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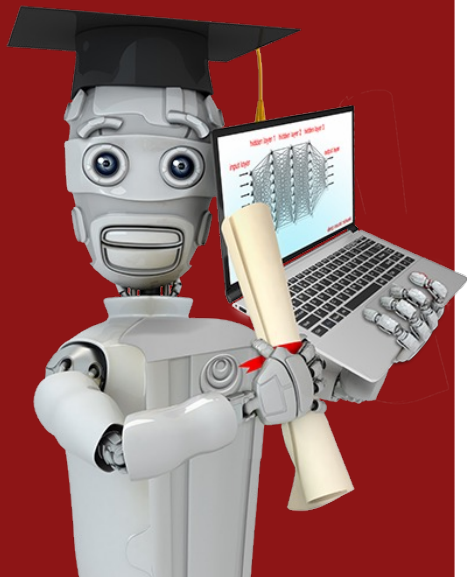
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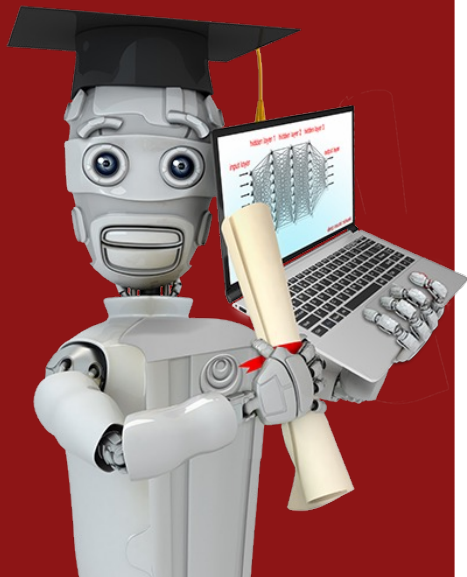
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Recommender Systems



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Recommender System

Making recommendations

Predicting movie ratings

User rates movies using one to five stars

Movie	Alice(1)	Bob(2)	Carol(3)	Dave(4)
Love at last	5	5	0	0
Romance forever	5	?	?	0
Cute puppies of love	?	4	0	?
Nonstop car chases	0	0	5	4
Swords vs. karate	0	0	5	?

Ratings				
★				
★	★			
★	★	★		
★	★	★	★	
★	★	★	★	★

n_u = no. of users
 n_m = no. of movies
 $r(i,j)=1$ if user j has rated movie i

$$n_u = 4$$

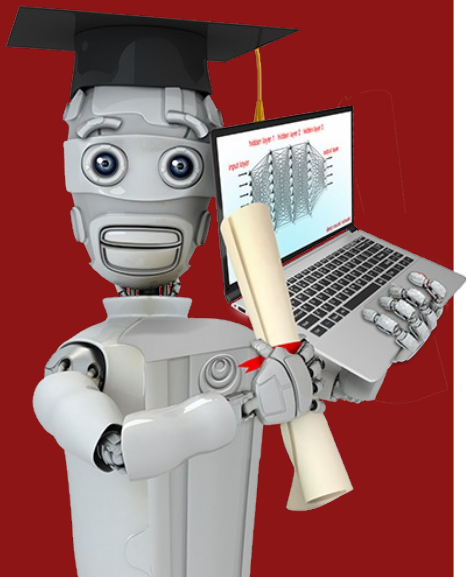
$$r(1,1) = 1$$

$$n_m = 5$$

$$r(3,1) = 0$$

$$y^{(3,2)} = 4$$

$y^{(i,j)}$ = rating given by user j to movie i
 (defined only if $r(i,j)=1$)



Collaborative Filtering

Using per-item features

What if we have features of the movies?

$$\begin{aligned} n_u &= 4 \\ n_m &= 5 \\ n &= 2 \end{aligned}$$

Movie	Alice(1)	Bob(2)	Carol(3)	Dave(4)	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

$$x^{(1)} = \begin{bmatrix} 0.9 \\ 0 \end{bmatrix}$$

$$x^{(3)} = \begin{bmatrix} 0.99 \\ 0 \end{bmatrix}$$

For user 1: Predict rating for movie i as: $w^{(1)} \cdot x^{(i)} + b^{(1)}$

just linear regression

$$w^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} \quad b^{(1)} = 0 \quad x^{(3)} = \begin{bmatrix} 0.99 \\ 0 \end{bmatrix}$$

$$w^{(1)} \cdot x^{(3)} + b^{(1)} = 4.95$$

→ For user j : Predict user j 's rating for movie i as

$$w^{(j)} \cdot x^{(i)} + b^{(j)}$$

Cost function

Notation:

- $r(i,j) = 1$ if user j has rated movie i (0 otherwise)
- $y^{(i,j)}$ = rating given by user j on movie i (if defined)
- $w^{(j)}, b^{(j)}$ = parameters for user j
- $x^{(i)}$ = feature vector for movie i

For user j and movie i , predict rating: $w^{(j)} \cdot x^{(i)} + b^{(j)}$

- $m^{(j)}$ = no. of movies rated by user j

To learn $w^{(j)}, b^{(j)}$

$$\min_{w^{(j)}, b^{(j)}} J(w^{(j)}, b^{(j)}) = \frac{1}{2m^{(j)}} \sum_{i:r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2m^{(j)}} \sum_{k=1}^n (w_k^{(j)})^2$$

Handwritten annotations: Blue arrows point from $w^{(j)}$ and $x^{(i)}$ in the prediction formula to the corresponding terms in the cost function. A blue circle highlights the summation index $i:r(i,j)=1$. A blue arrow points from the text "number of features" to the summation index $k=1$ to n in the regularization term.

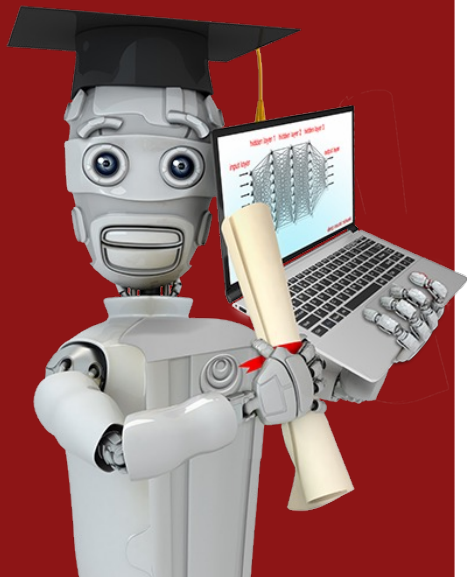
Cost function

To learn parameters $w^{(j)}, b^{(j)}$ for user j :

$$J(w^{(j)}, b^{(j)}) = \frac{1}{2} \sum_{i:r(i,j)} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^n (w_k^{(j)})^2$$

To learn parameters $w^{(1)}, b^{(1)}, w^{(2)}, b^{(2)}, \dots, w^{(n_u)}, b^{(n_u)}$ for all users :

$$J\left(\begin{matrix} w^{(1)}, & \dots, & w^{(n_u)} \\ b^{(1)}, & \dots, & b^{(n_u)} \end{matrix}\right) = \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} \underbrace{(w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2}_{f(x)} + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (w_k^{(j)})^2$$



Collaborative Filtering

Collaborative filtering algorithm

Problem motivation

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	0.9	0
Romance forever	5	?	?	0	1.0	0.01
Cute puppies of love	?	4	0	?	0.99	0
Nonstop car chases	0	0	5	4	0.1	1.0
Swords vs. karate	0	0	5	?	0	0.9

Problem motivation

Movie	Alice (1)	Bob (2)	Carol (3)	Dave (4)	x_1 (romance)	x_2 (action)
Love at last	5	5	0	0	?	?
Romance forever	5	?	?	0	?	?
Cute puppies of love	?	4	0	?	?	?
Nonstop car chases	0	0	5	4	?	?
Swords vs. karate	0	0	5	?	?	?

$$w^{(1)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, w^{(2)} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, w^{(3)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}, w^{(4)} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$

$$b^{(1)} = 0, b^{(2)} = 0, b^{(3)} = 0, b^{(4)} = 0$$

using $w^{(j)} \cdot x^{(i)} + b^{(j)}$

$$w^{(1)} \cdot x^{(1)} \approx 5$$

$$w^{(2)} \cdot x^{(1)} \approx 5$$

$$w^{(3)} \cdot x^{(1)} \approx 0$$

$$w^{(4)} \cdot x^{(1)} \approx 0$$

$$\rightarrow x^{(1)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Cost function

Given $w^{(1)}, b^{(1)}, w^{(2)}, b^{(2)}, \dots, w^{(n_u)}, b^{(n_u)}$

to learn $x^{(i)}$:

$$J(x^{(i)}) = \frac{1}{2} \sum_{j:r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{k=1}^n (x_k^{(i)})^2$$

→ To learn $x^{(1)}, x^{(2)}, \dots, x^{(n_m)}$:

$$J(x^{(1)}, x^{(2)}, \dots, x^{(n_m)}) = \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2$$

Collaborative filtering

Cost function to learn $w^{(1)}, b^{(1)}, \dots, w^{(n_u)}, b^{(n_u)}$:

$$\min_{w^{(1)}, b^{(1)}, \dots, w^{(n_u)}, b^{(n_u)}} \frac{1}{2} \sum_{j=1}^{n_u} \sum_{i:r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (w_k^{(j)})^2$$

→ $i = 1$
→ $i = 2$

	$j=1$	$j=2$	$j=3$
	Alice	Bob	Carol
Movie1	5	5	?
Movie2	?	2	3

Cost function to learn $x^{(1)}, \dots, x^{(n_m)}$:

$$\min_{x^{(1)}, \dots, x^{(n_m)}} \frac{1}{2} \sum_{i=1}^{n_m} \sum_{j:r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2$$

Put them together:

$$\min_{\substack{w^{(1)}, \dots, w^{(n_u)} \\ b^{(1)}, \dots, b^{(n_u)} \\ x^{(1)}, \dots, x^{(n_m)}}} J(w, b, x) = \frac{1}{2} \sum_{(i,j):r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (w_k^{(j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2$$

Gradient Descent

collaborative filtering

Linear regression (course 1)

repeat {

~~$$w_i = w_i - \alpha \frac{\partial}{\partial w_i} J(w, b)$$~~

~~$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$~~

$$w_i^{(j)} = w_i^{(j)} - \alpha \frac{\partial}{\partial w_i^{(j)}} J(w, b, x)$$

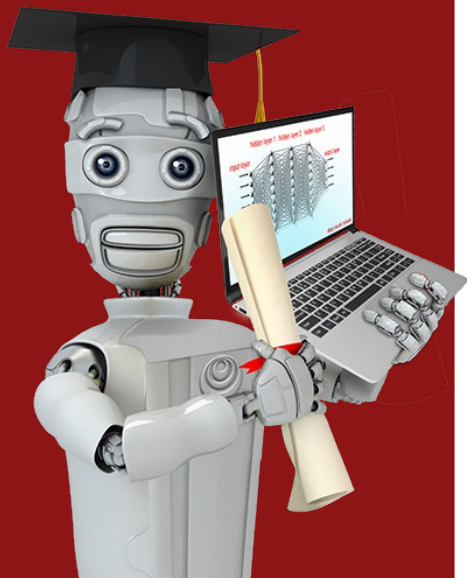
$$b^{(j)} = b^{(j)} - \alpha \frac{\partial}{\partial b^{(j)}} J(w, b, x)$$

$$x_k^{(i)} = x_k^{(i)} - \alpha \frac{\partial}{\partial x_k^{(i)}} J(w, b, x)$$

}

parameters w, b, x

x is also a parameter



Collaborative Filtering

Binary labels:
favs,
likes and clicks

Binary labels

Movie	Alice(1)	Bob(2)	Carol(3)	Dave(4)
Love at last	1	1	0	0
Romance forever	1	? ←	? ←	0
Cute puppies of love	? ←	1	0	? ←
Nonstop car chases	0	0	1	1
Swords vs. karate	0	0	1	? ←

1
0
?

Example applications

- 1. Did user j purchase an item after being shown? 1, 0, ?
- 2. Did user j fav/like an item? 1, 0, ?
- 3. Did user j spend at least 30sec with an item? 1, 0, ?
- 4. Did user j click on an item? 1, 0, ?

Meaning of ratings:

- 1 - engaged after being shown item
- 0 - did not engage after being shown item
- ? - item not yet shown

From regression to binary classification

- Previously:
 - Predict $y^{(i,j)}$ as $\underline{w^{(j)} \cdot x^{(i)} + b^{(j)}}$
 - For binary labels:
Predict that the probability of $y^{(i,j)} = 1$
is given by $\underline{g(w^{(j)} \cdot x^{(i)} + b^{(j)})}$
- where $\underline{g(z) = \frac{1}{1+e^{-z}}}$

Cost function for binary application

Previous cost function:

$$\frac{1}{2} \sum_{(i,j):r(i,j)=1} (\underbrace{w^{(j)} \cdot x^{(i)} + b^{(j)}}_{f(x)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (w_k^{(j)})^2$$

Loss for binary labels

$y^{(i,j)}$:

$$f_{(w,b,x)}(x) = g(w^{(j)} \cdot x^{(i)} + b^{(j)})$$

$$L(f_{(w,b,x)}(x), y^{(i,j)}) = -y^{(i,j)} \log(f_{(w,b,x)}(x)) - (1 - y^{(i,j)}) \log(1 - f_{(w,b,x)}(x))$$

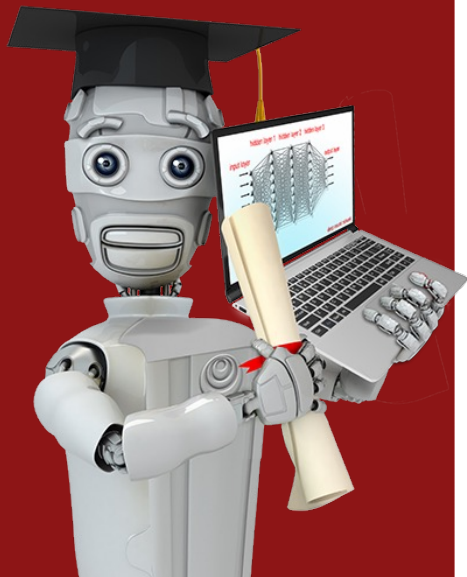
Loss for single example

$$J(w, b, x) = \sum_{(i,j):r(i,j)=1} L(\underbrace{f_{(w,b,x)}(x)}_{g(w^{(j)} \cdot x^{(i)} + b^{(j)})}, y^{(i,j)})$$

cost for all examples



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Recommender Systems implementation

Mean normalization

Users who have not rated any movies

Movie	Alice(1)	Bob (2)	Carol (3)	Dave (4)	Eve (5)
Love at last	5	5	0	0	?
Romance forever	5	?	?	0	?
Cute puppies of love	?	4	0	?	?
Nonstop car chases	0	0	5	4	?
Swords vs. karate	0	0	5	?	?

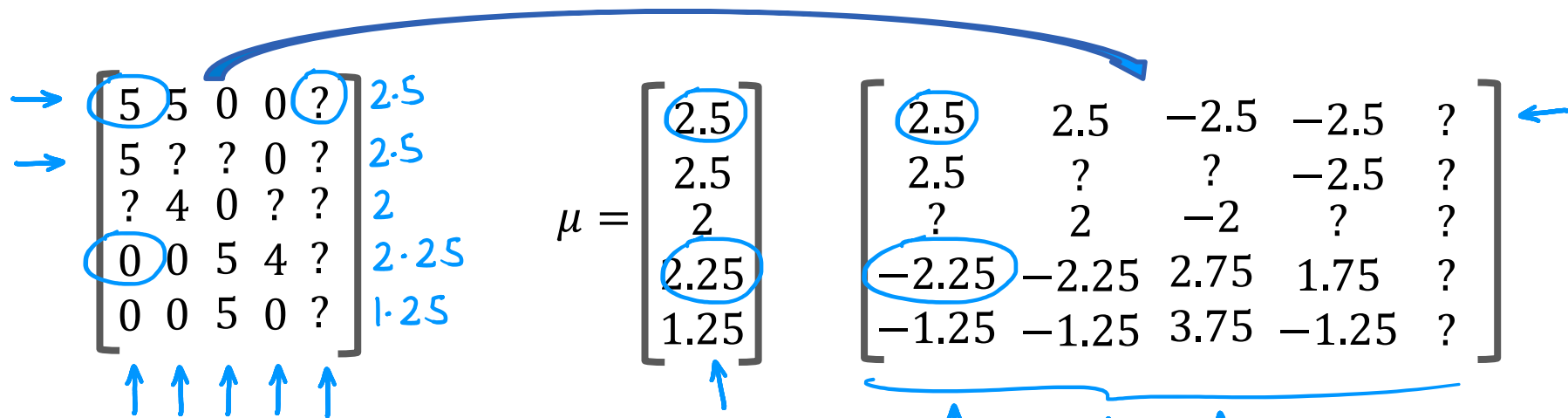
$$\begin{bmatrix} 5 & 5 & 0 & 0 & ? \\ 5 & ? & ? & 0 & ? \\ ? & 4 & 0 & ? & ? \\ 0 & 0 & 5 & 4 & ? \\ 0 & 0 & 5 & 0 & ? \end{bmatrix}$$



$\rightarrow \min_{\substack{w^{(1)}, \dots, w^{(n_u)} \\ b^{(1)}, \dots, b^{(n_u)} \\ x^{(1)}, \dots, x^{(n_m)}}} \frac{1}{2} \sum_{(i,j): r(i,j)=1} (w^{(j)} \cdot x^{(i)} + b^{(j)} - y^{(i,j)})^2 + \frac{\lambda}{2} \sum_{j=1}^{n_u} \sum_{k=1}^n (w_k^{(j)})^2 + \frac{\lambda}{2} \sum_{i=1}^{n_m} \sum_{k=1}^n (x_k^{(i)})^2$

$w^{(5)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad b^{(5)} = 0 \quad w^{(5)} \cdot x^{(i)} + b^{(5)}$

Mean Normalization



For user j , on movie i predict:

$$w^{(j)} \cdot x^{(i)} + b^{(j)} + \mu_i$$

$$y^{(i,j)} = w^{(j)} \cdot b^{(j)} \cdot x^{(i)}$$

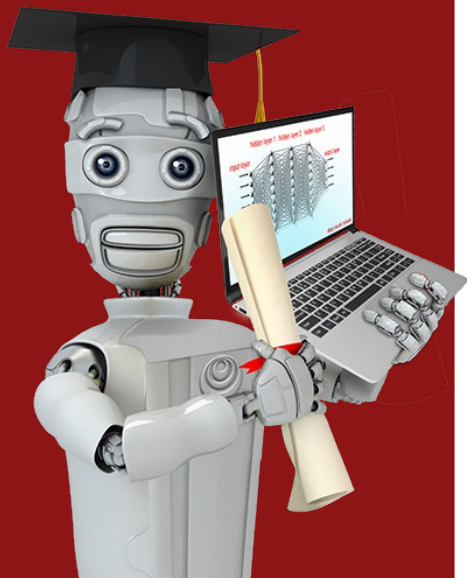
User 5 (Eve):

$$w^{(5)} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad b^{(5)} = 0$$

$$\underbrace{w^{(5)} \cdot x^{(1)} + b^{(5)}}_0 + \mu_1 = 2.5$$



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Recommender Systems implementational detail

TensorFlow implementation

Derivatives in ML

Gradient descent algorithm

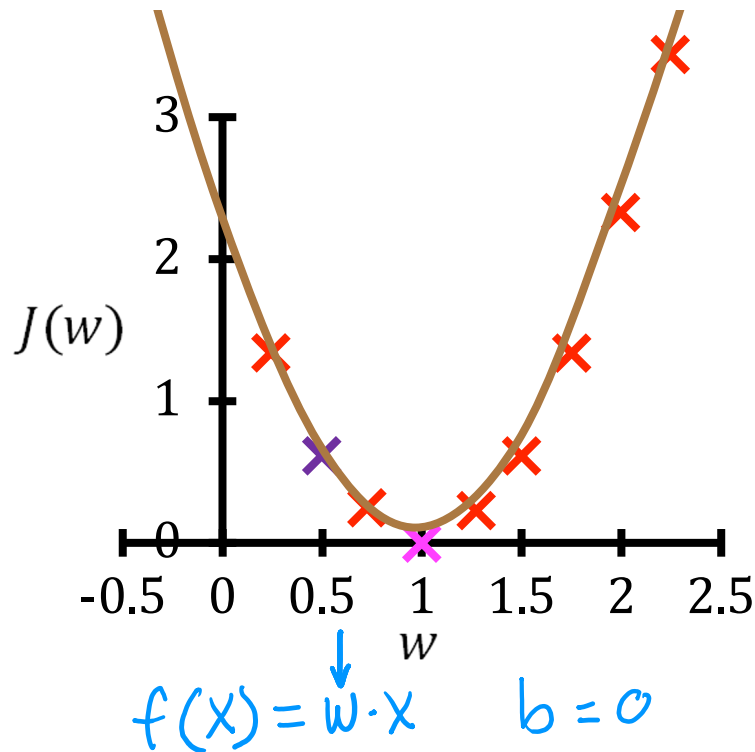
Repeat until convergence

$$\underline{w} = w - \alpha \frac{d}{dw} J(w, b)$$

Learning rate

Derivative

$$\underline{b} = b - \alpha \frac{d}{db} J(w, b) \quad \leftarrow \quad b = 0$$



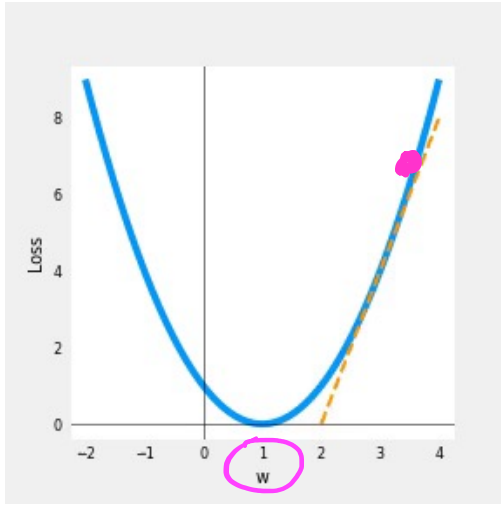
$$J = (\overbrace{wx}^{f(x)} - \overbrace{1}^y)^2$$

Gradient descent algorithm

Repeat until convergence

$$w = w - \alpha \frac{d}{dw} J(w, b)$$

Fix $b = 0$ for this example



Custom Training Loop

```
w = tf.Variable(3.0)
x = 1.0
y = 1.0 # target value
alpha = 0.01
```

Tf.variables are the parameters we want to optimize

Auto Diff

Auto Grad

```
iterations = 30
for iter in range(iterations):
    # Use TensorFlow's Gradient tape to record the steps
    # used to compute the cost J, to enable auto differentiation.
```

```
    with tf.GradientTape() as tape:
        fwb = w*x
        costJ = (fwb - y)**2
```

```
    # Use the gradient tape to calculate the gradients
    # of the cost with respect to the parameter w.
    [dJdw] = tape.gradient(costJ, [w])
```

```
    # Run one step of gradient descent by updating
    # the value of w to reduce the cost.
```

```
    w.assign_add(-alpha * dJdw)
```

tf.variables require special function to modify

$$\frac{\partial}{\partial w} J(w)$$

Implementation in TensorFlow

Gradient descent algorithm

Repeat until convergence

$$\begin{aligned} w &= w - \alpha \frac{\partial}{\partial w} J(w, b, X) \\ b &= b - \alpha \frac{\partial}{\partial b} J(w, b, X) \\ X &= X - \alpha \frac{\partial}{\partial X} J(w, b, X) \end{aligned}$$

```
# Instantiate an optimizer.
optimizer = keras.optimizers.Adam(learning_rate=1e-1)

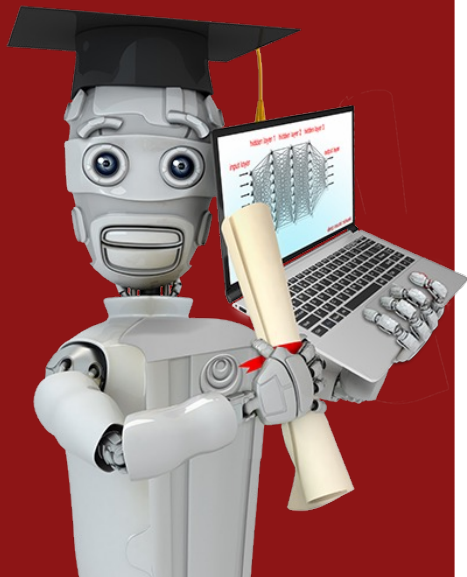
iterations = 200
for iter in range(iterations):
    # Use TensorFlow's GradientTape
    # to record the operations used to compute the cost
    # with tf.GradientTape() as tape:

        # Compute the cost (forward pass is included in cost)
        cost_value = cofiCostFuncV(X, W, b, Ynorm, R,
                                   num_users, num_movies, lambda)

        # Use the gradient tape to automatically retrieve
        # the gradients of the trainable variables with respect to
        # the loss
        grads = tape.gradient( cost_value, [X, W, b] )

        # Run one step of gradient descent by updating
        # the value of the variables to minimize the loss.
        optimizer.apply_gradients( zip(grads, [X, W, b]) )
```

Dataset credit: Harper and Konstan. 2015. The MovieLens Datasets: History and Context



Collaborative Filtering

Finding related items

Finding related items

The features $x^{(i)}$ of item i are quite hard to interpret.
To find other items related to it,
find item k with $x^{(k)}$ similar to $x^{(i)}$

i.e. with smallest
distance

$$\sum_{l=1}^n (x_l^{(k)} - x_l^{(i)})^2$$
$$\|x^{(k)} - x^{(i)}\|^2$$

romance
action
 x_1, x_2, x_3
 n

$x^{(k)}$ $x^{(i)}$

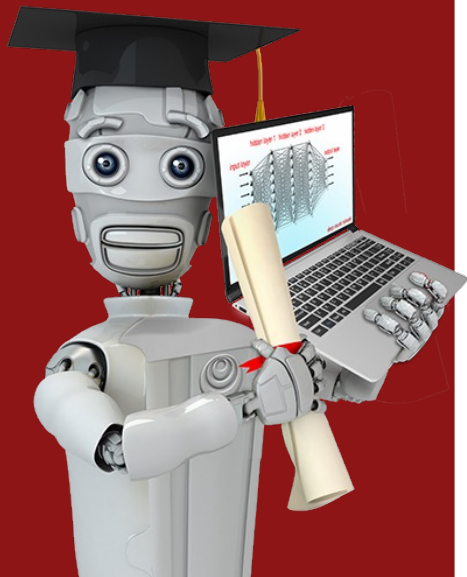
Limitations of Collaborative Filtering

→ Cold start problem. How to

- • rank new items that few users have rated?
- • show something reasonable to new users who have rated few items?

→ Use side information about items or users:

- • Item: Genre, movie stars, studio,
- • User: Demographics (age, gender, location), expressed preferences, ... }



Content-based Filtering

Collaborative filtering
vs
Content-based filtering

Collaborative filtering vs Content-based filtering

→ Collaborative filtering:

Recommend items to you based on rating of users who gave similar ratings as you

→ Content-based filtering:

Recommend items to you based on features of user and item to find good match

$r(i, j) = 1$ if user j has rated item i

$y(i, j)$ rating given by user j on item i (if defined)

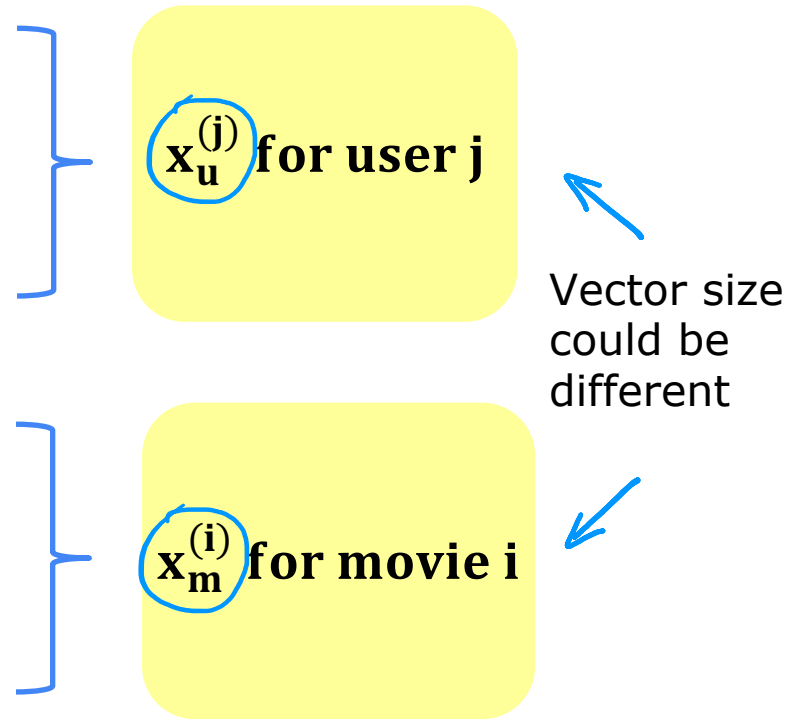
Examples of user and item features

User features:

- • Age
- • Gender (1 hot)
- • Country (1 hot, 200)
- • Movies watched (1000)
- • Average rating per genre
- ...

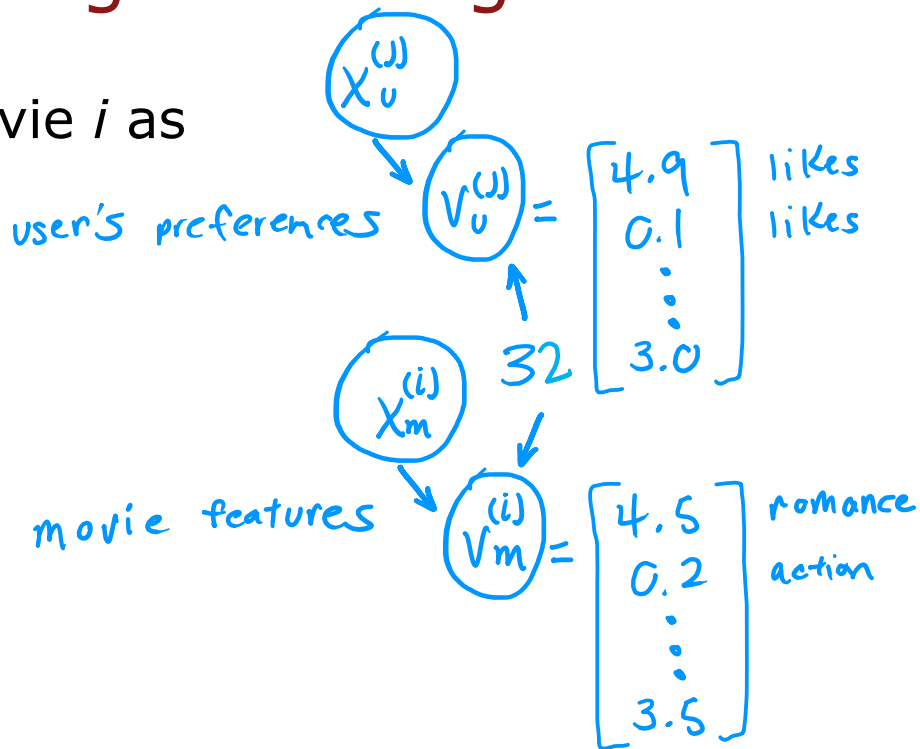
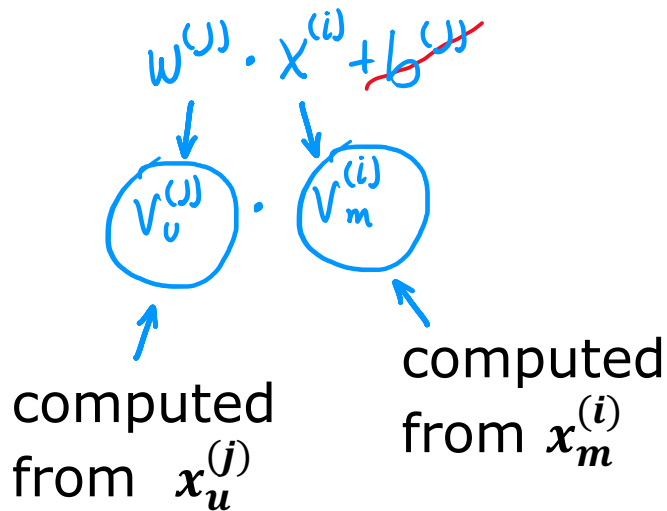
Movie features:

- • Year
- • Genre/Genres
- • Reviews
- • Average rating
- ...



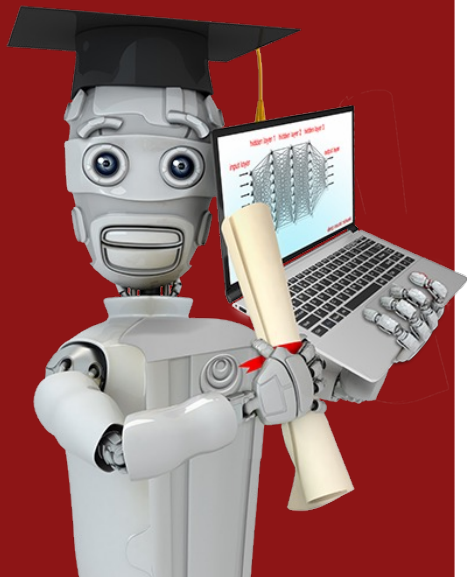
Content-based filtering: Learning to match

Predict rating of user j on movie i as





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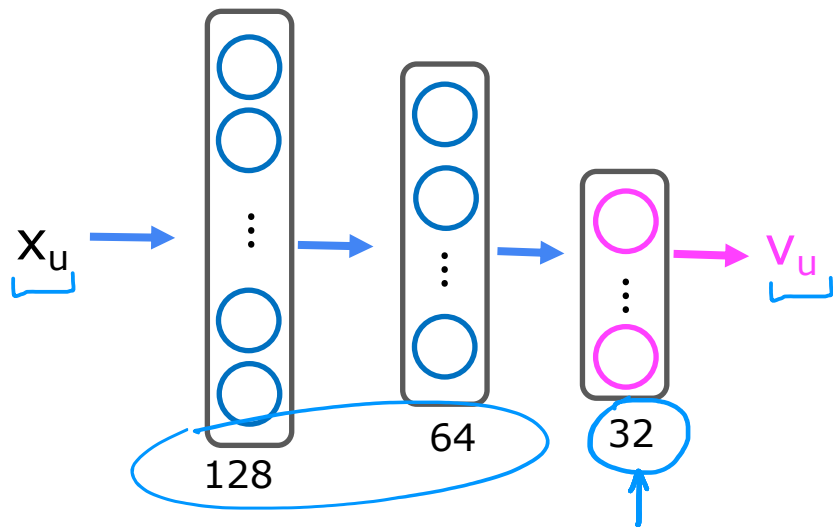


Content-based Filtering

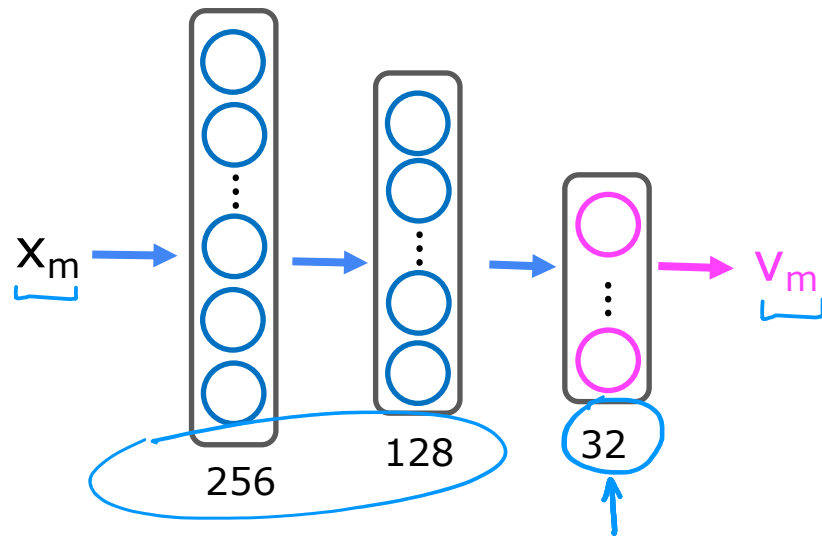
Deep learning for
content-based filtering

Neural network architecture

$X_u \rightarrow V_u$ User network

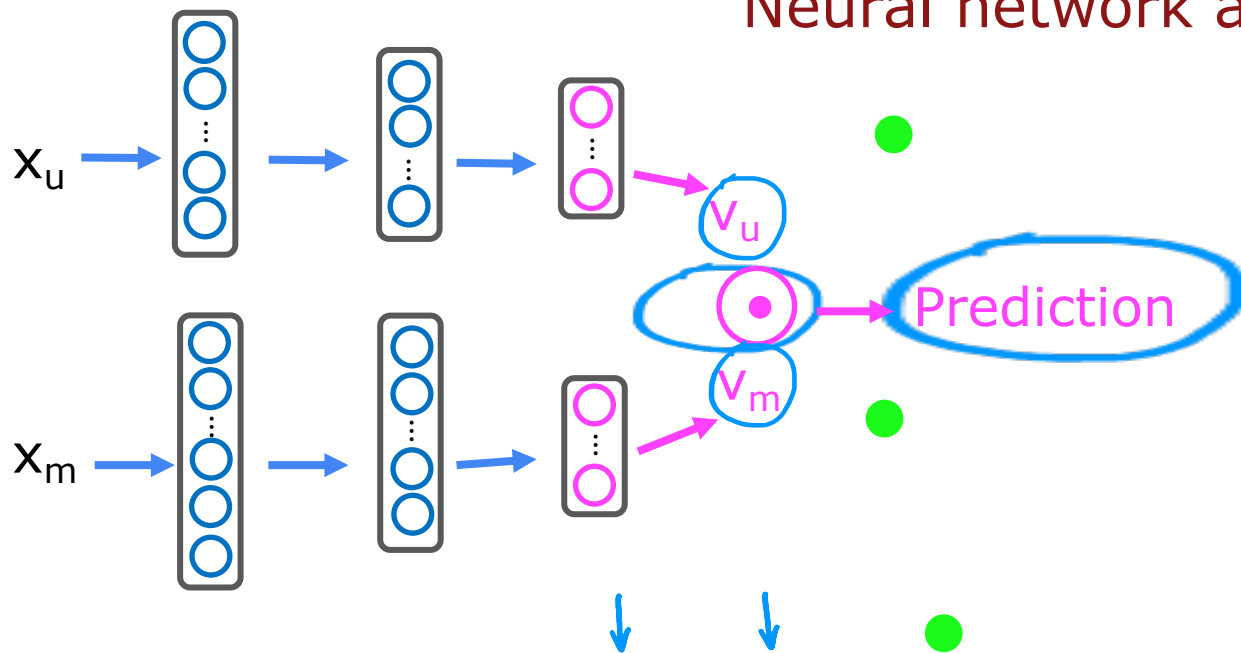


$X_m \rightarrow V_m$ Movie network



Prediction : $V_u^{(j)} \cdot V_m^{(i)}$
 $g(v_u^{(j)} \cdot v_m^{(i)})$ to predict the probability that $y^{(i,j)}$ is 1

Neural network architecture



Cost
function

$$J = \sum_{(i,j):r(i,j)=1} (v_u^{(j)} \cdot v_m^{(i)} - y^{(i,j)})^2 + \text{NN regularization term}$$

Learned user and item vectors:

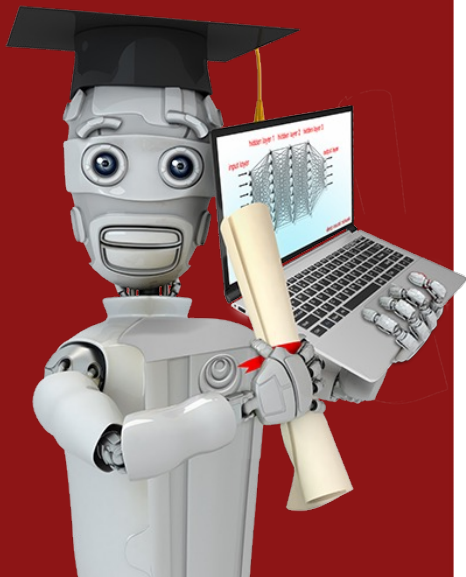
- $v_u^{(j)}$ is a vector of length 32 that describes user j with features $x_u^{(j)}$
- $v_m^{(i)}$ is a vector of length 32 that describes movie i with features $x_m^{(i)}$

To find movies similar to movie i :

$$\|v_m^{(k)} - v_m^{(i)}\|^2 \text{ small}$$

$$\|x^{(k)} - x^{(i)}\|^2$$

- Note: This can be pre-computed ahead of time

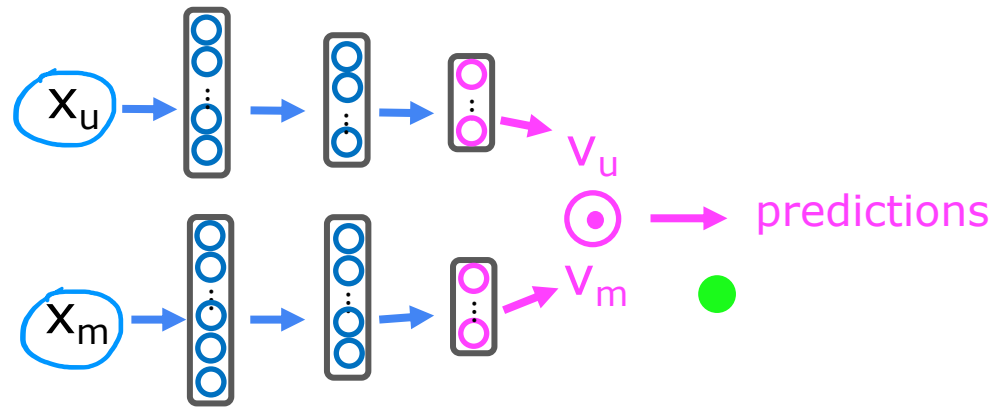


Advanced implementation

Recommending from a large catalogue

How to efficiently find recommendation from a large set of items?

- • Movies 1000+
- • Ads 1m+
- • Songs 10m+
- • Products 10m+



Two steps: Retrieval & Ranking

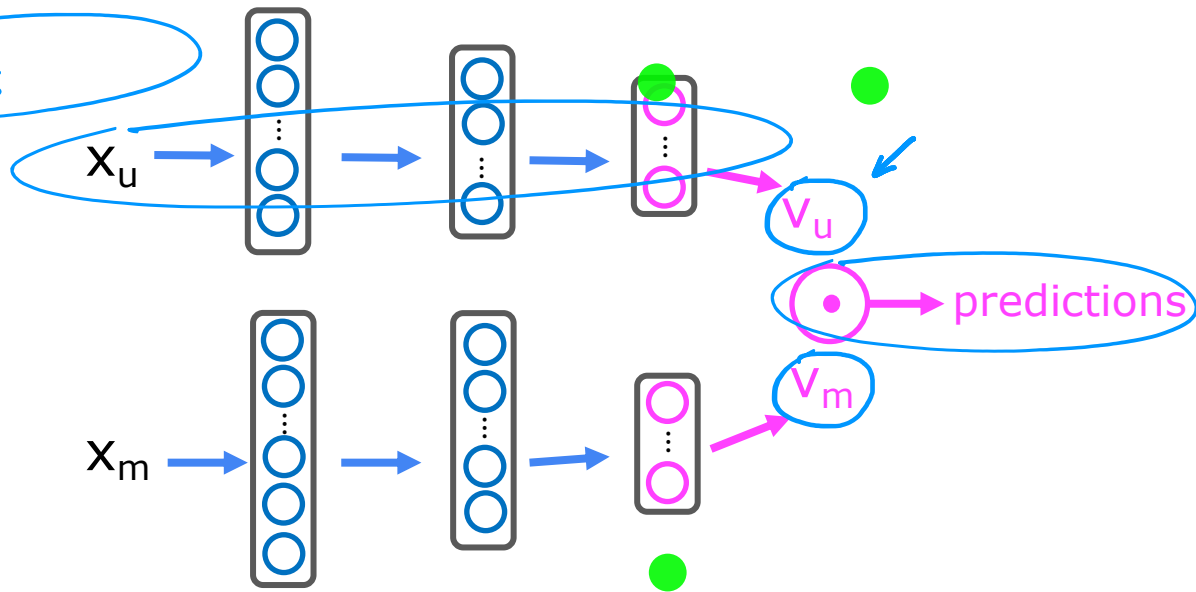
Retrieval:

- • Generate large list of plausible item candidates ~100s
 - e.g.
 - 1) For each of the last 10 movies watched by the user, find 10 most similar movies
$$\|v_m^{(k)} - v_m^{(i)}\|^2$$
 - 2) For most viewed 3 genres, find the top 10 movies
 - 3) Top 20 movies in the country
- • Combine retrieved items into list, removing duplicates and items already watched/purchased

Two steps: Retrieval & ranking

Ranking:

- Take list retrieved and rank using learned model



- Display ranked items to user

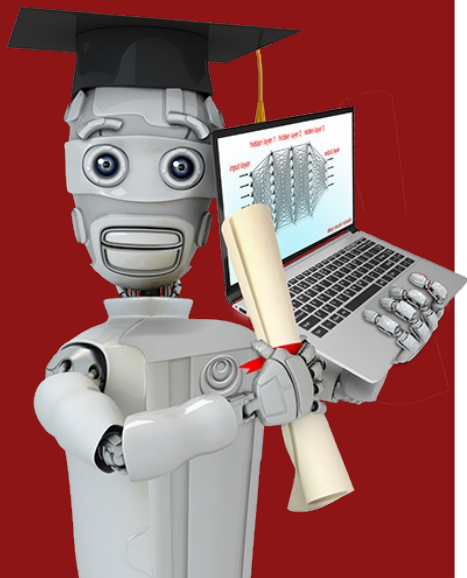
Retrieval step

- • Retrieving more items results in better performance, but slower recommendations.
- • To analyse/optimize the trade-off, carry out offline experiments to see if retrieving additional items results in more relevant recommendations (i.e., $p(y^{(i,j)}) = 1$ of items displayed to user are higher).

100 500



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Advanced implementation

**Ethical use of
recommender systems**

What is the goal of the recommender system?

Recommend:

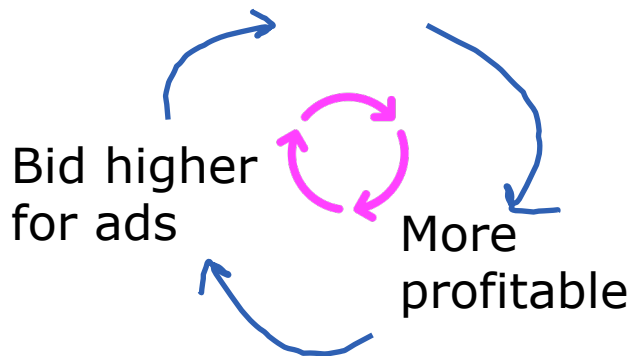
- • Movies most likely to be rated 5 stars by user
- • Products most likely to be purchased
- • Ads most likely to be clicked on *+high bid*
- • Products generating the largest profit
- • Video leading to maximum watch time



Ethical considerations with recommender systems

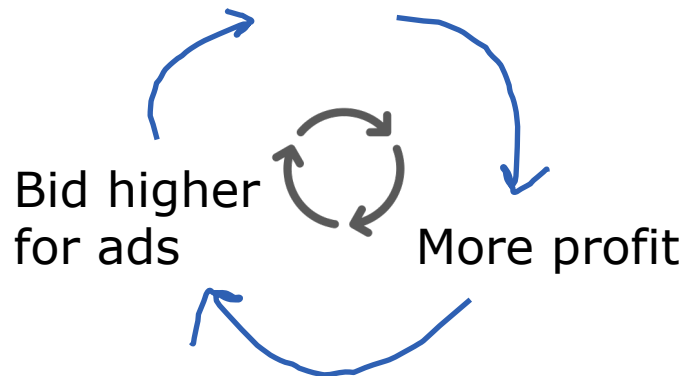
Travel industry

Good travel experience
to more users



Payday loans

Squeeze customers
more



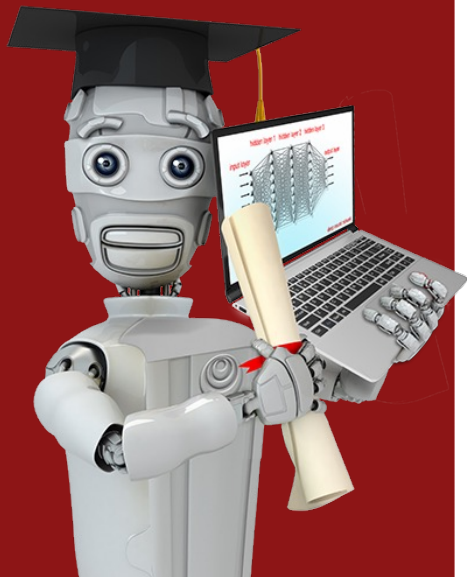
Amelioration: Do not accept ads from exploitative businesses

Other problematic cases:

- • Maximizing user engagement (e.g. watch time) has led to large social media/video sharing sites to amplify conspiracy theories and hate/toxicity
- Amelioration : Filter out problematic content such as hate speech, fraud, scams and violent content
- • Can a ranking system maximize your profit rather than users' welfare be presented in a transparent way?
- Amelioration : Be transparent with users

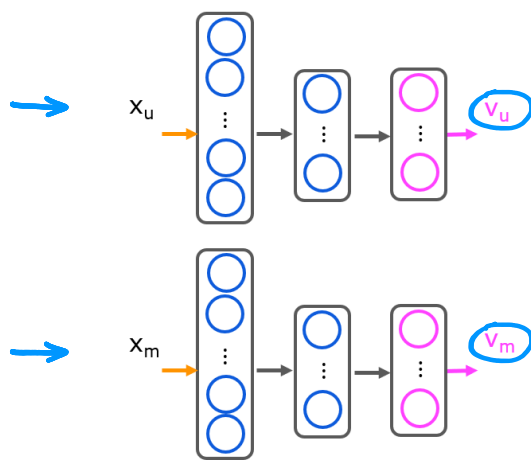


Stanford
ONLINE



Content-based Filtering

TensorFlow Implementation



```
user_NN = tf.keras.models.Sequential([
    tf.keras.layers.Dense(256, activation='relu'),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(32)
])
```

```
item_NN = tf.keras.models.Sequential([
    tf.keras.layers.Dense(256, activation='relu'),
    tf.keras.layers.Dense(128, activation='relu'),
    tf.keras.layers.Dense(32)
])
```

```
# create the user input and point to the base network
input_user = tf.keras.layers.Input(shape=(num_user_features))
vu = user_NN(input_user)
vu = tf.linalg.l2_normalize(vu, axis=1)
```

```
# create the item input and point to the base network
input_item = tf.keras.layers.Input(shape=(num_item_features))
vm = item_NN(input_item)
vm = tf.linalg.l2_normalize(vm, axis=1)
```

```
# measure the similarity of the two vector outputs
output = tf.keras.layers.Dot(axes=1)([vu, vm])
```

```
# specify the inputs and output of the model
model = Model([input_user, input_item], output)
```

```
# Specify the cost function
cost_fn = tf.keras.losses.MeanSquaredError()
```

