



# Excursion into Malaria Modeling

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**VACCINE  
IMPACT  
MODELLING  
CONSORTIUM**



# Background

- Professor of Biostatistics, University of Abomey-Calavi
- Coordinator, Master and PhD Programmes in Biostatistics
- Coordinator, Humboldt Research Hub on Socio-ecological modeling of COVID-19 (<https://hrh.semca-uac.org/en>)
- Group lead, Malaria modeling at the Vaccine Impact modeling Consortium

# Excursion into Malaria Modeling



The image shows a large group of approximately 40 people from diverse backgrounds, including men and women of various ethnicities, posing for a group photo. They are dressed in casual to semi-formal attire, with many wearing lanyards or name tags. The group is arranged in several rows, standing on a grassy lawn. In the background, there is a circular stone fountain with water flowing, surrounded by manicured green lawns, shrubs, and small flower beds with orange and yellow flowers. The overall atmosphere is professional and suggests a formal gathering or conference.

VACCINE IMPACT  
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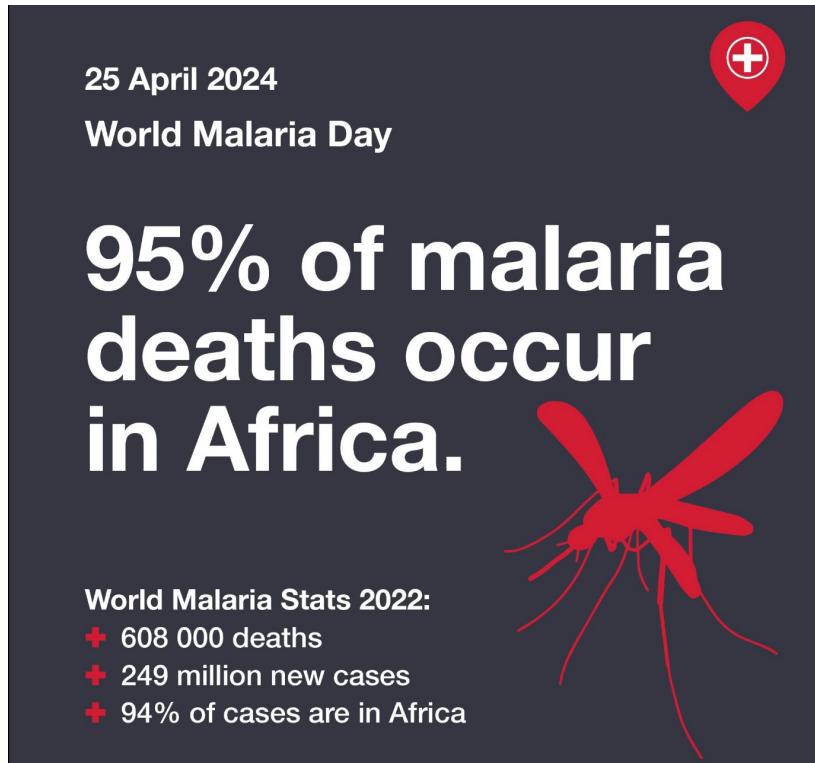
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Search...

VIMC is an international community of modellers providing high-quality estimates of the public health impact of vaccination, to inform and improve decision making.

# Background

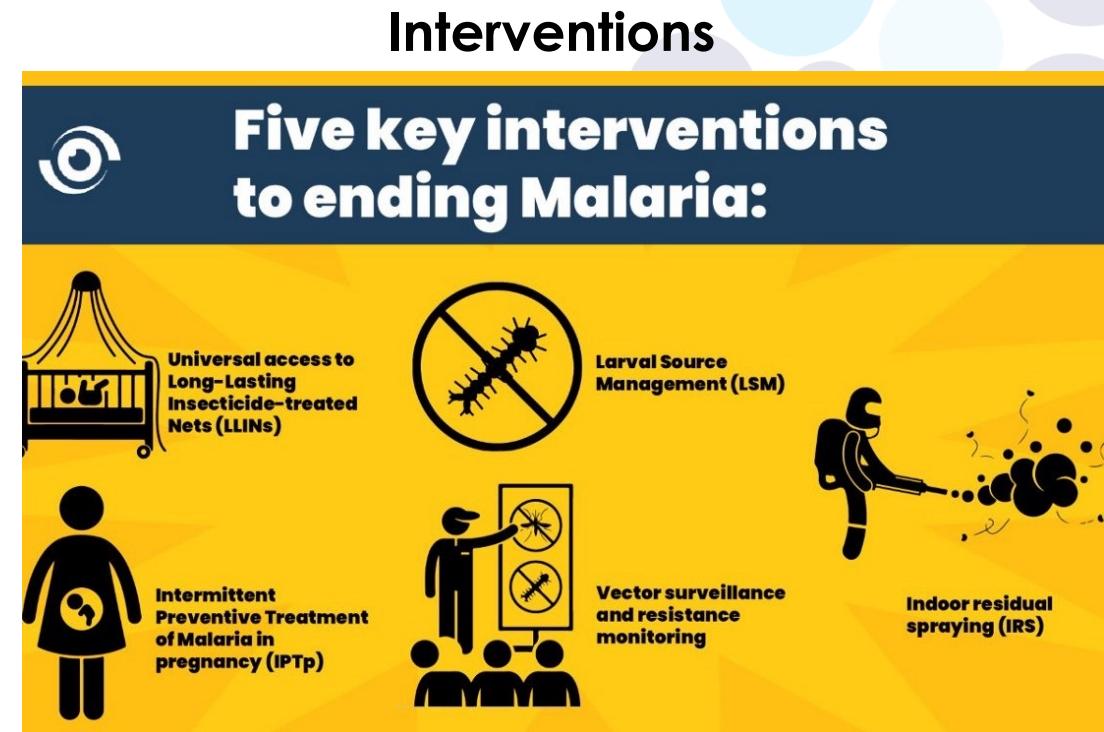
Malaria: vector-borne disease causing devastating health and economic burden



Source: Response-Med



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<https://allafrica.com/>

# Background

## Vaccination

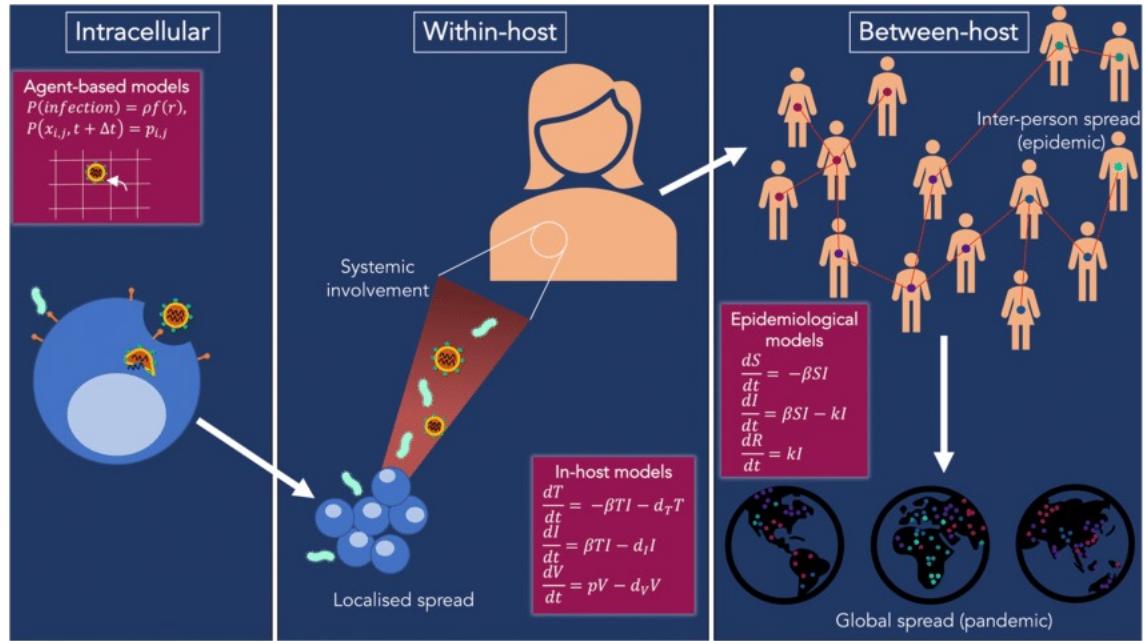


Source: <https://lanatayaise.com/>



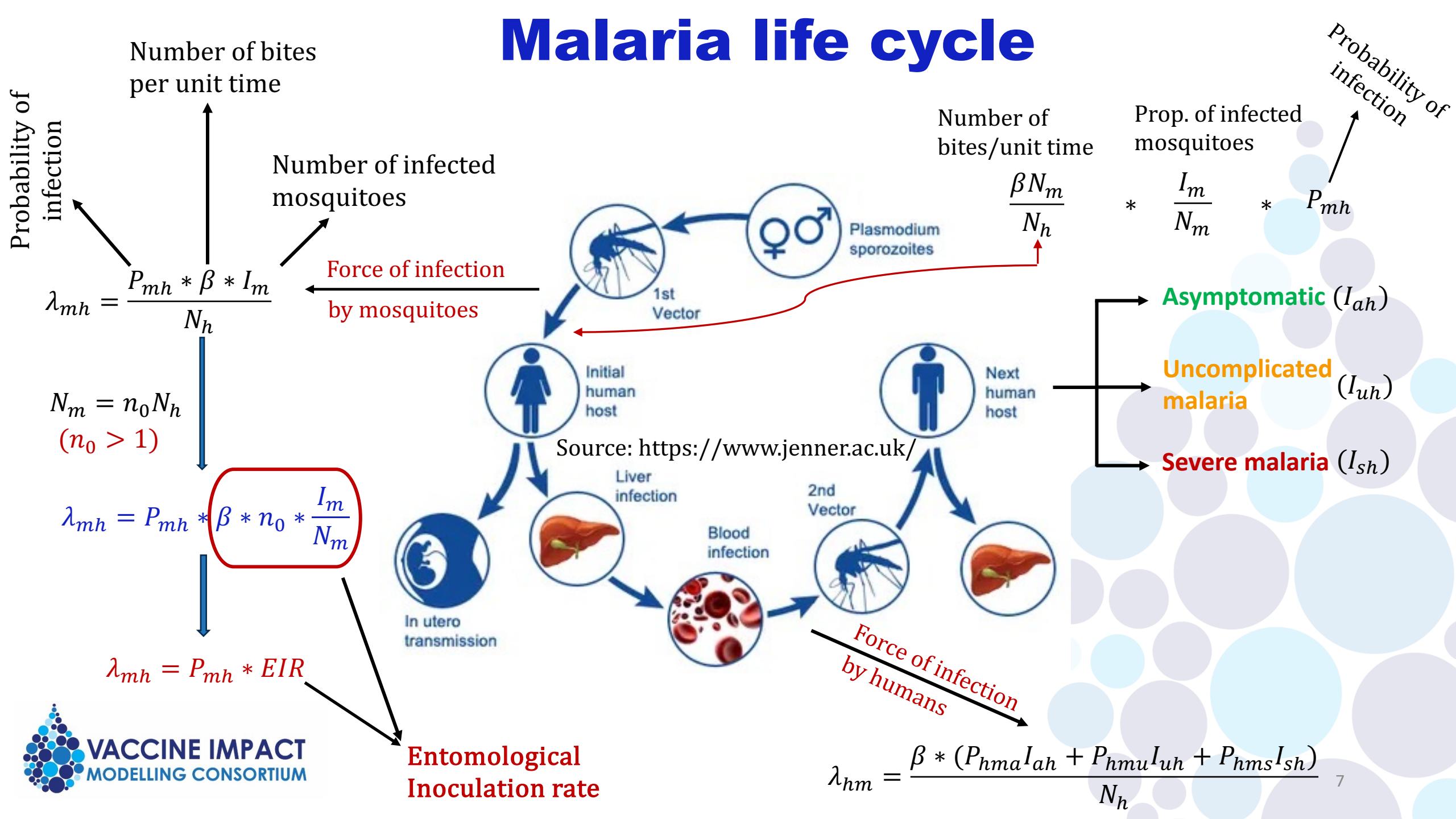
Source: <https://www.shutterstock.com/>

# Background

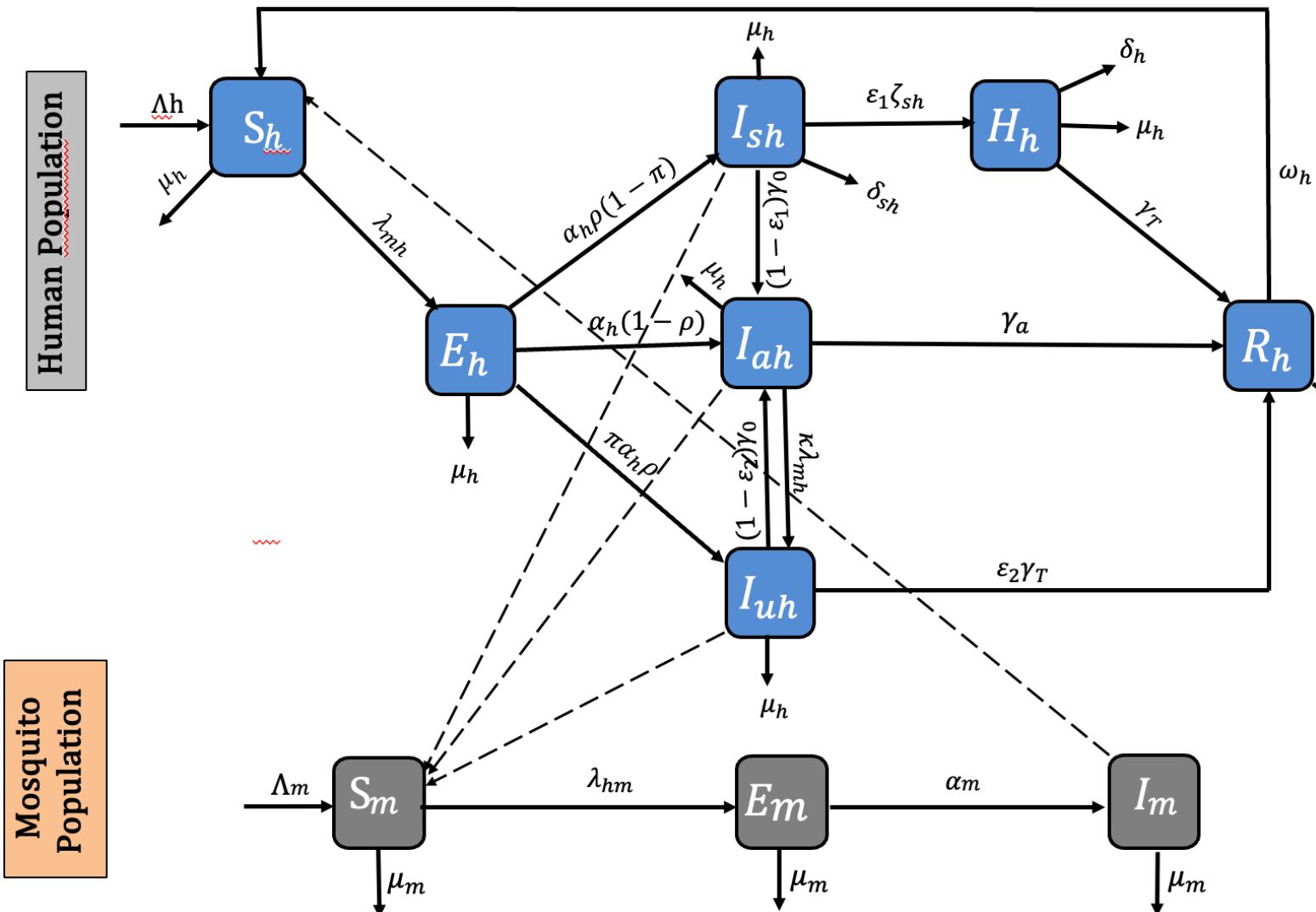


Source: <https://writeoddly.com/>

# Malaria life cycle



# Flowchart of Malaria dynamics



# Set of ODEs

## Set of ODEs linked to the model

$$\dot{S}_h = \Lambda + \omega_h R_h - (\lambda_{mh} + \mu_h) S_h, \quad (1a)$$

$$\dot{E}_h = \lambda_{mh} S_h - (\alpha_h + \mu_h) E_h, \quad (1b)$$

$$\dot{I}_{sh} = \alpha_h \rho (1 - \pi) E_h - (\varepsilon_1 \zeta_{sh} + (1 - \varepsilon_1) \gamma_0 + \mu_h + \delta_{sh}) I_{sh} - I_{sh}, \quad (1c)$$

$$\dot{I}_{ah} = \alpha_h (1 - \rho) E_h + (1 - \varepsilon_1) \gamma_0 I_{sh} + (1 - \varepsilon_2) \gamma_0 I_{uh} - a_0 I_{ah}, \quad (1d)$$

$$\dot{I}_{uh} = \alpha_h \rho \pi E_h + \kappa \lambda_{mh} I_{ah} - (\varepsilon_2 \gamma + (1 - \varepsilon_2) \gamma_0 + \mu_h) I_{uh}, \quad (1e)$$

$$\dot{H}_h = \varepsilon_1 \zeta_{sh} I_{sh} - (\delta_h + \gamma + \mu_h) H_h - H_h, \quad (1f)$$

$$\dot{R}_h = \gamma_a I_{ah} + \gamma_t H_h + \varepsilon_2 \gamma_t I_{uh} - (\omega_h + \mu_h) R_h, \quad (1g)$$

$$\dot{S}_m = \Lambda_m N_m - (\lambda_{hm} + \mu_m) S_m, \quad (1h)$$

$$\dot{E}_m = \lambda_{hm} S_m - (\alpha_m + \mu_m) E_m, \quad (1i)$$

$$\dot{I}_m = \alpha_m E_m - \mu_m I_m, \quad (1j)$$

where,  $a_0 = \gamma_a + \kappa \lambda_{mh} + \mu_h$ ,  $a_1 = \gamma_a + \kappa (1 - \tau_{v1}) \lambda_{mh} + \mu_h$ ,  
 $a_2 = \gamma_a + \kappa (1 - \tau_{v2}) \lambda_{mh} + \mu_h$ ,  $a_3 = \varepsilon_2 \gamma + (1 - \varepsilon_2) \gamma_0 + \mu_h$ .

# Modeling LLINs



Contents lists available at ScienceDirect

Journal of Theoretical Biology

journal homepage: [www.elsevier.com/locate/yjtbi](http://www.elsevier.com/locate/yjtbi)



Quantifying the impact of decay in bed-net efficacy  
on malaria transmission

Calistus N. Ngonghala <sup>a,b,\*</sup>, Sara Y. Del Valle <sup>c</sup>, Ruijun Zhao <sup>d</sup>, Jemal Mohammed-Awel <sup>e</sup>

## Long Lasting Insecticide nets (LLINs)

Protect from mosquito bites (reduction  
in the biting rate)

Kill mosquitoes (increase in the mosquito  
mortality rate)

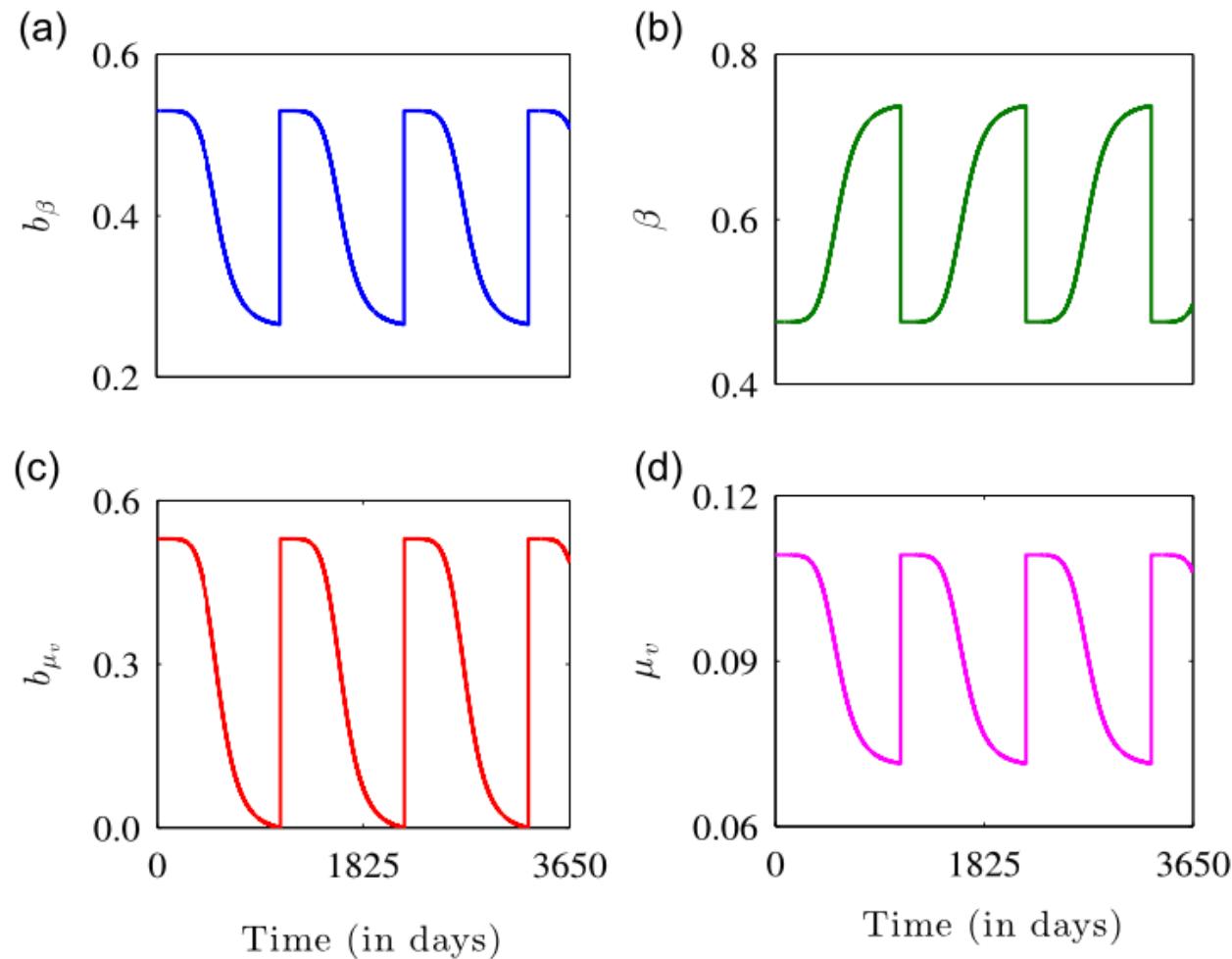
## Effect on the biting rate

$$\begin{aligned}\beta(\varepsilon_{\beta(t)}) &= \beta_{max} - (\beta_{max} - \beta_{min})\varepsilon_{\beta}(t), \\ \varepsilon_{\beta(t)} &= p + \left( \frac{2^n + 1}{2(n+1)} \right) \left( \frac{2^n - 1}{2^n + 1} + \frac{1}{1 + \left( \frac{t \bmod T}{\frac{T}{2}} \right)^n} \right) \varepsilon_0,\end{aligned}$$

## Effect on the mosquito mortality rate

$$\begin{aligned}\mu_m(\varepsilon_{\mu_m}(t)) &= \mu_{m0} + \bar{\mu}_m(t) = \mu_{m0} + \mu_{m1}\varepsilon_{\mu_m}(t), \\ \varepsilon_{\mu_m}(t) &= p + \left( 1 + \frac{1}{2^n} \right) \left( \frac{-1}{2^n + 1} + \frac{1}{1 + \left( \frac{t \bmod T}{\frac{T}{2}} \right)^n} \right) \varepsilon_0,\end{aligned}$$

# Modeling LLINs



Ngonghala et al. (2014)

Graphical representation of the effects of LLINs.

# Parameter estimation

## Some Fixed Parameters

$\kappa = 0.5$ . %Reduction factor of the force of infection from  $I_{ah}$  to  $I_{uh}$ . (Assumed)

$Prop = 0.5$ . %Proportion of uncomplicated malaria (Assumed)

$\gamma_0 = 2.02778$  %Progression rate of  $I_{uh}$  to  $I_{ah}$  (year $^{-1}$ ) (Chitnis et al. 2008)

$\gamma_T = 14.6$  %Recovery rate of treated individuals (year $^{-1}$ ) (Chitnis et al. 2008)

$\alpha_m = 36.5$  % Progress rate of Exposed mosquito to the Infectious mosquito class (year $^{-1}$ ) (Chitnis et al. 2008)

$\alpha_h = 24$  % Progress rate of Exposed humans to the Infectious human class (year $^{-1}$ ) (Chitnis et al. 2008)

$P_{mh0} = 0.1$  % Disease transmission probability from infectious mosquitoes to susceptible humans (Chitnis et al. 2008)

$n_s > 1$  % Dimensionless shape constant in the LLIN modeling (Ngonghala et al., 2014)

$\mu_{m0} = 20.27$  % baseline value of mosquito mortality rate (year $^{-1}$ ) (Ngonghala et al., 2014)

$\Lambda_m = 45.47$  % baseline value of the mosquito birth rate (Ngonghala et al., 2014)

$T = 3$  years % useful life or duration of LLIN efficacy (Ngonghala et al., 2014)

## Estimation techniques

- Maximum Likelihood Estimation
- Bayesian Inference
- Least Squares Fitting
- Etc.

# **Some key features in malaria modeling**

**Advanced concepts**



# Malaria deaths by age

## Malaria deaths by age, World

Estimated annual number of deaths from malaria.

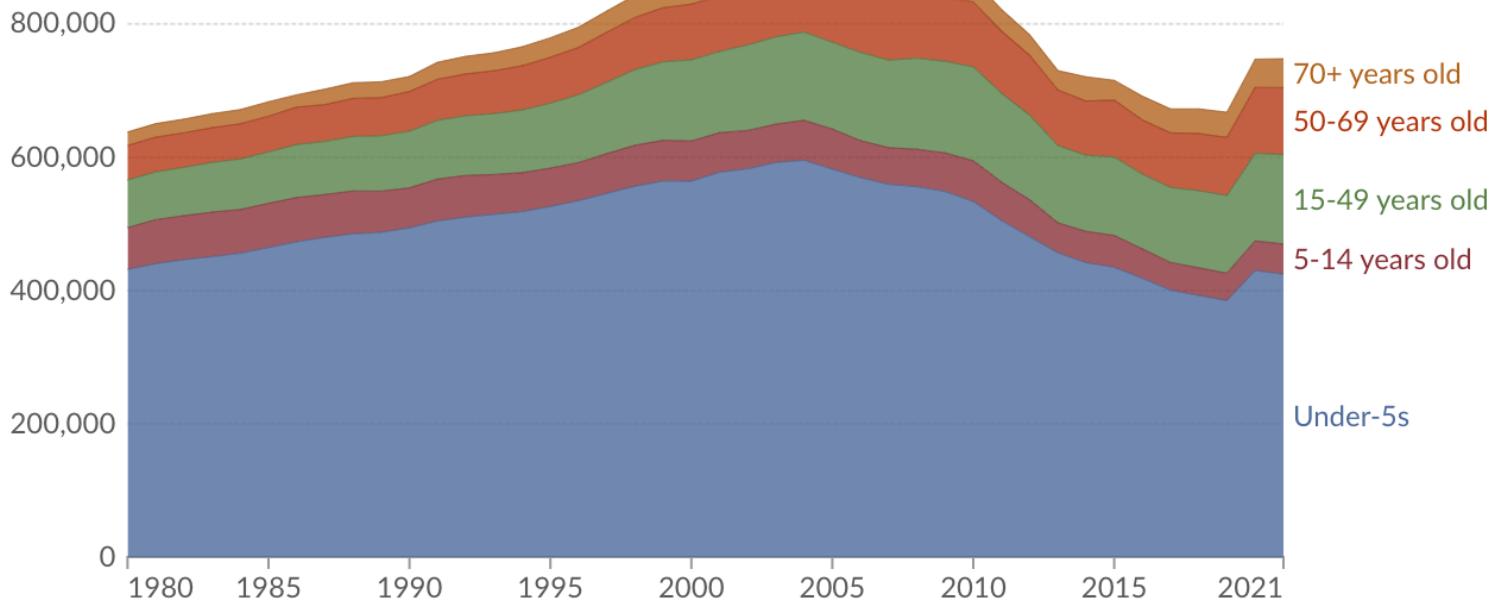
Our World  
in Data

Table

Chart

Edit countries and regions

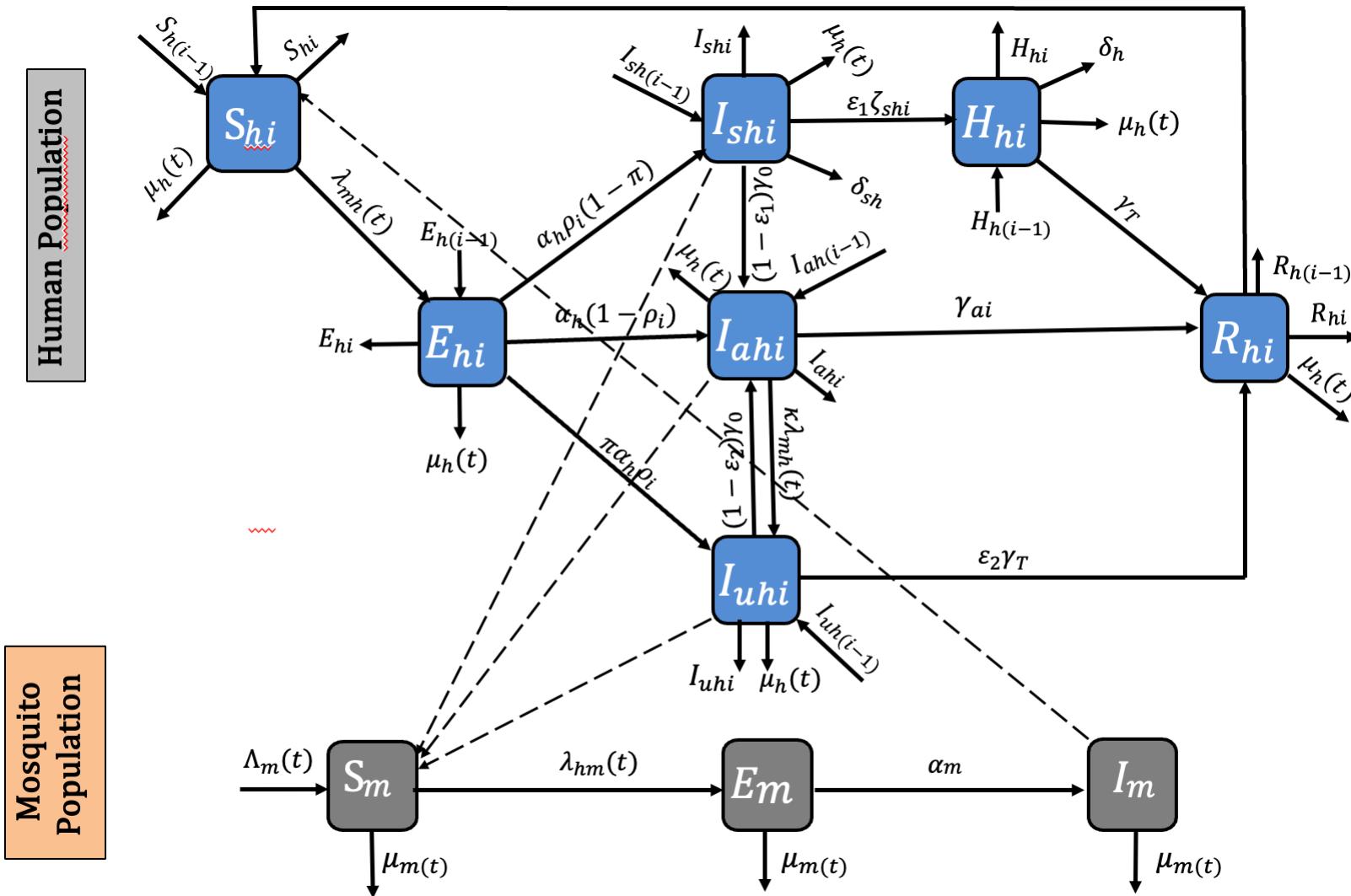
Settings



Data source: IHME, Global Burden of Disease (2024)  
[OurWorldInData.org/malaria](https://OurWorldInData.org/malaria) | CC BY

Malaria dynamic parameters are age-dependent

# Age structure malaria model



$$\lambda_{hm} = \frac{\beta(t) * (P_{hma} \sum_{i=1}^C I_{ahi} + P_{hmu} \sum_{i=1}^C I_{uhi} + P_{hms} \sum_{i=1}^C I_{shi})}{N_h}$$



# Age structure malaria model: ODEs

## Set of ODEs linked to the model

$$\dot{S}_{hi} = S_{h(i-1)} + \omega_{hi}R_{hi} - (\lambda_{mhi} + \mu_{hi})S_{hi} - S_{hi}, \quad (1a)$$

$$\dot{E}_{hi} = E_{h(i-1)} + \lambda_{mhi}S_{hi} - (\alpha_h + \mu_{hi})E_{hi} - E_{hi}, \quad (1b)$$

$$\dot{I}_{shi} = I_{sh(i-1)} + \alpha_h\rho_i(1 - \pi)E_{hi} - (\varepsilon_1\zeta_{sh} + (1 - \varepsilon_1)\gamma_0 + \mu_{hi} + \delta_{sh})I_{shi} - I_{shi}, \quad (1c)$$

$$\dot{I}_{ahi} = I_{ah(i-1)} + \alpha_h(1 - \rho)E_{hi} + (1 - \varepsilon_1)\gamma_0I_{shi} + (1 - \varepsilon_2)\gamma_0I_{uhi} - a_0I_{ahi} - I_{ahi}, \quad (1d)$$

$$\dot{I}_{uhi} = I_{uh(i-1)} + \alpha_h\rho\pi E_{hi} + \kappa\lambda_{mh}I_{ahi} - (\varepsilon_2\gamma + (1 - \varepsilon_2)\gamma_0 + \mu_{hi})I_{uhi} - I_{uhi}, \quad (1e)$$

$$\dot{H}_{hi} = H_{h(i-1)} + \varepsilon_1\zeta_{sh}I_{shi} - (\delta_h + \gamma + \mu_{hi})H_{hi}, \quad (1f)$$

$$\dot{R}_{hi} = R_{h(i-1)} + \gamma_aI_{ahi} + \gamma_tH_{hi} + \varepsilon_2\gamma_tI_{uhi} - (\omega_{hi} + \mu_{hi})R_{hi} - R_{hi}, \quad (1g)$$

$$\dot{S}_m = \Lambda_mN_m - (\lambda_{hm} + \mu_m)S_m, \quad (1h)$$

$$\dot{E}_m = \lambda_{hm}S_m - (\alpha_m + \mu_m)E_m, \quad (1i)$$

$$\dot{I}_m = \alpha_mE_m - \mu_mI_m, \quad (1j)$$

# Modeling the force of infection as a function of age

Epidemiol. Infect. (2017), 145, 2545–2562. © Cambridge University Press 2017  
doi:10.1017/S0950268817001297

## Estimating age-time-dependent malaria force of infection accounting for unobserved heterogeneity

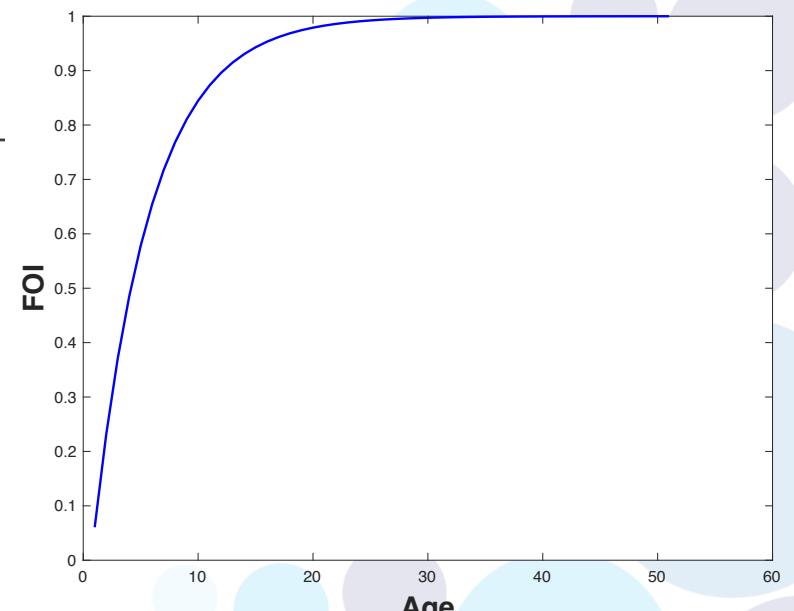
L. MUGENYI<sup>1,2\*</sup>, S. ABRAMS<sup>2</sup> AND N. HENS<sup>2,3</sup>

$$\text{For age}=0 : \lambda_{mh}(0) = \lambda_{mh}\left(1 - e^{-\frac{age_0}{\alpha_0}}\right)$$

$$\text{For age}>0 : \lambda_{mh}(\text{age}) = \lambda_{mh}\left(1 - e^{-\frac{\text{age}}{\alpha_0}}\right)$$

$\alpha_0$ : age of half increase in exposure

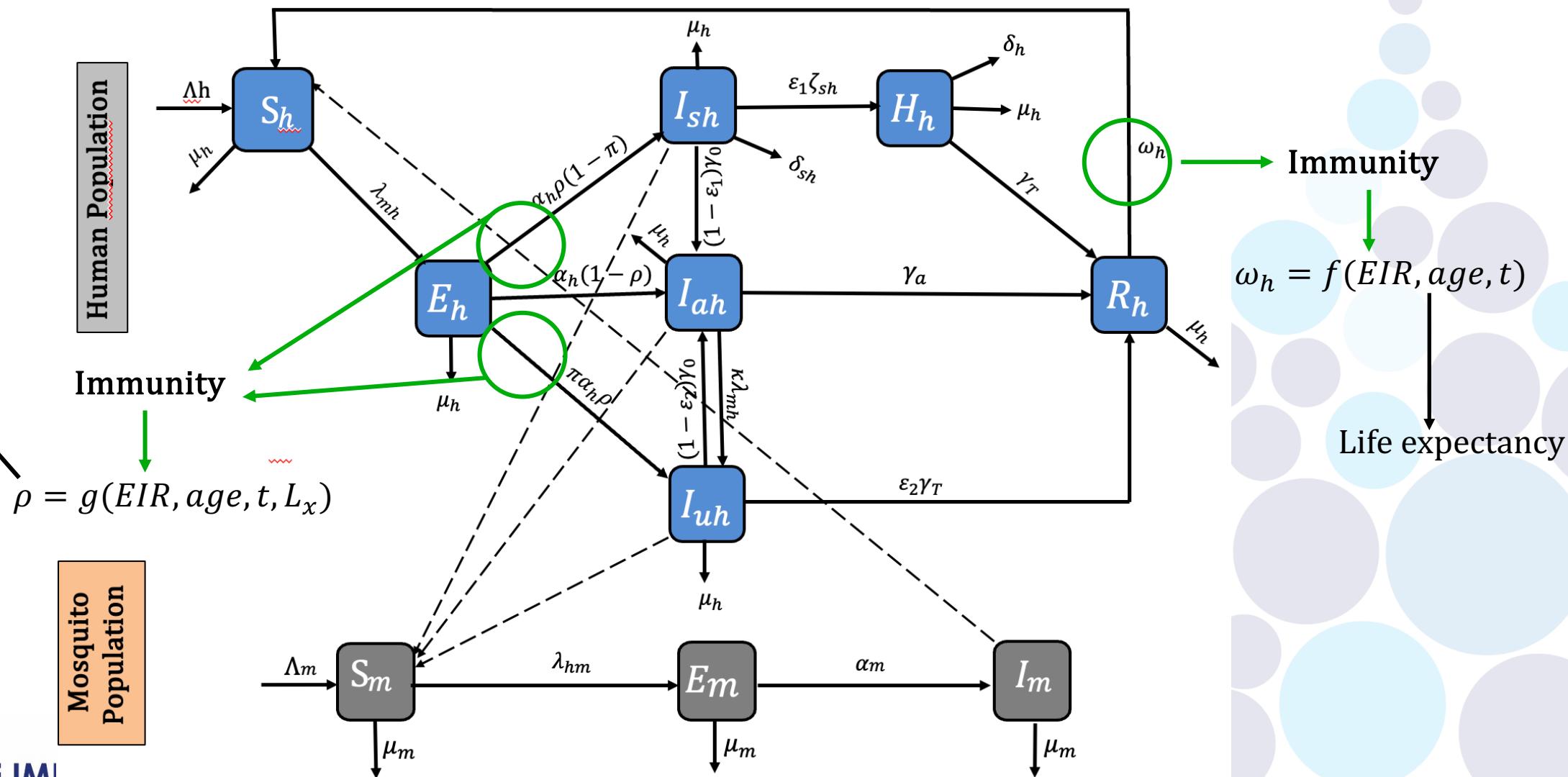
$\alpha_0$  and  $age_0$  are estimated by the model.



$$\lambda_{mh} = P_{mh} * \beta * n_0 * \frac{I_m}{N_m} \text{ (FOI)}$$

# Modeling immunity to Malaria infection

Probability to develop symptoms



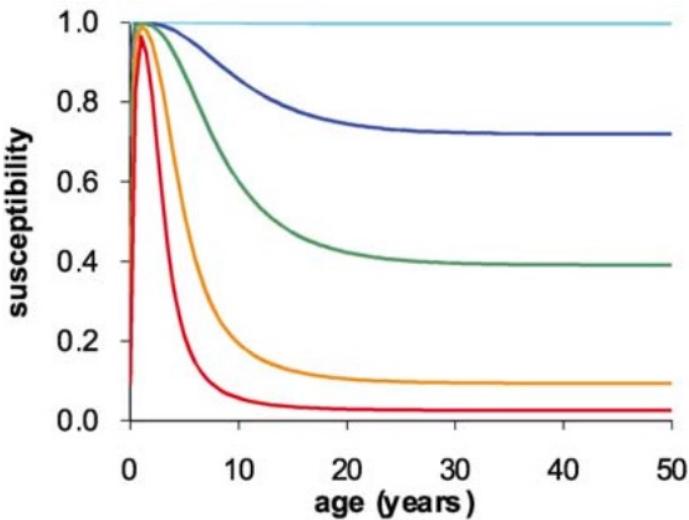
# Modeling immunity to Malaria infection in adults

OPEN  ACCESS Freely available online

PLOS COMPUTATIONAL BIOLOGY

## Determination of the Processes Driving the Acquisition of Immunity to Malaria Using a Mathematical Transmission Model

João A. N. Filipe<sup>1<sup>xx</sup>a</sup>, Eleanor M. Riley<sup>2</sup>, Christopher J. Drakeley<sup>2</sup>, Colin J. Sutherland<sup>2</sup>, Azra C. Ghani<sup>1<sup>xx</sup>b\*</sup>

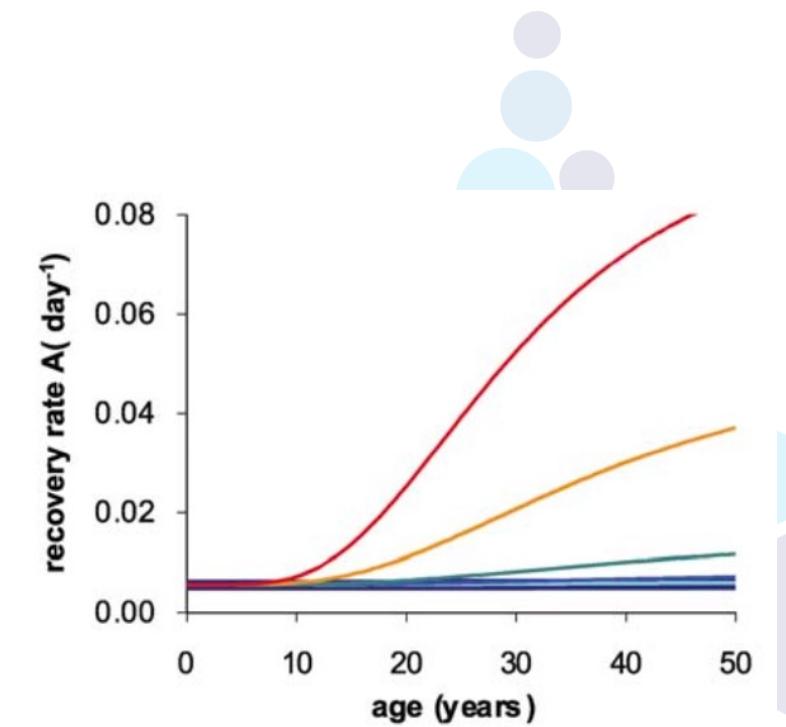


### Example of the immunity function

$$\rho(\text{age}, \text{EIR}, L_x) = \frac{\rho_{\max}}{1 + \exp(-\frac{A}{B})}$$

$$A = \text{age} - \text{age}_0$$

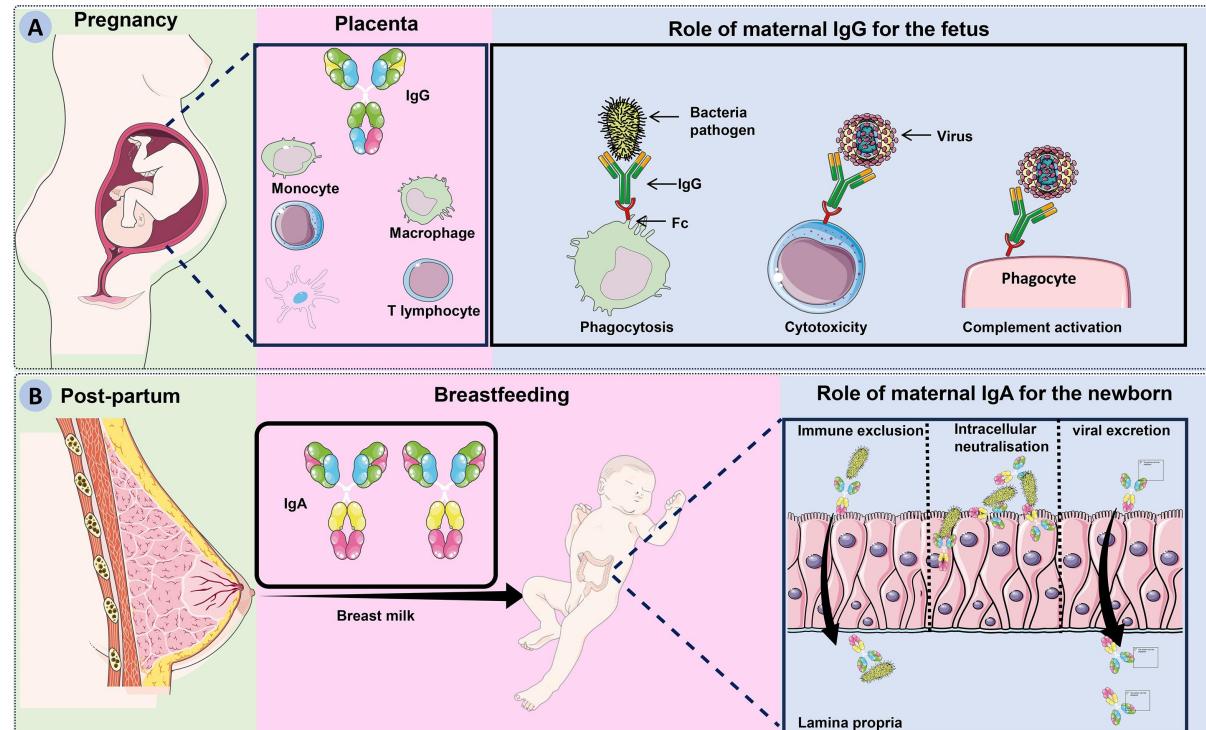
$$B = \theta_0 \frac{L_x}{L - \text{age}_0} (1 + c_k \text{EIR})^{ns}$$



$$\omega_h(\text{EIR}, \text{age}) = 365 * (\omega_0 + \frac{0.038 - \omega_0}{(1 + \frac{\text{age}}{1.5})^{1.5}})$$
$$\omega_0 = 0.022 * \exp(-0.311 * \text{EIR})$$

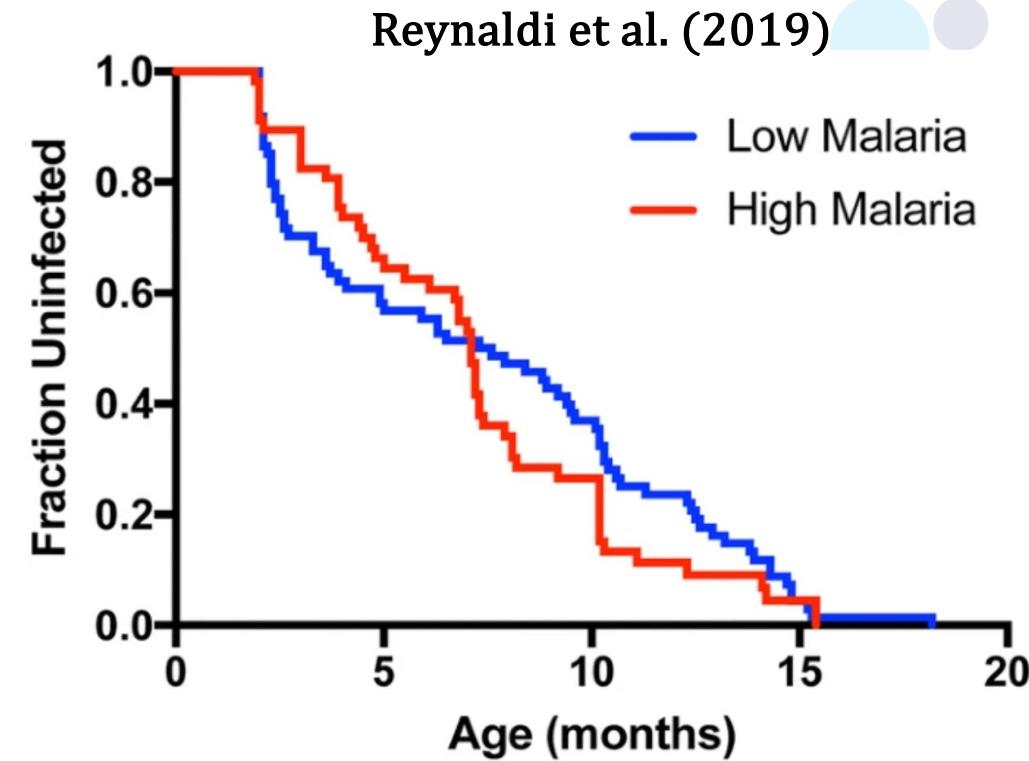
# Modeling immunity to Malaria infection

## Maternal immunity transferred to newborns



Sereme et al. (2024)

Immunity in newborns



$$\rho(\text{age}) = \begin{cases} \left(\frac{1}{p}\right) * \rho_{\text{mother}} * \exp(-k * \text{age}), & \text{age} \leq \text{age}_0 \\ 0, & \text{age} > \text{age}_0 \end{cases}$$



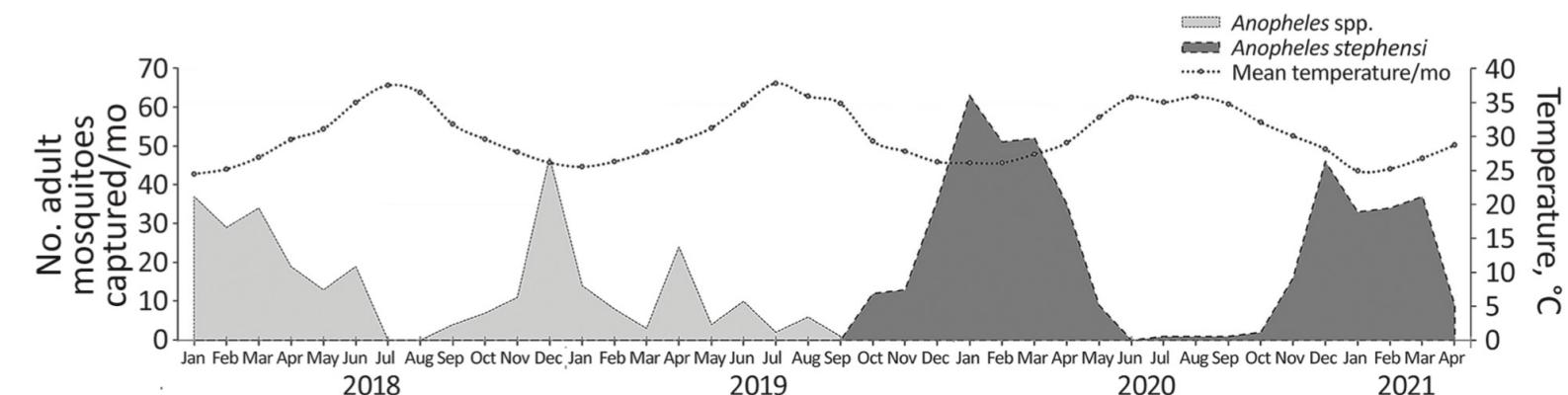
# Modeling the effect of seasonality

[Emerg Infect Dis.](#) 2023 Apr; 29(4): 801–805.  
doi: [10.3201/eid2904.220549](#)

PMCID: PMC10045708  
PMID: [36958009](#)

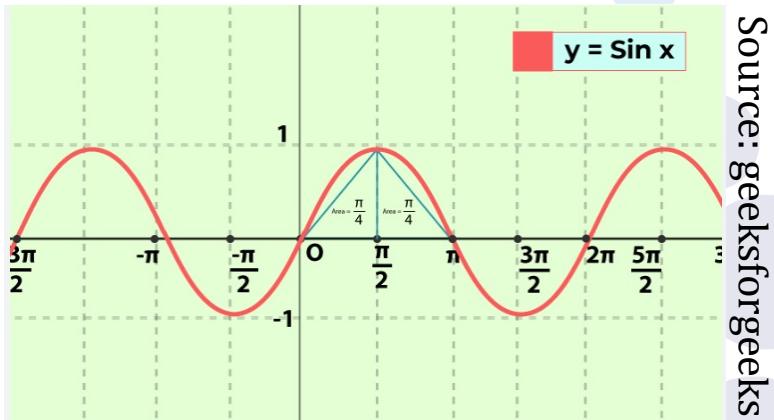
## Effects of Seasonal Conditions on Abundance of Malaria Vector *Anopheles stephensi* Mosquitoes, Djibouti, 2018–2021

Alia Zayed,<sup>✉</sup> [Manal Moustafa](#), [Reham Tageldin](#), and [James F. Harwood](#)



**Figure 1.** Associations between numbers of adult mosquitoes captured and mean temperature, by month, US military base, Djibouti, September 2019–August 2020. (We began identifying *Anopheles stephensi* mosquitoes specifically in October 2019.)

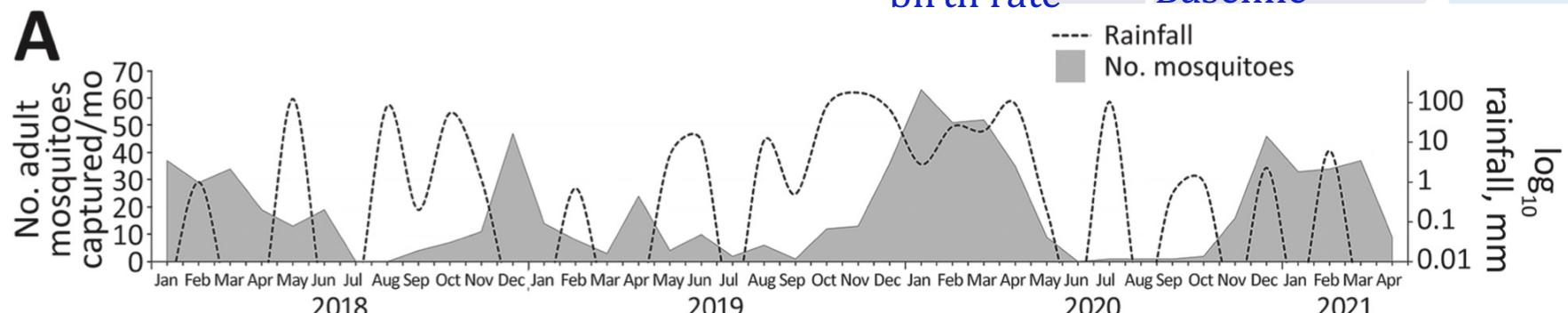
Seasonality driven by rainfall (or temperature)



$$\Lambda_m = \Lambda_{m0}(1 + a_1 \sin\left(\frac{2\pi}{12}(t + \phi_1)\right))$$

Mosquito birth rate  
Baseline

Phase shift



# Modeling the effect of seasonality

Seasonality driven by rainfall (or temperature)

$$\Lambda_m = \Lambda_{m0} \left(1 + a_1 \sin\left(\frac{2\pi}{12}(t + \phi_1)\right)\right)$$

↓  
Mosquito birth rate      ↓  
Baseline                  Phase shift

Linking Peak month and  $\phi_1$

$$\sin\left(\frac{2\pi}{12}(t + \phi_1)\right) \text{ is max if } \frac{2\pi}{12}(M_{Peak} + \phi_1) = \frac{\pi}{2} \Rightarrow \phi_1 = 3 - M_{Peak} \Rightarrow$$

Force of infection

$$\lambda_{mh}(t) = \lambda_{mho} \left(1 + a_2 \sin\left(\frac{\pi}{6}(t - M'_{Peak} + 3)\right)\right)$$

Force of infection

Baseline

Peak month in rainfall

Mosquito birth rate

$$\boxed{\Lambda_m(t) = \Lambda_{m0} \left(1 + a_1 \sin\left(\frac{\pi}{6}(t - M_{Peak} + 3)\right)\right)}$$

# Modeling Seasonal Malaria Chemoprevention



- Administration of drugs to under-five children
- Period: Peak transmission period (3 months)
- Reduction of the force of infection ( $\lambda_{mh}^{SMC}$ )

Normal seasonal force of infection

$$\lambda_{mh}(t) = \lambda_{mh0} \left(1 + a_1 \sin\left(\frac{\pi}{6}(t - M'_{Peak} + 3)\right)\right)$$

$$\lambda_{mh0} = P_{mh} * \beta * n_0 * \frac{I_m}{N_m}$$

$$\lambda_{mh}^{SMC}(t, age) = \begin{cases} (1 - \varphi_0) * \lambda_{mh} + (1 - \varphi_0 \xi_0 e^{-k*(t-t_{SMC})}) * \lambda_{mh}, & age \leq 60 \text{ and } start \leq m(t) \leq end \\ \lambda_{mh}, & age > 60 \end{cases}$$

SMC initial efficacy

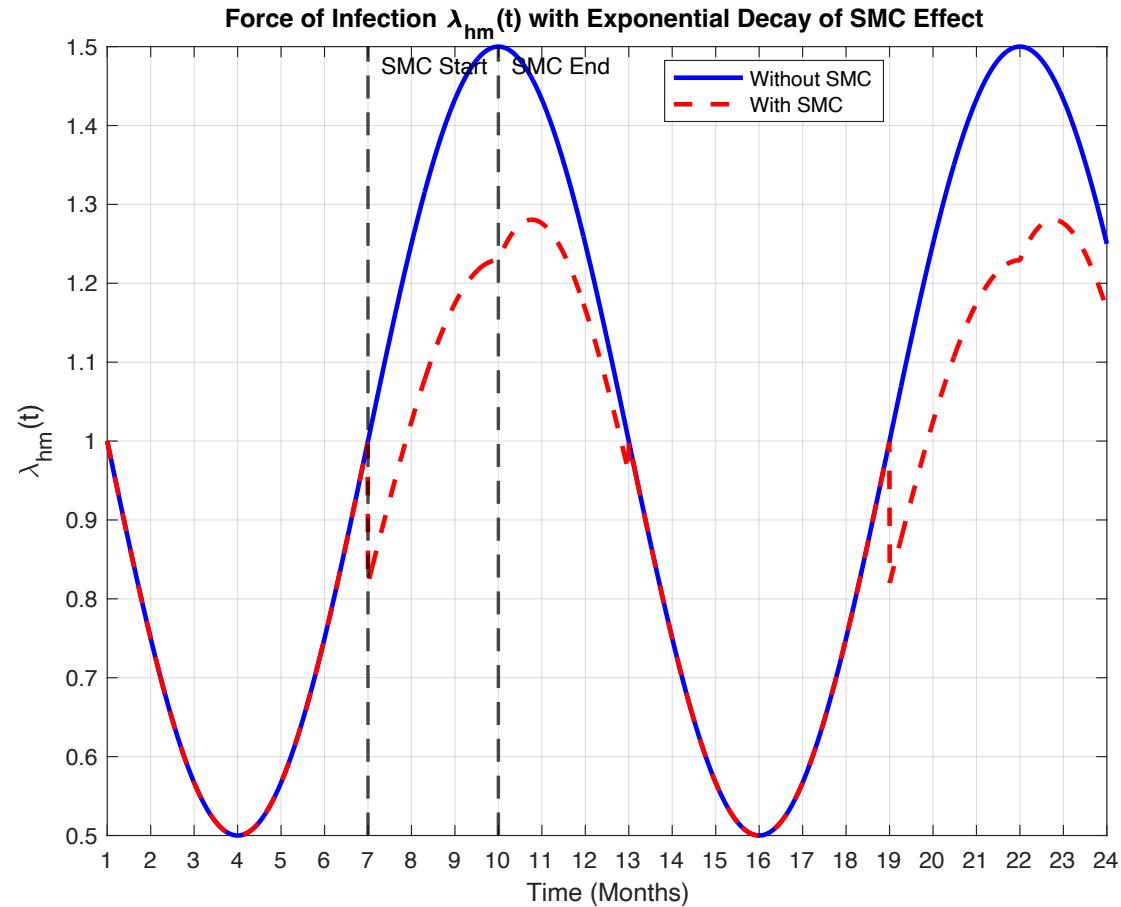
$$\lambda_{hm}^{total}(t) = \sum_{c=1}^{n_c} p_a \lambda_{mh}^{SMC}(t, age)$$

$$m(t) = (\text{mod}(t - 1), T) + 1$$

$$M_{Peak} - 1 \quad M_{Peak} + 2$$

# Modeling Seasonal Malaria Chemoprevention

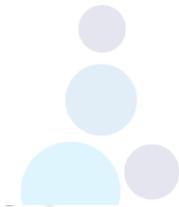
Graphical illustration



# Vaccine impact modeling

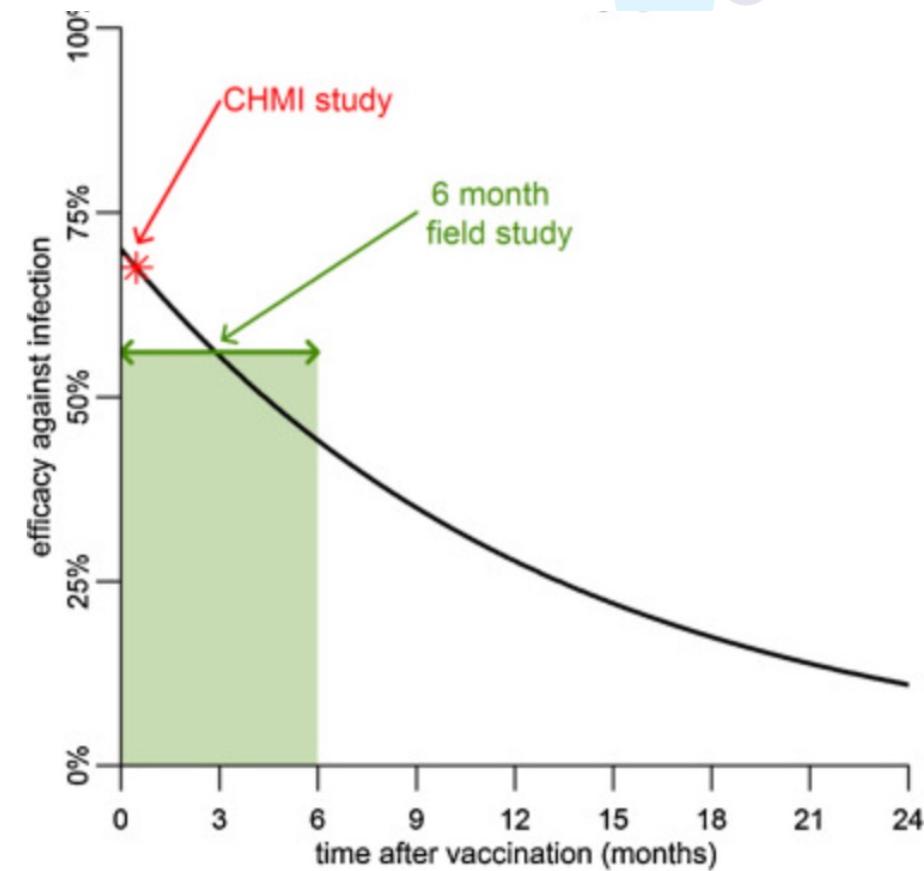
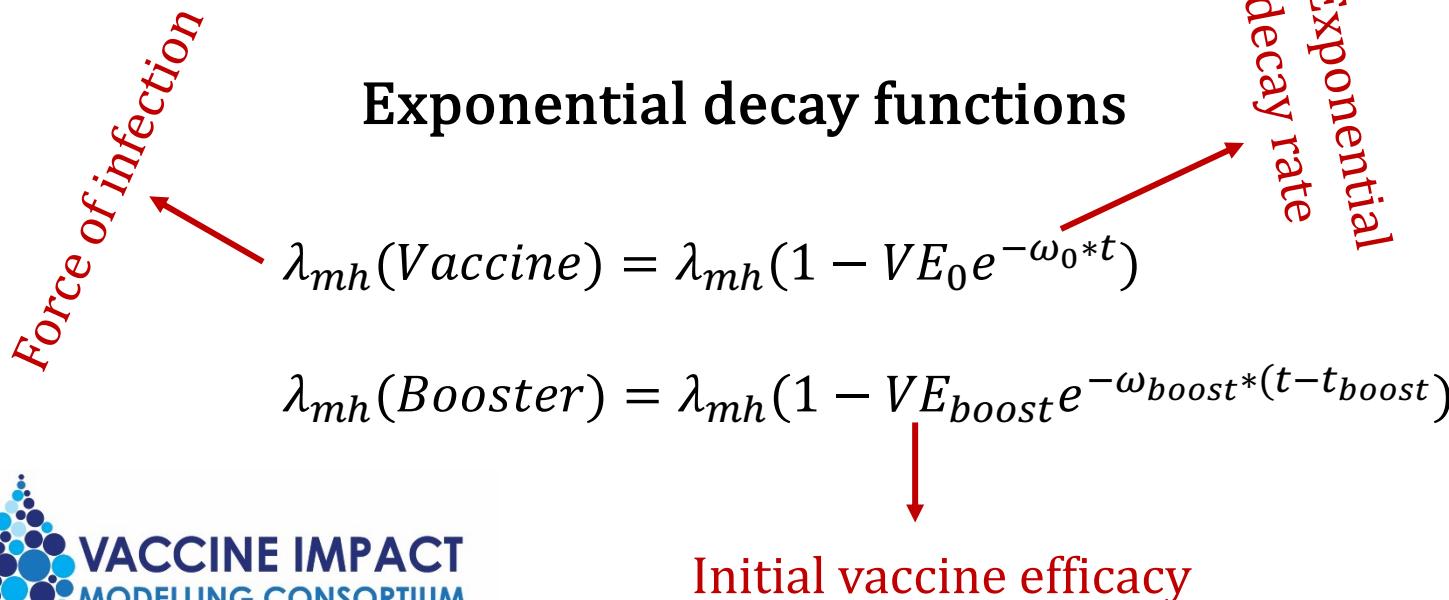


Vaccine  
Volume 33, Issue 52, 22 December 2015, Pages 7544-7550



## Vaccine approaches to malaria control and elimination: Insights from mathematical models ☆

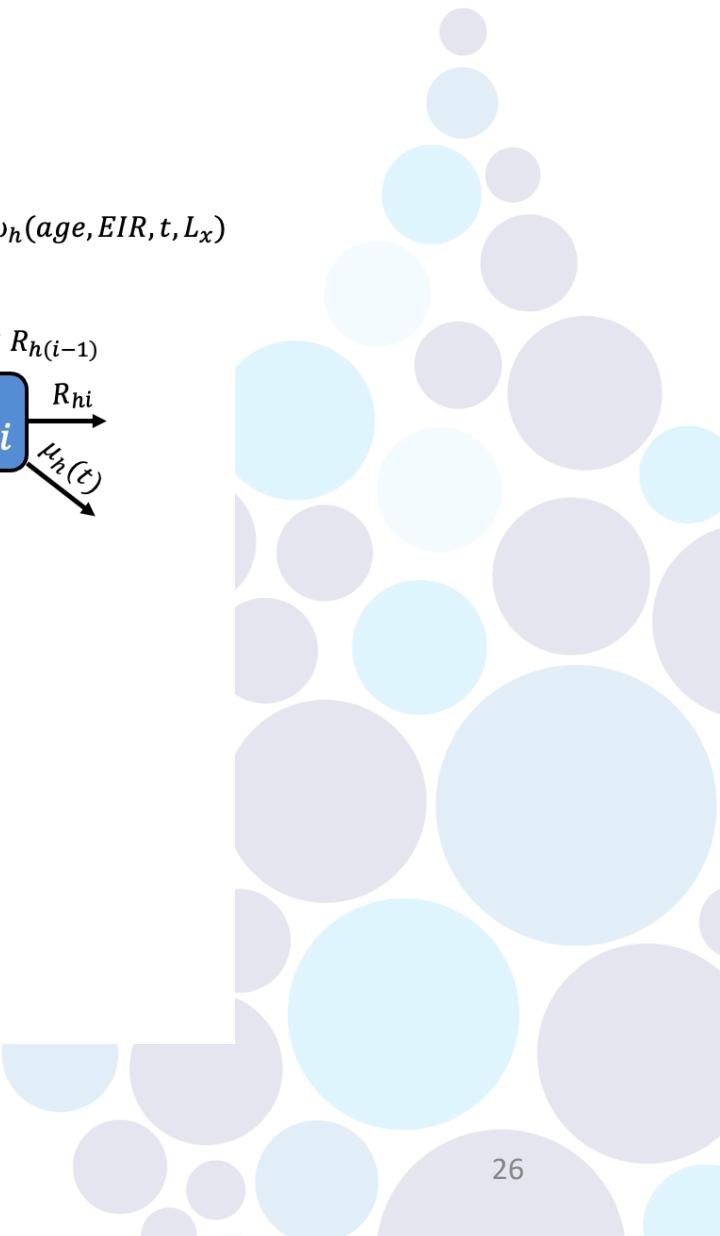
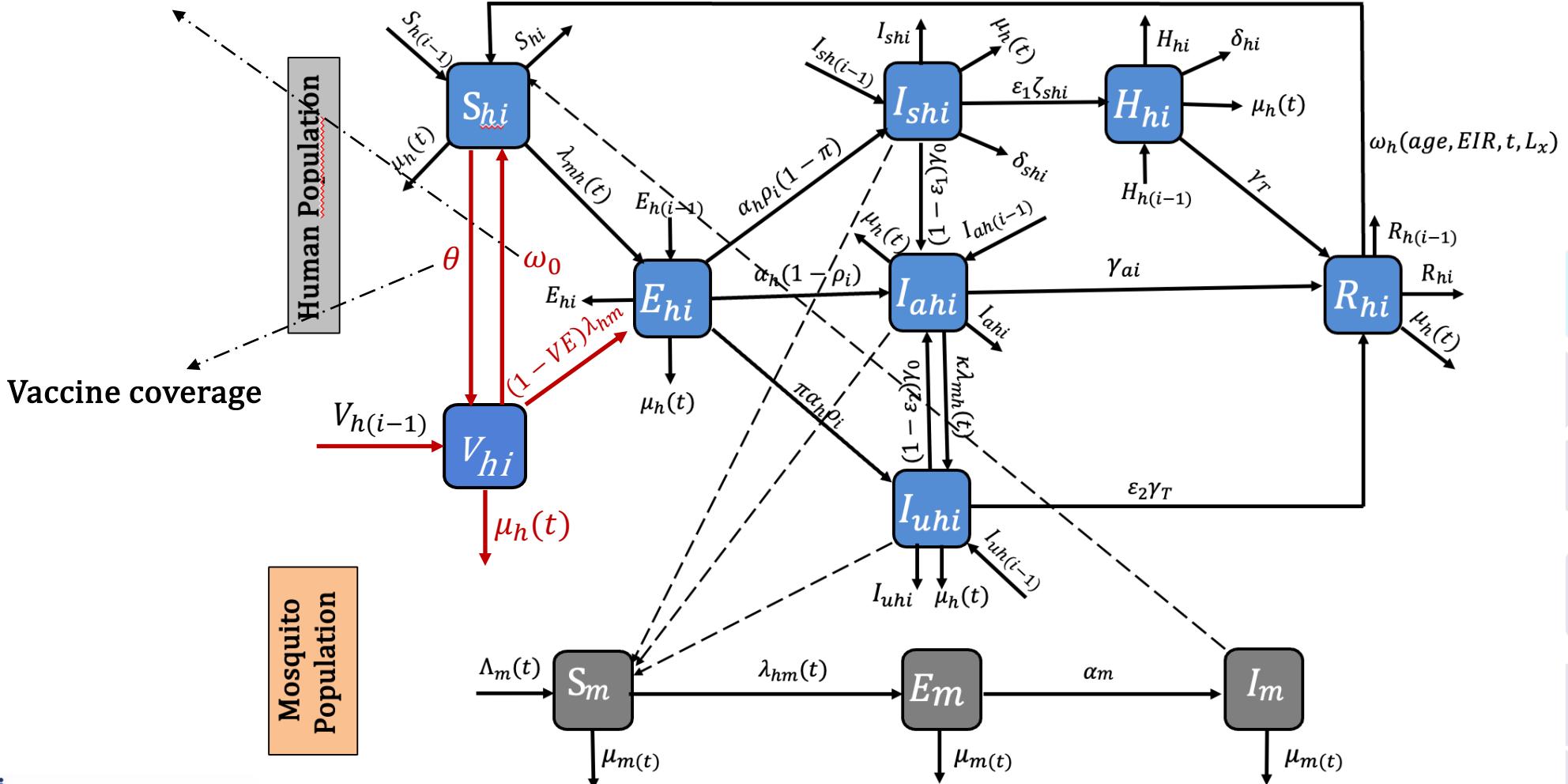
Michael T. White , Robert Verity, Thomas S. Churcher, Azra C. Ghani



# Accounting for vaccination

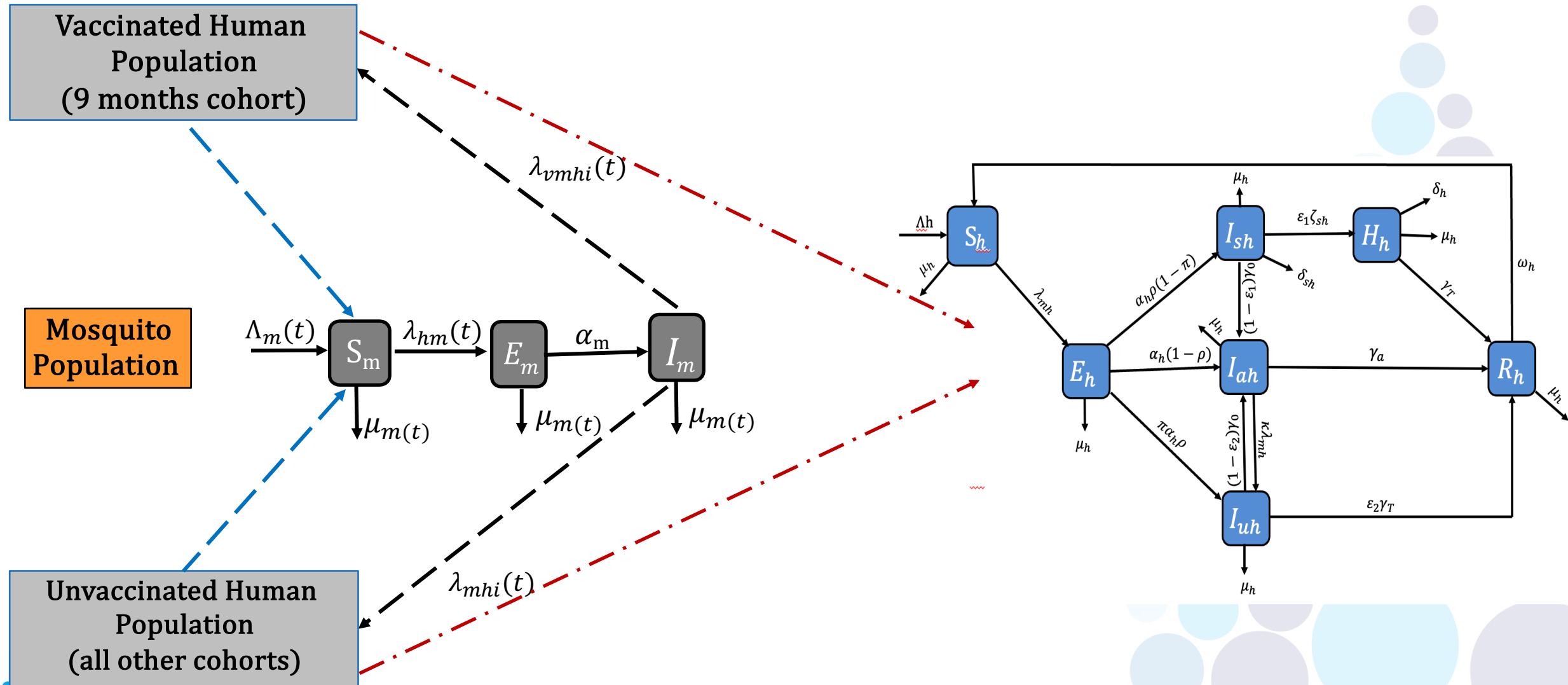
## (Model1)

Wanning rate



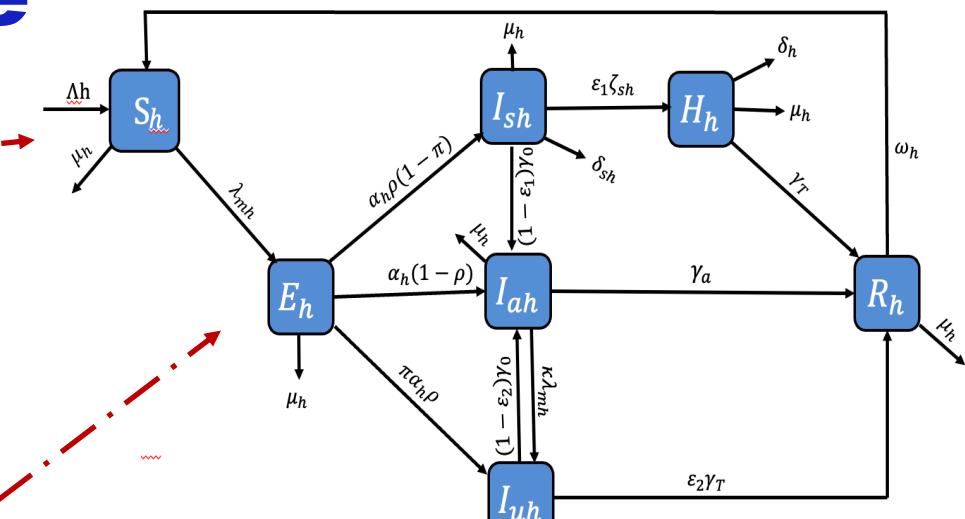
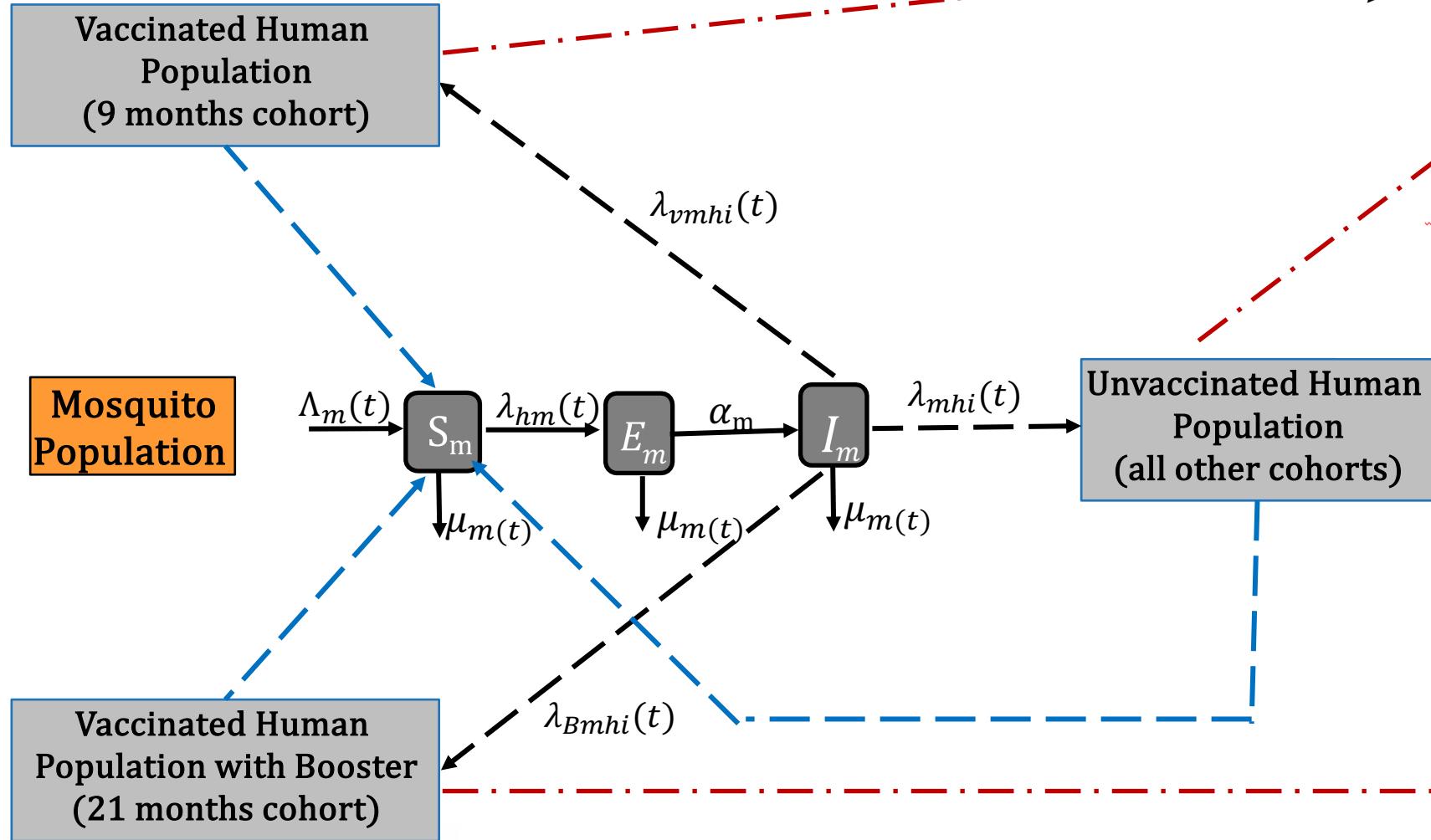
# Accounting for vaccination

## (Model2)



# Accounting for a booster dose

(Model3)



# Associated computer code

## Model code

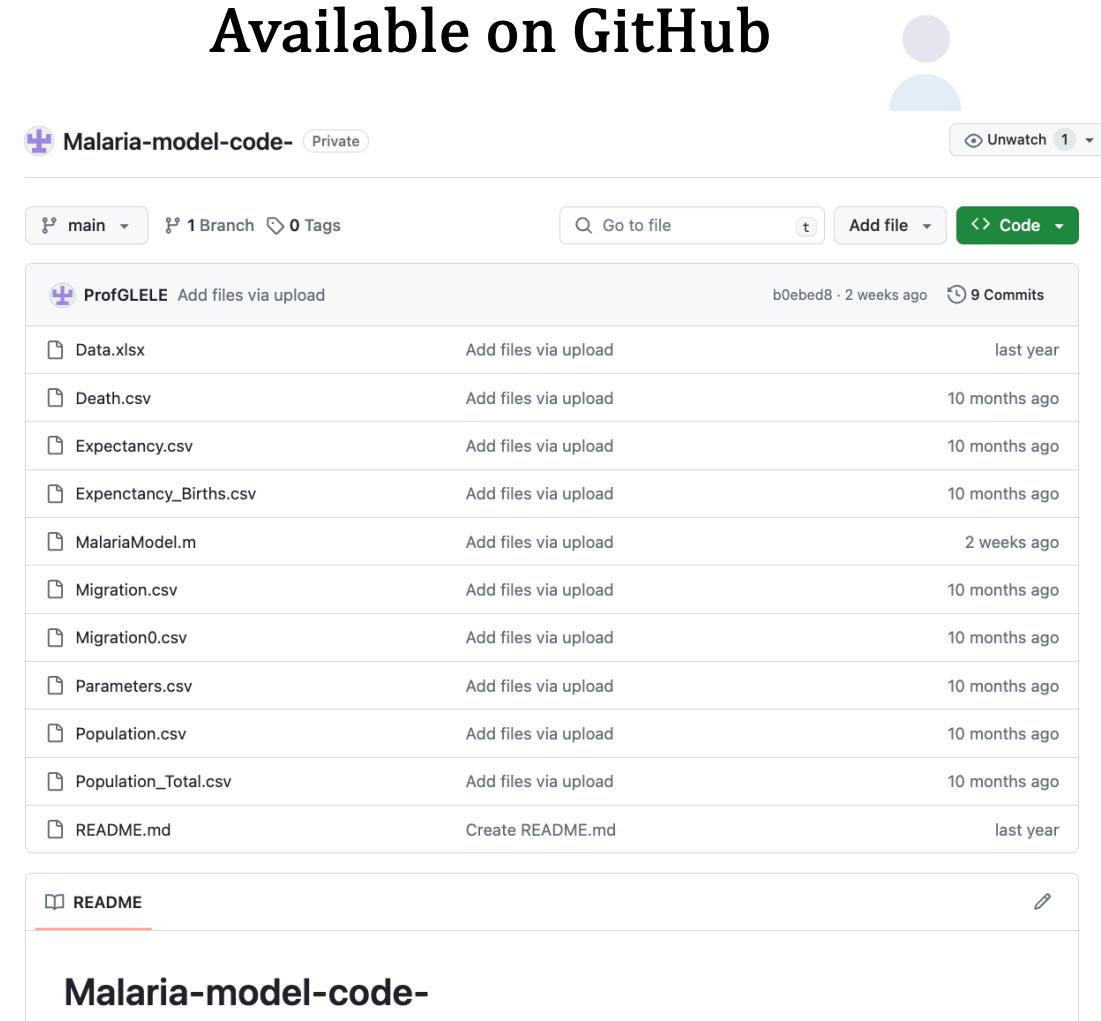
```
function MalariaModel
tstart=cputime;

% This function computes for each age cohort (0 to 100) of a country, the
% number of individuals (CohortSize), the number of severe Malaria cases (Cases),
% the number of DALYs (DALYs) and the number of deaths due to Malaria (Deaths)
% from year 2000 to year 2100 using a mechanistic model.
% The function imports variables from the following .csv files:
% - Total population of the country from 2000 to 2100 ('PopT'; Population_Total.csv)
% - Total births of the country from 2000 to 2100 ('Births'; Population.csv)
% - Size of each cohort from 2000 to 2100 ('Ncohort'; Population.csv)
% - Net migration rate for cohort 0 from 2000 to 2100 ('Migrat'; Migration0.csv)
% - Net migration rate for other cohorts from 2000 to 2100 ('Migrationr'; Migration.csv)
% - Death rate for cohort classes ('Tdeaths'; Death.csv)
% - The Expected remaining years of life for cohort 0 ('Exp0'; Expenctancy_Births)
% - The Expected remaining years of life for other cohorts ('Exp'; Expenctancy.csv)
% We consider:
% - theta1: vaccination coverage for cohort 0 in 2023=0.8
% - theta2: vaccination coverage for cohort 2 in 2025=0.64
% - r: number of stochastic runs of the model; here, we consider r=30.

% Author: Prof Romain GLELE KAKAI, Laboratory of Biomathematics and
% Forest estimations, University of Abomey-Calavi; 04BP1525, Cotonou, Benin
% Email: roman.glelekakai@fsa.uac.bj/glele.romain@gmail.com

% Importing the total population of the country from 2000 to 2100
Population = readmatrix('Population_Total');
% Importing death rates from 2000 to 2100
TDeath = readmatrix('Death');
```

## Available on GitHub

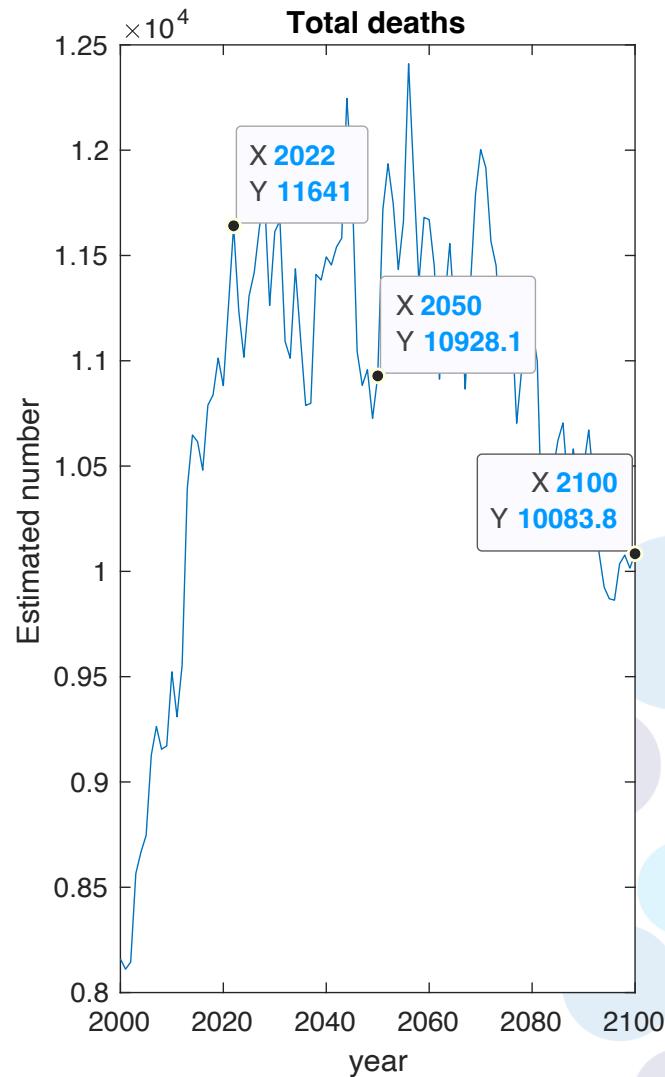
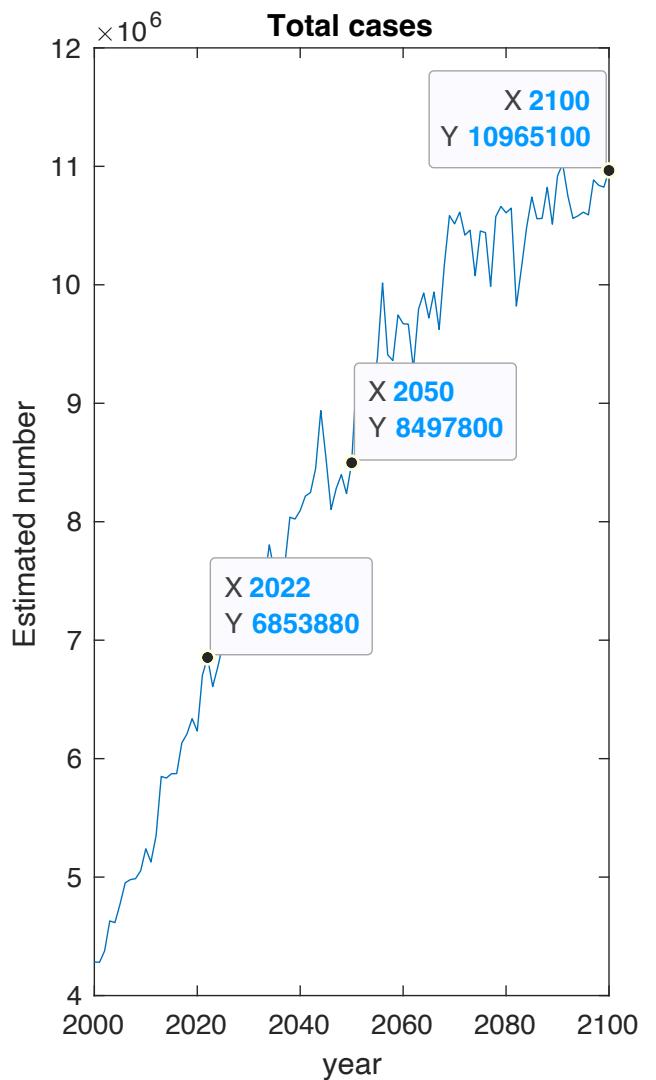


A screenshot of a GitHub repository page titled "Malaria-model-code-". The repository is private and was created by "ProfGLELE" via upload. It contains 1 branch, 0 tags, and 9 commits. The files listed include Data.xlsx, Death.csv, Expectancy.csv, Expenctancy\_Births.csv, MalariaModel.m, Migration.csv, Migration0.csv, Parameters.csv, Population.csv, Population\_Total.csv, and README.md. The README file contains the text "Malaria-model-code-". The repository was last updated 2 weeks ago.

# Applications

## Malaria projection: Ghana

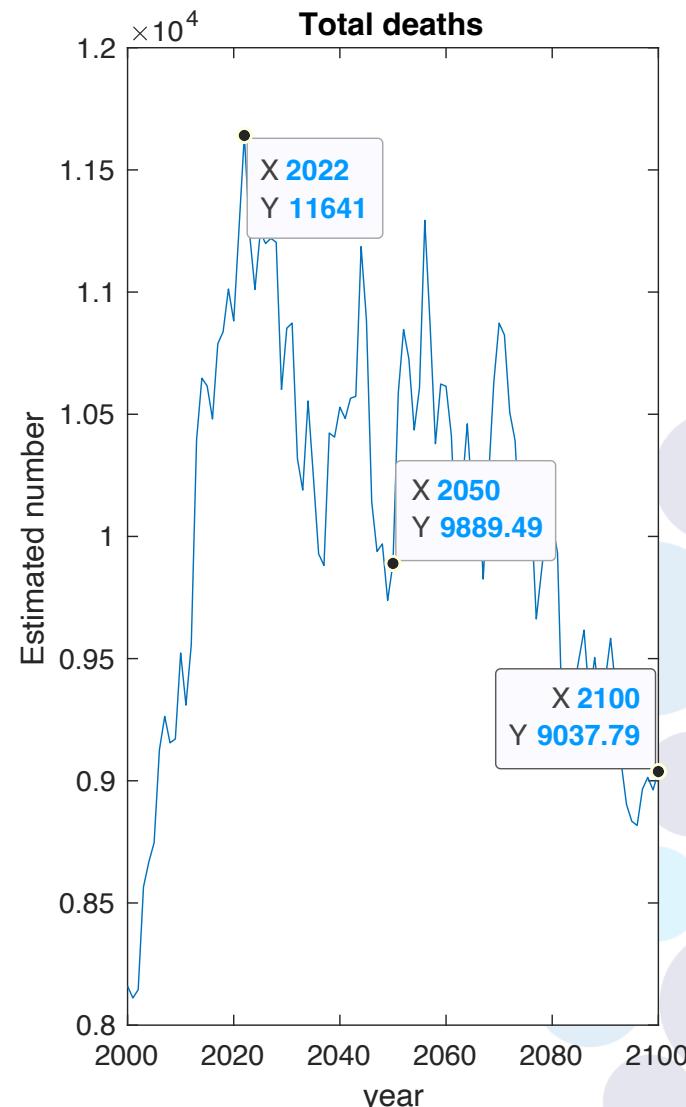
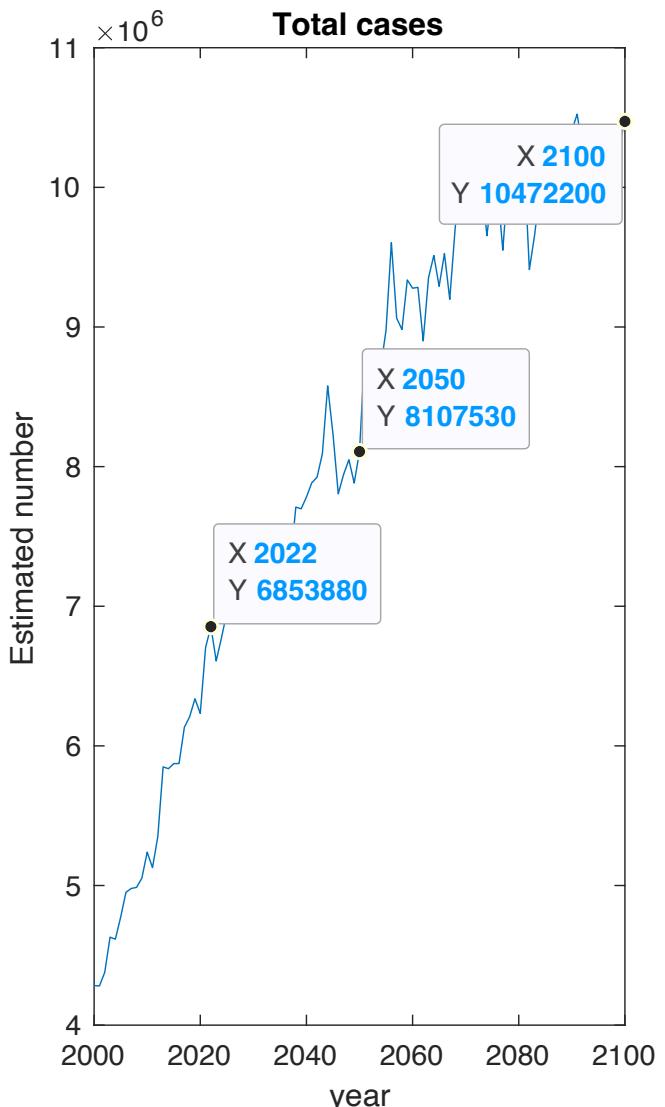
Without vaccination



# Applications

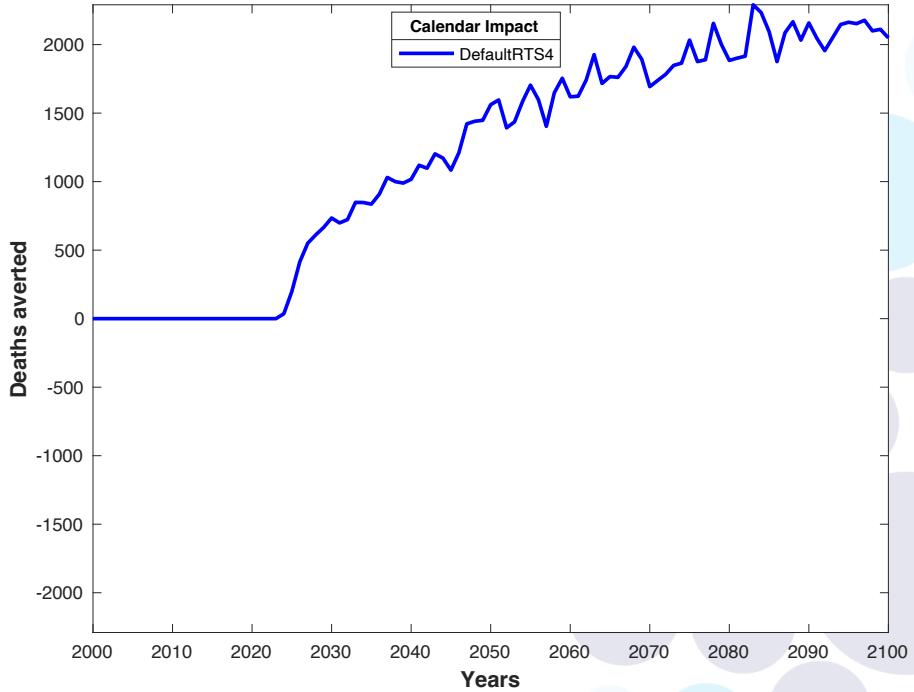
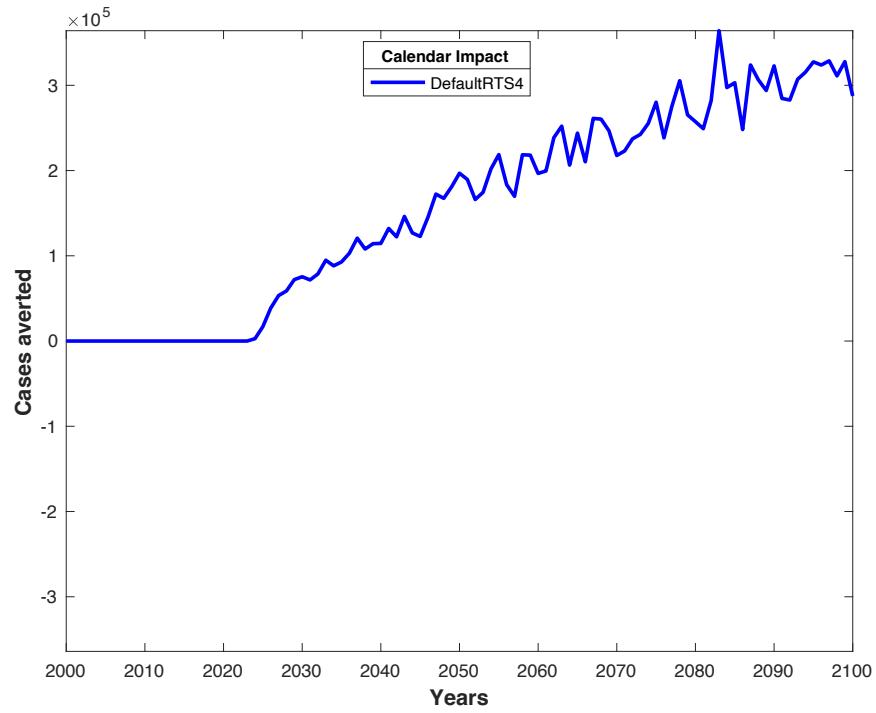
## Malaria projection: Ghana

With vaccination  
(RTS,S with a  
booster dose)



# Applications

## Malaria projection: Ghana





# **Thank you for your attention**