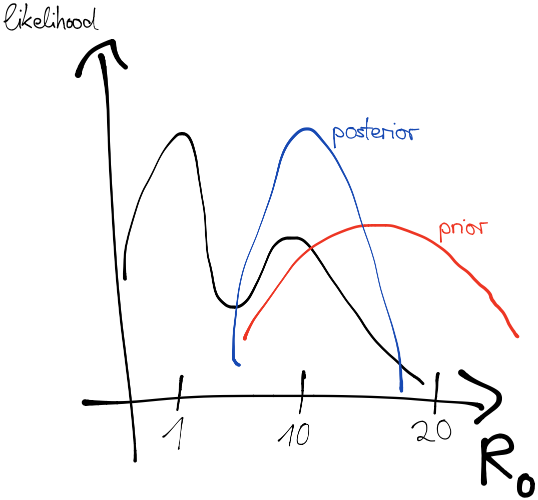
Outline

1Revision

The point of all of this

- posterior probabilities $p(\theta|\text{data}) = \frac{p(\text{data}|\theta)p(\theta)}{p(\text{data})}$



Sampling from the posterior

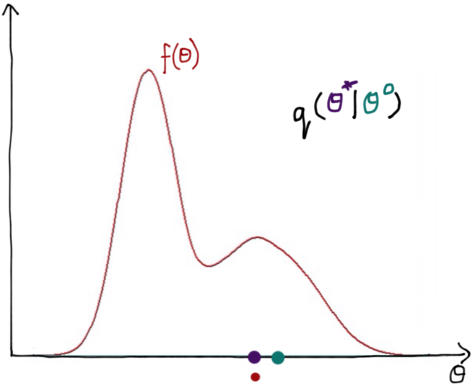
We interpret $p(\theta|\mathrm{data})$ as the probability distribution of a random variable θ, from which we sample (via MCMC)

Why sample?

1. explore parameter space
2. samples can be useful
   - explore interventions, forecasts

MCMC: Sampling from a distribution

- We can calculate (in a deterministic model) $p(\theta|\text{data})$ given any θ – how do we sample?



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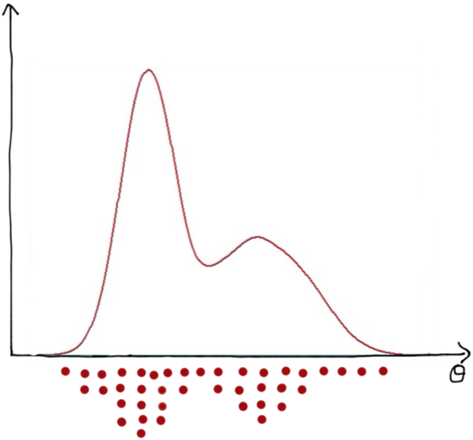
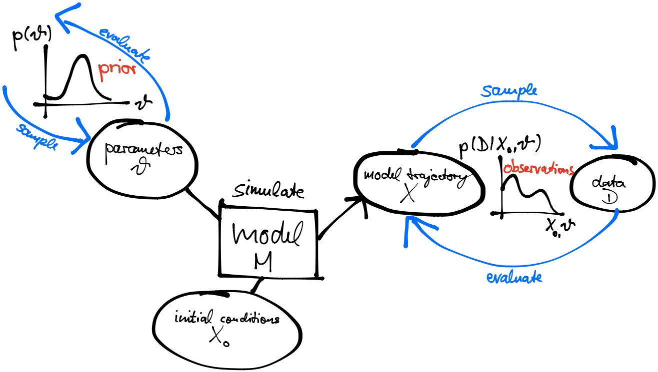


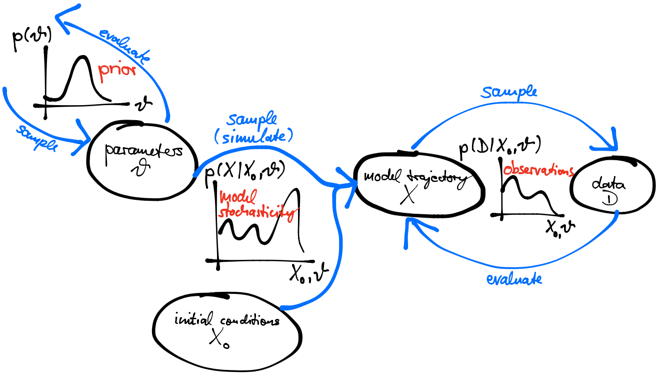
Deterministic models



$p(\theta|\text{data}) \propto p(\text{data}|\theta)p(\theta)$

Use MCMC to get samples from it: θ_1, θ_2, θ_3, $\ldots$

Stochastic models

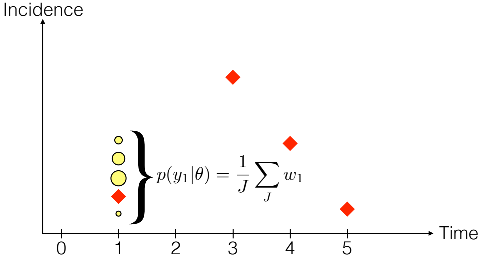


Stochastic models

- one θ can lead to many possible outcomes X
- we can
  1. sample from $p(X|\theta)$ (via simulation)
  2. evaluate the trajectory likelihood $p(\mathrm{data}|X, \theta)$
- we can't directly evaluate the likelihood $p(\mathrm{data}|\theta)$
  $p(\mathrm{data}|\theta) = \sum_X p(\mathrm{data}|X, \theta)p(X|\theta)$
- The number of possible trajectories X for one value of θ is large (usually infinite)
- We replace the sum above with a Monte Carlo (random) sample

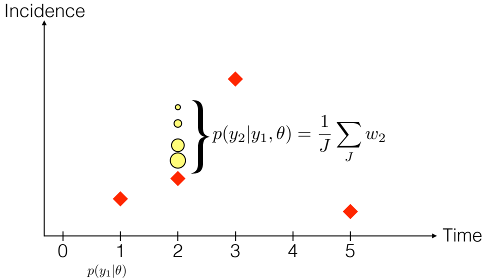
Sequential Monte Carlo (SMC) / Particle Filter I

We sample J trajectories X_J from $p(X_{J,1}|\theta)$ and sum over $p(y_1|X_{J,1},\theta)$



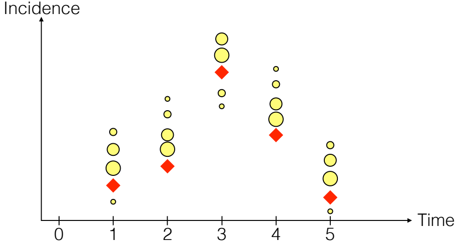
Sequential Monte Carlo (SMC) / Particle Filter II

We then sample J trajectories X_J from $p(X_{J,2}|y_1,\theta)$ and sum over $p(y_2|X_{J,2},\theta)$



Sequential Monte Carlo (SMC) / Particle Filter III

The sum of all these (logged) values is $\sum_J p(y_{1:T}|X_J,\theta)p(X_J|\theta)$ which is a sample estimate of the likelihood.

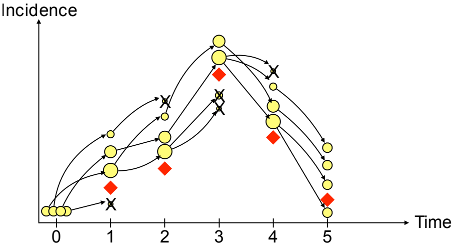


$p(y_1|\theta) \times p(y_2|y_1,\theta) \times p(y_3|y_{1:2},\theta) \times p(y_4|y_{1:3},\theta) \times p(y_5|y_{1:4},\theta)$

Log-Likelihood: $\log\{p(y_{1:T}|\theta)\} = \sum_T \log\{p(y_t|y_{1:t-1},\theta)\}$

Sequential Monte Carlo (SMC) / Particle Filter IV

We can also retrieve filtered trajectories, that is samples from $p(X|\mathrm{data})$ by following the particles from the last point backwards.



pMCMC

- Once we can estimate $p(\mathrm{data}|\theta)$, we can combine this with the prior to evaluate the posterior $p(\theta|\mathrm{data})$ for any θ.
- We can then use MCMC to sample from this -> pMCMC

Sampling from the posterior

MCMC

Sampling from the posterior

Estimating the likelihood

MCMC

SMC

PMCMC ✓

Sampling from the posterior

Estimating the likelihood

	MCMC	SMC
SMC	PMCMC ✓	SMC^2

Sampling from the posterior

Estimating the likelihood

	MCMC	SMC
SMC	PMCMC	SMC^2
ABC	ABC-MCMC	ABC-SMC