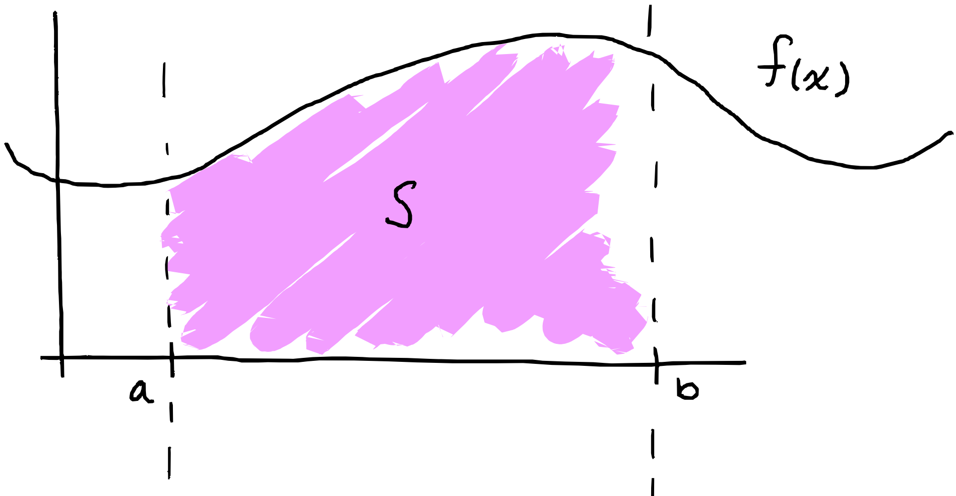
5.1 Area and Distance

Area under a curve



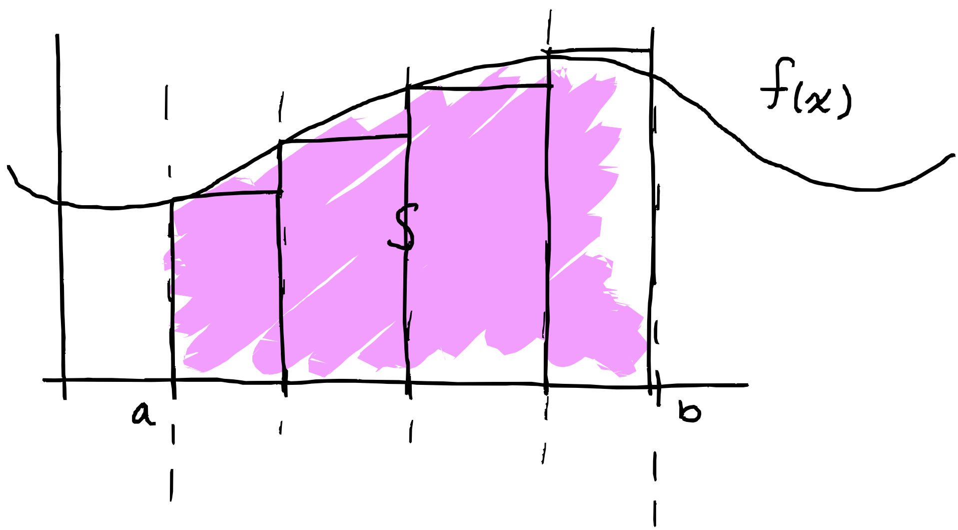
$$S = \{(x, y) : a \leq x \leq b,\ 0 \leq y \leq f(x)\}$$

S is the region bounded to the left and right by $x = a$ and $x = b$, and above and below by $y = 0$ and $y = f(x)$.

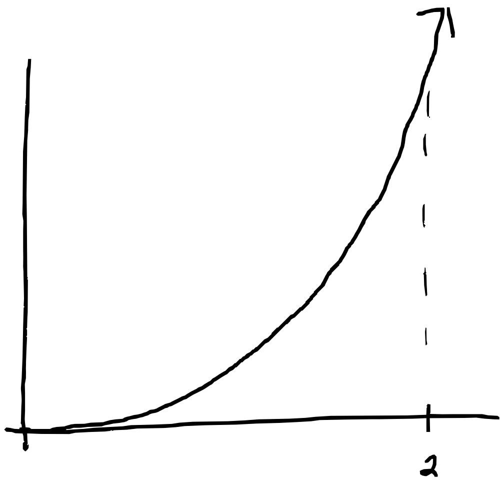
We will develop tools for finding the area of S.

Approximating the area under a curve

General idea: Use rectangles of equal width to approximate the area.



E.g. $f(x) = x^2$ on $[0,2]$



Use four rectangles to approximate the area under the curve.

The four rectangles are on the intervals

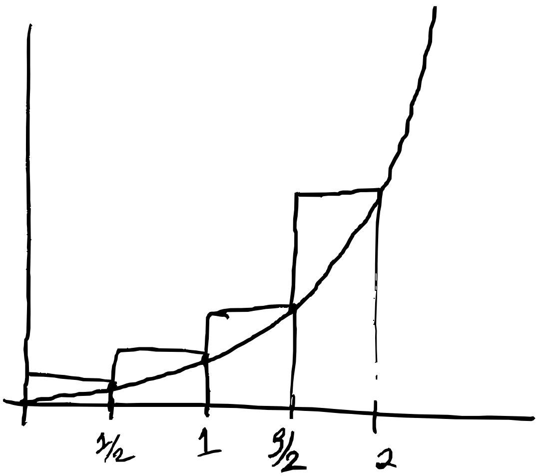
$[0, \frac{1}{2}]$, $[\frac{1}{2}, 1]$, $[1, \frac{3}{2}]$, $[\frac{3}{2}, 2]$.

Widths are each $\frac{1}{2} = \frac{2-0}{4}$ ← interval length

↑ # rectangles

Heights? We can choose either the left sides or right sides.

Lets try right side first



$$R_4 = \tfrac{1}{2}\cdot f(\tfrac{1}{2}) + \tfrac{1}{2}\cdot f(1) + \tfrac{1}{2}\cdot f(\tfrac{3}{2}) + \tfrac{1}{2} f(2)$$

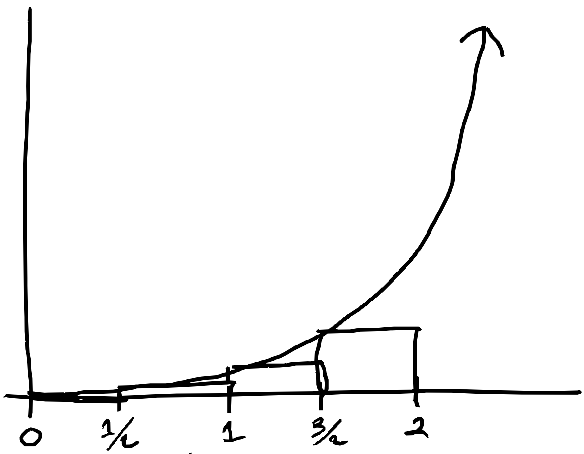
right-side approximation

$$= \tfrac{1}{2}\left(f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2}) + f(2)\right)$$

$$= \tfrac{1}{2}\left(\tfrac{1}{4} + 1 + \tfrac{9}{4} + 4\right)$$

$$= \tfrac{30}{8}$$

Now let's try left side



$$L_4 = \tfrac{1}{2}\left(f(0) + f(\tfrac{1}{2}) + f(1) + f(\tfrac{3}{2})\right)$$

$$= \tfrac{1}{2}\left(0 + \tfrac{1}{4} + 1 + \tfrac{9}{4}\right)$$

$$= \tfrac{13}{8}$$

In-class examples

1) $f(x) = 1/x$ on $[0,3]$

find R_3

2) $g(x) = 4-x^2$ on $[-2,0]$

find L_3

as the number of rectangles increases, the approximate area approaches the actual area

Distance

Premise: distance = velocity × times

works when velocity is constant

When velocity isn't constant, we break up the time into smaller chunks, and approximate over those chunks as if velocity were constant.

time s	0	5	10	15	20	25	30
velocity ft/s	15	20	18	19	12	6	0

$$R_6 = 5\cdot 20 + 5\cdot 18 + 5\cdot 19 + 5\cdot 12 + 5\cdot 6 + 5\cdot 0$$
$$= 5(75) = 375$$

$$L_6 = 5\cdot 15 + 5\cdot 20 + 5\cdot 18 + 5\cdot 19 + 5\cdot 12 + 5\cdot 6$$
$$= 5(90) = 450$$

In - Class Problem :

t	0	0.5	1	1.5	2	2.5	3
v	20	19	17	14	10	3	0

Find the upper and lower estimates for distance travelled.