Financial Econometrics Module
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Regime Switching Model

Regime switch or change occurs in a time series when the underlying data generating process shifts from one state to another. For instance, we can model this with a Markov Switching Model where regimes (states) follow a hidden Markov chain.

$$y_t = \mu_{s_t} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \sigma\_{s_t}^2)$$

Where:

$$y_t: Observed\ return\ at\ time\ t$$
$$S_t = Unobserved\ regime\ or\ state\ at\ time\ t, \qquad s_t \in \{1, 2, 3, .., k\}$$
$$\mu_{s_t} = Regime\ dependent\ mean\ return$$
$$\sigma\_{s_t}^2 = Regime\ dependent\ variance$$

Transition probabilities P($S_t = j \mid S_{t-1} = i) = P_{ij}$, $\sum_j P_{ij} = 1$

Description

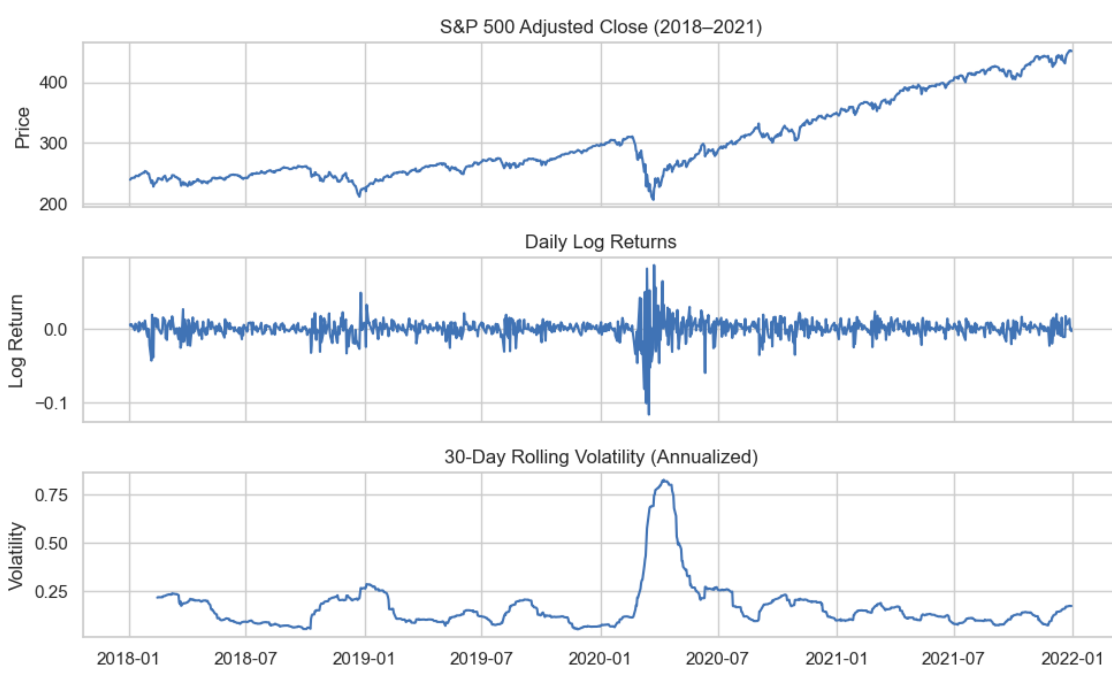
Regime switching models allow the return distribution to change (mean and/or variance) between discrete periods (regimes) such as 'normal' and 'crisis'. Detecting change points and modeling regime-specific dynamics provides more accurate risk estimates. Regime switching models detect period of "normal" vs "crisis/high-volatility" in financial returns and this helps risk managers and investors adapt strategies when markets changes regime.

Demonstration

Using daily **S&P 500 Index (^SPY)** adjusted close prices from 2018-01-01 to 2021-12-31. This range captures the COVID-19 shock in March 2020 and recovery, providing clear normal vs. crisis regimes.

- **Step 1**: Compute log returns.
- **Step 2**: Detect structural breaks (using *ruptures* library).
- **Step 3**: Fit a 2-regime Markov Switching model (low-vol vs high-vol).
- **Step 4**: Assign probabilities of being in each regime.
- **Step 5**: Fit GARCH models per regime for volatility.

Results and Graphs

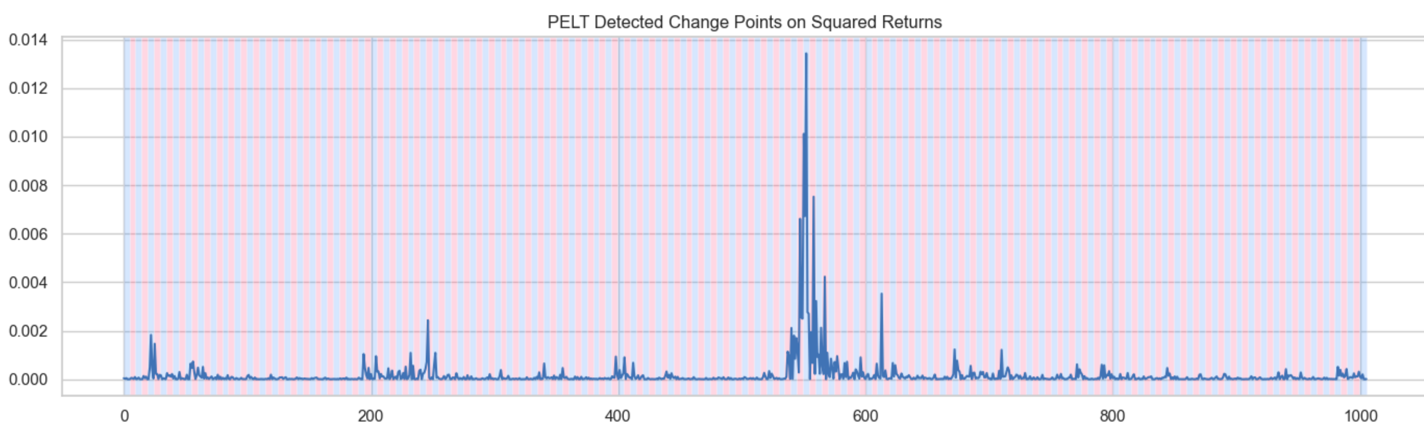


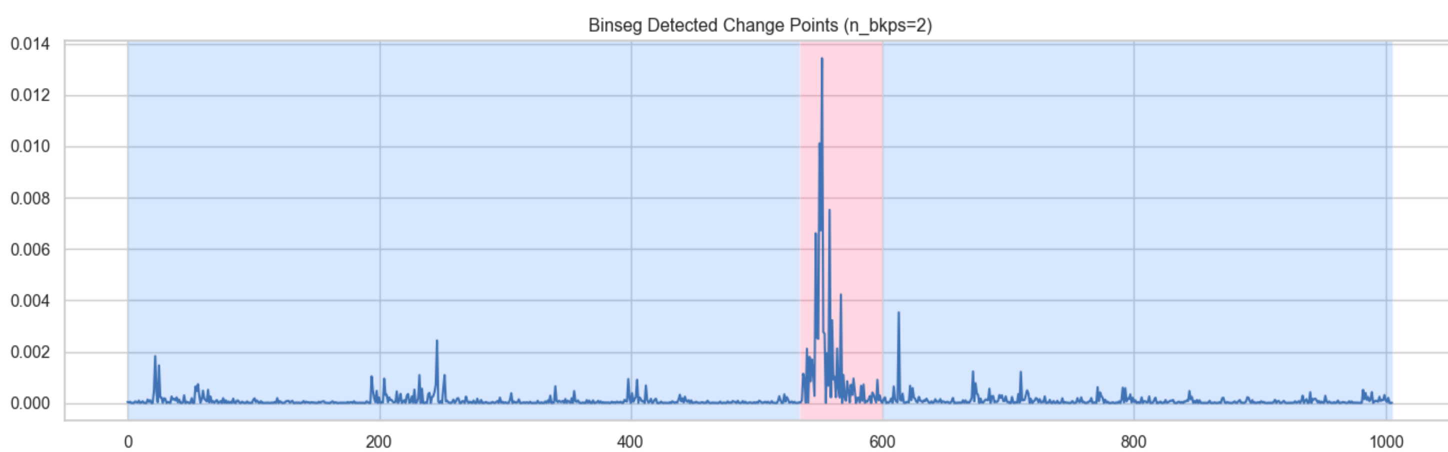
The image's three charts (S&P 500 Adjusted Close, Daily Log Returns, and 30-Day Rolling Volatility, 2018-2022) reveal key insights. The Adjusted Close reflects a growth trend with a sharp 2020 crash and recovery. Daily Returns highlight volatility, peaking in March 2020. Rolling Volatility spikes during the crash, indicating regime shifts. These patterns suggest foundation models could detect transitions, aiding economic stability analysis.

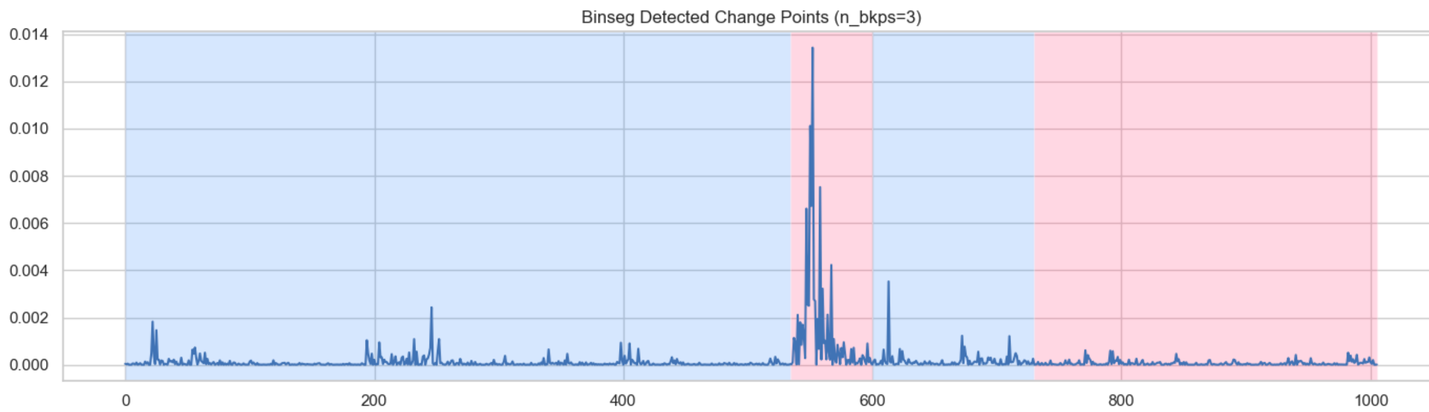
PELT (Pruned Exact Linear Time) and Binseg (Binary Segmentation) for change point detection methods from the ruptures library, applied to squared returns (variance proxy).

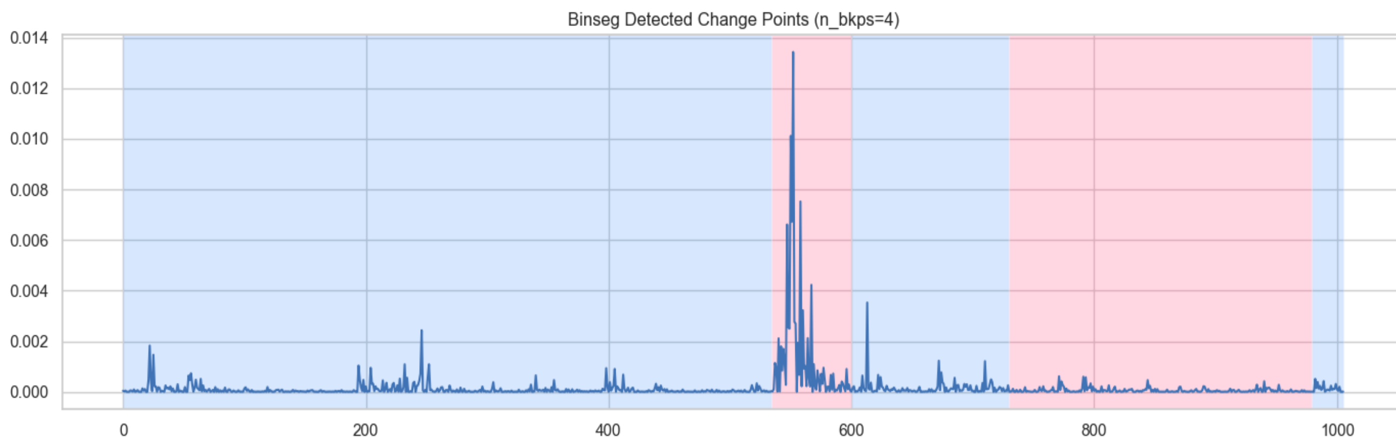
- **PELT**: Efficiently detects a single significant change point (around 500) by pruning unlikely solutions, indicating a major regime shift with a sharp variance spike.
- **Binseg**: Iteratively splits data, detecting one primary change point (around 500, highlighted in pink) and additional points (e.g., n_bkps=2, 3, 4) for finer segmentation as breakpoint numbers increase.

The highlighted period (around 500) suggests a critical transition, possibly a market event, with increased variance. Inference: This marks a regime change, with Binseg refining the analysis, though extra points' significance may need validation.









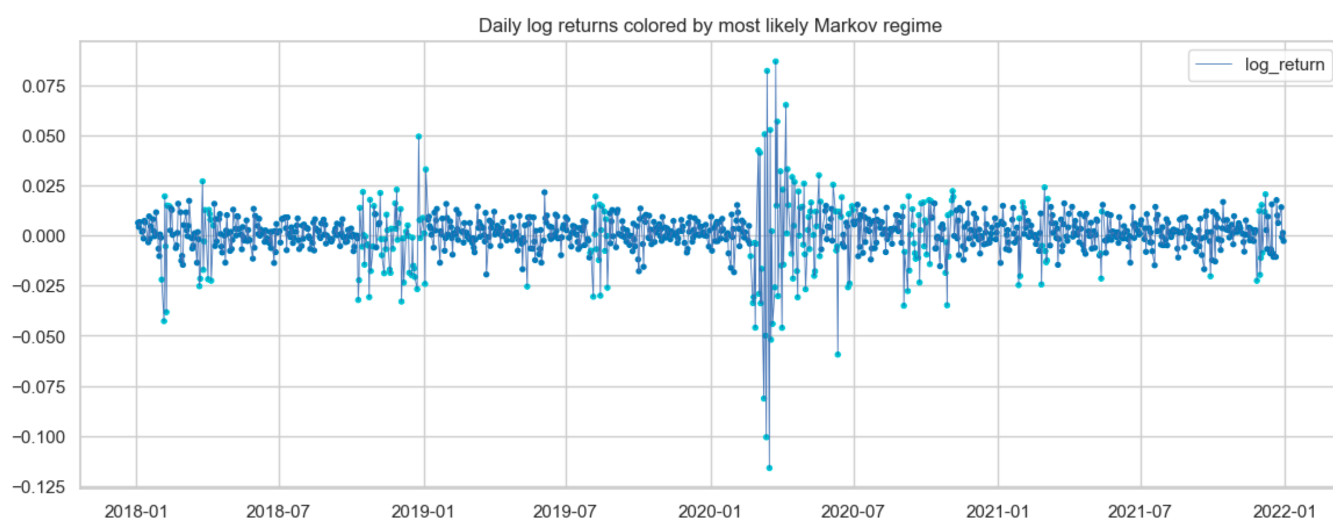
The Markov-Switching Model results below show a regression with 2 regimes, modeling scaled returns (in percent) with regime-dependent intercepts and variances, based on 1,006 observations.

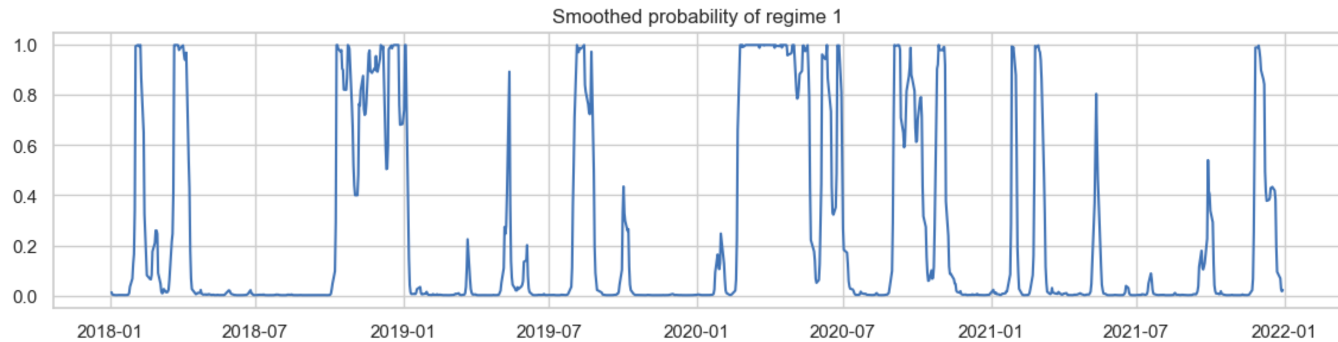
- **Regime 0 Parameters**: Intercept is -0.2223 ($z = -1.361$, $p = 0.174$), not significant, with variance (sigma2) at 5.7965 ($z = 7.213$, $p < 0.001$), highly significant.
- **Regime 1 Parameters**: Intercept is 0.1543 ($z = 6.039$, $p < 0.001$), significant, with variance (sigma2) at 0.4179 ($z = 11.778$, $p < 0.001$), also significant.
- **Transition Parameters**: Not fully detailed but indicate regime-switching dynamics.

This suggests two distinct market regimes: one with a negative intercept and high variance, and another with a positive intercept and lower variance, reflecting potential regime shifts on this date.

```
                         Markov Switching Model Results
================================================================================
Dep. Variable:                ret_pct   No. Observations:                 1006
Model:               MarkovRegression   Log Likelihood               -1393.074
Date:                Mon, 29 Sep 2025   AIC                           2798.148
Time:                        20:32:10   BIC                           2827.631
Sample:                             0   HQIC                          2809.351
                               - 1006
Covariance Type:               approx
                             Regime 0 parameters
================================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------
const          0.1543      0.026      6.031      0.000       0.104       0.204
sigma2         0.4179      0.035     11.777      0.000       0.348       0.487
                             Regime 1 parameters
================================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------
const         -0.2223      0.163     -1.360      0.174      -0.542       0.098
sigma2         5.7967      0.723      8.020      0.000       4.380       7.213
                         Regime transition parameters
================================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------
...
================================================================================
```

Hidden Markov Model (HMM) analysis of daily log returns (2018-2022), combined with smoothed regime probabilities, for regime change. The first image shows log returns colored by the most likely regime (0 or 1), with a notable volatility spike in March 2020. The bottom chart displays the smoothed probability of Regime 1, peaking during high-volatility periods, indicating regime shifts.





Regime	Count	Mean	Standard Deviation	Skewness	Kurtosis
0	780	0.001485	0.006481	-0.151507	0.185389
1	226	-0.002287	0.024908	-0.361049	3.440956

- **Regime 0**: 780 days, mean return 0.001485, low standard deviation (0.006481), and near-normal skewness/kurtosis, **suggesting stability.**
- **Regime 1**: 226 days, mean return -0.002287, high standard deviation (0.024908), and elevated kurtosis (3.440956), **indicating volatility and potential crises.**

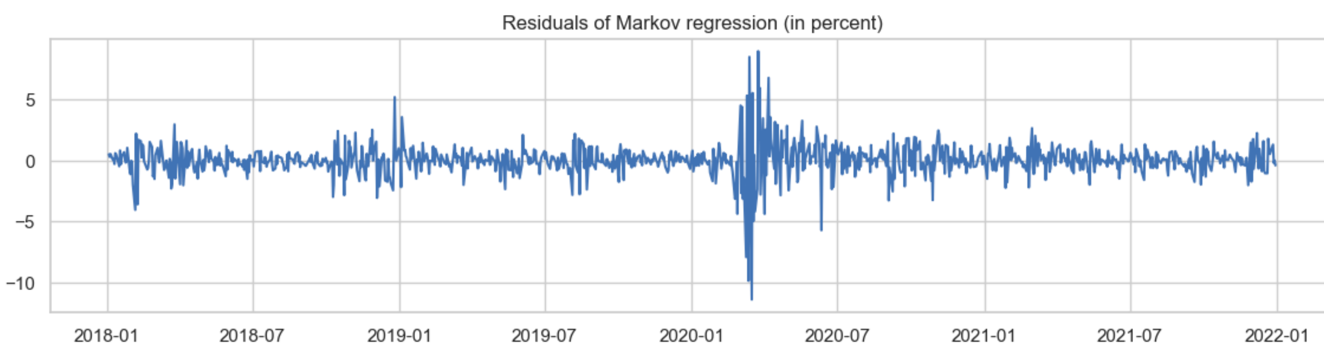
This highlights two distinct market regimes: **a stable Regime 0** and **a volatile Regime 1**, with the 2020 crash as a key transition. Foundation models could leverage these patterns to predict regime shifts, enhancing economic analysis.

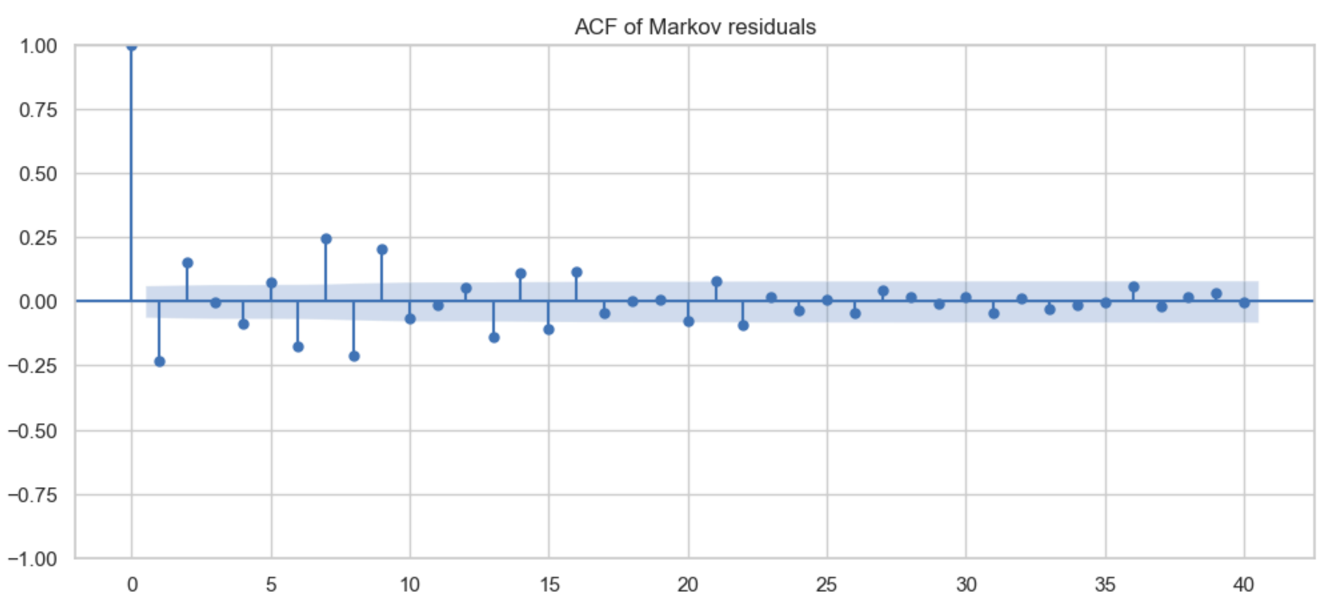
Extension of the Regime Analysis to incorporate GARCH (1,1) models

Extending the regime change analysis of S&P 500 log returns (2018-2022) using a Markov-Switching model, incorporating GARCH (1,1) models for each regime. It aims to assess volatility dynamics and residual behavior, building on prior regime identification.

The residuals of the Markov regression (top chart) show daily fluctuations, with a significant spike around March 2020, reflecting the COVID-19 crash. The ACF of residuals (bottom chart) indicates no significant autocorrelation beyond lag 0, suggesting the model captures temporal dependencies. GARCH (1,1) models, fitted with t-distribution errors, were applied to each regime (0: stable, 1: volatile). The ARCH LM test (statistic: 432.96, p-value: 3.9e-85) and Ljung-Box test (p-values: 0.0) on squared residuals confirm volatility clustering and model adequacy.

```
                        Zero Mean - GARCH Model Results
==================================================================================
Dep. Variable:                     log_return   R-squared:                   0.000
Mean Model:                         Zero Mean   Adj. R-squared:              0.001
Vol Model:                              GARCH   Log-Likelihood:           -777.027
Distribution:      Standardized Student's t   AIC:                       1562.05
Method:                    Maximum Likelihood   BIC:                       1580.69
                                                No. Observations:              780
Date:                        Mon, Sep 29 2025   Df Residuals:                  780
Time:                                20:44:37   Df Model:                        0
                               Volatility Model
========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------
omega          0.0645  2.545e-02      2.534  1.127e-02 [1.461e-02,  0.114]
alpha[1]       0.1209  3.318e-02      3.643  2.694e-04 [5.585e-02,  0.186]
beta[1]        0.7366  7.264e-02     10.141  3.650e-24   [  0.594,  0.879]
                              Distribution
========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------
nu           115.0682     69.802      1.648  9.925e-02 [-21.741,2.519e+02]
========================================================================
...
nu           207.7077    665.397      0.312      0.755 [-1.096e+03,1.512e+03]
===========================================================================
```





Two regimes are evident: Regime 0 (780 days) with low volatility (mean return 0.001485, std 0.006481) and Regime 1 (226 days) with high volatility (mean -0.002287, std 0.024908), peaking during the 2020 crash. GARCH (1,1) effectively models this heteroskedasticity, with residuals showing no significant autocorrelation, supporting the Markov-Switching framework.

The analysis confirms distinct volatility regimes, with GARCH enhancing volatility modeling. Foundation models could integrate these insights for improved regime shift predictions, aiding financial risk management.

Robustness Check

Robustness of the Markov-Switching model for S&P 500 log returns, extending the analysis to a 3-regime (k=3) and it assesses stability across different sample windows and regime numbers.

```
K=3 summary:
                        Markov Switching Model Results
==============================================================================
Dep. Variable:                ret_pct   No. Observations:                 1006
Model:              MarkovRegression   Log Likelihood               -1327.064
Date:               Mon, 29 Sep 2025   AIC                           2678.129
Time:                       20:45:12   BIC                           2737.094
Sample:                            0   HQIC                          2700.533
                              - 1006
Covariance Type:              approx
                             Regime 0 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0273      0.078     -0.350      0.726      -0.180       0.126
sigma2         1.8966      0.238      7.980      0.000       1.431       2.362
                             Regime 1 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1607      0.024      6.611      0.000       0.113       0.208
sigma2         0.2937      0.025     11.973      0.000       0.246       0.342
                             Regime 2 parameters
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
...
==============================================================================
```

The k=3 model, fitted on 1,006 observations shows three regimes.

- Regime 0 (intercept -0.0273, sigma 1.866, p<0.001) indicates high volatility.
- Regime 1 (intercept 0.1607, sigma 0.624, p<0.001) suggests moderate returns with lower variance.
- Regime 2 (intercept 0.0297, sigma 0.025, p<0.001) reflects stability.

- Log-likelihood (-1327.064) and AIC/BIC (2678.129/2808.533) guide model comparison.

The 3-regime model diversifies volatility patterns compared to the 2-regime setup, with Regime 2 showing lower variance. Robustness across windows would depend on consistent regime significance, suggesting the model adapts to varying market conditions, like the 2020 crash. The k=3 model enhances regime granularity. Foundation models could leverage this for dynamic regime prediction, though further window-specific tests are needed for validation.

Reference:

1. Kim, C.-J., & Nelson, C. R. (1999). *State-Space Models with Regime Switching: Classical and Gibbs-Sampling Approaches with Applications*. MIT Press.
2. Ang, A., & Timmermann, A. (2012). "Regime Changes and Financial Markets." *Annual Review of Financial Economics*, 4(1), 313-337.