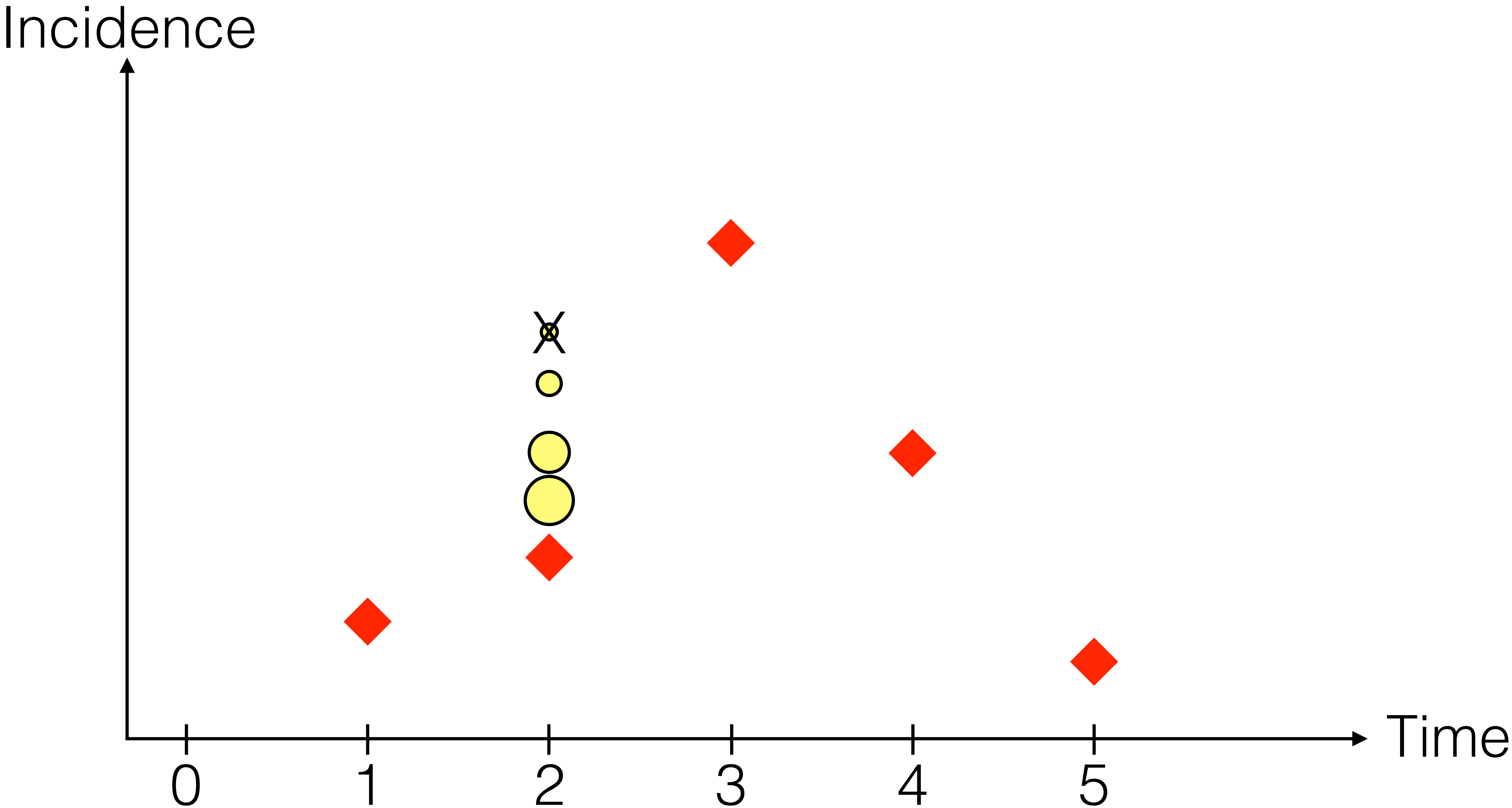

Propagate

$$\begin{cases} x_2 \sim p(.|x_1, \theta) \\ \dots \end{cases}$$



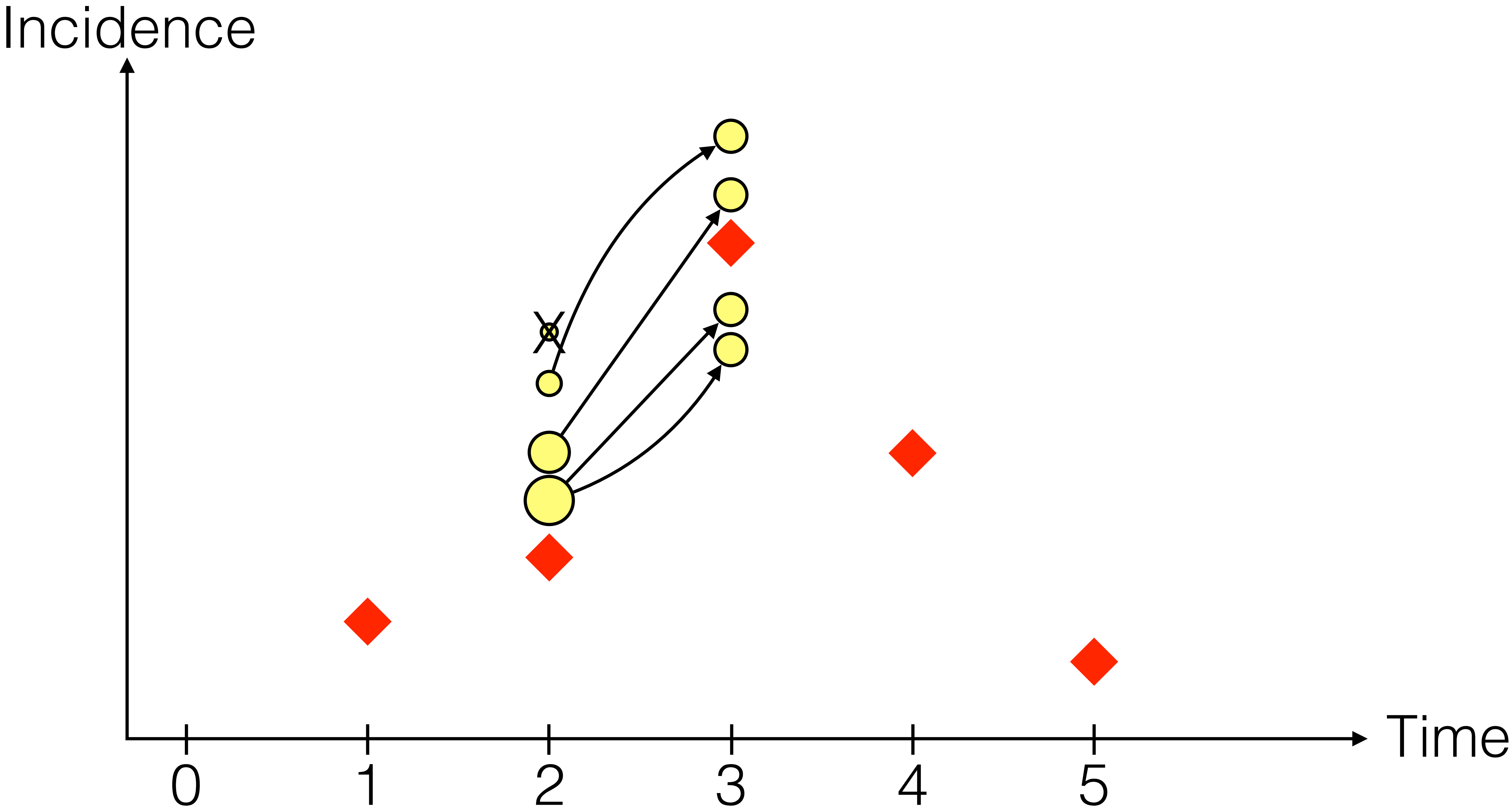

Weight

$$\begin{cases} x_2 \sim p(.|x_1, \theta) \\ w_2 = p(y_2|x_2, \theta) \end{cases}$$



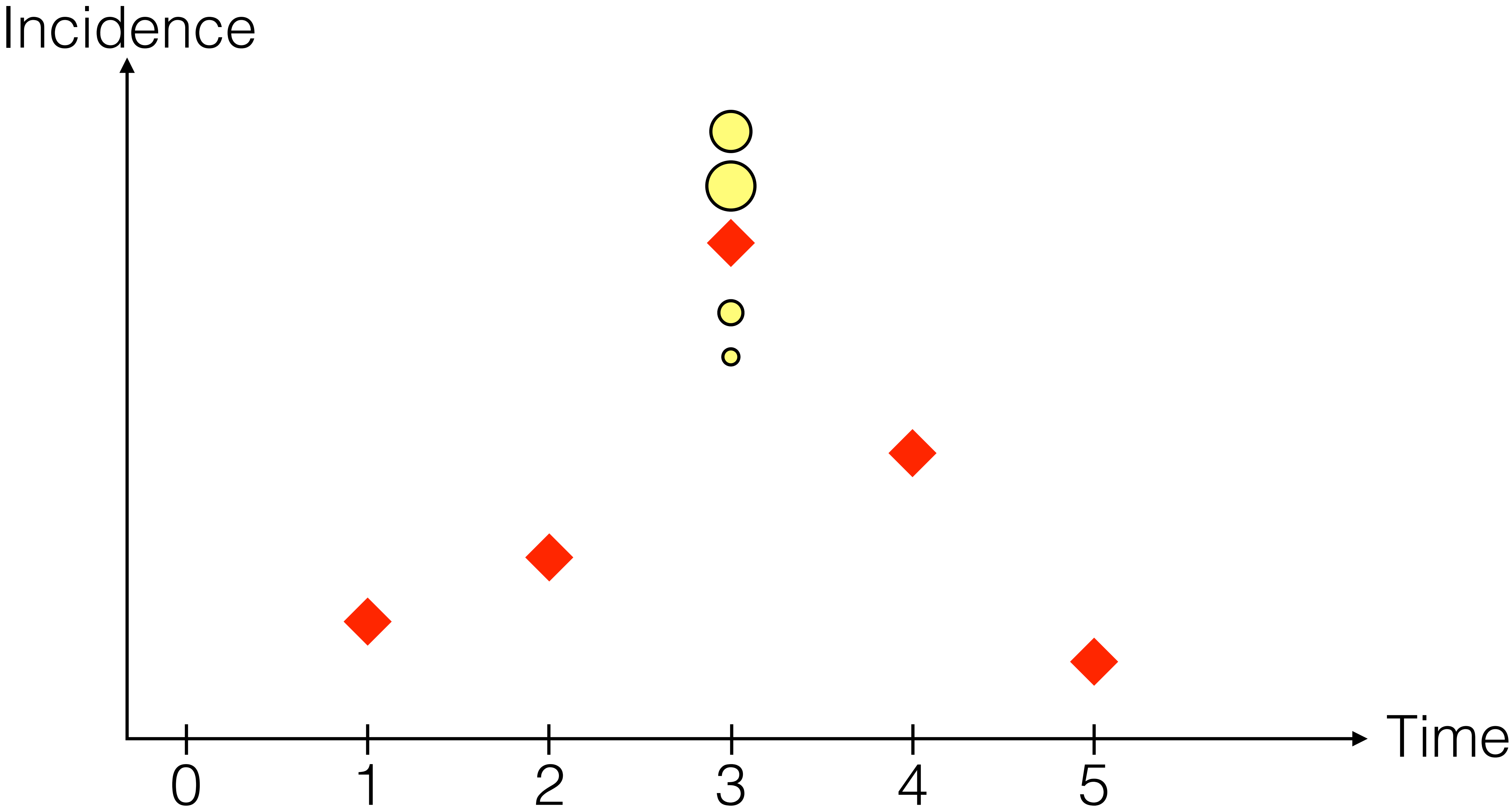

Resample

$\propto w_2$



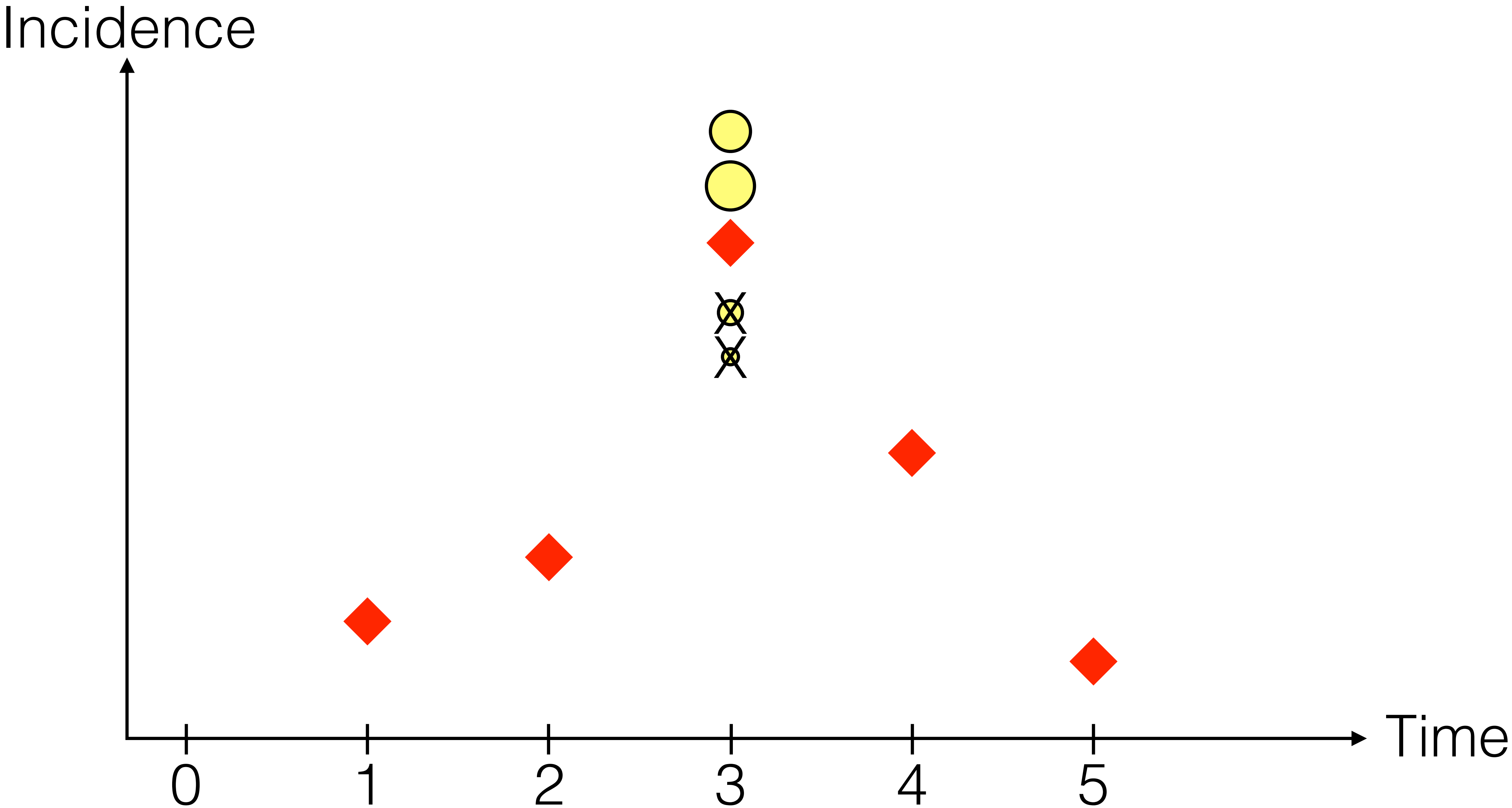

Propagate

$$\begin{cases} x_3 \sim p(.|x_2, \theta) \\ \dots \end{cases}$$



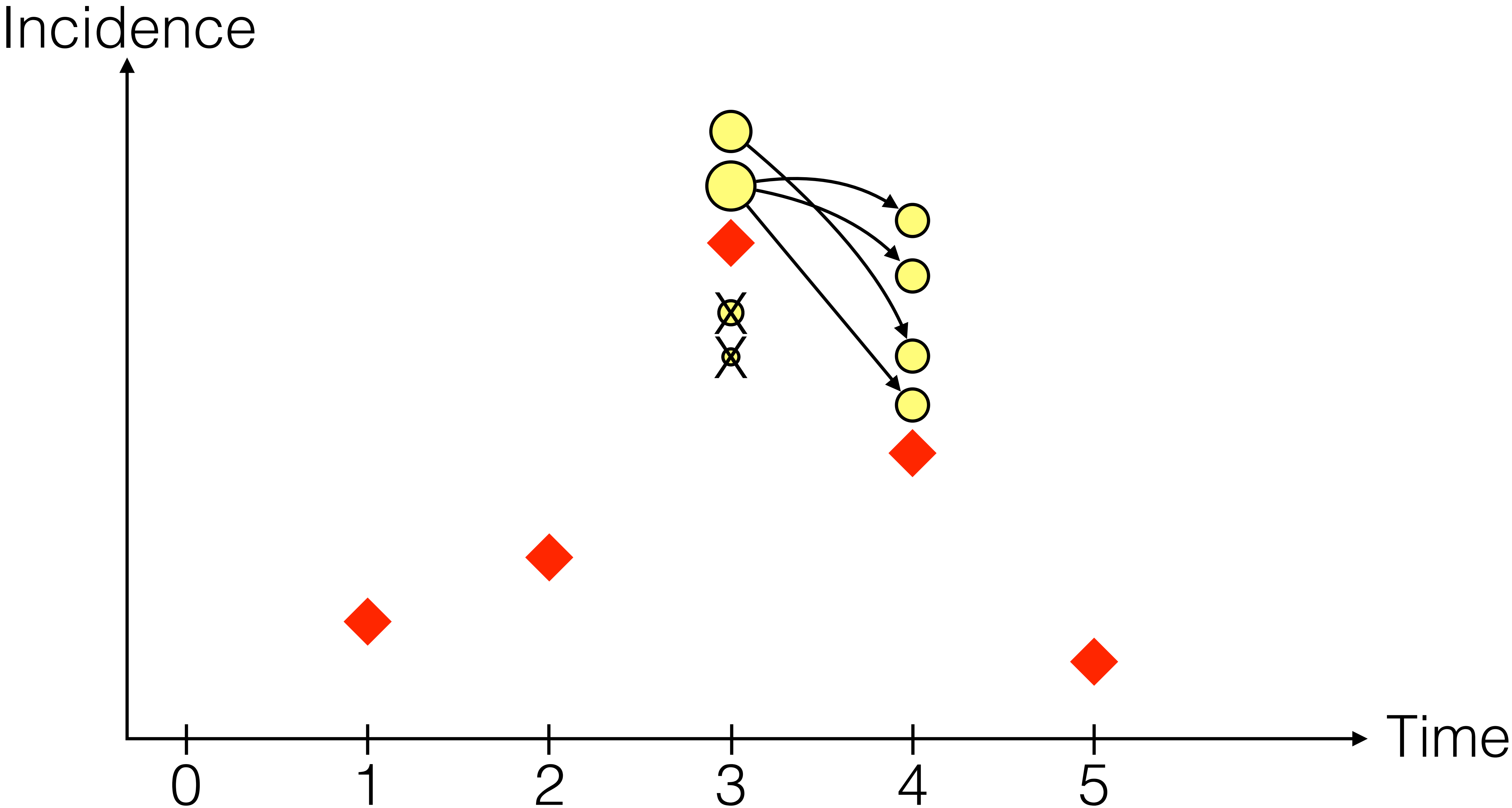

Weight

$$\begin{cases} x_3 \sim p(.|x_2, \theta) \\ w_3 = p(y_3|x_3, \theta) \end{cases}$$



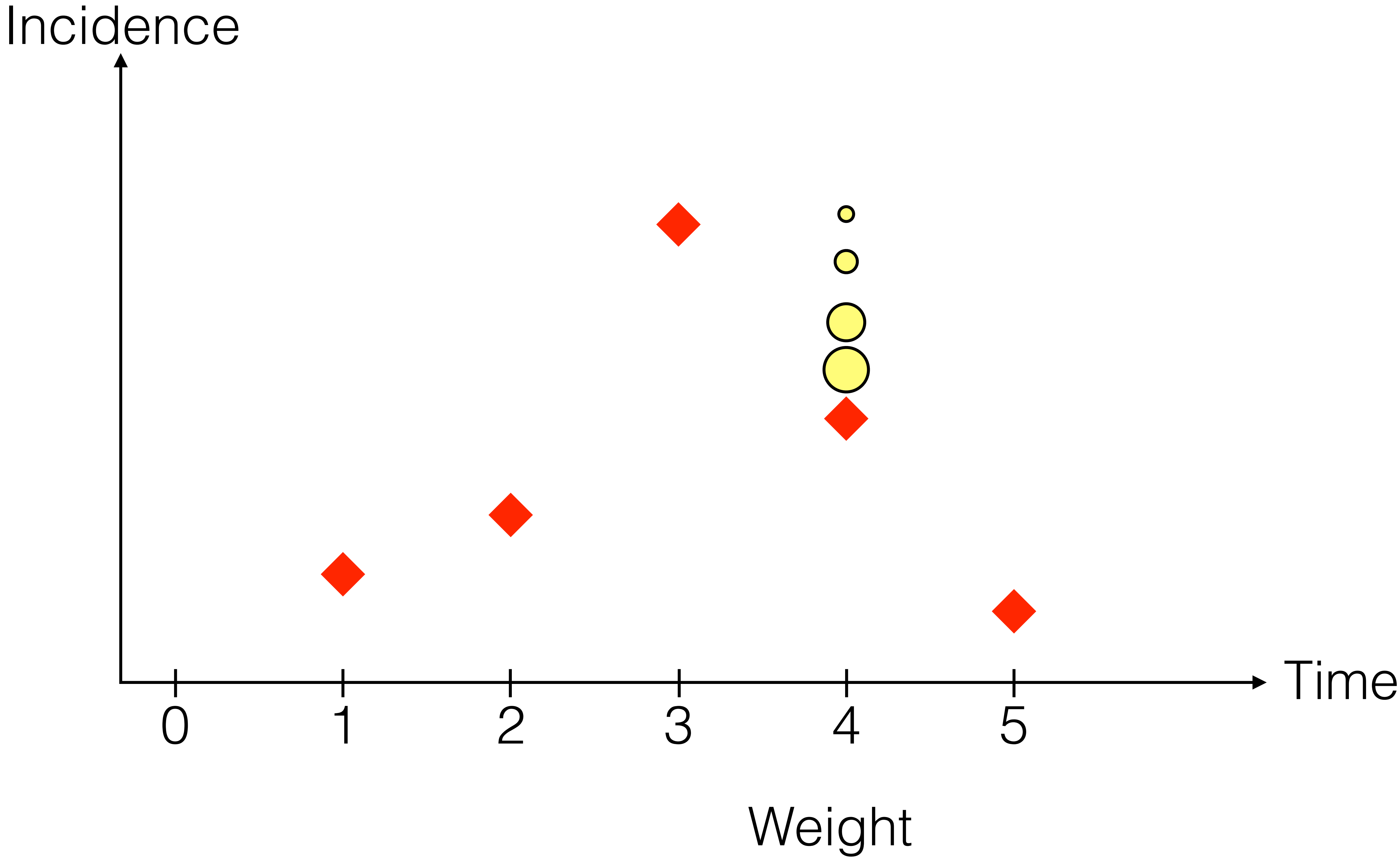

Resample

$\propto w_3$

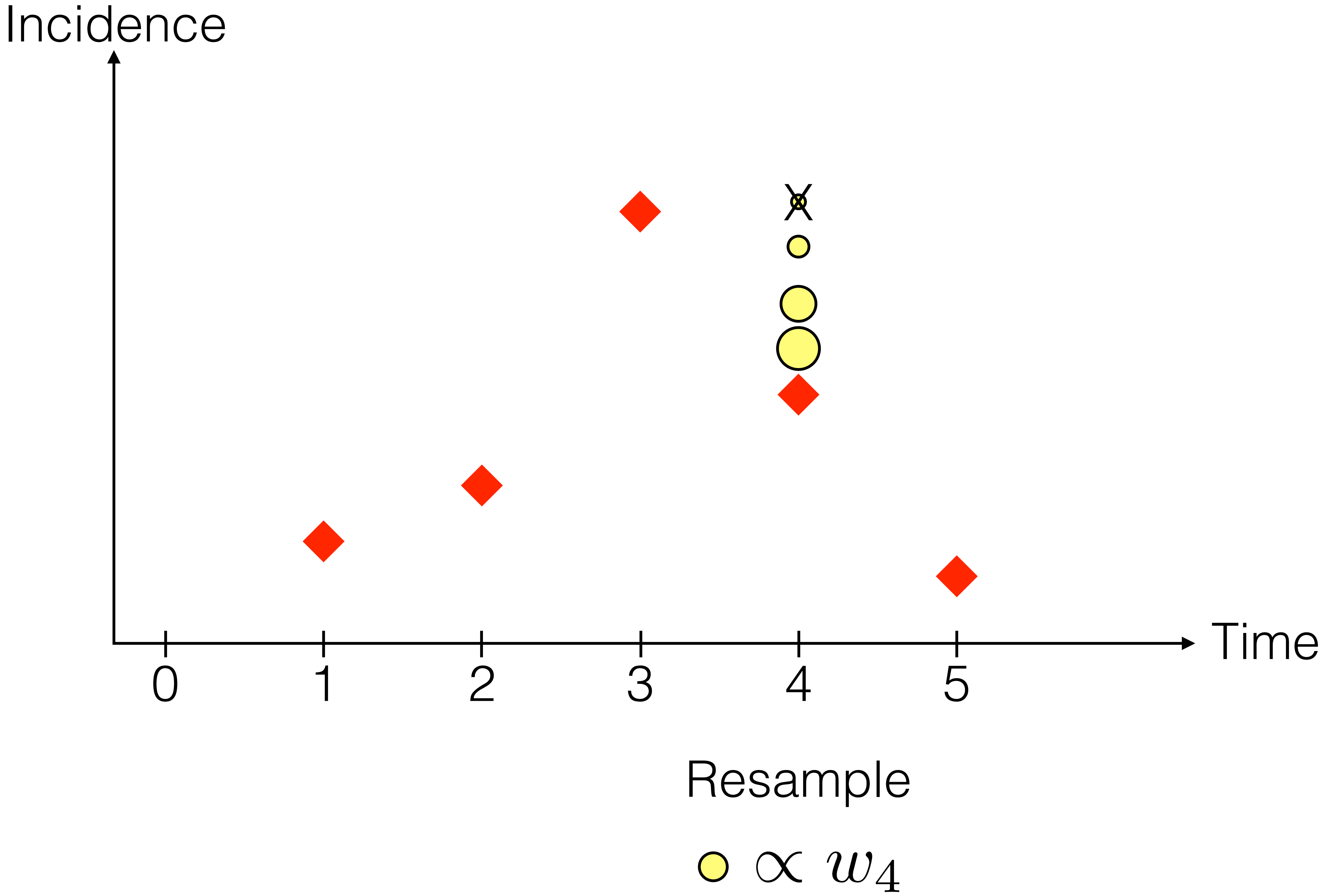


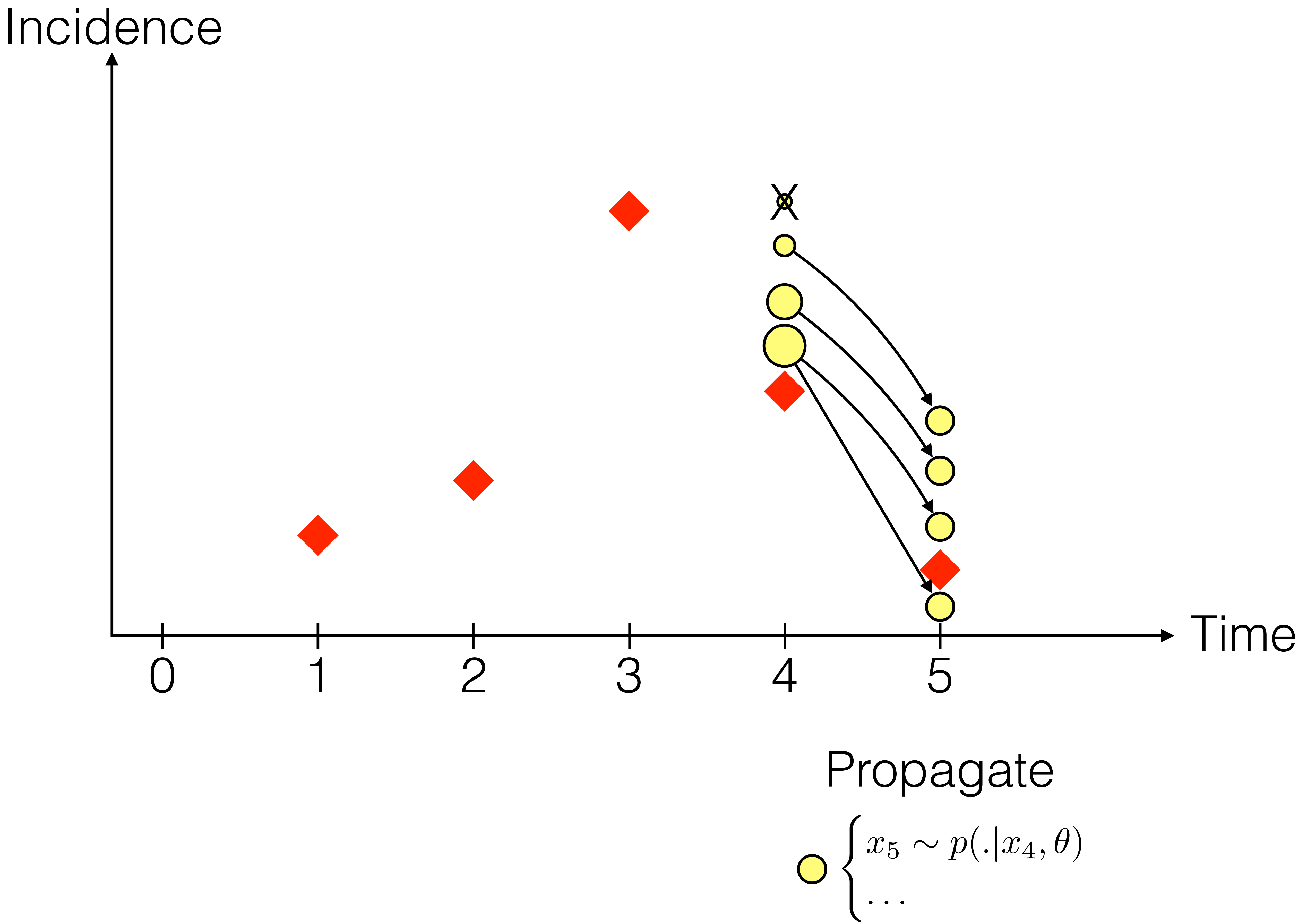

Propagate

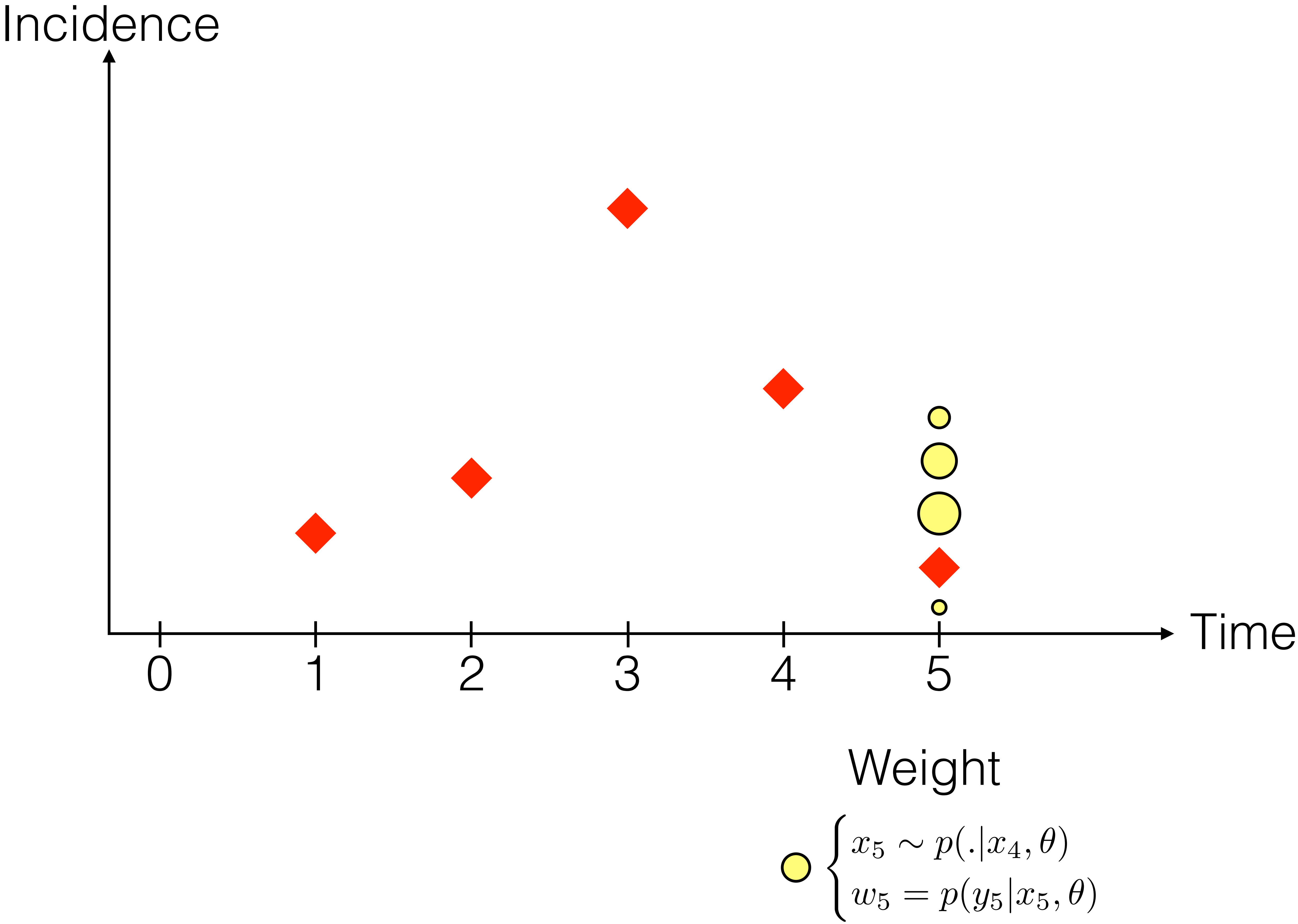
$$\begin{cases} x_4 \sim p(.|x_3, \theta) \\ \dots \end{cases}$$



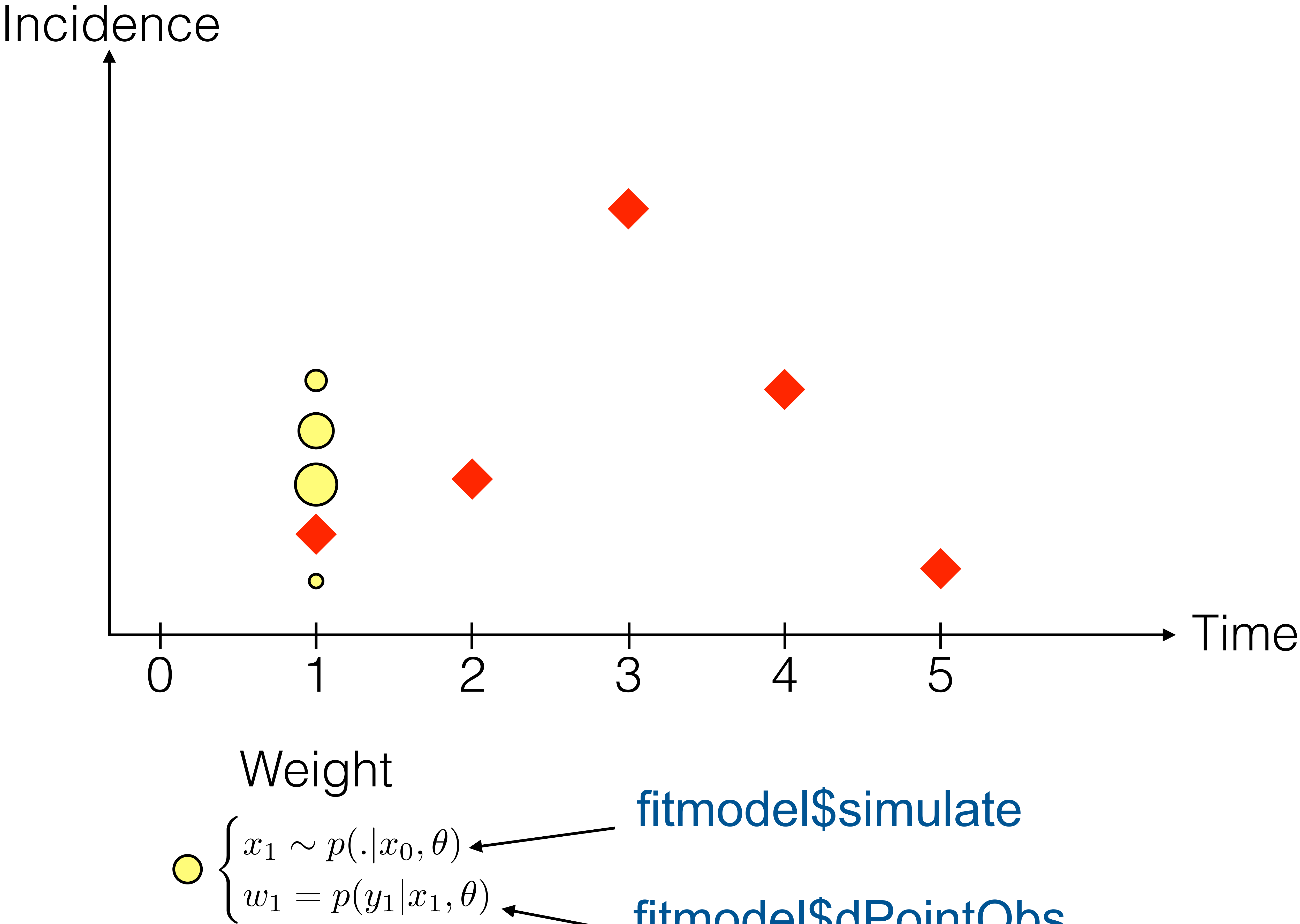

$$\begin{cases} x_4 \sim p(.|x_3, \theta) \\ w_4 = p(y_4|x_4, \theta) \end{cases}$$


Incidence
Time
0
1
2
3
4
5
Resample
$\propto w_4$

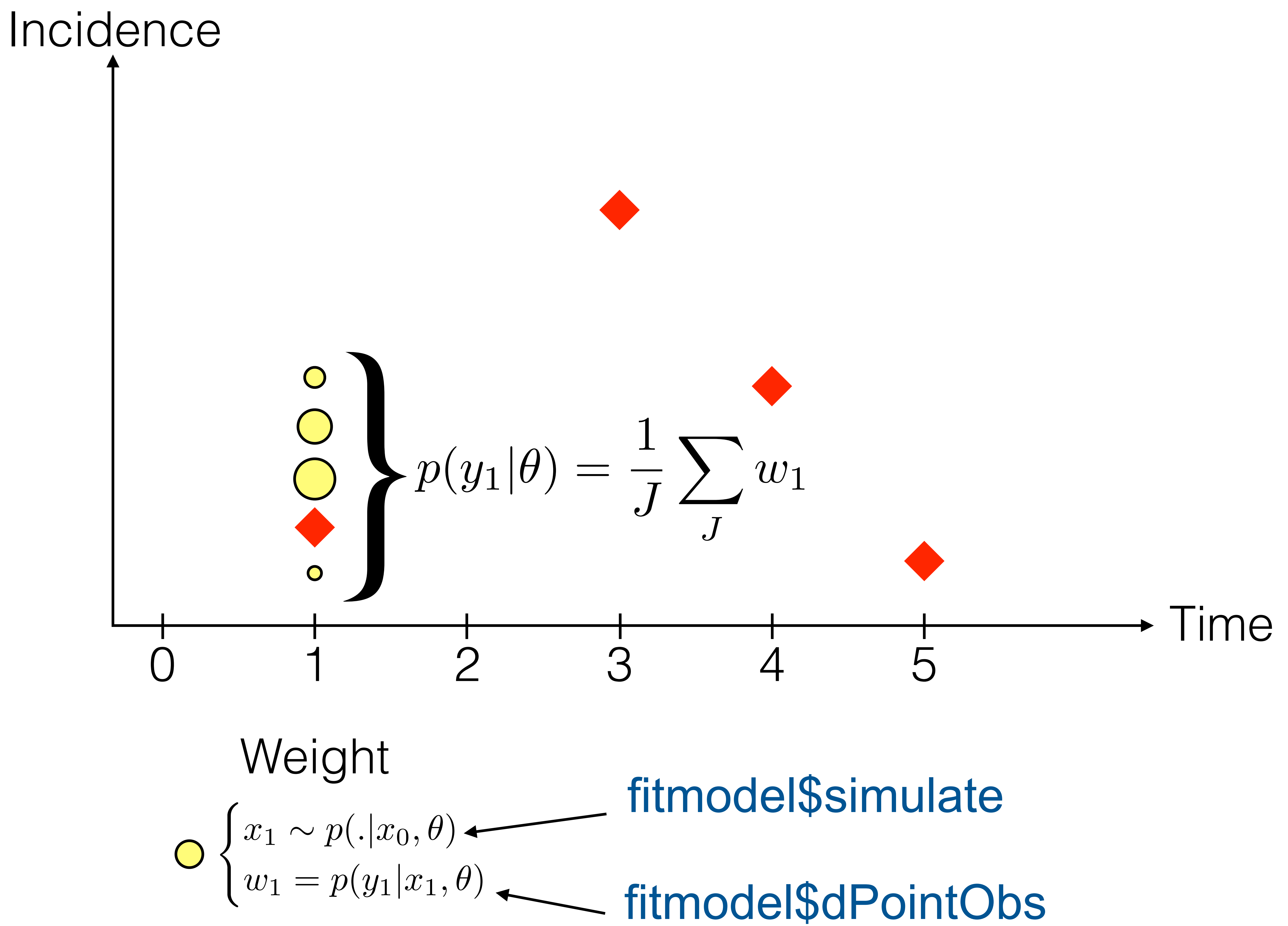

Incidence
Time
0
1
2
3
4
5
Propagate
$x_5 \sim p(.|x_4, \theta)$
$\dots$


Incidence
Time
0
1
2
3
4
5
Weight
$x_5 \sim p(.|x_4, \theta)$
$w_5 = p(y_5|x_5, \theta)$

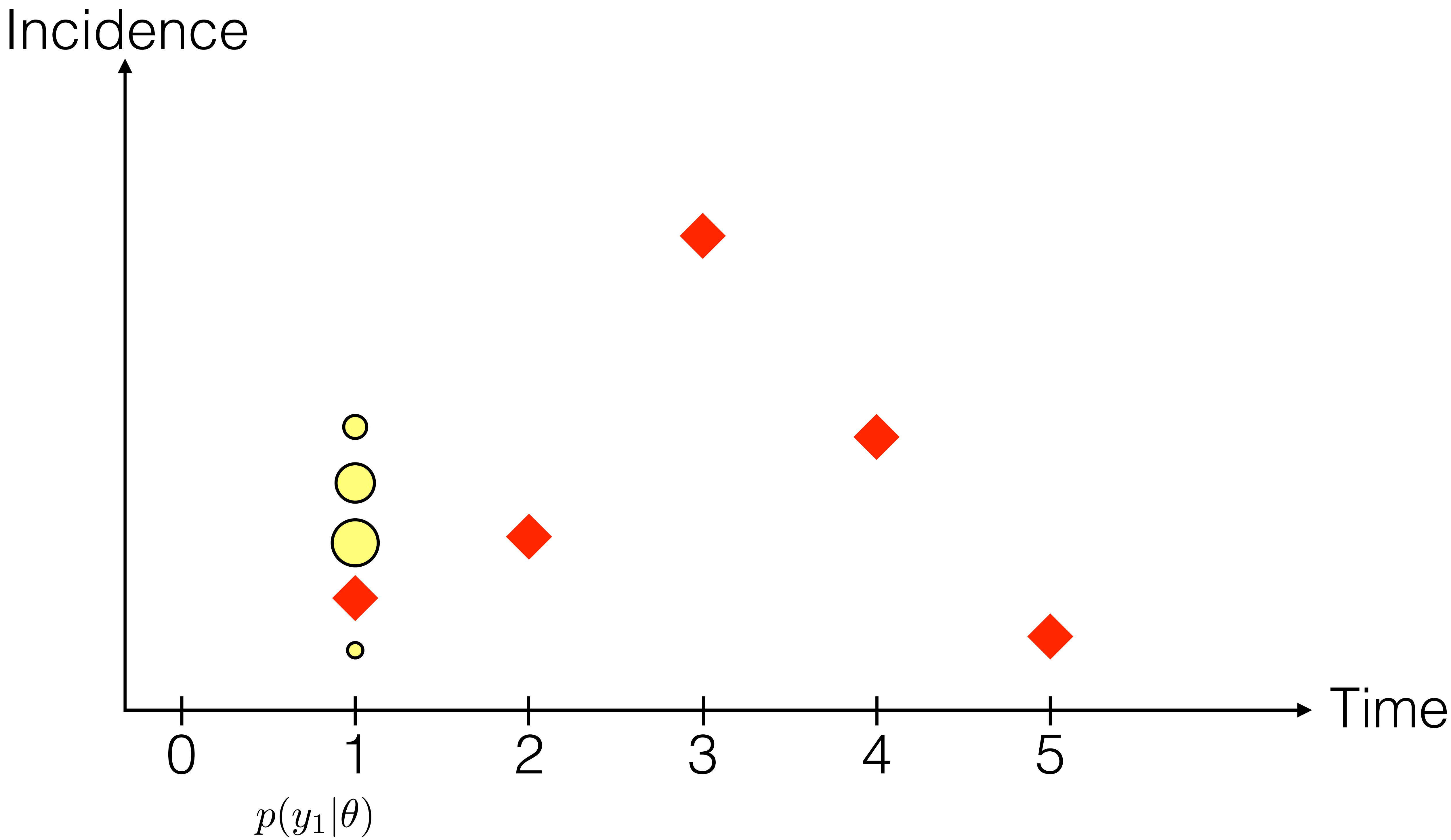
So how can I get the likelihood from this particle filter?

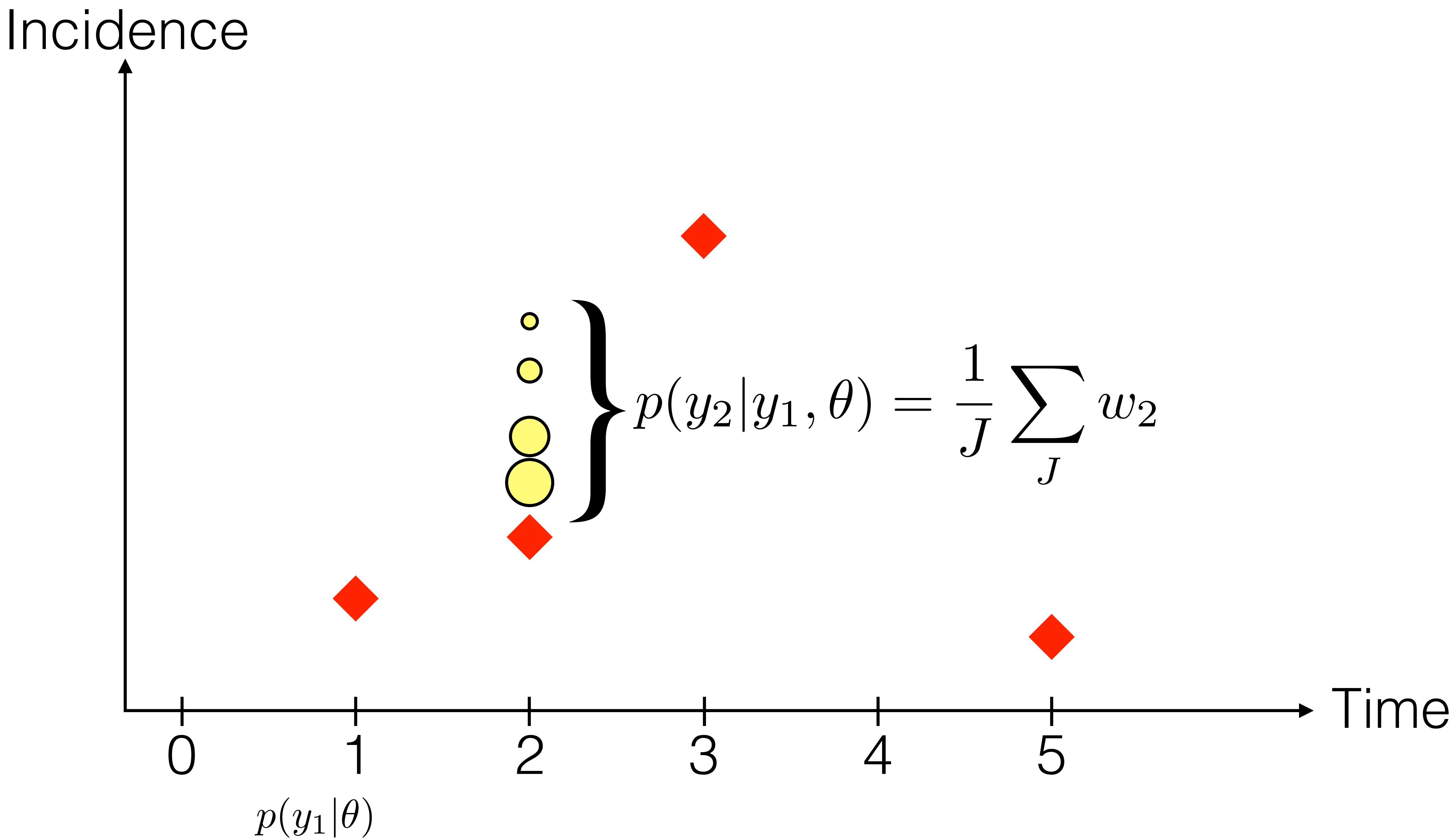


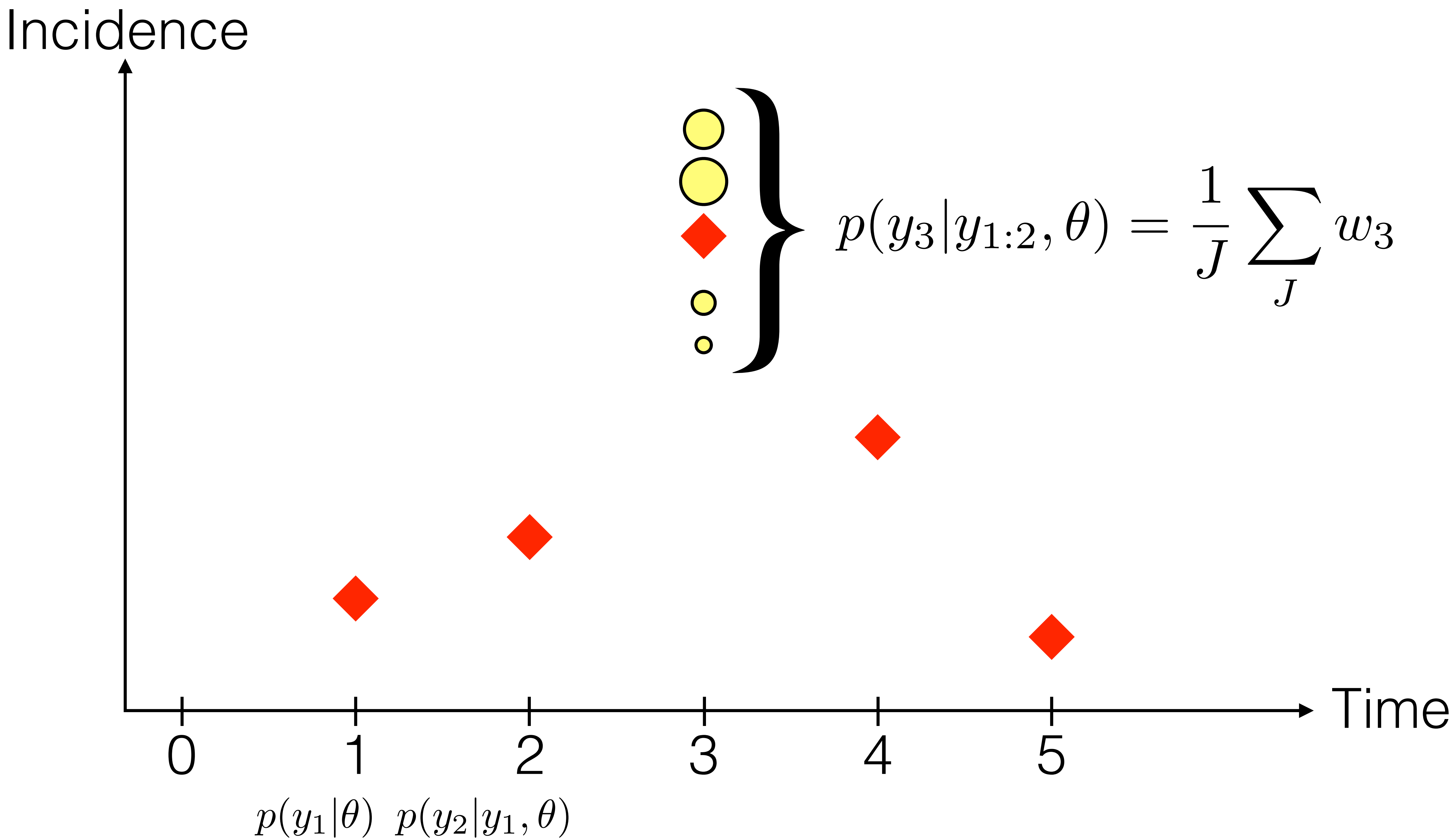
Incidence
Time
0
1
2
3
4
5
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate
fitmodel$dPointObs



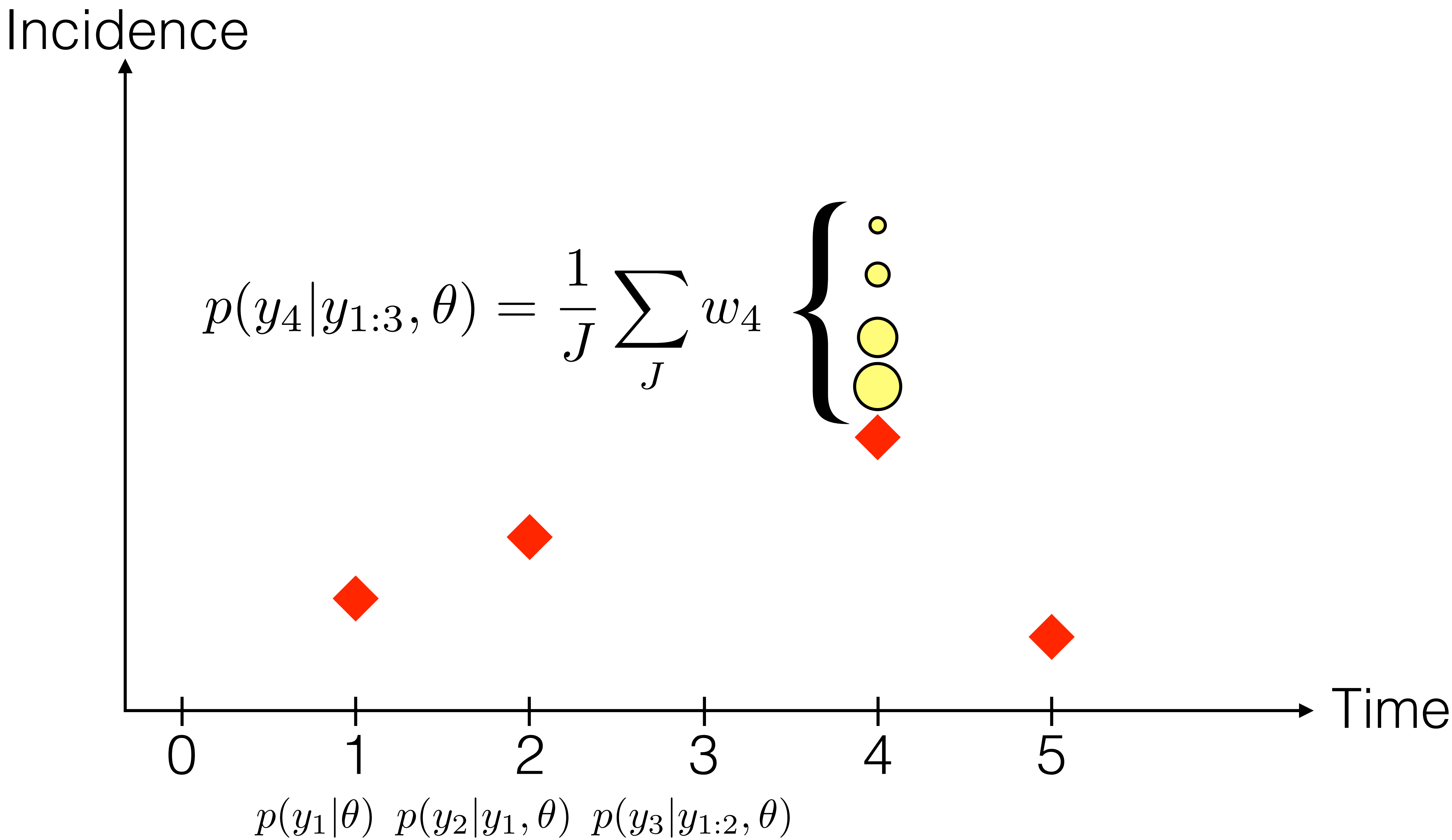
Incidence
Time
0
1
2
3
4
5
$p(y_1|\theta) = \frac{1}{J}\sum_J w_1$
Weight
$x_1 \sim p(.|x_0, \theta)$
$w_1 = p(y_1|x_1, \theta)$
fitmodel$simulate
fitmodel$dPointObs


Incidence
Time
0
1
2
3
4
5
$p(y_1|\theta)$

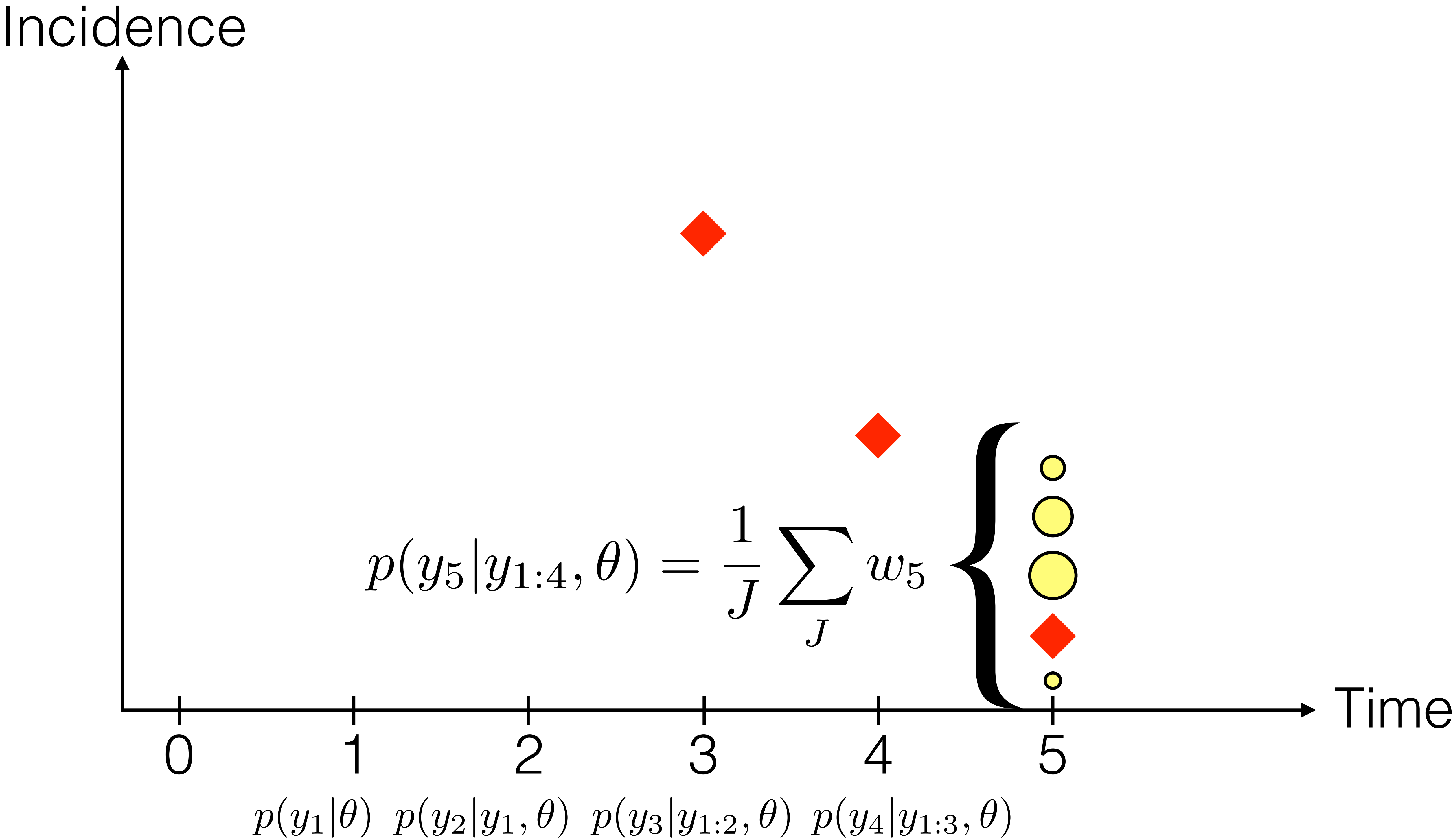

Incidence
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta) = \frac{1}{J}\sum_J w_2$

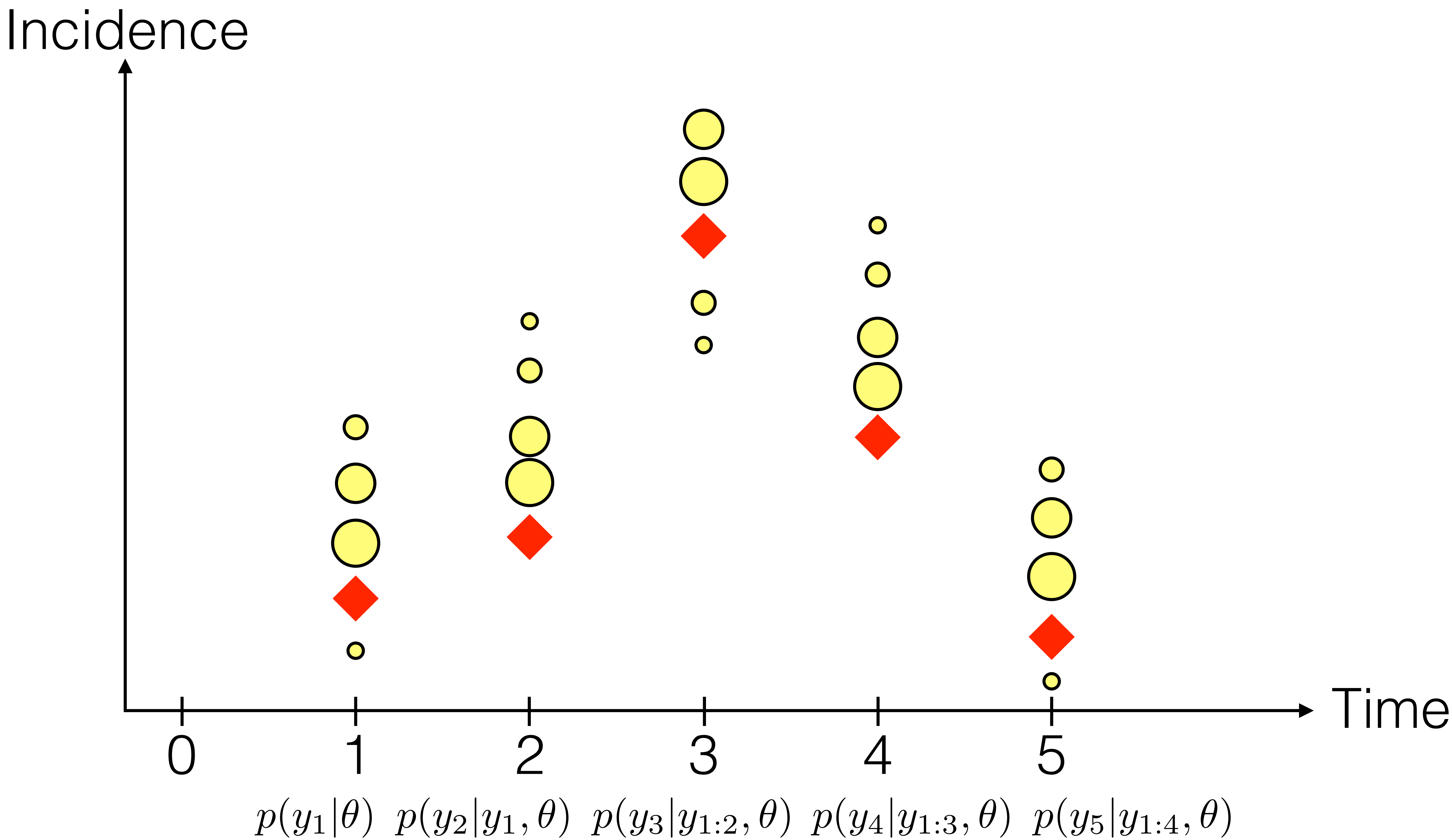


Incidence
$p(y_3|y_{1:2},\theta) = \frac{1}{J}\sum_J w_3$
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$

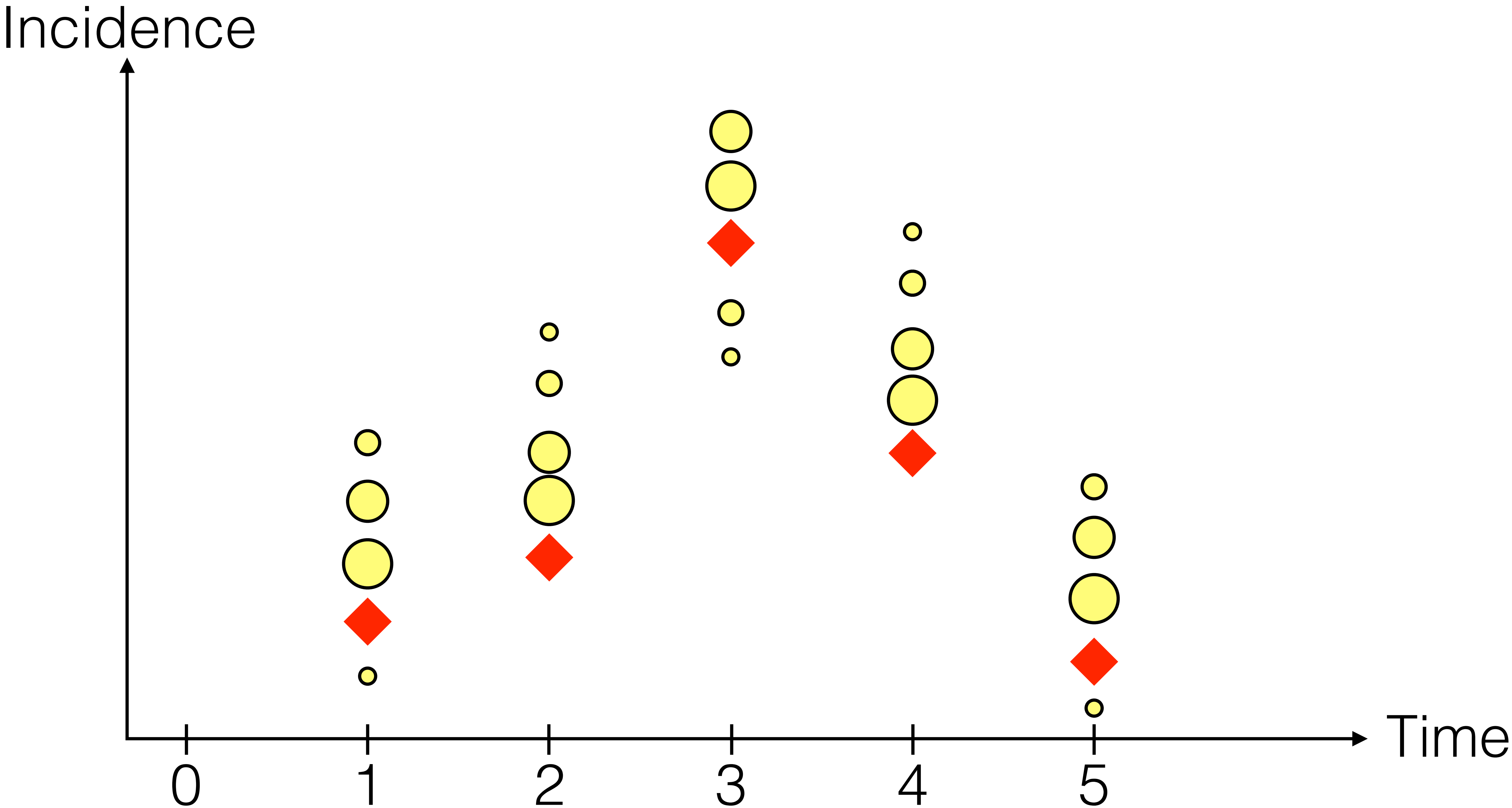


Incidence
$p(y_4|y_{1:3},\theta) = \frac{1}{J}\sum_J w_4$
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$
$p(y_3|y_{1:2},\theta)$


Incidence
Time
0
1
2
3
4
5
$p(y_5|y_{1:4},\theta) = \frac{1}{J}\sum_J w_5$
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$
$p(y_3|y_{1:2},\theta)$
$p(y_4|y_{1:3},\theta)$

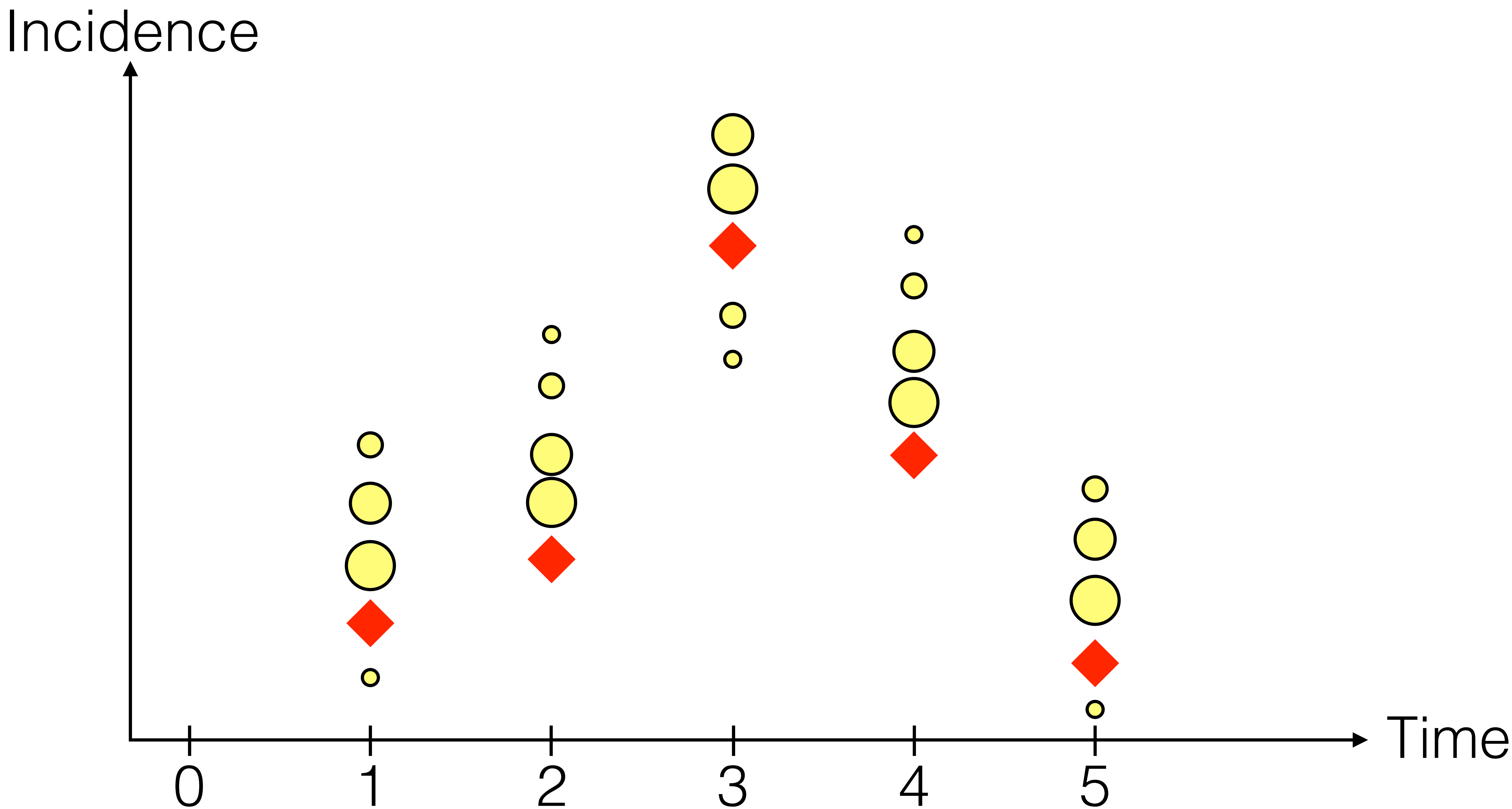


Incidence
Time
0
1
2
3
4
5
$p(y_1|\theta)$
$p(y_2|y_1,\theta)$
$p(y_3|y_{1:2},\theta)$
$p(y_4|y_{1:3},\theta)$
$p(y_5|y_{1:4},\theta)$




$$p(y_1|\theta) \times p(y_2|y_1,\theta) \times p(y_3|y_{1:2},\theta) \times p(y_4|y_{1:3},\theta) \times p(y_5|y_{1:4},\theta)$$

*Likelihood:* $p(y_{1:T}|\theta) = \prod_{T} p(y_t|y_{1:t-1},\theta)$




$$p(y_1|\theta) \times p(y_2|y_1,\theta) \times p(y_3|y_{1:2},\theta) \times p(y_4|y_{1:3},\theta) \times p(y_5|y_{1:4},\theta)$$

*Log-Likelihood:* $\log\{p(y_{1:T}|\theta)\} = \sum_{T} \log\{p(y_t|y_{1:t-1},\theta)\}$

# Implement your own particle filter

**Go to the pMCMC practical**

# Pseudocode for the particle filter

1. **For each particle j = 1 … J**
2. initialise the sate of particle j
3. initialise the weight of particle j
4. **For each observation time t = 1 … T**
5. resample particles
6. **For each particle j = 1 … J**
7. propagate particle j to next observation time
8. weight particle j