

Review

Centre for the Mathematical Modelling of Infectious Diseases
London School of Hygiene & Tropical Medicine

LONDON
SCHOOL of
HYGIENE
& TROPICAL
MEDICINE

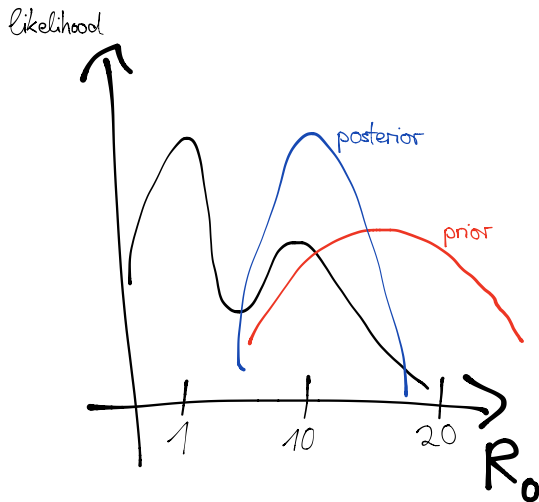


centre for the
mathematical
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The point of all of this

- posterior probabilities

$$p(\theta|\text{data}) = \frac{p(\text{data}|\theta)p(\theta)}{p(\text{data})}$$



Sampling from the posterior

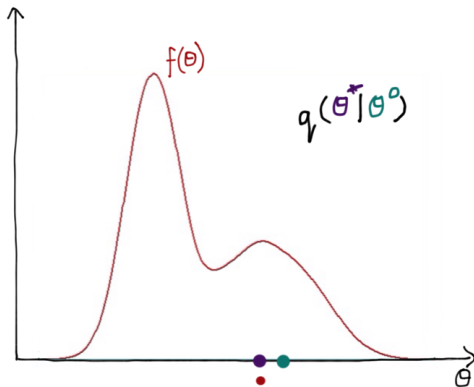
We interpret $p(\theta|\text{data})$ as the probability distribution of a random variable θ , from which we ***sample*** (via MCMC)

Why sample?

1. explore parameter space
2. samples can be useful
 - ▶ explore interventions, forecasts

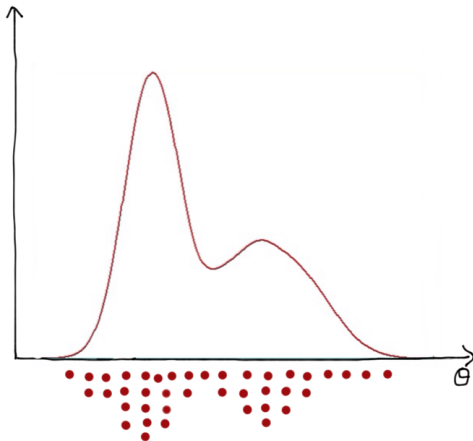
MCMC: Sampling from a distribution

- We can calculate (in a deterministic model) $p(\theta|\text{data})$ given any θ – how do we sample?



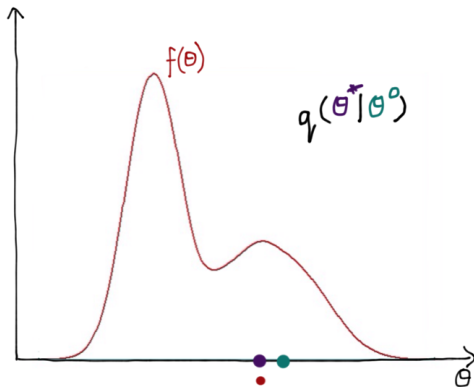
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Stochastic models

- ▶ one θ can lead to many possible outcomes X
- ▶ we can
 1. sample from $p(X|\theta)$ (via simulation)
 2. evaluate the trajectory likelihood $p(\text{data}|X, \theta)$
- ▶ we can't directly evaluate the likelihood $p(\text{data}|\theta)$

$$p(\text{data}|\theta) = \sum_X p(\text{data}|X, \theta)p(X|\theta)$$

- ▶ The number of possible trajectories X for one value of θ is large, potentially infinite

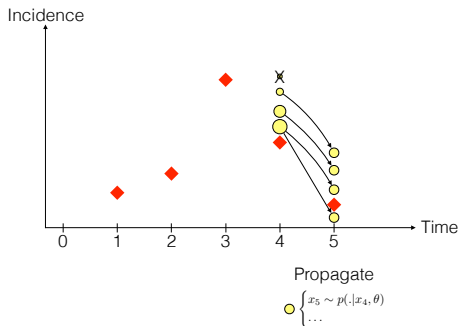
Sequential Monte Carlo (SMC) / Particle Filter I

To approximate

$$p(\text{data}|\theta) = \sum_X p(\text{data}|X, \theta)p(X|\theta)$$

we **sample** n trajectories X_n from

$$p(Y_{1:(T-1)}|X, \theta)p(X|\theta)$$



Sequential Monte Carlo (SMC) / Particle Filter II

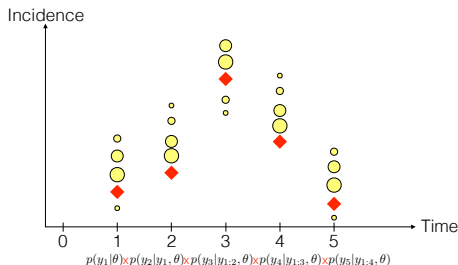
We then calculate

$$p(Y_T | X_n, \theta)$$

for each of the particles. The sum of these values is

$$\sum_{X_n} p(Y_{1:T} | X_n, \theta) p(X_n | \theta)$$

which is a **sample estimate** of the likelihood.



Log-Likelihood: $\log\{p(y_{1:T}|\theta)\} = \sum_T \log\{p(y_t|y_{1:t-1}, \theta)\}$

pMCMC

- ▶ Once we can estimate $p(\text{data}|\theta)$, we can combine this with the prior to evaluate the **posterior** $p(\theta|\text{data})$ for any θ .
- ▶ We can then use MCMC to sample from this -> pMCMC

Sampling from the posterior

MCMC

Estimating the
likelihood

Sampling from the posterior

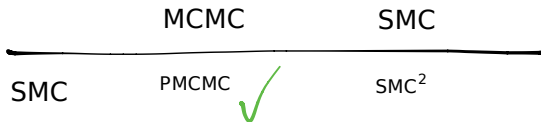
SMC

MCMC

PMCMC ✓

Sampling from the posterior

Estimating the
likelihood



Sampling from the posterior

Estimating the
likelihood

	MCMC	SMC
SMC	PMCMC ✓	SMC ²
ABC	ABC-MCMC	ABC-SMC