

Import statement

```
1 from math import pi
2 tau = 2 * pi
```

Assignment statement

Code (left):
Statements and expressions
Red arrow points to next line.
Gray arrow points to the line just executed

Frames (right):
A name is bound to a value
In a frame, there is at most one binding per name

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(-2)
```

Built-in function
func mul(...) [parent=Global]
func square(x) [parent=Global]

User-defined function
f1: square [parent=Global]
Formal parameter bound to argument: x = -2
Return value: 4
Return value is not a binding!

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(square(3))
```

A name evaluates to the value bound to that name in the earliest frame of the current environment in which that name is found.

Evaluation rule for call expressions:

1. Evaluate the operator and operand subexpressions.
2. Apply the function that is the value of the operator subexpression to the arguments that are the values of the operand subexpressions.

Applying user-defined functions:

1. Create a new local frame with the same parent as the function that was applied.
2. Bind the arguments to the function's formal parameter names in that frame.
3. Execute the body of the function in the environment beginning at that frame.

Execution rule for def statements:

1. Create a new function value with the specified name, formal parameters, and function body.
2. Its parent is the first frame of the current environment.
3. Bind the name of the function to the function value in the first frame of the current environment.

Execution rule for assignment statements:

1. Evaluate the expression(s) on the right of the equal sign.
2. Simultaneously bind the names on the left to those values, in the first frame of the current environment.

Execution rule for conditional statements:

- Each clause is considered in order.
1. Evaluate the header's expression.
 2. If it is a true value, execute the suite, then skip the remaining clauses in the statement.

Evaluation rule for or expressions:

1. Evaluate the subexpression <left>.
2. If the result is a true value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for and expressions:

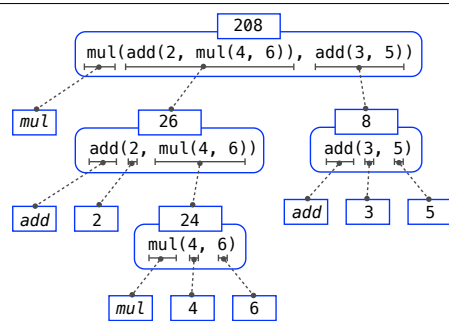
1. Evaluate the subexpression <left>.
2. If the result is a false value v, then the expression evaluates to v.
3. Otherwise, the expression evaluates to the value of the subexpression <right>.

Evaluation rule for not expressions:

1. Evaluate <exp>; The value is True if the result is a false value, and False otherwise.

Execution rule for while statements:

1. Evaluate the header's expression.
2. If it is a true value, execute the (whole) suite, then return to step 1.



Defining:
Formal parameter: x
Return expression: mul(x, x)
Body (return statement): return mul(x, x)
Def statement: >>> def square(x):
Call expression: square(2+2)
operator: square
function: func square(x)
operand: 2+2
argument: 4

Calling/Applying:
Argument: 4
Intrinsic name: square
Return value: 16

```
1 def f(x, y):
2     return g(x)
3
4 def g(a):
5     return a + y
6
7 result = f(1, 2)
```

Error: "y" is not found

Global frame:
f: f [parent=Global]
g: g [parent=Global]
x: 1
y: 2

Local frames:
f1: f [parent=Global]
f2: g [parent=Global]
a: 1

- An environment is a sequence of frames
- An environment for a non-nested function (no def within def) consists of one local frame, followed by the global frame

```
1 from operator import mul
2 def square(x):
3     return mul(x, x)
4 square(4)
```

Global frame:
mul: mul
square: square
f1: square [parent=Global]
square: 4
Return value: 16

A call expression and the body of the function being called are evaluated in different environments

```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1 # Zeroth and first Fibonacci numbers
    k = 1 # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (not called term)
A formal parameter that will be bound to a function
The cube function is passed as an argument value
The function bound to term gets called here

0 + 1³ + 2³ + 3³ + 4³ + 5³

Pure Functions

```
-2 > abs(number): 2
2, 10 > pow(x, y): 1024
```

Non-Pure Functions

```
-2 > print(...): None
display "-2"
```

Compound statement
Clause
Suite
Header: <statement>
Suite: <statement>
Separating header: <statement>
Suite: <statement>
...

```
def abs_value(x):
    1 statement,
    3 clauses,
    3 headers,
    3 suites,
    2 boolean
    contexts
    if x > 0:
        return x
    elif x == 0:
        return 0
    else:
        return -x
```

Higher-order function: A function that takes a function as an argument value or returns a function as a return value

Nested def statements: Functions defined within other function bodies are bound to names in the local frame

```
square = lambda x,y: x * y
```

A function

with formal parameters x and y
that returns the value of " $x * y$ "

Evaluates to a function.
No "return" keyword!

Must be a single expression

```
def make_adder(n):
```

A function that returns a function

Return a function that takes one argument k and returns $k + n$.

```
>>> add_three = make_adder(3)
```

```
>>> add_three(4)
```

The name `add_three` is
bound to a function

```
7 def adder(k):
```

A local def statement

```
    return k + n
```

```
    return adder
```

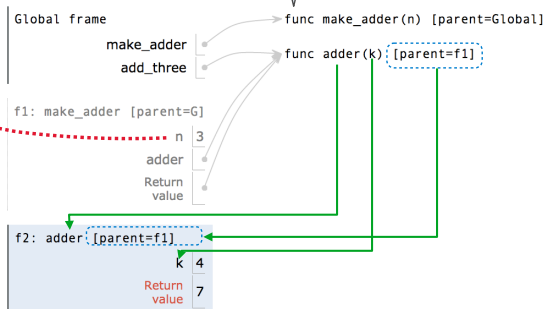
Can refer to names in
the enclosing function

- Every user-defined function has a **parent frame** (often global)
- The parent of a function is the frame in which it was **defined**
- Every local frame has a **parent frame** (often global)
- The parent of a frame is the parent of the function **called**

A function's signature
has all the information
to create a local frame

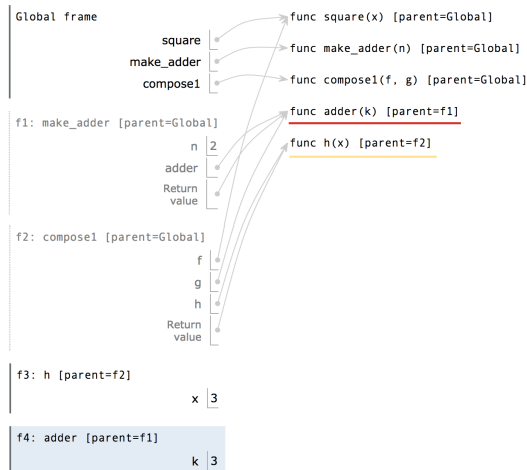
```
1 def make_adder(n):
2   def adder(k):
3     return k + n
4   return adder
5
6 add_three = make_adder(3)
7 add_three(4)
```

Nested def



```
1 def square(x):
2   return x * x
3
4 def make_adder(n):
5   def adder(k):
6     return k + n
7   return adder
8
9 def compose1(f, g):
10  def h(x):
11    return f(g(x))
12  return h
13
14 compose1(square, make_adder(2))(3)
```

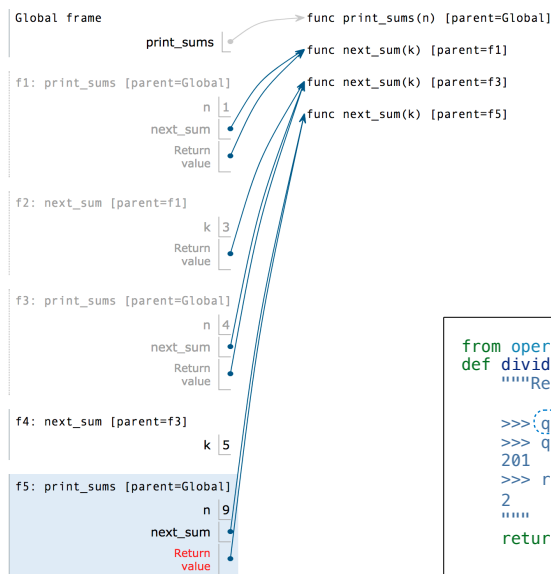
Return value of `make_adder` is an
argument to `compose1`



```
1 def print_sums(n):
2   print(n)
3   def next_sum(k):
4     return print_sums(n+k)
5   return next_sum
6
7 print_sums(1)(3)(5)
```

Printed output:

```
1
4
9
```



square = lambda x: x * x

VS

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both functions have as their parent the environment in which they were defined.
- Both bind that function to the name `square`.
- Only the `def` statement gives the function an intrinsic name.

When a function is defined:

1. Create a **function value**: `func <name>(<formal parameters>)`
2. Its parent is the current frame.

```
f1: make_adder      func adder(k) [parent=f1]
```

3. Bind **<name>** to the **function value** in the current frame (which is the first frame of the current environment).

When a function is called:

1. Add a **local frame**, titled with the **<name>** of the function being called.
2. Copy the parent of the function to the **local frame**: `[parent=<label>]`
3. Bind the **<formal parameters>** to the arguments in the **local frame**.
4. Execute the body of the function in the environment that starts with the **local frame**.

```
>>> min(2, 1, 4, 3)    >>> 2 + 3
1                      5
>>> max(2, 1, 4, 3)   >>> 2 * 3
4                      6
>>> abs(-2)           >>> 2 ** 3
2                      8
>>> pow(2, 3)         >>> 5 / 3
8                      1.6666666666666667
>>> len('word')       >>> 5 // 3
4                      1
>>> round(1.75)       >>> 5 % 3
2                      2
>>> print(1, 2)       1 2
```

```
def search(f):
    """Return the smallest non-negative
    integer x for which f(x) is a true value.
    """
```

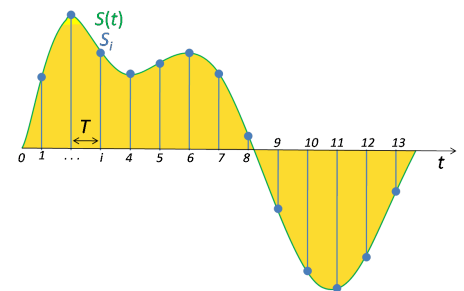
```
    x = 0
    while True:
        if f(x):
            return x
        x += 1
```

```
def is_three(x):
    """Return whether x is three.
    """
```

```
>>> search(is_three)
3
"""
return x == 3
```

```
def inverse(f):
    """Return a function g(y) that returns
    x such that f(x) == y.
    """
```

```
>>> sqrt = inverse(lambda x: x * x)
>>> sqrt(16)
4
"""
return lambda y: search(lambda x: f(x)==y)
```



```
from operator import floordiv, mod
def divide_exact(n, d):
    """Return the quotient and remainder of dividing N by D.
    """
```

```
>>> q, r = divide_exact(2012, 10)
```

```
>>> q
```

```
>>> r
```

```
2
"""
```

```
return floordiv(n, d), mod(n, d)
```

Multiple assignment
to two names

Two return values,
separated by commas