The Hall effect in an extrinsic semiconductor - EM11

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The relationship of the Hall voltage with magnetic field and current was investigated and verified with the theoretical equation. A value for the Hall coefficient of the doped Germanium sample was calculated as $-5.3(\pm 0.4) \times 10^{-3}\ m^3C^{-1}$. The dominant charge carriers were determined to be electrons, in this sample, and the charge concentration was calculated to be $1.18(\pm 0.09) \times 10^{21}\ m^{-3}$. The Hall mobility of the sample was calculated to be $-9.7(\pm 0.7) \times 10^{-2}\ m^2V^{-1}s^{-1}$. The temperature dependence of the Hall voltage and the conductivity of the sample was tested and verified with theory. The number of impurity atoms in the sample was estimated as $2.4(\pm 0.2) \times 10^{14}$.

INTRODUCTION

In this investigation the Hall effect in an extrinsic semiconductor was studied by applying a magnetic field and passing a current through a sample of doped germanium. The Hall voltage relation was tested first, and then, from the consequences of this relation, the concentration and mobility of the charge carries was determined.

The dependence of the Hall effect and the conductivity with temperature were then investigated by applying heat to the sample and measuring the temperature via a built-in thermocouple.

Semiconductors were used in radio sets as early as 1906 in the "Cat's whisker detector", however it was not until the development of the transistor in 1947 that their behaviour and potential was fully understood[1]. Since then semiconductors have become ubiquitous and modern life would be impossible without the transistors and integrated circuits that power our computers.

The Hall effect was discovered by Edwin Hall in 1879 whilst he was working on his doctoral degree. His detection of the small effect was excellent for the time considering it was 18 years prior to the discovery of the electron[2].

THEORY

Semiconductors

Semiconductors are materials with an intermediate electrical conductivity, but their most useful property is that the conductivity can be varied by introducing small amounts of impurities to the semiconductor in a controlled manner (doping) and that the conductivity will increase as the temperature of the material increases or light is shone upon the semiconductor (a phonomenon termed photoconductivity)[3].

An intrinsic semiconductor is a pure semiconductor, an extrinsic semiconductor is a doped semiconductor[3].

Band Gaps

Semiconductors have this property because they have an even number of electrons per primitive unit cell in the lattice (germanium has four). This means that the wavevectors of the electrons lie on a Brillouin zone boundary where a band gap is present due to the production of a standing wave with two possible energies due to Bragg scattering of the electrons from the lattice[4].

This means that the Fermi energy lies halfway between two allowed bands as shown in Figure 1.

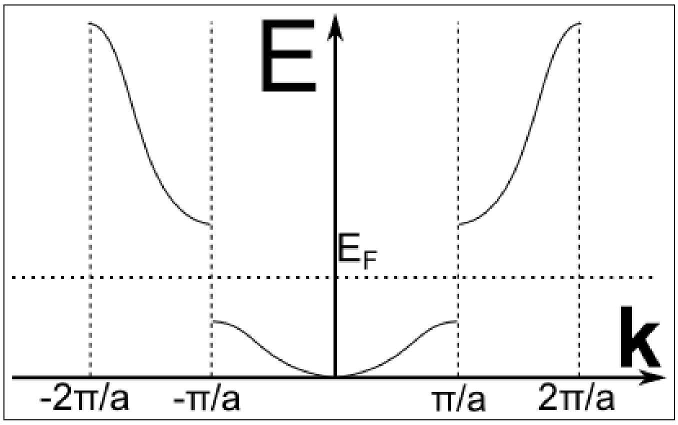

FIG. 1: A diagram of the dispersion relation for the semiconductor. Note that the even number of electrons puts the Fermi energy between two allowed energy bands.

This means that the electrons must exist in one of the energy bands and can jump between the bands by obtaining some thermal energy, k_BT, and so the conductivity increases with temperature as more charge carriers are released.

The release of charge carriers is found to be an exponential increase with temperature, this effect of increasing the conductivity dominates the effect of the increased lattice vibrations (causing greater electron scattering) decreasing the mobility of the electrons and so the conductivity[3].

Charge Carriers and Holes

In semiconductors there are two types of charge carriers: electrons and holes. Holes (also known as vacancies) are produced as the excitation of valence electrons to the conduction band leaves a vacancy in the valence band with a positive charge due to the missing electron. The electron hole can move by accepting an electron from a nearby bond and so it acts as a positively charged charge carrier [2].

This leads to two types of semiconductors, n-type semiconductors which have mostly electrons as charge carriers and p-type semiconductors which have mostly holes as charge carriers. These types are produced by doping the semiconductor with n-type and p-type impurities respectively[1].

Doping

Doping is the process of adding very small amounts of impurities (dopants) to the semiconductor to modify its conductivity. Donor impurities produce n-type regions in the semiconductor while acceptor impurities produce p-type regions[2].

Donor impurities create allowed energy states near the conduction band, while acceptor impurities near the valence band. This has the effect of either lowering or raising the Fermi level of the semiconductor respectively, and increasing or decreasing the conductivity (as at a higher Fermi level, less thermal energy is required to produce more charge carriers).

The Hall Effect

The Hall effect is the phenomenon whereby a voltage difference is produced transverse to the direction of the current when a strong magnetic field is applied in a direction perpendicular to the current.

Derivation of the Hall voltage

This potential difference arises because of the Lorentz force acting on the charge carriers. The Lorentz force is described by the Equation (1).

$$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \tag{1}$$

where $\vec{F}$ is the Lorentz force, q is the charge of the charge carriers, $\vec{v}$ is the velocity of the charge carriers and $\vec{B}$ is the applied magnetic field.

After equilibrium has been reached, there is no net transverse force on the charge carriers, and so the Lorentz force must be equal to the repulsion the charge carriers experience from the charge carriers already gathered on the edge. This is shown mathematically in equations (2) and (3).

$$F_{x_{net}} = q\vec{E} + q\left(\vec{v} \times \vec{B}\right) = 0 \tag{2}$$

Since the current is solely directed in the y-axis and the magnetic field is solely directed in the z-axis, as shown in Figure 2, then this can be simplified to Equation (3).

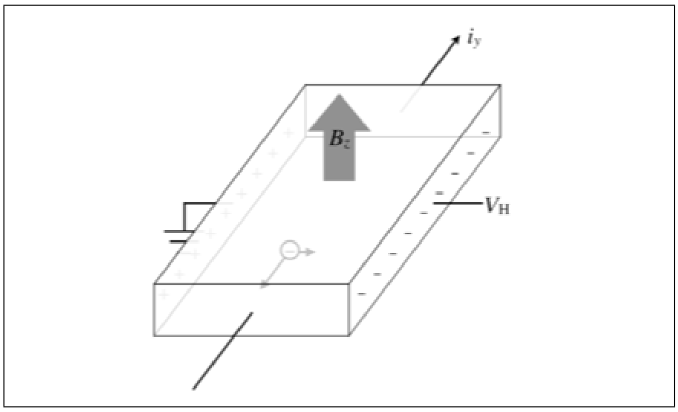

FIG. 2: A diagram showing the directions of the magnetic field and current across the material. Taken from [5]

$$E_x = -v_y B_z \tag{3}$$

Assuming there is only one type of charge carrier (this is acceptable for a doped semiconductor since either the electrons or holes will be the dominant charge carriers). Then the current density, $\vec{j}$, is given by equation (4).

$$\vec{j} = nq\vec{v}_d \tag{4}$$

Where n is the density of the charge carriers (the number per unit volume) and $\vec{v}_d$ is their drift velocity due to the electric field in the direction of the current, $\vec{E}_y$ (this is *not* the transverse electric field from the Hall effect).

So the drift velocity is the velocity of the charge carriers in the y-direction (the direction of the current), so let $\vec{v}_d = v_y$. And so rearranging equation (4) obtains:

$$v_y = \frac{1}{nq} j_y = \frac{1}{nq}\frac{i_y}{bt} \tag{5}$$

Where b is the width of the material and t is the thickness of the material.

From the definition of the electric field, the electric field in the transverse direction can be written as:

$$E_x = \frac{-V_H}{b} \tag{6}$$

And so substituting Equations (5) and (6) in to Equation (3) obtains an expression for the Hall voltage:

$$V_H = \frac{1}{nq}\frac{i_y B_z}{t} \tag{7}$$

The Hall Coefficient

The Hall coefficient is defined by Equation (8). it is a measure of the magnitude of the Hall effect.

$$R_H = \frac{E_x}{j_y B_z} \tag{8}$$

This can be simplified as shown in Equation (9).

$$R_H = \frac{E_x}{j_y B_z} = \frac{V_H t}{i_y B_z} = \frac{1}{nq} \tag{9}$$

And by measuring R_H , the charge carrier density n can be calculated. The sign of the Hall coefficient gives the sign of the charge carriers (which will either be positive for p-type semiconductors or negative for n-type semiconductors due to the dominance of hole and electron conduction respectively).

The electrical conductivity, σ, is defined by the ratio of the current density to the electric field.

$$\sigma = \frac{\vec{j}}{\vec{E}} = \frac{j_y}{E_y} \tag{10}$$

The drift mobility is defined as the drift velocity per unit of applied electric field.

$$\mu = \frac{\vec{v}_d}{\vec{E}} = \frac{v_y}{E_y} \tag{11}$$

And so substituting Equation (11) in to Equation (4) obtains:

$$j_y = nq\mu_H E_y \tag{12}$$

And substituting Equation (12) in to Equation (10) obtains an expression for the conductivity.

$$\sigma = nq\mu_H \tag{13}$$

Which can be written in terms of the Hall coefficient.

$$R_H \sigma = \mu_H \tag{14}$$

Note that this means the conductivity (in the absence of a magnetic field) must also be measured to calculate a value for the Hall mobility.

The conductivity is calculated experimentally using Equation (15), where L is the length of the sample, A is the area of the sample and R is the resistance of the sample.

$$\sigma = \frac{L}{AR} \tag{15}$$

The Effect of Temperature

Of the variables in the expression for the Hall voltage (Equation (7)) both the density of charge carriers n and the current i_y are dependent on the temperature.

The Hall voltage will increase with the resistance due to increased electron scattering as the lattice vibrates more due to the higher temperature, as the current is kept constant across the sample.

The density of charge carriers will increase with temperature, as more electrons will be able to jump to the conduction band with the increased thermal energy.

The release of charge carriers will far outweigh the increased resistance, until such a temperature that the majority of charge carriers have been released, while the lattice vibrations continue to increase.

METHOD

A sample of doped germanium (of dimensions 10mm x 20mm x 1mm) was used as the semiconductor in this experiment. The sample was placed in to a plug-in board which contains a heater, thermocouple and an electromagnet.

This base unit was connected to a computer and the results were recorded using the CASSY Lab software.

Note that the contacts across the semiconductor can never be precisely opposite to each other and so even when there is no magnetic field there will be a small transverse voltage across the sample due to this asymmetry. This voltage was adjusted for by measuring the transverse voltage when no magnetic field is applied, and then subtracting it from further measurements, the integrated board had a compensation function to do this.

The Hall effect was first investigated by varying the magnetic field and current (the maximum current for the sample is 25 mA). This data was used to prove the Hall voltage equation in Equation (7).

The sign and concentration of the charge carriers were then calculated from the observed value for the Hall coefficient using Equation (9).

The conductivity of the semiconductor in the absence of a magnetic field was then measured and used with the experimental value of the Hall coefficients to calculate the Hall mobility as shown in Equation (14).

The relationship of the Hall voltage with temperature was then investigated using the thermocouple and heater on the integrated board.

Then, assuming that each charge carrier is produced by a single impurity atom, the number of impurity atoms in the sample was estimated.

Consideration of Errors

The error in the Hall coefficient can be estimated by taking the mean of the result from the two sets of data, and then using the bounds as the absolute error in the value. This error can the be used to calculate the error in the derived quantities, by taking it as a percentage of the Hall coefficient, as the error in the conductivity and current measurements can be assumed to be negligible in comparison to this error.

RESULTS

The Hall voltage relationship with Current and Magnetic Field

Plots of the relationship of the Hall voltage with a varying magnetic field and current are shown in Figure 3 and Figure 4 respectively.

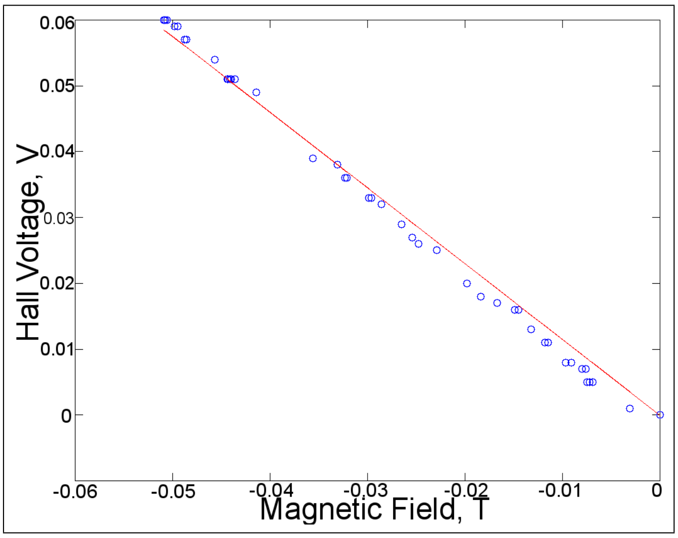

FIG. 3: A plot of the Hall voltage against varying magnetic field, at a constant current of 20mA. The gradient of the fit is -0.11499.

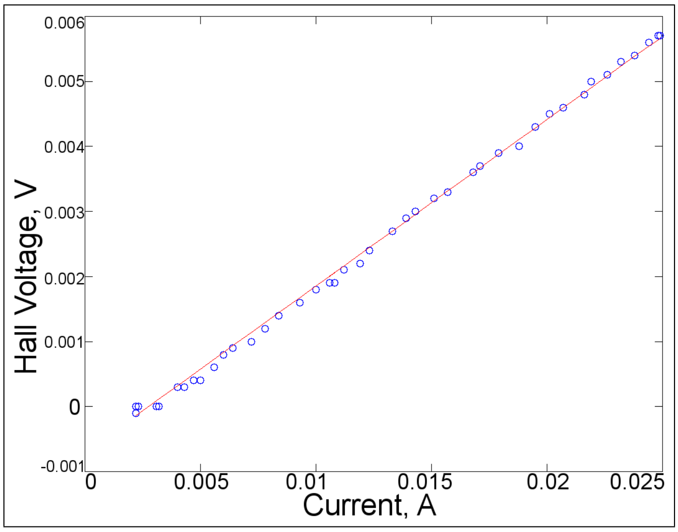

FIG. 4: A plot of the Hall voltage against varying current, at a constant magnetic field of -52mT. The gradient of the fit is 0.25584.

By dividing the gradient of the straight-line fits by the constant current (20mA) for the varying magnetic field plot and the constant magnetic field (-52mT) for the varying current plot, and multiplying by the thickness of the sample (1mm), two values for the Hall coefficient (defined in Equation (9)) were calculated. These were -5.7495×10^{-3} for the varying magnetic field and -4.92×10^{-3} for the varying current. By taking the mean of these two values, and the error as the bounds, this gives a value of the Hall coefficient as $-5.3(\pm0.4)\times10^{-3}$ m^3C^{-1}. This value of the Hall coefficient is of similar to magnitude to the value of $-1.91(\pm0.07)\times10^{-2}$ m^3C^{-1}, measured for a different sample of a Germanium-based semiconductor by S. Chandramouli[6]. The values are not consistent with eachother, however the samples would likely have been doped differently and so this is not significant.

This value suggested that the charge carriers were negatively charged, and so the sample was an n-type semiconductor. Assuming that the charge carriers have an elementary negative charge (-1.6×10^{-19} C) then the charge density of the sample, n, was $1.18(\pm0.09)\times10^{21}$ m^{-3}.

A plot of the voltage applied against current measured, at room temperature and in the absence of a magnetic field, for the sample is given in Figure 5. The gradient of the plot, and so resistance of the sample, was 109.35Ω.

Using Equation (15) with the area of the sample as (10mm x 1mm) and the length of the sample as 20mm, the conductivity was calculated to be 18.29 $m^{-1}\Omega^{-1}$.

The Hall mobility is the product of the Hall coefficient and the conductivity, this was calculated to be $-9.7(\pm0.7)\times10^{-2}$ $m^2V^{-1}s^{-1}$ assuming negligible error in the conductivity at room temperature.

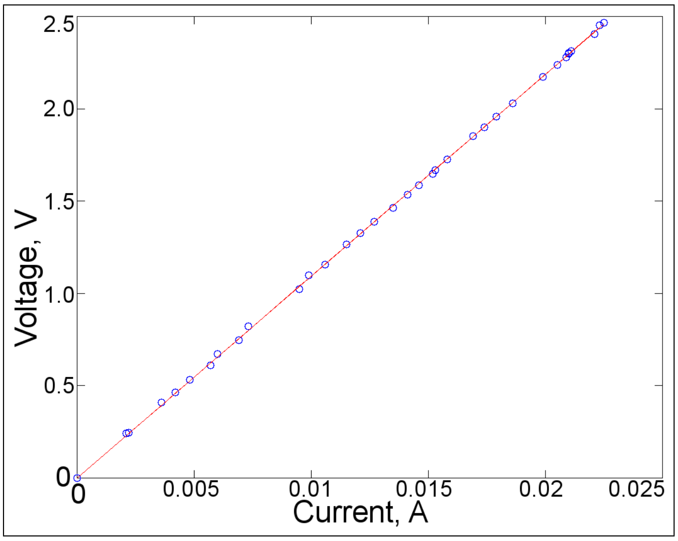

FIG. 5: A plot of the voltage against current for the sample, to determine the resistance and calculate the resistivity of the sample. The measurements were made at room temperature and without an applied magnetic field. The gradient of the fit is 109.35.

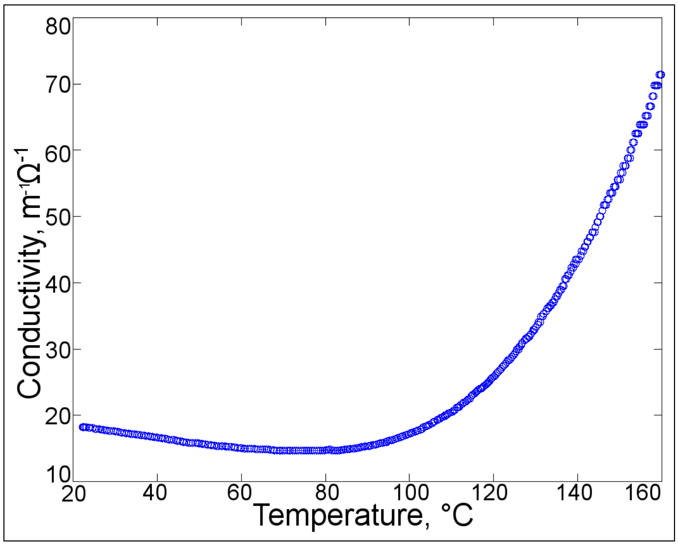

FIG. 6: A plot of the conductivity of the sample against temperature. The conductivity decreases at first due to the increased resistance from the lattice vibrations, however then the temperature reaches the level where many more electrons can jump to the conduction band, and the conductivity rapidly increases.

Temperature Dependence

In the investigation of the variance of the conductivity of the sample with temperature, the variance of the current was found to be negligible, and so the current was assumed to be constant (at 15mA) so the conductivity could be easily calculated. A graph of the conductivity against temperature is shown in Figure 6.

The variation of the Hall Voltage with temperature was also investigated, a plot of the Hall Voltage against temperature at a constant current of 20mA and a constant magnetic field of -49.9mT is shown in Figure 7.

From the charge concentration of the sample, $1.18(\pm 0.09) \times 10^{21}\ m^{-3}$, and the volume of the sample, $2 \times 10^{-7}\ m^3$, the number of impurity atoms can be estimated assuming there is one charge carrier per impurity atom, as the product of these quantities. Using this assumption, the number of impurity atoms in the sample was estimated to be $2.4(\pm 0.2) \times 10^{14}$.

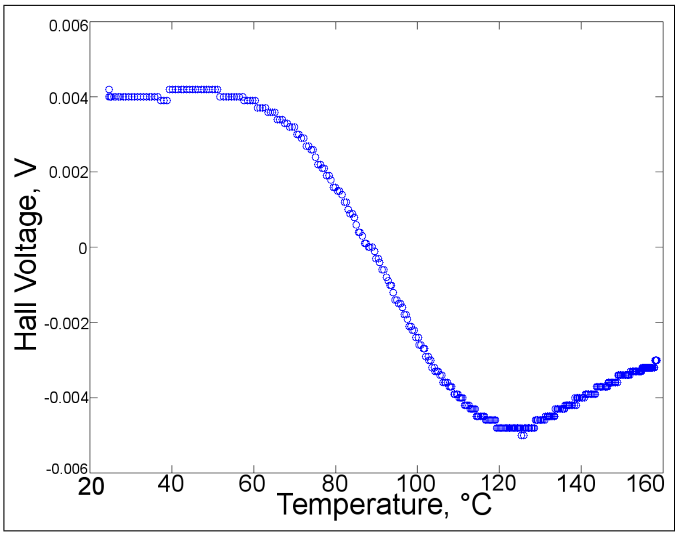

FIG. 7: A plot of the Hall Voltage against temperature, at a constant current of 20mA and a constant magnetic field of -49.9 mT.

DISCUSSION

The conductivity of the sample initally decreases with temperature, because the increased temperature increases the vibrations of the lattice and so increases the scattering of charge carriers and lowers the conductivity. It then increases rapidly after it reaches a certain temperature, such that electrons can jump to the conduction band, and so the conductivity rapidly increases as the effect of the exponential growth of the charge carriers far outweights the effect of the increased lattice vibrations. The electrons can jump to the conduction band when they are within k_BT of the Fermi energy, where k_B is the Boltzmann constant, T is the temperature (in Kelvin). The Fermi energy is the energy at which all the energy states below are filled by electrons (at 0 Kelvin), as the temperature increases, more electrons are within k_BT of the Fermi energy and so more charge carriers are released, lowering the conductivity.

The Hall voltage initially increases slightly with temperature, as the resistance is increased due to the increased lattice vibrations, however the Hall voltage is then decreased greatly by the rapid release of charge carriers, it then levels out as the effect of the increased resistance begins to outweigh the release of charge carriers, as the majority have already been released.

Possible sources of error in the experiment include the magnetic field not being constantly aligned through-out the measurements, as small vibrations will cause this to change, and so the effective strength of the magnetic field to change. Imperfections in the Germanium crystal may also have been responsible for minor discrepancies in the results. There is also the error measured in the Hall voltage due to the connections on either end of the sample not being perfectly aligned, however this is largely mitigated through the use of the offset voltage.

A possible improvement to the experiment would be to constantly use the Hall probe to ensure the magnetic field is constant, and fix the board and electromagnetic in place using stands to prevent vibrations affecting the alignment. This would help to mitigate the error from the varying magnetic field, however it was not possible in this investigation due to the limited number of ports on the CASSY module.

CONCLUSIONS

The Hall Voltage relation given in Equation (7) was successfully verified. The Hall coefficient was calculated as $-5.3(\pm0.4)\times10^{-3}\ m^3C^{-1}$, with the negative sign suggesting that electrons are the predominate charge carriers in the sample, and this was on the same order of magnitude as published results for the Hall coefficient of germanium. The charge density was calculated to be $1.18(\pm0.09)\times10^{21}\ m^{-3}$, and the Hall mobility was calculated as $-9.7(\pm0.7)\times10^{-2}\ m^2V^{-1}s^{-1}$ with the negative sign due to the negative charge carriers.

The temperature dependence of the conductivity and Hall voltage was investigated, and it was found that the effect of the release of charge carriers far outweighed the increase in resistance due to lattice vibrations with increasing temperature, and so the conductivity rapidly increased with temperature and the Hall voltage decreased until the resistance effects became dominant again after the majority of charge carriers were released.

The number of impurity atoms in the sample was estimated to be $2.4(\pm0.2)\times10^{14}$, assuming that each charge carrier originated from a single impurity atom.

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