Solid State II

Drude Model, etc.

Applies Kinetic Theory of gasses to electrons in metals: four assumptions of Drude theory:

1. Electrons are solid spheres moving in straight lines until collision;
2. Time taken up by a collision is negligible;
3. No force acts on the particles between collisions.
4. Electrons achieve thermal equilibrium through collisions.

I.e. Free electrons, fixed ions, no electron-electron interaction, classical treatment.

DC Electrical Conductivity:

$$\vec{E} = \rho \vec{j}$$

where ρ is the resistivity.

$$\vec{j} = -ne\vec{v}$$

$$\vec{v}_{\text{avg}} = \frac{-e\vec{E}\tau}{m} \ , \ \vec{j} = \left(\frac{ne^2\tau}{m}\right)\vec{E}$$

$$\vec{j} = \sigma\vec{E} \ , \ \sigma = \frac{ne^2\tau}{m} = ne\mu$$

Mobility:

$$\mu = \frac{e\tau}{m}$$

AC Derivation from equation of motion:

$$\frac{d\vec{p}(t)}{dt} = -\frac{\vec{p}(t)}{\tau} + \vec{f}(t)$$

Force acting on electrons is $-e\vec{E}$
Electric field takes form: $\vec{E}(t) = Re(\vec{E}(\omega)e^{-i\omega t})$
Seek steady state solution of form: $\vec{p}(t) = Re(\vec{p}(\omega)e^{-i\omega t})$
$\vec{p}(\omega)$ must satisfy:

$$-i\omega\vec{p}(\omega) = -\frac{\vec{p}(\omega)}{\tau} - e\vec{E}(\omega)$$

$$\vec{j}(t) = Re(j(\omega)e^{-i\omega t}) \ . \ \vec{j} = \frac{-ne\vec{p}}{m}$$

$$\vec{j}(\omega) = -\frac{ne\vec{p}(\omega)}{m} = \frac{(ne^2/m)\vec{E}(\omega)}{(1/\tau) - i\omega} = \sigma(\omega)\vec{E}(\omega)$$

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau} \ , \ \sigma_0 = \frac{ne^2\tau}{m}$$

Complex dielectric constant:

$$-\nabla^2 \vec{E} = \frac{\omega^2}{c^2}\varepsilon(\omega)\vec{E} \ , \ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2}$$

For plasmon propagation require:

$$\frac{\omega^2}{c^2}\varepsilon(\omega)\vec{E} \ , \ \varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} = 0$$

Plasma Frequency: At high frequencies:

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \ , \ \omega_p^2 = \frac{ne^2}{m\varepsilon_0}$$

BCC has 2 atoms per cell, FCC has 4 atoms per cell.

Mean free path:

$$MFP = v_F \tau$$

Hall Effect: Force due to perpendicular magnetic field leads to movement of electrons in y-direction (transverse to applied E field) and formation of opposing E_y Hall Field.

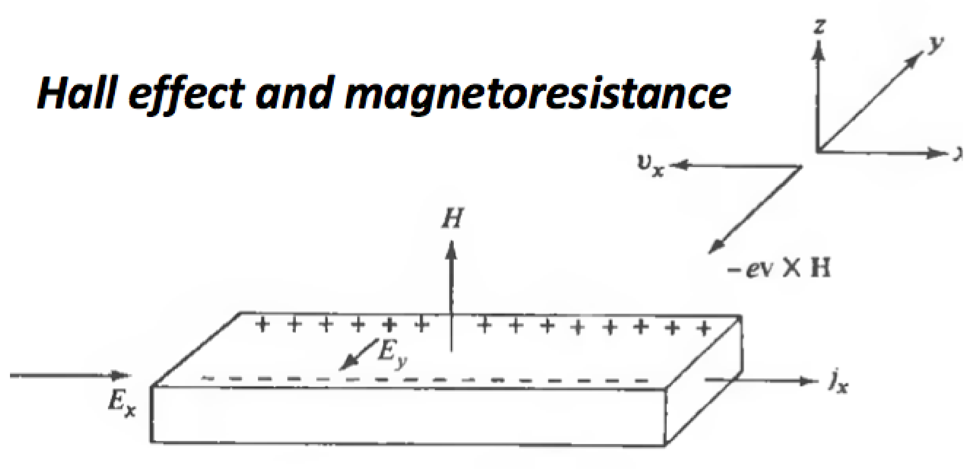

Figure 1: The Hall Effect

Hall coefficient (in SI):

$$R_H = \frac{-1}{ne}$$

Hall Field:

$$E_y = R_H J_x B$$

As n (charge carrier concentration) is an intensive property it will remain the same even if the system size is changed therefore the Hall Coefficient depends only on the carrier concentration and not the size of the sample. The sign of n and thus R_H can identify whether carriers are electrons or holes. R_H is independent of the Magnetic Field.

The Drude Model is classical electron gas, addition of quantum statistics (Fermi-Dirac) leads to the Sommerfeld Model, both are Free Electron models. The addition of a periodic potential (due to the ionic lattice) leads to **Bloch Theory**.

The Drude Theory is a purely classical theory with describes the particles as solid spheres and therefore does not explain quantum interference effects. It predicted an electronic contribution to the specific heat of $\frac{3}{2}nk_b$ which was not observed and is far too large, the Lorenz Number $\frac{\kappa}{\sigma T}$ was half the typical observed value. In the Seebeck Effect (Thermoelectric Field) it calculated a thermopower coefficient a factor of 100 too large.

Key equation of Bloch Theory:

$$\psi_{nk}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{nk}(\vec{r})$$

where $u_{nk}(\vec{r}+\vec{R}) = u_{nk}(\vec{r})$ (i.e. is periodic)
Tight binding approximation:

$$E(k) = E_0 + 2\gamma\cos(ka) \approx E_0 + 2\gamma - \gamma a^2k^2$$

$$m^* = \hbar^2\left[\frac{d^2E}{dk^2}\right]^{-1} = \frac{\hbar^2}{2|\gamma|a^2}$$

Band Theory (i.e. use Bloch Theory with Schrodinger Equation)

Assumptions for Band Theory:

Rigid lattice approximation: The nuclei are taken as fixed at their equilibrium positions; the large difference between the masses of the electrons and the nuclei is the basic justification for this.

One-electron approximation in local form: In essence, the complicated many-body electron problem is simplified to a single one-electron problem with an appropriate local potential.

Relativistic effects are neglected: Whenever necessary, one should replace the Schrodinger equation with the Dirac equation. Often one includes relativistic terms of interest (such as spin-orbit coupling) by perturbation theory.

Degenerate 2-d Electron Gas

In a 2D system the electrons may only travel in one plane and may not move perpendicular to this plane, due to confining potentials or physical limitations. This effects the energy levels as the k_z state becomes discretised and the density of states becomes 2D and thus independent of energy.

In 2D:

$$k_F = \sqrt{\frac{4\pi n}{g_s}}, \qquad \rho(k) = \frac{g_sL_xL_y}{4\pi^2} = \frac{g_sA}{4\pi^2}, \qquad \rho(E) = g_SA\frac{M}{2\pi\hbar^2}$$

The presence of a magnetic field modifies the continuous kinetic energy of the Free Electron Gas into discrete **Landau Levels**:

$$E_{nk} = \left(n + \frac{1}{2}\right)\hbar\omega_c$$

$$\frac{m^* v_F^2}{R_c} = q v_F B$$

$$\therefore \omega_c = \frac{v_F}{R_c} = \frac{qB}{m^*}$$

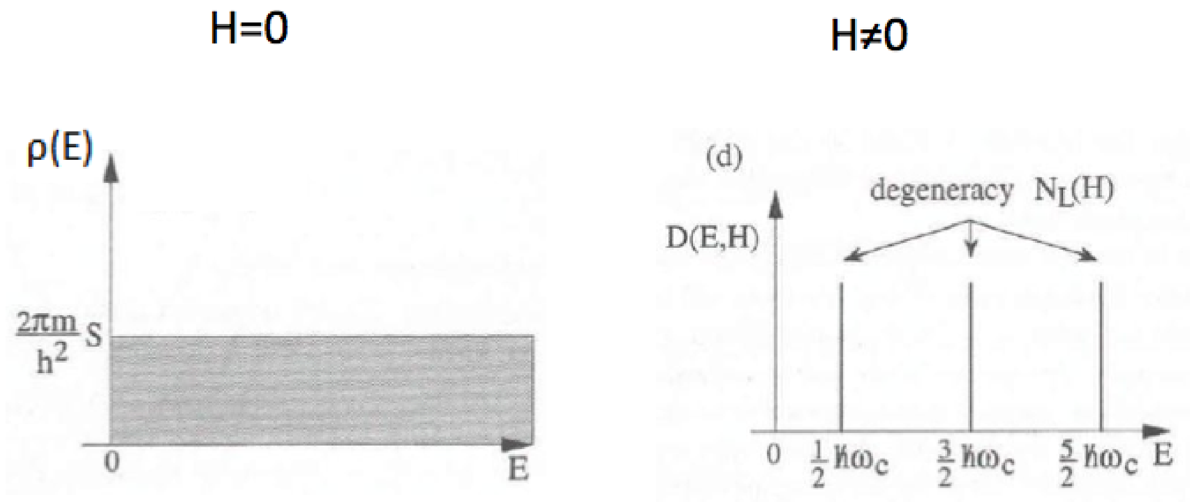

Figure 2: The density of states of 2D-FEG and it's dependency on the applied magnetic field

$$\text{No. of Occupied Landau Levels} = \frac{n}{2eB/h} = \frac{nh}{2eB}$$

The Fermi Energy depends on the applied Magnetic field.
Zeeman splitting of Landau Levels ($\pm g\mu_B B$) at very high fields.

Unconfined Electrons (U=0) in non-zero magnetic field ($B \neq 0$)

The Hall Resistance, R_{xy}, exhibits flat plateaus from which a Universal Constant may be determined which is used in Metrology as the Primary Standard of Resistance. That Universal Constant is:

$$\frac{h}{e^2} = 25812.806\Omega$$

The Conductance is quantised in units of $\frac{e^2}{h}$:

$$G = N_{max}\frac{g_s e^2}{h}$$

where N_{max} is given by the number of edge states in the system.

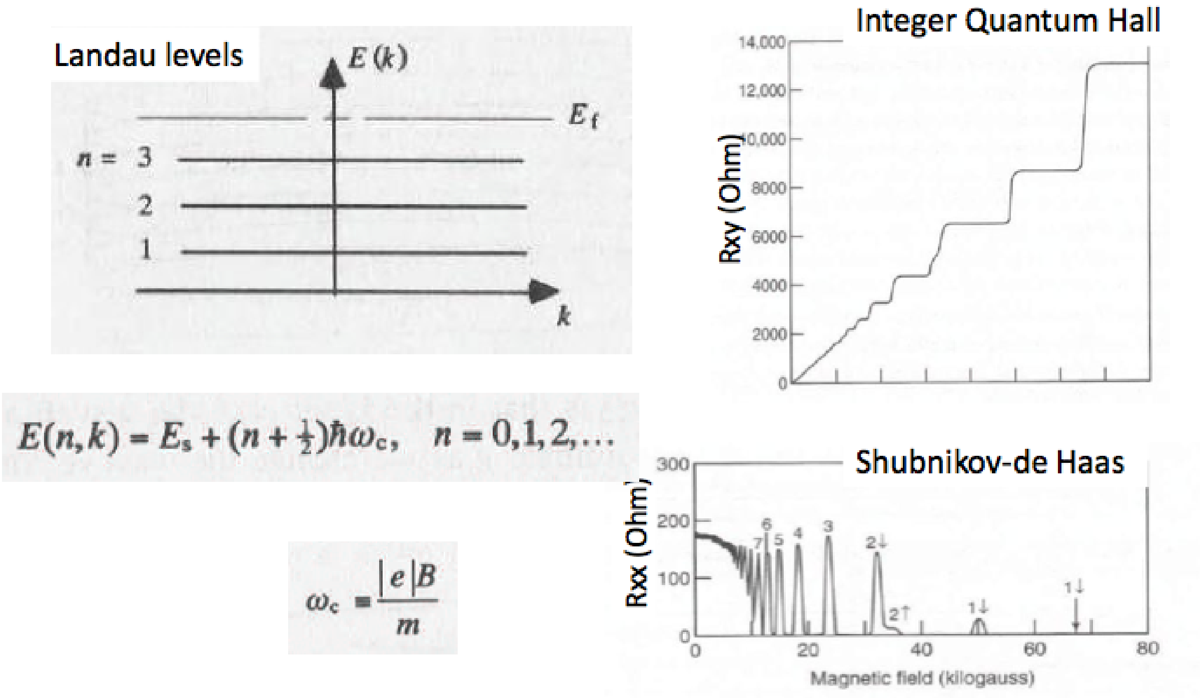

Figure 3: Unconfined Electrons (U=0) in non-zero magnetic field ($B \neq 0$)

The Shubnikov-de Haas Oscillations are periodic in $\frac{1}{\Delta B}$.
Recall: No. of Occupied Landau Levels $= \frac{nh}{2eB}$

R_{xx} is maximum every time the number of occupied Landau Levels is half-integer and the Fermi Energy lies at the centre of a Landau level.

For Landau Levels to exist we require:

- An electron to be able to complete several orbits before scattering: $\omega_c^{-1} << \tau_m$
- Equivalently the spacing of states must be greater than the broadening (due to

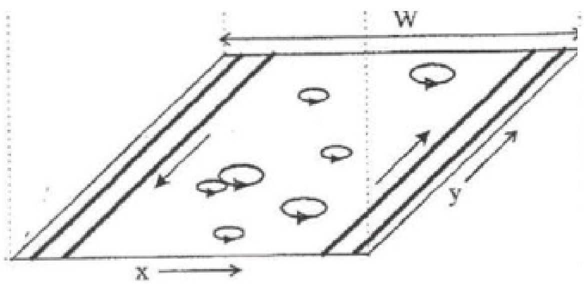

Figure 4: Quantum Hall effect edge states: Edge currents flow in opposite directions at the sides of the channel

Heisenberg Uncertainty Principle):

$$\hbar\omega_c >> \frac{\hbar}{\tau_m}$$

$$\therefore \quad \because \omega_c = \frac{eB}{m} \text{ and } \mu = \frac{|e|\tau}{m}$$

$$\therefore B >> \mu^{-1}$$

- Thermal Broadening means that we also have a limit on temperature: $\hbar\omega_c >> k_b T$

Mesoscopic Physics is the bridge between the world of bulk materials and the microscopic world of atoms and molecules.

Length Scales

- Size of system in 1d, L
- (In)elastic Mean Free Path,$l_{i/e}$, distance between (In)elastic scattering events:

$$\tau_{i/e} = \frac{l_{i/e}}{v_F}$$

- Fermi Wavelength:

$$\lambda_F = \frac{h}{\sqrt{2ME_F}}$$

- Phase Coherence Length, l_ϕ - distance electron can travel before phase is randomised.
- Cyclotron Orbit:

$$\omega_c = \frac{v_F}{R_c} = \frac{eB}{m^*} \quad \therefore R_c = \frac{m^* v_F}{eB} = \frac{\hbar k_F}{eB}$$

Energy Scales

- Thermal Energy, $k_b T$
- Fermi Energy, E_F
- Single Electron Charging Energy: Electrostatic energy required to position an electron on an isolated island with capacitance C to the external world:

$$E_c - \frac{e^2}{2C}$$

- Thouless Energy: Provides a measure between the Energy of a state and the phase evolution in a restricted area of space of size L.

Transport Regimes

Diffusive Classical: $\lambda_F, l_i, l_e <<$ L
Diffusive Quantum: $\lambda_F, l_e <<$ L$, l_i$
Ballistic Classical: $\lambda_F <<$ L $< l_i, l_e$
Ballistic Quantum: $\lambda_F,$ L $< l_e < l_i$

1D Ballistic Conductor

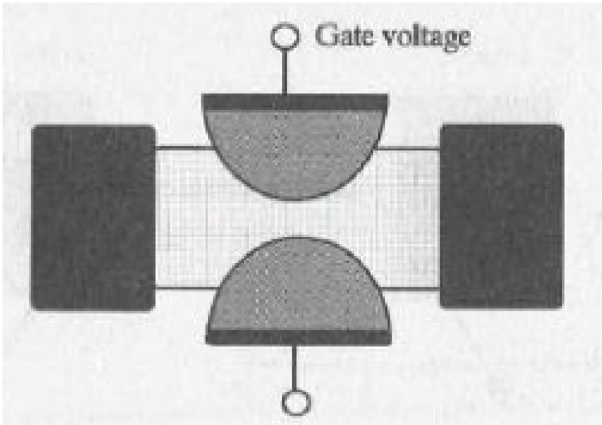

Figure 5: Experimental set-up of 1D ballistic conductor: Conductance depends on width, controlled experimentally via gate voltage.

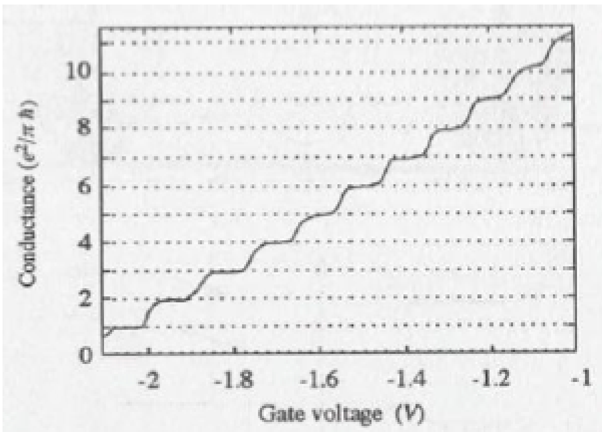

Figure 6: 1D ballistic conductor: Although the width of the conductor changes continuously the number of modes changes in discrete steps.

Naively we would expect that as the 1D conductor is made shorter the conductance would tend to infinity:

$$G = \lim_{L \to 0} \sigma \frac{W}{L} = \infty$$

But experimentally G approaches a finite value as the length becomes smaller than the mean free path i.e. as L<< l_m so the ballistic conductor does not have the expected zero

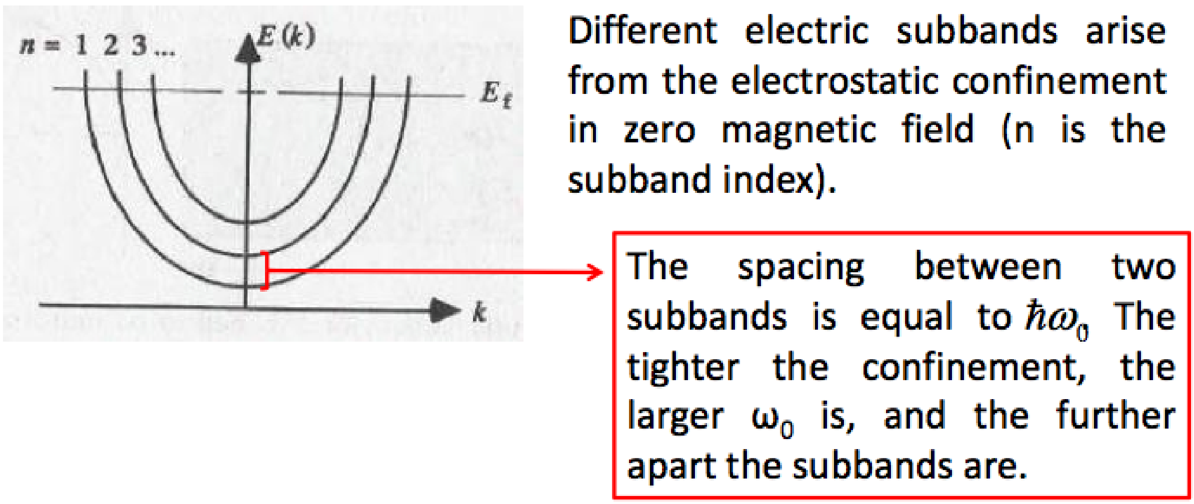

Figure 7: Confined electrons ($U \neq 0$) in zero magnetic field (B=0)

resistance (expected as there should be no scattering). (Note 1D means W $< \lambda_F$)

States in the narrow conductor belong to different modes/subbands, current carried per mode per unit energy by an occupied state is $\frac{2e}{h}$

$$\therefore G_c = \frac{2e^2}{h} M$$

$$\therefore \text{ resistance is } G_c^{-1} = \frac{h}{2e^2 M} \approx \frac{12.9k\Omega}{M}$$

$$\text{M is number of modes} = int\left[\frac{k_F W}{\pi}\right]$$

where W is the width and $\hbar k_f = \sqrt{2M_e E_f}$ (From $E = \frac{p^2}{2M} = \frac{\hbar^2 k^2}{2M}$)
The discrete steps in the conductance are smeared out when the Thermal Energy is comparable to that separating the modes i.e. when $k_b T \approx \hbar\omega_0$ (ω_0 is the Confinement Parameter).

Landauer Formula: In a 1D Ballistic conductor we go from:

$$G = \sigma \frac{W}{L} \text{ to } G = \frac{2e^2}{h} MT$$

So the interface resistance is independent of the length L and does not decrease linearly with the width W. Therefore the conductance scales in steps, the factor T is the transmission probability that an electron injected at one end of the conductor will transmit to the other end.

The magnetic field increases the mass by a factor that depends on the relative magnitudes of the Confinement Parameter, ω_0, and the Cyclotron Frequency, ω_c:

$$m \rightarrow m\left[1 + \frac{w_c^2}{w_0^2}\right]$$

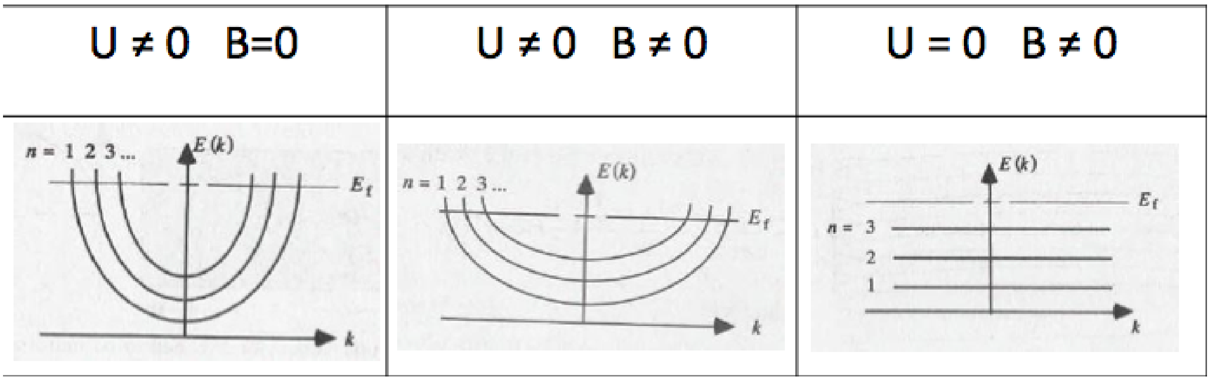

Figure 8: Summary of dispersion relations for 2DEG: Parabolas, broader parabolas, and discrete Landau Levels

Superconductivity

Meissner-Ochsenfeld Effect: A superconductor expels a weak magnetic field - superconductors are perfect diamagnets.

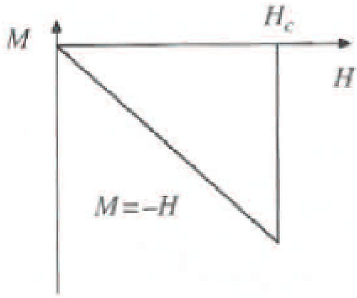

Figure 9: Type 1 Superconductor: Above critical field H_c the Magnetisation, M, collapses to zero and the superconductivity is destroyed

Ginzburg-Landau Free Energy: $f_s(T) = f_N(T) + a(T)|\psi|^2 + \frac{1}{2}b(T)|\psi|^4...$
The GL Theory is a Mean Field Theory. These fail to explain the critical field region as the assumption that all sample regions are the same fails. They also ignore correlations and fluctuations which become very important near to T_c where large fluctuations are observed in the order parameter. E.g. Critical Opalescence - Density fluctuations give rise to large variations in refractive index.

Josephson Effect

The Josephson Effect is a direct physical test of the macroscopic quantum coherence in the superconducting state - it is seen at an SIS Interface (AKA Josephson Junction).

Further superconductivity

London Equation: $\vec{J} = \frac{-n_s e^2}{m_e}\vec{A} = \frac{-1}{\mu_0 \lambda^2}\vec{A}$

Three different length scales:

type II superconductor: there are two different critical fields, denoted Hc1 the lower critical field, and Hc2 the upper critical field.

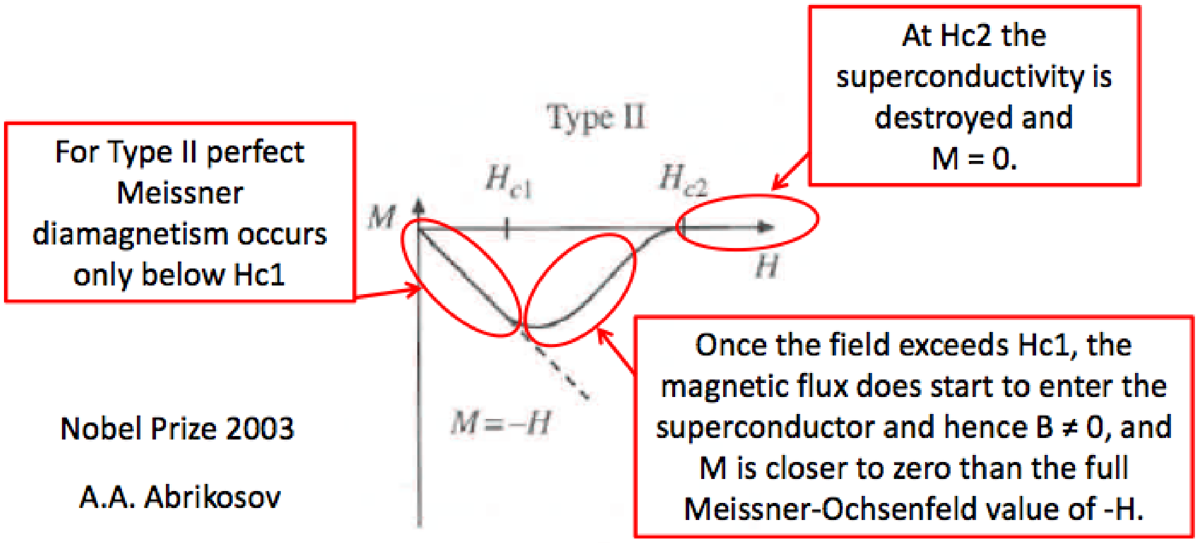

Figure 10: Type 2 superconductor: between H_{c1} and H_{c2}, M declines as the magnetic field starts to penetrate the superconductor in the form of vortices and destroy the superconductivity

- Mean free Path $l = v_F \tau$
- Coherence length $\xi = \frac{\hbar v_f}{\pi \Delta}$
- Penetration Depth $\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}}$

Type I superconductors have $0 < \frac{\lambda}{\xi} < \frac{1}{\sqrt{2}}$
Type 2 superconductors have $\frac{\lambda}{\xi} > \frac{1}{\sqrt{2}}$

Isotope Effect: $T_c \propto M^{-\alpha}$, where α is some constant.

Cooper pairs: displacement of ions (i.e. phonon interaction) may attract electrons resulting in a weak bond between two electrons of opposite spin.

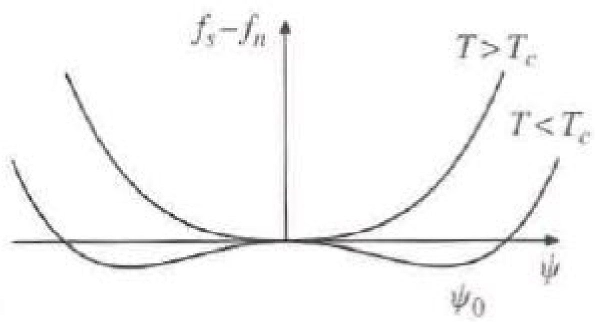

Figure 11: GL Theory: Free Energy wrt. order parameter

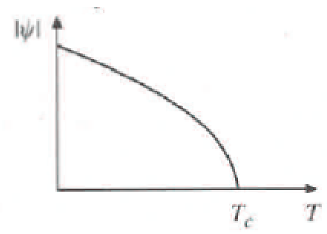

Figure 12: Order parameter magnitude, $|\psi|$ as a function of temperature in the GL model.

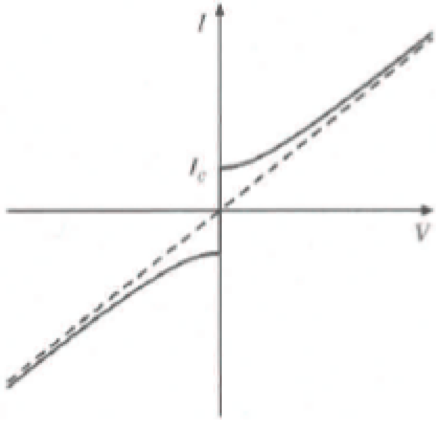

Figure 13: The I-V characteristics of an SIS Interface (Josephson Junction)

Spatial range of cooper pair:

$$\xi_0 \approx \frac{\hbar}{\delta p} \approx \frac{\hbar v_F}{\Delta} \approx \frac{1}{k_F}\frac{E_F}{\Delta}$$

Superconducting Energy Gap:

$$\frac{\Delta(0)}{k_b T_c} = 1.76$$

$$\frac{\Delta(T)}{\Delta(0)} = 1.74\left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}},\ \text{given } T \approx T_c$$

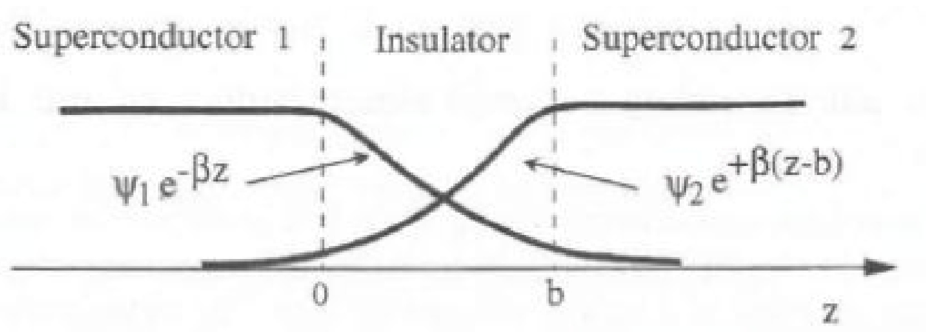

Figure 14: The variation of the order parameter across the Josephson Junction (SIS Interface).

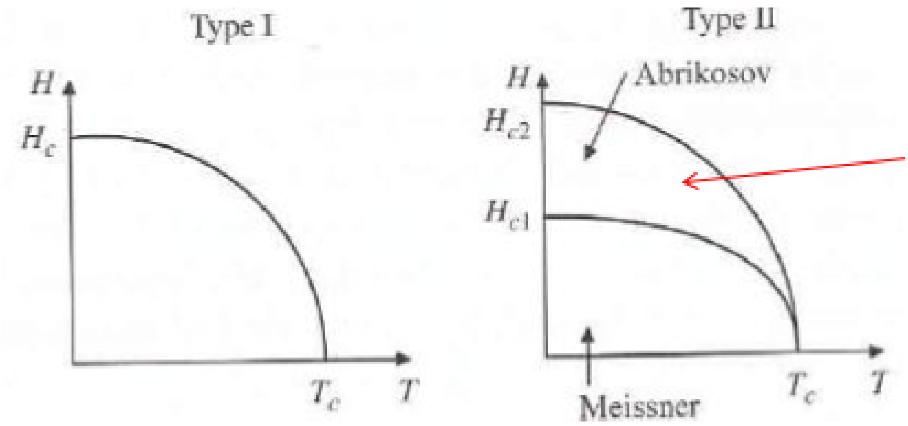

Figure 15: The field vs. temperature phase diagrams of type 1 and type 2 superconductors. In the Abrikosov region the magnetic field may enter the superconductor in the form of vortices.

Critical field:

$$\text{Far from } T_c: \quad \frac{H_c(T)}{H_c(0)} \approx 1 - 1.06 \left(\frac{T}{T_c}\right)^2$$

$$\text{Near } T_c: \quad \frac{H_c(T)}{H_c(0)} \approx 1.74 \left[1 - \frac{T}{T_c}\right]$$

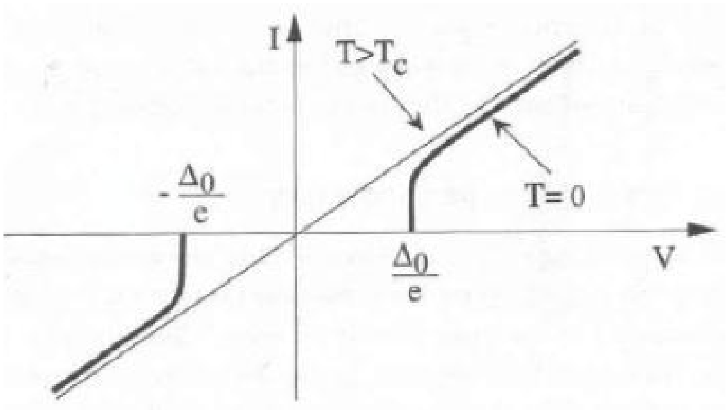

Figure 16: I-V Characteristics of an NIS interface: Electron tunnelling (Giaever Tunnelling) observed. Zero Current, unless voltage exceeds superconducting energy gap.

Ferromagnetism

$$\text{Free Energy}: F(M) = F_0 + a(T)M^2 + bM^4$$

Minimise F to determine the ground states (i.e. calculate M for $\frac{dF}{dM} = 0$
The expression for the Free Energy only contains even powers of M because the parallel and antiparallel states are both minima and should therefore be treated equivalently (i.e. F must be symmetric).

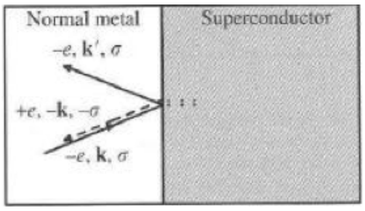

Figure 17: Andreev Reflection: An electron with an energy less than the superconducting gap may still pass by combining with another electron to form a Cooper pair which will pass freely into the superconductor. Conservation of charge and momentum requires that a hole is left behind with momentum equal and opposite to the original electron.

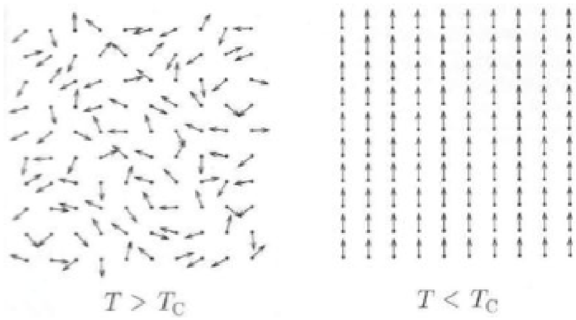

Figure 18: Above T_c there is no net magnetisation (i.e. the spin is randomly orientated) this is the Paramagnetic State. Below T_c there are domains of aligned spins (net magnetisation, as spin states in a given domain have the same alignment)

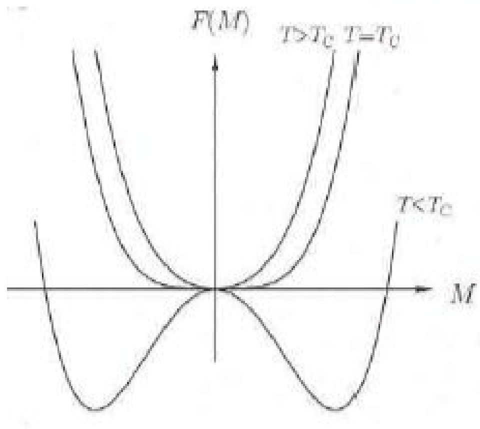

Figure 19: The Free Energy versus the Magnetisation of the Ferromagnetic state.

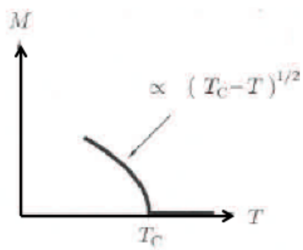

Figure 20: How the Magnetisation varies with Temperature in the Ferromagnetic state.