Finite volume method for a scalar transport

In this exercise we want to get practical experience with the finite volume method. We are going to study the transport of a scalar quantity, T, in a known velocity field. In the absence of source terms for the scalar the steady-state equation reduces to

$$\int_S \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{S} = 0,$$

where the flux of the scalar quantity is a combination of convection and diffusion

$$\boldsymbol{F} = T\boldsymbol{u} - \kappa \nabla T.$$

In the previous expression κ is the diffusivity coefficient and $\boldsymbol{u}$ is the velocity, which is given by $\boldsymbol{u} = (u_x, u_y) = (x, -y)$. The 2D computational domain is $x \in [0, 1]$ and $y \in [0, 1]$. The boundary conditions are

$$\begin{aligned} T &= 1 - y \quad \text{at } x = 0 \text{ (west)}, \\ \frac{\partial T}{\partial x} &= 0 \quad \text{at } x = 1 \text{ (east)}, \\ \frac{\partial T}{\partial y} &= 0 \quad \text{at } y = 0 \text{ (south)}, \\ T &= 0 \quad \text{at } y = 1 \text{ (north)}. \end{aligned}$$

1. Evaluate both the convective fluxes and the diffusive fluxes with a central method. Compute the solution with $\kappa = 0.01$ and with $\kappa = 0.001$.

2. Evaluate now the convective fluxes with an upwind method and the diffusive fluxes with a central method, compare your results to the previous case.