Modern Physics Departmental Exam

James Natoli
January 2021

Problem 1

The muon is an unstable particle that spontaneously decays into an electron and two neutrinos. If the number of muons at $t = 0$ is N_0, the number at time t is given by $N = N_0 e^{\frac{t}{\tau}}$, where τ is the mean lifetime, equal to 2.2 μs. Suppose the muons move at a speed of $0.95c$ and there are $5.0 \cdot 10^4$ muons at $t = 0$

a) What is the observed lifetime of the muons?

b) How many muons remain after traveling a distance of 3.0 km?

Solution

a) This is a time dilation problem, just need to know the formula

$$t' = \gamma t$$

where $\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$, t' is the dilated time, and t is the regular time

$$t' = 2.2 \times \frac{1}{\sqrt{1-(0.95)^2}}$$

$$t' = 7.0456\ \mu s$$

To help check your answer, it should always be a longer time than the regular time

b) The muons will see a shorter distance, due to length contraction

$$L' = \frac{L}{\gamma}$$

where γ is same as before, L' is the contracted length, and L is the regular length

$$L' = 3 \times 10^3 \cdot \sqrt{1 - 0.95^2}$$

$$L' = 936.75 \text{ m}$$

We need a time for the formula given in the prompt, $N = N_0 e^{\frac{t}{\tau}}$, so we need to find the time it takes for the muons to travel L' at $0.95c$

$$\frac{L'}{v} = \text{ time } = \frac{936.75}{0.95c} = 3.28 \cdot 10^{-6} \text{ s}$$

$$N(t) = N_0 e^{\frac{t}{\tau}} = 5 \cdot 10^4 \cdot e^{\frac{3.28 \cdot 10^{-6}}{2.2 \cdot 10^{-6}}}$$

$$N(3.28 \cdot 10^{-6}) = 1.1223 \cdot 10^4 \text{ muons}$$

Problem 2

X-ray photons of wavelength 0.02580 nm are incident on a target and the Compton-scattered photons are observed at 90°

a) What is the wavelength of the scattered photons?

b) What is the momentum of the incident photons? Of the scattered photons?

c) What is the kinetic energy of the scattered electrons?

d) What is the momentum (magnitude and direction) of the scattered electrons?

Solution

a)

Problem

a)

Solution

a)