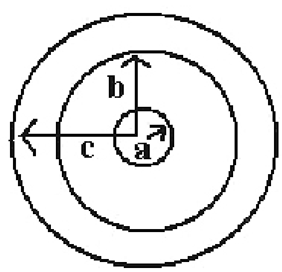
Electricity & Magnetism
Study Questions for the Fall 2020 Department Exam
June 4, 2020

1. a. Find the capacitance of a spherical capacitor with inner radius ℓ_i and outer radius ℓ_0 filled with dielectric of permittivity ε .
   b. Find the capacitance if the dielectric fills only the lower half of the capacitor.

2. A conducting sphere of radius "a" is placed in a uniform electrostatic field $\vec{E}_o$. If the sphere is well insulated, find the expressions for potential and electric fields V and $\vec{E}$ at a point P(r, θ, ϕ) lying outside the sphere. Also obtain the expression for the induced surface charge density "σ" and show that it forms a dipole distribution. Specify your boundary conditions clearly.

3. A long co-axial cable (shown in cross section) has uniform current 'I' flowing in the center conductor into the paper and the same current 'I' flowing in the outer cylinder out of the paper. Find the magnetic field in the following regions - (i) r<a (ii) a<r<b (iii) b<r<c (iv) r>c where 'r' is the distance of any point (in the specified regions) from the center of the co-axial cable.



4. A sphere of radius R has total charge Q uniformly distributed within its volume.

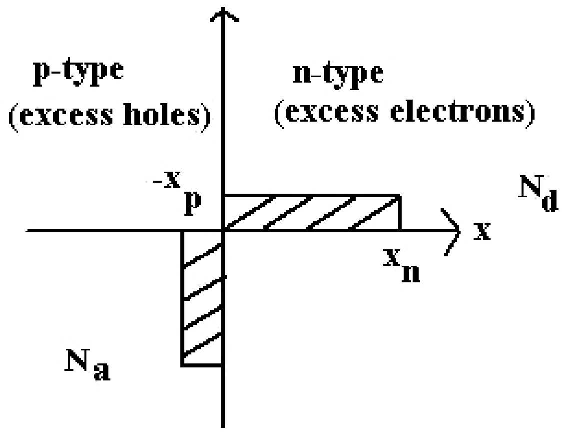
   a. Find the total electrostatic energy of the system, expressed in terms of Q and R.
   b. Find the net electric force that the southern hemisphere exerts on the northern hemisphere, expressed in terms of Q and R.

5. Give short answers.
   a. If one wants to define uniquely the electric field $\vec{E}$ in a given region, is it enough to specify divergence of $\vec{E}$?
   b. How will you express mathematically the law of conservation of charges?
   c. What is the physical significance of the Maxwell's equation
   $$\nabla \cdot \vec{B} = 0.$$
   d. If the magnetic vector potential is zero at the center of a current carrying loop, then does it mean that $\vec{B}$ is zero at that point?
   e. Are advanced potentials physically acceptable solutions to Maxwell's equations?
   f. Which of the four Maxwell's equations is the consequence of Faraday's law of induction?
   g. State clearly the Coulomb gauge and the Lorentz gauge.
   h. What is a plasma frequency in a medium of free charges?

6. A current I flows into a parallel plate capacitor with circular plates of radius R separated by d. The current was 0 before $t = 0$ and $I \neq 0$ after.

   a. What is the charge on the plates as a function of time?
   b. What is the electric field between the plates?
   c. What is the displacement current density between the plates?
   d. What is the magnetic field between the plates at $r = R/2$ from the center of the plates.
   e. What is the Poynting vector between the plates at R/2?

7. A semiconductor junction may often be treated as a sharp interface. Electrons and holes diffuse across the interface creating a depletion region which grows until the potential change across it equals (essentially) E_g (which is also called the semiconductor band gap). Let the semiconductor have a density $N_d = 1 \times 10^{18}/\text{cm}^3$ of donors and $N_a = 1 \times 10^{16}/\text{cm}^3$ of acceptor ions (see sketch).



   a. Use Poisson's equation to calculate the <u>electric field</u> and the <u>potential</u> as a function of x across the interface. (Note that for $x<-x_p$ and $x>x_n$ free carriers exist and therefore $E = 0$ in these regions).
   b. Calculate the depletion layer width $W = x_n + x_p$.

8. A plane wave is propagating in a conductor which has a dielectric constant epsilon and conductivity sigma. The wave is plane polarized along the x-direction and is propagating along the z-direction. Starting with Maxwell's equations, calculate the electric and magnetic fields of the plane wave inside the conductor. In the case of a good conductor calculate the depth to which the plane wave can penetrate (skin depth).

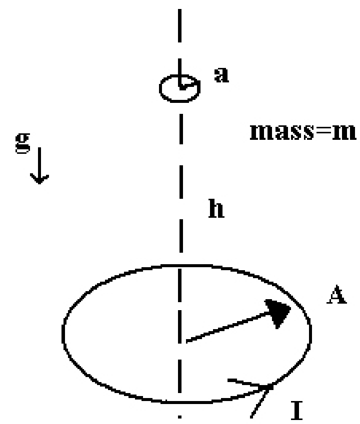
9. a. State Maxwell's equations. Show that the speed of propagation of an electromagnetic wave in free space is

$$\sqrt{1/\mu_o\varepsilon_o}.$$

   b. A plane wave in free space falls on a thick sheet of a material characterized by μ and ε.
      1. State the boundary conditions on $\vec{B}(\vec{H})$ and $\vec{E}(\vec{D})$ which must be satisfied at the interface.
      2. For normal incidence, find and $\vec{E}$ in $\vec{B}$ the free space region.
      3. Sketch the form of $<|E^2|>$ versus the distance from the material in the free space region. The average is over a full time cycle.

10. Two long coaxial cylindrical metal tubes (inner radius a, outer radius b) stand vertically in a dielectric oil (dielectric constant ε, density ρ). The inner one is maintained at potential V, and the outer one is grounded. To what height does the oil rise in the space between the tubes? (Hints: Find the electric field, then the electric energy and then the force.)

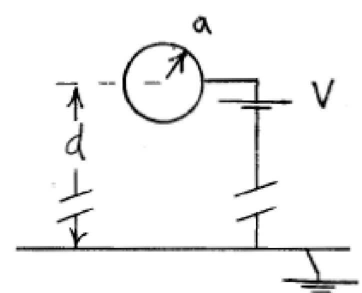
11. The small loop of wire (radius a, resistance R) falls under gravity towards the larger loop (radius A), which has a constant current I? The small loop is constrained to move along the axis of the large loop and remain parallel to the large loop.



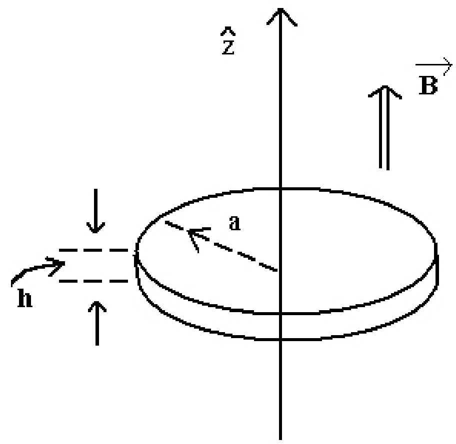
a. Explain (in words) what happens to the small loop.
b. For a particular height h and velocity v, what is the induced emf in the small loop? Assume $a \ll A$.
c. What is the electromagnetic force acting on the small loop?

12. A grounded conducting plane is connected to a conducting sphere with radius *a* through a battery of voltage *V*. The sphere is a distance $d \gg a$ above the plane.

a. Find the charge on the sphere.
b. Find the force between the plane and the sphere.



13. A thin non-magnetic, conducting disk of thickness h, radius a, and conductivity σ is placed in a region where the net, spatially uniform alternating magnetic field $\vec{B} = \hat{z} B_o \sin \omega t$ is parallel to the z axis, as shown in the figure.



a. Find the induced current *density* as a function of radial distance from the axis of the disk.
b. What is the direction of this current at any instant in the first quarter of the period $\pi/(2\omega)$?
c. Find the total induced current at any instant in the above period.
d. Calculate the average Joule heating *i.e.*, the electromagnetic energy that is converted to heat.

14. An off-centered hole of radius *a* is bored parallel to the axis of an infinite-length right circular cylinder of radius *b* (and contained within the circumference of the cylinder). The two axes are a distance *d* apart. A current *I* flows in the solid cylinder. Compute the magnetic field at the center of the hole.

15. A (non-radiating) particle of mass m, charge q, moves under the influence of a uniform electric field $\vec{\mathbf{E}} = \mathbf{E}\hat{\mathbf{y}}$. The particle has initial momentum $\vec{\mathbf{P}}_\mathbf{o} = \mathbf{P}_\mathbf{o}\hat{\mathbf{x}}$. Using *relativistic* dynamics, determine,

a. the kinetic energy as a function of time,
b. the position of the particle as a function of time,
c. the shape of the path followed by the particle.

16. a. What is the minimum magnetization in each bar magnet of length L=40 cm to be able to pick up an identical iron bar magnet? The density of iron is 7.9 g cm^{-3}.

b. What minimum magnetization would be required if the second iron bar was instead a paramagnet with permeability μ=4?

17. A conducting sphere of radius *a* and total charge Q is surrounded by a spherical shell of dielectric material (with permittivity ε) of inner radius *a* and outer radius *b*. Find the electrostatic energy of the system.

18. A center-fed antenna consists of two short segments, each of length d/2, each carrying a current in the same direction so that

$$I(z,t) = I_o \left(1 - \frac{2|z|}{d}\right) e^{-i\omega t}$$

a. Use the continuity of charge equation to show that

$$\lambda(z) = \pm \frac{2iI_o}{\omega d} e^{-i\omega t}$$

where λ is the linear charge density.

b. Find expressions for the vector and scalar potentials, (Lorentz gauge) and show that only the vector potential is needed to find $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ far from the antenna.

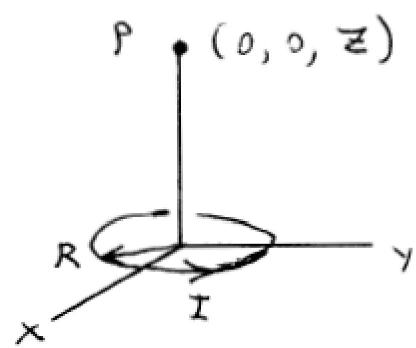
c. Find and $\vec{\mathbf{E}}$ far $\vec{\mathbf{B}}$ from the antenna.

d. Find the total power radiated.

19. A loop of wire of radius R lies in the xy plane centered on the origin carrying a current I as drawn.

a. Find the magnetic field at point P at (0, 0, z).

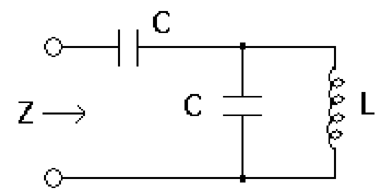
b. Expand your result to two terms for z >> R. Identify the poles.



20. A magnetically "hard" material in the shape of a right circular cylinder of radius *a* and length L has magnetization M uniform throughout its volume and parallel to its axis. Find $H_Z(z)$ on the axis of the magnet by treating it as two sheets of magnetic charge located at z = +L/2 and z = -L/2.

21. Two thin, parallel, infinitely long, non-conducting rods, a distance *a* apart, with identical constant charge density λ per unit length in their rest frame, move with a velocity v, not necessarily small compared to the speed of light, in a direction parallel to the rods' length. Calculate the force per unit length between them in a frame of reference that is at rest, and in a frame of reference moving with the rods, and compare the results.

22. In the accompanying circuit find $Z(j\omega)$ and the frequencies for which $|Z| \to 0$, and for which $|Z| \to \infty$.



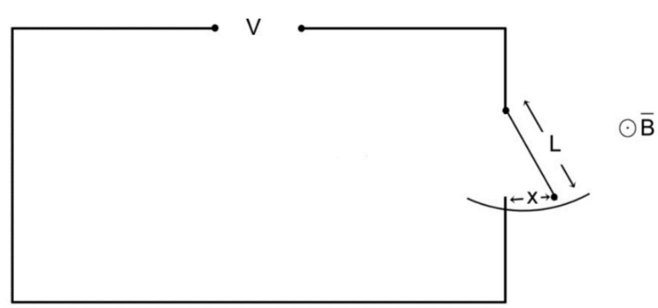
23. The intensity of sunlight at the earth's surface is 1.2×10^6 erg $cm^{-2}s^{-1}$.

    a. Find the electric field of this radiation at the earth's surface in units of V m^{-1}.
    b. Find the radiation pressure (in dyne cm^{-2}) if the sunlight is fully reflected at normal incidence.
    c. Find the radiation pressure if the sunlight is absorbed without reflection.

24. A copper disk of radius 5 cm rotates at 20 revolutions per second, in a magnetic field B=0.5 T perpendicular to the disk. The rim and center are connected electrically by a fixed wire with sliding contacts. The total resistance is $10\,\Omega$. Calculate the induced current.

25. A cylinder with length l and radius r_0 contains a uniform volume charge density ρ_0. When set rotating at angular speed ω around it axis, how large is its magnetic dipole moment?

26. As shown in the figure, a wire pendulum of length L performs small oscillations with velocity $\dot{x} = \omega D\cos(\omega t)$ where D is the maximum horizontal deflection of the pendulum. A constant magnetic field of amplitude B points out of the plane of oscillation of the pendulum. What is the induced voltage V?



27. Most of the electromagnetic energy in the Universe is in the cosmic microwave background radiation, a remnant of the Big Bang. This radiation was discovered by A. Penzias and R. Wilson in 1965, by observations with a radio telescope. The radiation is electromagnetic waves with wavelengths around 1.1mm. The energy density is $4.0 \times 10^{-14} J/m^3$. (This is $2.5 \times 10^5 eV/m^3$, half the rest energy of an electron in each cubic meter of the Universe.)

    a. What is the RMS electric field strength of the cosmic microwave background radiation?
    b. How far from a 1000 W transmitter would you have to go to have the same field strength? Assume the power from the transmitter is isotropic.

28. An inverted hemispherical bowl of radius R has a uniform surface charge density σ. Suppose that the pole of the hemisphere lies at $z=+R$, and the center is at the origin of a standard coordinate system.

a. Find the electrostatic potential at the pole of the hemisphere, $\vec{r} = (0,0,+R)$.

b. Find the electric field strength at the pole.

29. A linear molecule with a permanent electric dipole moment p_o and moment of inertia I (for example, HCl) is placed in a uniform electric field E.

a. Make a sketch showing the charges that make up the dipole, the dipole moment and the electric field when the dipole is in its equilibrium orientation.

b. Derive an expression for the frequency ω_o of small amplitude oscillations about the equilibrium orientation.

c. Describe the polarization and power of the radiation produced by this oscillating dipole, assuming it has been initially displaced by angle $\theta << 90$ degrees from its equilibrium orientation. Make a sketch of the angular distribution of this power

d. Over what time scale will it continue radiating appreciably, and therefore what range of frequencies are emitted.

30. A point dipole sits at the origin of a spherical coordinate system. It points in the z-direction and has a dipole moment $\vec{m}_1 = m\hat{z}$. A second dipole sits at a point $(r,\theta,\phi) = (d,90°,0°)$. It also has a dipole moment $\vec{m}_1 = m\hat{z}$.

a. What is the vector potential due to the dipole located at the origin?

b. What is the magnetic induction due to the dipole located at the origin?

c. What is the value of $\vec{\mathbf{B}}$ at the position of the second dipole?

d. What is the magnitude of the magnetic dipole-dipole interaction energy for these two dipoles?

e. What are the translational force and torque on the second dipole?