

PHY1003 – Properties of Matter

Young's Modulus, Y

$$Y = \frac{(\text{tensile stress})}{(\text{tensile strain})} = \frac{\sigma}{\varepsilon} = \frac{F_{\perp} / A}{(\Delta L / L_0)} = \frac{(F_{\perp} L_0)}{(A \Delta L)} \quad \text{Note it is the force perpendicular to the area}$$

Poisson's ratio

$$\frac{\Delta w}{w_0} = -\sigma \frac{\Delta l}{l_0} \quad \text{where } \sigma \text{ is some constant typically between 0.1 and 0.4, } w \text{ is width and } l \text{ is length}$$

Bulk stress

$$\text{Bulk stress} = \text{Pressure} = p = \frac{F_{\perp}}{A}$$

Bulk Strain

$$\text{Bulk strain} = \text{Volume strain} = \frac{\Delta V}{V_0}$$

Bulk Modulus

$$\text{Bulk Modulus} = B = \frac{-\Delta p}{(\Delta V / V_0)}$$

Compressibility

$$\text{Compressibility} = k = \frac{(1)}{B} = \frac{-\Delta V / V_0}{\Delta p}$$

Shear stress

$$\text{Shear Stress} = \text{tangential force per unit area} = \frac{F_{\tan}}{A}$$

Shear strain

$$\text{Shear Strain} = \frac{x}{h} = \tan \varphi \approx \varphi \quad \text{approx. may be used when } x \ll h$$

Shear Modulus

$$\eta = \frac{(\text{Shear Stress})}{(\text{Shear Strain})} = \frac{(F_{\tan} / A)}{\varphi} \quad \eta \text{ is usually } Y/2 \text{ to } Y/3 \text{ (Y is the Young's Modulus)}$$

Isotropic Materials

For isotropic materials (I.e all directions are equal) the bulk and shear moduli may be expressed in terms of the Young's Modulus and Poisson's Ratio.

$$B = \frac{Y}{(3(1-2\sigma))} \quad \text{And} \quad \eta = \frac{Y}{(2(1+\sigma))} \quad \text{Note that to test if a material is isotropic it must pass}$$

BOTH of these tests.

Elastic Energy

$$E_{el} = \frac{1}{2} k_s X^2 \quad \text{where} \quad k_s = \frac{YA}{L}$$

Hooke's Law is not followed in substances where Elastic Hysteresis occurs such as vulcanised rubber.

Variation of pressure with depth

$p = p_0 + \rho gh$ Pressure in a fluid of uniform density depends only on the depth and is equal for any two points at the same level.

Hydraulic Lift

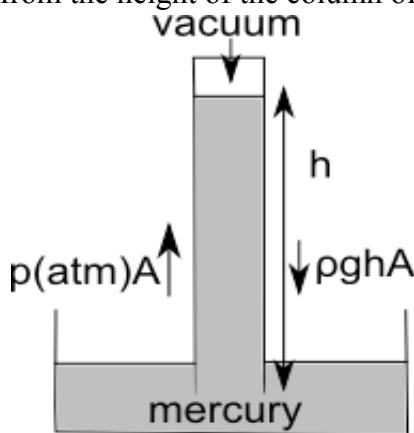
$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{therefore} \quad F_2 = \frac{A_2}{A_1} F_1$$

Gauge Pressure

$p = p_{atm} + \rho gh$ but gauge pressure is the excess pressure above atmospheric pressure so
 $p_{gauge} = \rho gh$

Barometer:

Used to measure atmospheric pressure, a column is filled with mercury and tipped upside down into a container of mercury – a small vacuum will form at the top, the atmospheric pressure is derived from the height of the column of mercury.



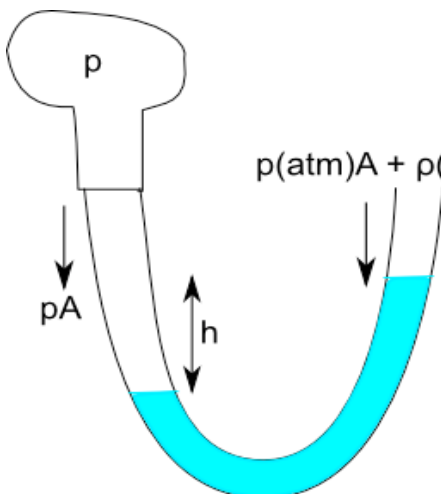
From the diagram, $p_{atm} A = \rho gh A$

So: $p_{atm} = \rho gh$

also, 1mm Hg = 1 torr

Open-tube Manometer:

Used to measure the pressure inside an object, such as a balloon. One side of the manometer is connected to the object whilst the other side is left open to the air.



From the diagram:

$$pA = p_{atm} A + \rho_f gh A$$

So:

$$p = p_{atm} + \rho_f gh$$

So gauge pressure:

$$p_g = \rho_f gh$$

Where h is the difference in height between the columns.

Buoyancy

Archimedes' principle: When a fluid is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.

$$F_{buoyant} = \rho_f gV$$

$$F_{net} = (\rho_s - \rho_f) gV$$

Surface Tension

Surface tension is the force of contraction acting across a line of unit length on the surface. The line and the force are perpendicular to each other and both are in the plane of the liquid surface.

$$\Upsilon = \frac{\text{Force}}{\text{Length}} = \frac{F}{l} \quad \text{Units Nm}^{-1}$$

Surface tension usually decreases as temperature increases.

Surface Free Energy

$$\Upsilon = \frac{W}{\Delta A} \quad \text{Therefore, the surface tension is the work done to create a unit surface area}$$

Pressure inside a bubble

Force due to surface tension:

$$2\Upsilon * \text{Circumference} = 2\Upsilon (2\pi R)$$

Excess force due to air pressure difference:

$$(p - p_{atm})\pi R^2$$

Therefore at equilibrium:

$$p - p_{atm} = \frac{(4\Upsilon)}{R}$$

For a liquid drop (only one surface not two):

$$p - p_{atm} = \frac{(2\Upsilon)}{R}$$

Capillarity

$$\Upsilon = \frac{\text{Force}}{\text{Length}} = \frac{F}{l} \quad \text{and} \quad \Upsilon = \frac{\text{Work}}{\text{Area}} = \frac{W}{\Delta A}$$

$$h = \frac{(2\Upsilon \cos(\theta))}{(\rho_{liq} g R)} \quad \text{but for water/glass/air } \theta \approx 0 \text{ so } h = \frac{2\Upsilon}{\rho g R}$$

Fluids under negative pressure will **pull inwards** on their container, negative pressure is often found at the tops of tall trees.

Interatomic Interactions

There are different types of potential equations, asymmetric potentials and harmonic/symmetric potentials. Harmonic potentials can be represented by the quantum oscillator equations, however fail to explain some properties such as thermal expansion.

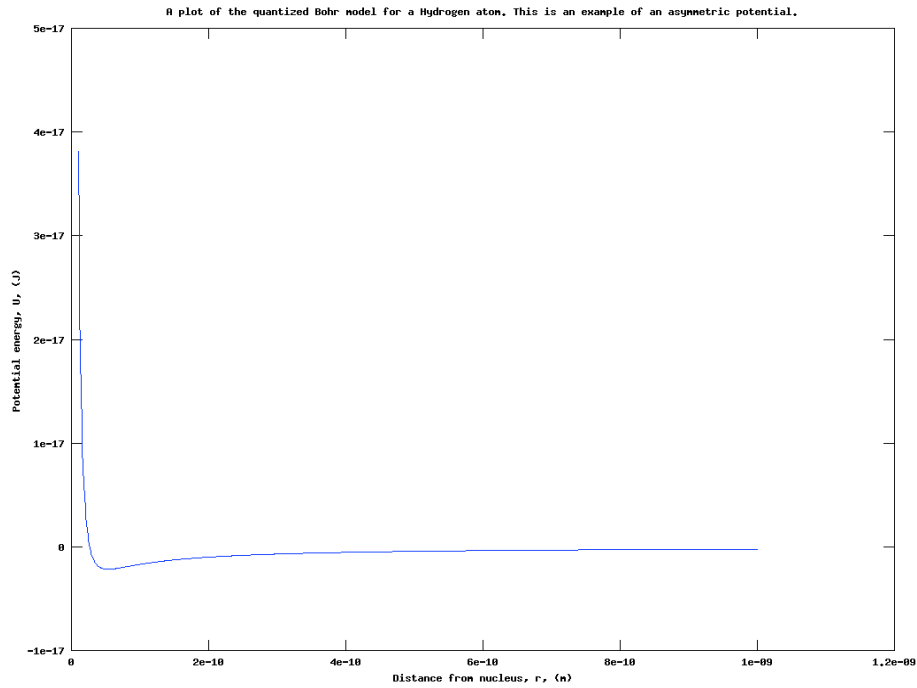


Illustration 1: An asymmetric potential fitted to the Bohr model of the Hydrogen atom.

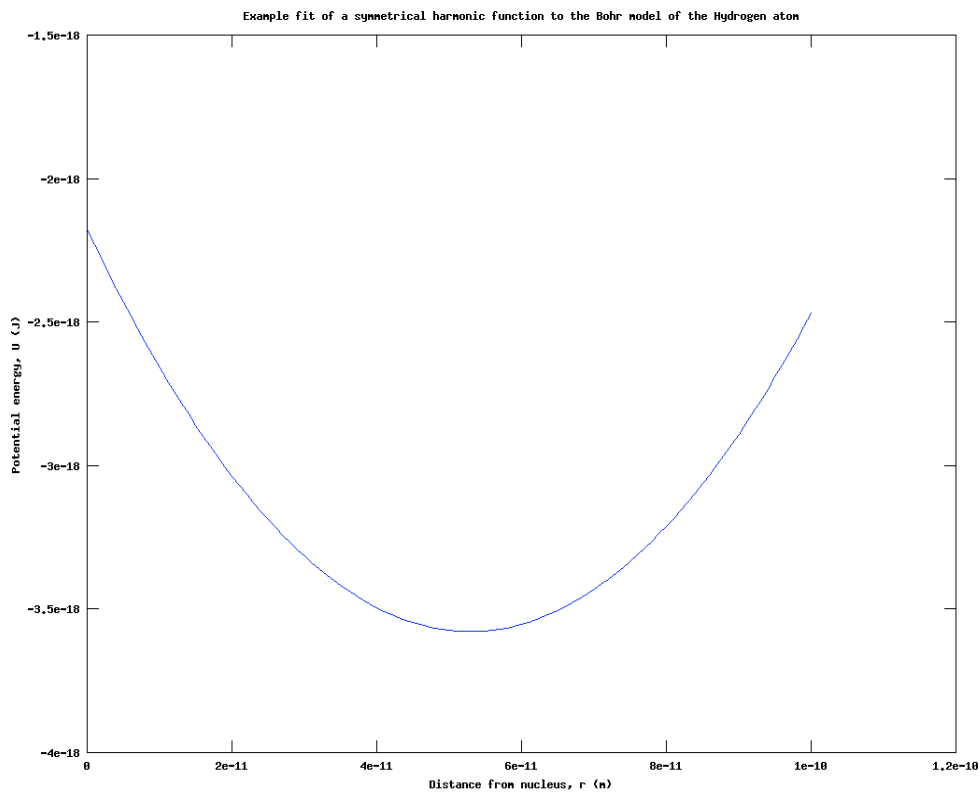


Illustration 2: A harmonic, symmetric fit to the Bohr model of the Hydrogen atom.

$$\mathbf{F} = -\nabla U$$

Lennard-Jones potential: $U(r) = \frac{-A}{r^6} + \frac{B}{r^{12}}$

This can be written as: $U(r) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right]$

where $\epsilon = \frac{A^2}{4B}$ [energy] and $\sigma = \left(\frac{B}{A} \right)^{\frac{1}{6}}$ [length]

Equilibrium interatomic distance: $r_0 = 2^{\frac{1}{6}} \sigma = 1.1225\sigma$

Equilibrium potential energy: $U_{min} = U(r_0) = -\epsilon$

Interatomic Force: $F = \frac{24\epsilon}{\sigma} \left[2 \left(\frac{\sigma}{r} \right)^{13} - \left(\frac{\sigma}{r} \right)^7 \right]$

Born-Meyer Potential (ionic lattices): $U(r) = \frac{-e^2}{4\pi\epsilon_0 r} + \frac{B}{r^{12}}$ where B is a constant like that in the Lennard-Jones Potential.

The depth of the potential energy well determines characteristics such as 'binding energy' latent heat, surface tension and characteristic temperature. The interatomic equilibrium distance determines the density at low temperature. The slope of the repulsive part of the curve determines the collision cross-section.

Emergence of Bulk Modulus: $B = \frac{C}{9Nr_0} \frac{d^2 U}{dr^2} \bigg|_{r=r_0}$ where N is the no. of atoms and C is a constant

Therefore for Lennard-Jones: $B = \frac{2^{5/2} C \epsilon}{N \sigma^3}$

Atomic vibrations:

$F = -Ax$ so atom undergoes SHM with freq $\omega = \sqrt{\frac{A}{m}}$ so $f = \frac{1}{2\pi} \sqrt{\frac{A}{m}}$

where $A = \frac{d^2 U}{dr^2} \bigg|_{r=r_0}$

This is because the harmonic part approximation is the quantum harmonic oscillator which has a potential:

$U(x) = \frac{1}{2} m \omega^2 x^2$ where m is the mass, ω the angular frequency and x the distance from the

centre of the well. Therefore $m\omega^2 = k$ and so $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$ as above. For diatomic molecules the reduced mass must be used in place of the mass, this is described below. The energy levels of the quantum harmonic oscillator are given by $E_n = (n + \frac{1}{2}) \hbar \omega$ which you will recognise as the vibrational energy levels, this is because the atomic vibrations are described by the harmonic part of the potential well.

Speed of sound in fluid: $v = \sqrt{\frac{B}{\rho}}$

Speed of longitudinal wave in solid rod: $v_L = \sqrt{\frac{Y}{\rho}}$

Speed of transverse wave on a string with tension F and mass per unit length μ :

$$v_T = \sqrt{\frac{F}{\mu}}$$

Thermal expansion can only be explained by the asymmetric properties of the L-J potential and thus it can not be explained by the parabolic approximation.

The harmonic part can explain compressibility, sound waves and vibrational frequencies by the anharmonic part is needed to explain thermal expansion and final thermal conductivity.

Thermal Expansion

Linear expansion: $L = L_0 + \alpha L_0 \Delta T$ where $\alpha = \frac{\Delta L}{L_0 \Delta T}$

Coefficient of volume expansion: $V = V_0 + \beta V_0 \Delta T$ where $\beta = \frac{\Delta V}{V_0 \Delta T}$

It can be shown that $\beta = 3\alpha$

Thermal Stress

$$\frac{F}{A} = Y \alpha \Delta T \quad \text{Thermal stress (tensile, compressive)}$$

$$\Delta P = B \beta \Delta T \quad \text{Volume stress}$$

Molecular spectra

Molecular bonds:

- 1) Ionic bond – transfer of electrons of one atom to another. Strongest bonding.
- 2) Covalent bond – sharing of electrons/directional bond. Weaker than Ionic bonding.
- 3) Van der Waal's bonds – interaction between electric dipole moments of atoms and molecules. Weakest bonding.
- 4) Hydrogen bond – caused by polarisation due to H^+ . Stronger than Van der Waal's but weaker than ionic and covalent bonds.

Rotational energy levels:

$$E = \frac{1}{2} I \omega^2 \quad \text{and} \quad L = I \omega$$

$$E = \frac{L^2}{2I} = \frac{l(l+1)\hbar^2}{2I} \quad l=0,1,2,\dots$$

$$\text{where } I = m_r r_0^2 \quad \text{so} \quad E = \frac{l(l+1)\hbar^2}{2I} = \frac{l(l+1)\hbar^2}{2m_r r_0^2}$$

Vibrational energy levels:

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega = \left(n + \frac{1}{2}\right) \hbar \sqrt{\frac{A}{m_r}} \quad n=0,1,2,\dots$$

$$m_r = \frac{m_1 m_2}{m_1 + m_2}$$

Structure of Crystals

A lattice is a regular arrangement of points in space so that all the points have identical surroundings.

A Primitive Cell is the smallest cell which, when repeated endlessly throughout space, will reproduce the lattice.

NOTE: Lattice points are NOT atoms, they are simply a set of points in space.

There are 7 crystal systems (cubic, tetragonal, hexagonal, orthorhombic, rhombohedral, monoclinic and triclinic) and 14 Bravais lattices (these are derived from the above crystal systems by considering their face centred and body centred etc. varieties in the cases where such varieties may exist.)

The cubic system may be a simple cubic, which only has lattice points at the eight vertices, a face centred cubic which has lattice points at the centre of each face, as well as at the vertices, or a body centred cubic which has a point in the centre of the cube as well as a point at each vertex. The distance between adjacent lattice points is a , $\frac{a}{\sqrt{2}}$, $\frac{\sqrt{3}}{2}a$ respectively.

In a crystal structure, each lattice point is associated with an atom or a group of atoms, such an atom or group is called the **basis**. A complete description of the crystal structure includes both the lattice and the basis.

For example, NaCl is Face-Centred-Cubic with a basis of two ions, Na⁺ at (0,0,0) and Cl⁻ at (0.5a,0.5a,0.5a) where a is the separation between lattice points (I.e distance between Na⁺ in this case), it has a co-ordination number of 6 as each ion interacts with 6 other ions.

Metallic crystals are held together by delocalised valence electrons which are free to move over the crystal. Metallic crystals exhibit close packing, they are often Face Centred Cubic or Hexagonal Close Packed with a co-ordination number of 12.

Ionic crystals are held together by electrostatic (Coulomb's law) forces. These forces are not directional hence the arrangement of the ions is partly determined by the relative sizes of the two ions.

Stable crystals always have a lower total energy than their separated constituents.

Covalent solids are formed from covalent crystals, these bonds are strongly directional because of the asymmetrical electron distributions. Molecular solids are held together by Van Der Waals forces arising from (induced) dipoles.

Hydrogen bonds form between the OH⁻ and H⁺ parts of water molecules, these lead to the ordered crystalline structure of ice and its important property of being less dense than water.

Liquid Crystals

Liquid crystals are unlike regular liquids in that they exhibit long-range order, though less than solids. Nematic has preferred direction but still exhibits positional disorder. Smectic A has same orientation as Nematic but has positional ordering. Smectic C also has positional ordering but a different orientation. Isotropic has no positional or directional ordering.