

THESIS OF  
1948  
.0 75

A STUDY OF PRESSURE DROP  
AND FLOW CHARACTERISTICS  
IN TWO PHASE FLOW

BY  
JAMES E. ORR, JR.

SEPT. 1948

A Study of Pressure Drop and Flow Characteristics in  
Two Phase Flow

By

James E. Orr Jr

A thesis submitted to the faculty of the University  
of Delaware in partial fulfillment of the require -  
ments for the degree of Bachelor of Chemical Engin-  
eering.

September, 1948

Newark, Delaware

Approved: J. A. Guerin  
Professor in Charge of Thesis

Approved: A. L. Diford  
Head of Department of Chemical Engineering

Approved: David L. Gray  
Dean of Engineering

110128

#### Acknowledgement

The writer wishes to express his appreciation  
and thanks to Mr. S. A. Guerrieri for his helpful  
suggestions and criticisms.

## Index

<u>Subject</u>	<u>Page</u>
Summary	1
Introduction (Theory)	3
Equipment	9
Figure 1 Apparatus for Pressure Drop Study	10
Figure 2 Particle Feed Apparatus	11
Experimental Procedure	20
Experimental Data	
Table I Orifice Calibration	23
Table II Pressure Drop due to Pipe	24
Table III Pressure due to Sand and Air	25
Calculated Data	
Table I Orifice Calibration	26
Table II Miscellaneous Calculations	27
Table III Miscellaneous Calculations	28
Table IV Miscellaneous Calculations	29
Miscellaneous Dimensions	32
Discussion of Results	33
Figure 3 Orifice Calibration Curve	38

<u>Subject</u>	<u>Page</u>
Figure 4 Pressure Drop without Sand	39
Figure 5a - 7 Pressure Drop with Sand	40
Figure 8 - 9 Gasterstadt Plots	45
Figure 10 - 11 Favorable Transport Velocities	46
Figure 12 Horizontal Conveying of Wheat and Oats with Air	48
Sample Calculations	51
Nomenclature	55
References	57

## Illustrations

<u>Picture Number</u>		<u>Pages</u>
1 - 12	Apparatus	1 <sup>4</sup> - 19
13 - 20	Visual Study of Particle Flow	49 - 50

## Summary

The object of this investigation was the study of flow characteristics of a two phase system using an air-solid flow mixture.

The conveyor was made up of three inch galvanized iron spouting with a gravity type feed apparatus. One section of three inch glass tubing was placed in the unit to permit visual inspection and photographing the flow. Washed white sand was the solid used.

Runs were made varying the velocity of the air and the amount of sand admitted into the air stream. The pressures at various points along the pipe were plotted against the distance from the feed point (in pipe diameters) and the pressure loss came to a constant rate at about 25 diameters in all cases. Plots of linear velocity against the amount of material transported per unit time showed a maximum transport at approximately 20 feet per second, and decreasing as the velocity increased. The same general curve was obtained when the loading ratio was plotted against the Reynolds number.

The previously established equations for pressure drop calculations did not hold for these investigations. Plots made of  $\Delta P_f / \Delta P_a$  versus R and put into the Gasterstadt equation,  $\Delta P_f / \Delta P_a = 1 + RS$ , gave good results. (Figures 8 and 9) The best results and closer correlation was obtained with the lighter loads. Comparisons made with the horizontal conveying of wheat and oats with air as reported by Vogt and White indicate the sand to behave in a similar fashion but at

lower velocities and pressures.

The minimum transport velocities were found by visual methods to be between 10.16 and 23.0 feet per second (Average 16.9). Calculation by the use of Dalle Valle's (2) equation gave a value of 17.35 feet per second. The experimental value was, in all probability, in the greatest error.

Photographic studies made of the flow pattern show a definite grading in the flow density from the top to bottom of the tube, the more dense flow being at the bottom. Better distribution was indicated at increased velocities.

## Introduction

Two phase flow is one of the most important types of flow study that is now being conducted. The amount of work previously done on solid-vapor systems is limited and old but recent work (7) indicates a reawakening in this field. There are few formulas or expressions now in existence that have been fully investigated but the work by Vogt and White (7) is an attempt to get started. Pneumatic conveyor and allied design problems depend on many variables--pipe size, amount of fluid and solid flowing, fluid properties and friction losses. This work was done in connection with the last variable in an effort to expand the use of existing expressions and if necessary present new correlations.

Pressure losses in a system must be determined and accounted for in the estimation of the power requirements necessary to keep the system operating. The elements that make up the total pressure loss are those due to friction in the pipe and fittings, those resulting from the acceleration of the air to carrying velocity and those due to the acceleration of the solids. (1)

Wood and Bailey<sup>(1)</sup> state that the pressure gradient is the greatest near the point of admittance of the solid particles and that the slope of a line representing pressure decreases as the distance from that point increases until at approximately thirty to forty pipe diameters it becomes constant. The figures thirty to forty pipe diameters are given for a feed

mechanism in which the particles are sucked into the stream.  
Other types of apparatus would show other distances.

The pressure drop resulting from the acceleration of  
the air may be determined from the following expression:<sup>(1)</sup>

$$(1) \Delta P_{\text{accel}} (\text{air}) = \rho_a \frac{V_a^2}{2g_c}$$

where:  $\Delta P_{\text{accel}}$  = pressure drop, lbs. force/sq. ft.

$\rho_a$  = density of air, lbs./cu. ft.

$V_a$  = superficial velocity of air ft./sec.

$g_c$  = conversion factor,  $\frac{\text{lbs.}}{\text{lbs. force}} \frac{\text{ft.}}{\text{sec.}^2}$

pressure due to the solid particle acceleration may  
be calculated from a current in balance:

$$(2) \Delta P_{\text{accel}} (\text{solids}) = \frac{W_s}{g_c A} V_s$$

where:  $\Delta P_{\text{accel}} (\text{solid})$  = pressure drop, lbs. force/sq. ft.

$W_s$  = weight of solids, lbs.

$g_c$  = conversion factor

$V_s$  = velocity of solids, ft./sec.

$A$  = cross-sectional area of pipe, sq. ft.

i.e., and  $V_s = V_a + V_p$

$V_p$  = minimum transport velocity ft./sec.

A conservative value for the pressure drops due to  
pipe friction and the recovery of the loss of momentum required  
to keep the particles in suspension can be obtained by the use

of the Fanning equation for pressure loss.

$$(4) \quad \Delta P_F = \frac{4 f L Q^2}{2 g_c D}$$

where:  $\Delta P_F$  = pressure drop lbs. force/sq. ft.  
 $f$  = friction factor, dimensionless  
 $L$  = length of pipe, ft.  
 $Q$  = mass velocity of air in pipe, lbs./sec./sq. ft.  
 $g_c$  = conversion factor  
 $D$  = diameter of pipe, ft.

The friction factor,  $f$ , is obtained by using the Reynolds number and a standard friction factor plot.

$$(5) \quad N_{Re} = \frac{DV_a D}{\mu_a}$$

where:  $N_{Re}$  = Reynolds number  
 $D$  = diameter of pipe, ft.  
 $V_a$  = velocity of air, ft./sec.  
 $\rho$  = density of mixture, lbs./cu. ft.  
 $\mu_a$  = viscosity of air, absolute, lb./(ft.)(sec.)

From the investigations of Alves<sup>(1)</sup> these equations were used at what are considered to be high loadings (the loading is the ratio of the weight of solid transported per unit time to the weight of air moved per unit time) and gave values from thirty to forty percent higher than experimental values. By using the feed loading and not the actual loading to compute the mixture density values in better agreement with

experimental work were obtained. The modified equations are as follows:

$$(6) \quad N_{Re} = \frac{DS}{\mu_a} (1+R) \quad (6)$$

$$(7) \quad \Delta P_f = \left( \frac{4fL}{D} \right) \left( \frac{G^2}{2g_c} \right) (1+R) \left( \frac{1}{\rho_a} + \frac{R}{\rho_s} \right)$$

where:  $N_{Re}$  = Reynolds number

$D$  = pipe diameter, ft.

$G$  = mass velocity of air in pipe, lbs./sec., sq.ft.

$\mu_a$  = viscosity of air, absolute lb.(ft.)(sec.)

$R$  = loading

$\Delta P_f$  = pressure drop, lbs. force/sq. ft.

$f$  = friction factor

$L$  = length of pipe, ft.

$\rho_a$  = density of air, lb./cu.ft.

$\rho_s$  = density of solid, (true), lb./cu.ft.

The sum of these various pressure drops may be represented as follows:

$$(8) \quad \Delta P = \Delta P_{\text{accel}}(\text{air}) + \Delta P_{\text{accel}}(\text{solid}) + \Delta P_f$$

where:  $\Delta P$  = total pressure drop, lbs. force/ sq. ft.

The limitation of these expressions is the fact that they have only been shown to hold for linseed and have not been used for other materials.

Gasterstadt<sup>(3)</sup> investigated the flow of wheat in horizontal pipes and found that the ratio of the pressure drop for the mixture to that of the air was linear with the loading:

$$(9) \quad \frac{\Delta P_f}{\Delta P_a} = 1 + RS$$

where:  $\Delta P_f$  = pressure drop due to friction of mixture,  
 lbs. force/sq. ft.  
 $\Delta P_a$  = pressure drop for air alone, lbs. force/sq.ft.  
 $R$  = loading  
 $s$  = slope

The value  $s$  is the slope of a line obtained by plotting  $\Delta P_f/\Delta P_a$  vs.  $R$ . Again the limitation of the equation is in its application to wheat only in the original tests.

Dalje Valle<sup>(2)</sup> found that the minimum transport velocities, or the lowest velocity at which movement of the particles will occur, can be calculated from the following:

$$(10) \quad V_t = 6000 \left( \frac{s}{s+1} \right)^{0.498} d$$

where:  $V_t$  = minimum transport velocity, ft./sec.  
 $s$  = specific gravity  
 $d$  = particle diameter, inches

Application of Stokes law to a functional relation of the Fanning equation,<sup>6</sup> Vogt and White gave the following equation:

$$(11) \quad \frac{\alpha - 1}{r} = \frac{\rho_s}{Re \left( \frac{d}{D} \right)^2 \frac{\omega}{\rho}}$$

This equation plotted as  $\frac{\alpha - 1}{r}$  vs.  $Re \left( \frac{d}{D} \right)^2 \frac{\omega}{\rho}$  gave excellent agreement with the data of Gasterstadt and Segler. A refinement of this equation representing the pressure drop for wheat is reported to agree well with experimental data.<sup>(7)</sup>

$$(12) \propto -1 \cdot \left(\frac{\rho}{d}\right)^2 \left(\frac{\rho}{\omega_{\text{res}}}\right)^k$$

where  $k = 2 \sqrt{\frac{1/5 (\mu - \rho)}{\rho}}$

## Equipment

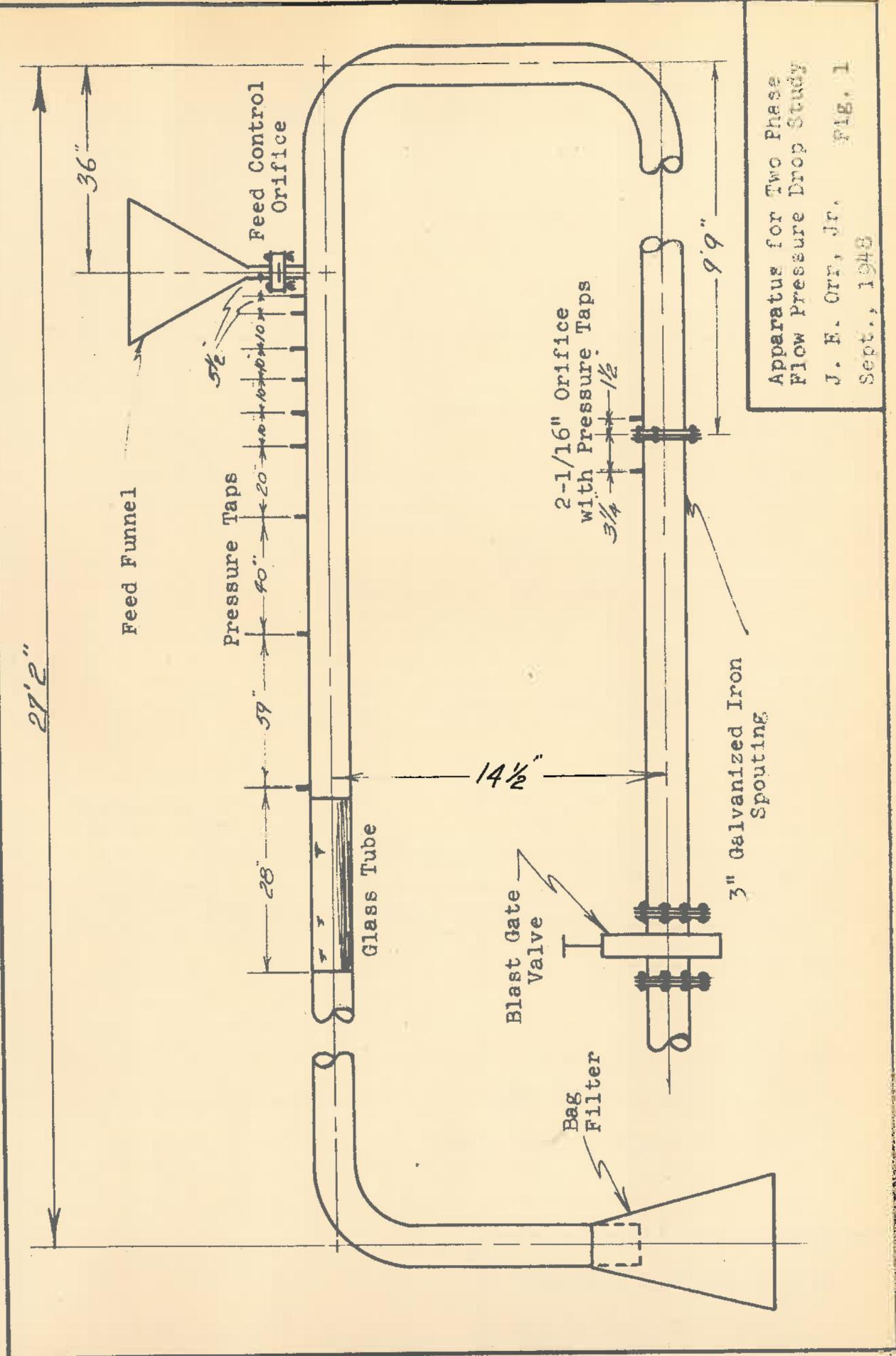
The apparatus used in this investigation is illustrated in Figure 1.

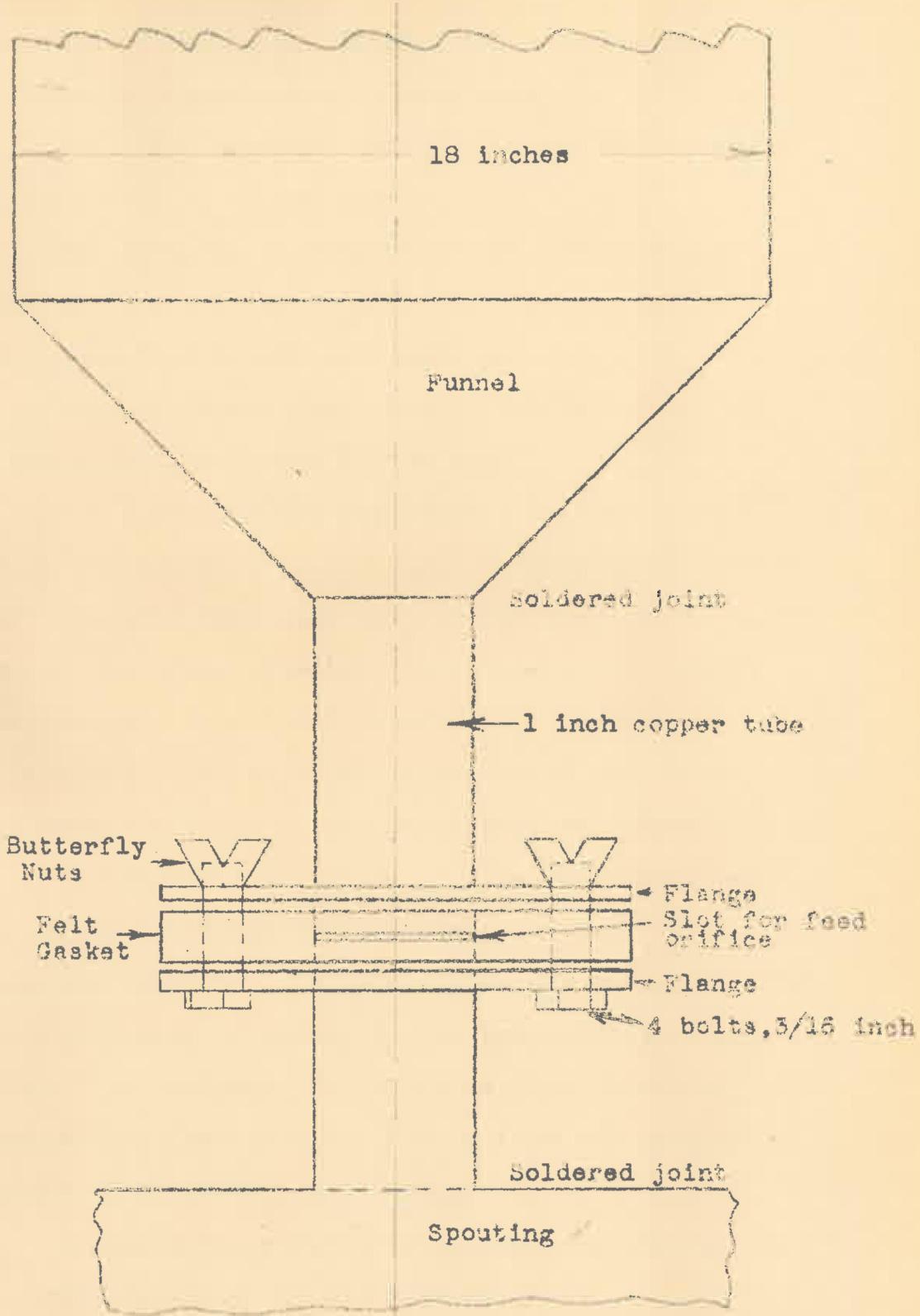
A Spenser turbo-compressor, powered by a five horsepower induction motor and capable of producing a gage pressure of twenty ounces per square inch, was used to move the air through the pipe. The air entered the unit through a blast gate valve which was used to control the rate of flow.

The pipe was made of several sections of galvanized iron spouting three inches in diameter and one section of glass tubing of the same size. The joints were wrapped with friction tape and then shellacked, giving a satisfactory air and dust tight seal. The sections between the blast valve and the orifice, and the bends between the orifice and the particle feed funnel were made more rigid by fastening the joints with small sheet metal screws. Sheetmetal straightening vanes, six inches long and three inches wide were placed between the particle feed funnel and the proximal bend in the pipe.

The glass tubing used was common soft glass, twenty-eight inches long and was located below the last pressure top of the unit so that a visual and photographic study of the flow pattern could be made.

The fluid meter was a sharp-edged, concentric orifice, 2 - 1/16 inches in diameter. Pressure tops, located 1.08 pipe diameters above and 0.5 pipe diameters below the





Particle Feed Apparatus  
J. E. Orr Jr  
July, 1948 Fig. 2

orifice, were connected to differential pressure and to static pressure manometers by rubber tube.

The solid material transported was washed white sand ranging from 16 to 100 mesh.

The sand was admitted into the air stream by gravity feed from a large funnel, type feed apparatus. This device is illustrated in Figure 2.

The air transported sand was collected at the end of the unit in a cotton filter bag.

The pipe pressure taps were 1/16 inch holes drilled in the spouting at the top. A sharp drill must be used for this work to insure that no burrs are left in the pipe around the hole. The taps were located to include pressure readings from 1.835 to 56.7 pipe diameters downstream from the point of entry of the solid particles. The taps were connected to the manometers by rubber tubing.

Eleven vertical, glass, U-type manometers were mounted on the manometer board. Ten read the static pressures and one read the orifice differential pressure. The scale was graduated in inches to the nearest five-hundredths of an inch. An inclined, differential type manometer (.01 inch graduations) was used for the orifice calibration in conjunction with a five cubic foot gasometer.

The sand was weighed on an Ohaus balance of about ten kilograms capacity (to the nearest gram).

The flow pictures were taken with a Speed Graphic, 4 x 5 inch camera using Eastman Kodak Super xx film. A

poloroid filter was used on the camera lens to eliminate as much glare as possible. The light was supplied by one 500 watt spotlight and one Number I Photoflood light in a reflector.

The pictures of the equipment were taken on a Leica Model III using Eastman Kodak Super xx film at various exposures.

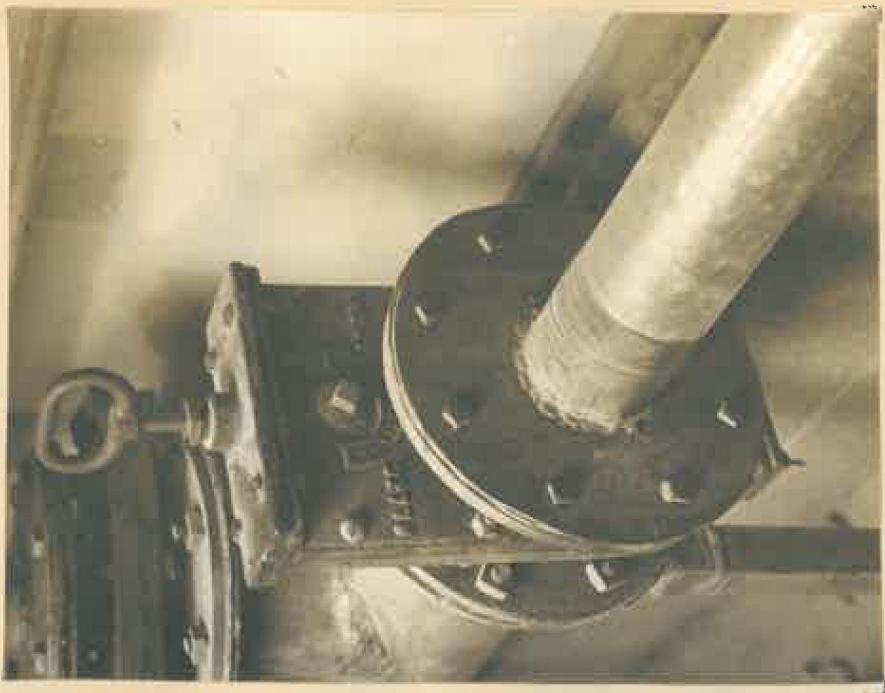


Illustration 2: Blast - Gate Valve

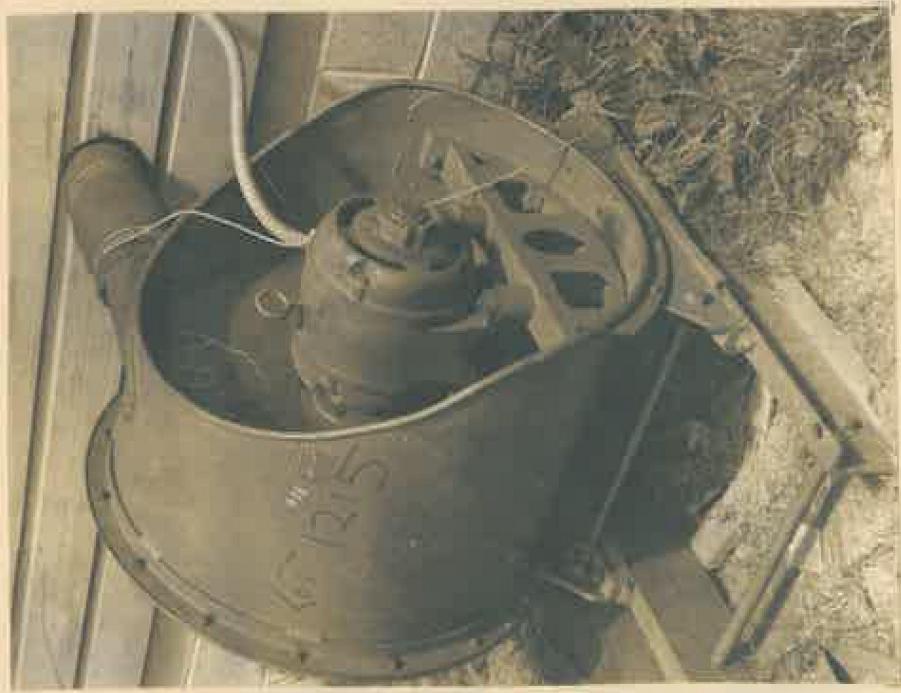


Illustration 1: Turbo Compressor

Illustration 4 : Glass Section

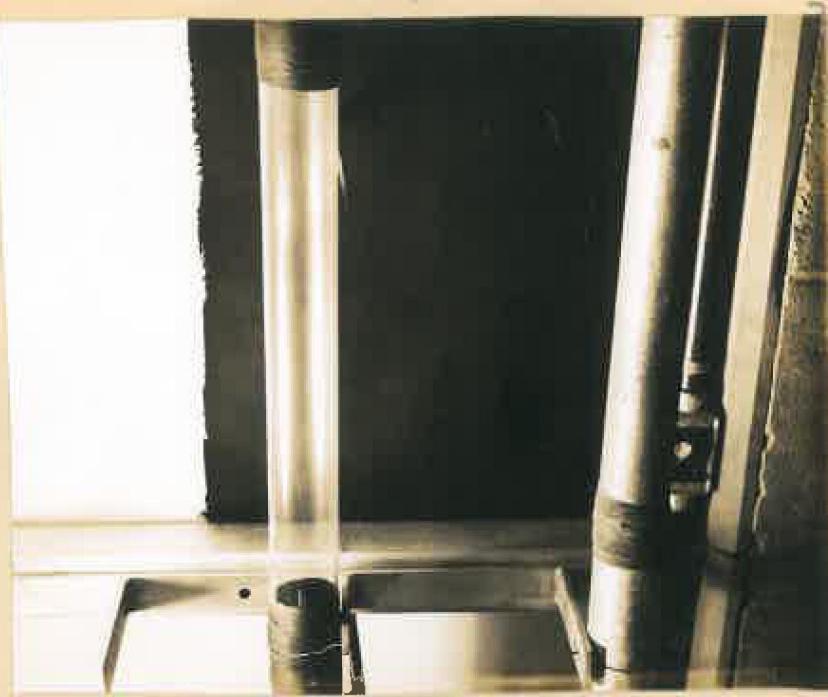


Illustration 3: Pipe Section  
showing particle feed funnel and  
pressure taps.

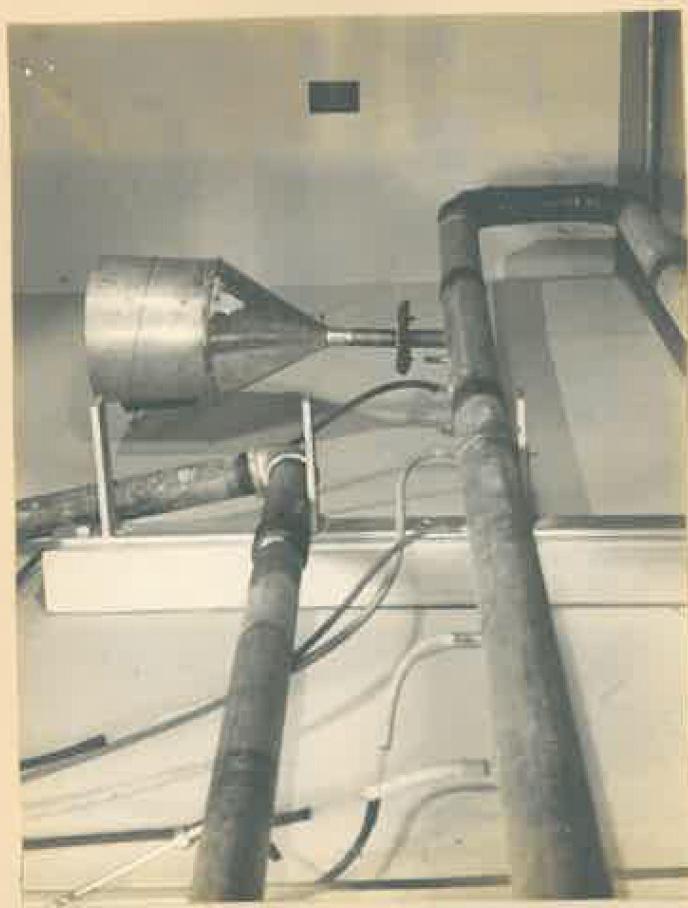




Illustration 6: Particle Feed  
Funnel



Illustration 5: Flanged Orifice  
Mounting showing pressure taps

Illustration 8: Manometer Board

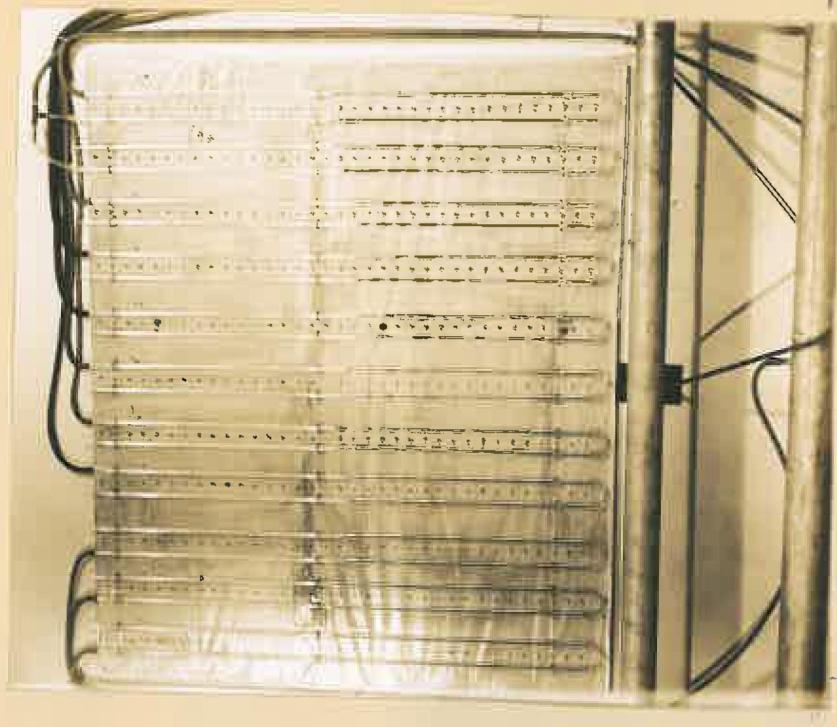
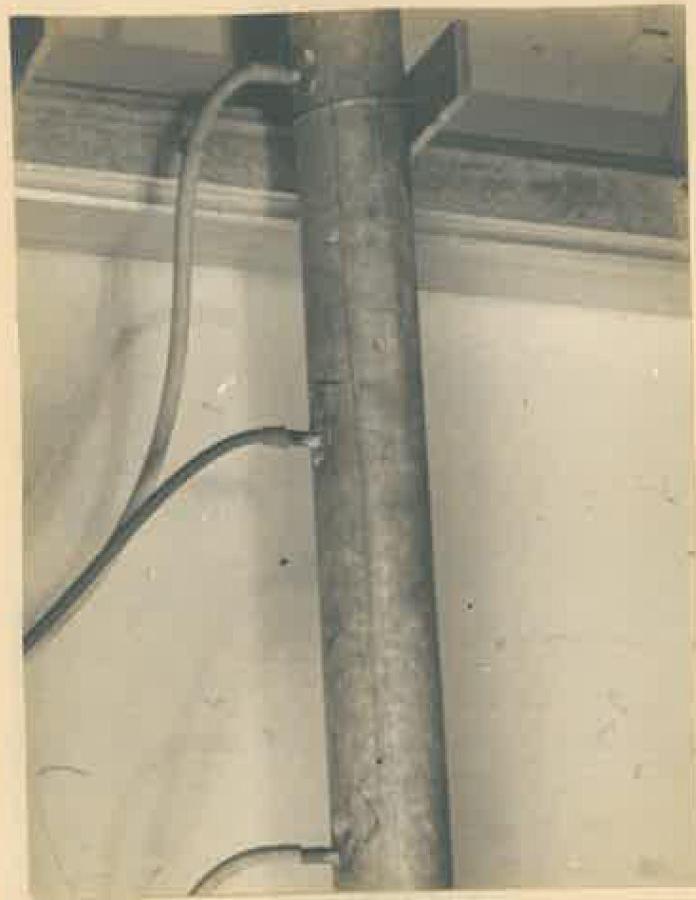


Illustration 7: Pipe Pressure Taps



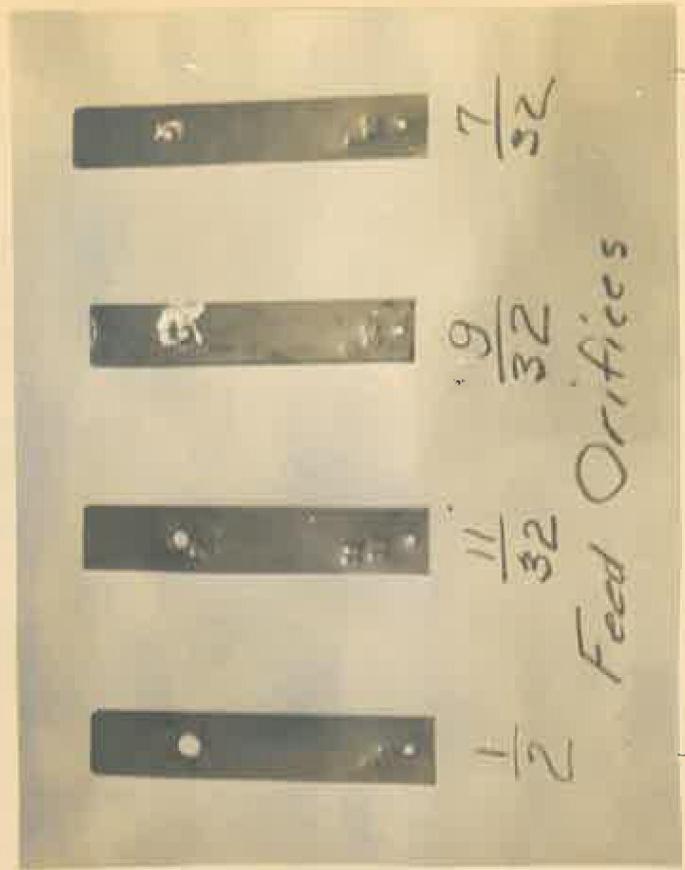


Illustration 10: Feed Orifices



Illustration 9: Balance showing  
sand being weighed

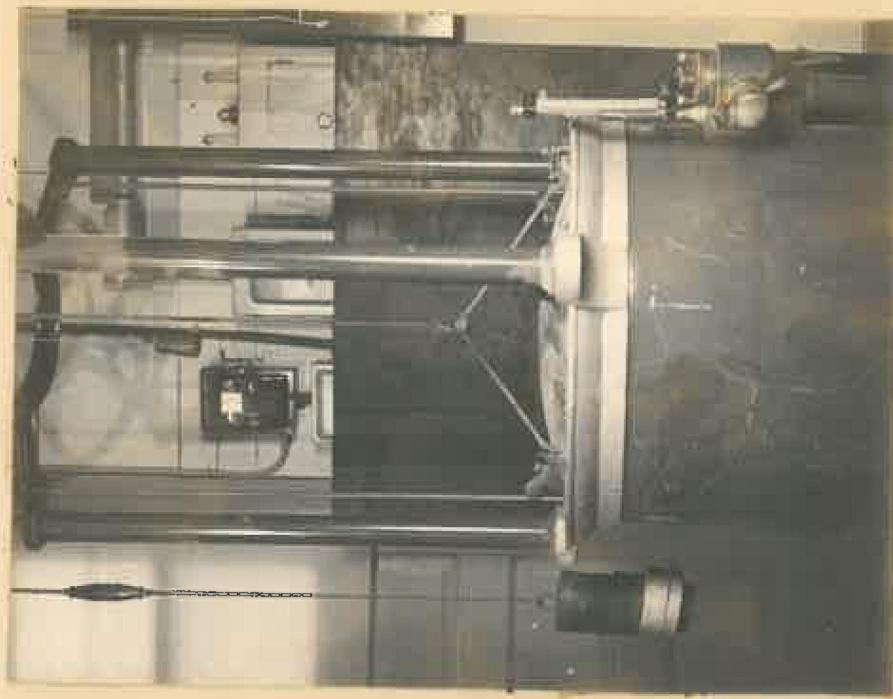


Illustration 12: Geissler tube used to calibrate orifice

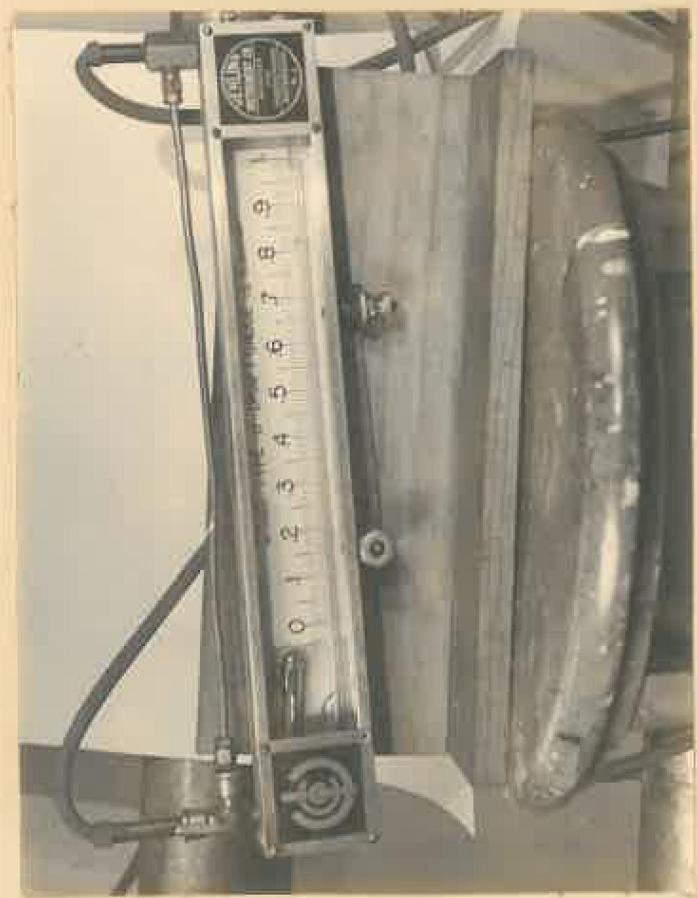


Illustration 11: Inclined Gage used to calibrate orifice

### Experimental Procedure

To provide a means of determining the velocity of the air in the pipe it was first necessary to calibrate the orifice. This was accomplished by measuring, in a gasometer, the amount of air passing through the orifice in a given period time. By plotting the quantity per unit time against the orifice differential pressure, a curve was obtained that gave the quantity of air flowing for any observed pressure differential. Seven runs were made over a very limited range of air rates, the limiting factor being the gasometer. The curve was extrapolated into a usable range by assuming an orifice coefficient of 0.612. (Figure 3)

Seven more runs were made without sand to determine the pressure losses due to pipe friction. Air orifice velocities used ranged from approximately 50 feet per second to 110 feet per second. During these runs the static pressure at each tap was observed and recorded. The differential pressures (the difference between the pressure at tap 1 and at the tap in question) were plotted against the orifice differential pressures and the connecting curve drawn.

The experimental runs with the sand were made by varying different factors. Various air velocities were used with each feed orifice. The range covered was from the minimum transport velocity to the velocity that gave discontinuous

flow due to the formation of a fluid seal in the neck of the particle feed funnel. The data taken for the record runs included the orifice and pipe manometer readings, the time required for the run and the weight of sand transported. An outline of the procedure for making a run is as follows:

- (1) Choose orifice differential pressure desired;
- (2) Open feed orifice, simultaneously starting watch;
- (3) Read all manometers;
- (4) Close feed orifice, simultaneously stopping watch;
- (5) Weight bag with transported sand.

As can be seen from the above outline the run included all time from the moment the feed orifice was opened until the moment the feed orifice was withdrawn from position shutting off the flow of sand.

The total weight of the sand transported during the run was obtained by subtracting the predetermined base weight of the bag from the total weight of the collected sand and the bag.

The minimum transport velocities were obtained by observing the flow in the glass tube. The air flow rate that gave a minimum movement of the sand was taken as the choking or minimum transport velocity. Because the sand used contained particles of more than one size, the flow rate chosen was an approximate mean rate, high for the smallest and low for the largest particles.

The pictures of the flow pattern were made during the record runs with the lights placed to minimize reflection.

from the tube. The lens of the camera was level with the tube so that the picture was a cross section view of the flow.

The density of the sand was determined by measuring the amounts of water displaced by a weighted amount of sand. This was done by putting a known weight of sand in a beaker full of water and measuring the overflow in a graduate.

## Experimental Data

Table I

## Orifice Calibration Data

Run Number	Orifice Differential Pressure In. of H <sub>2</sub> O	Orifice Static Pressure In. of H <sub>2</sub> O	Tank Pressure In. of H <sub>2</sub> O	Time of Run Sec.	Volume of Air at Tank Pressure Cu. Ft.
1a	0.08	3.40	1.70	10.3	4
2a	0.10	3.40	1.75	9.9	4
3a	0.19	5.31	2.15	8.1	4
4a	0.39	8.35	3.00	6.3	4
5a	0.28	5.55	2.20	8.0	4
6a	0.18	4.95	2.00	8.5	4
7a	0.150	4.35	1.85	9.2	4

Air Temperature: 80° F.

Barometric Pressure: 407.2 In. of H<sub>2</sub>O

## Experimental Data

Table II  
Pressure Drop due to Pipe

Run Number	Orifice Differential Pressure In. of H <sub>2</sub> O	Orifice Static Pressure In. of H <sub>2</sub> O	Pressure along Pipe at Taps						
			1	2	3	4	5	6	7
1aa	2.40	3.00	0.85	0.95	0.90	0.85	0.80	0.90	0.80
2aa	6.65	7.90	2.25	2.45	2.05	2.00	2.00	1.90	1.65
3aa	10.40	12.80	3.45	3.48	3.15	3.10	3.10	2.95	2.55
4aa	14.85	17.45	4.90	4.85	4.45	4.40	4.30	4.30	4.00
5aa	16.75	19.50	6.05	5.50	5.45	5.50	5.35	5.40	5.00
6aa	22.75	26.55	7.30	7.30	6.60	6.65	6.45	6.40	6.10
7aa	25.80	30.10	7.80	8.20	7.45	7.50	7.30	7.30	6.85
									6.05
									4.90

## Experimental Data

Table III  
Pressures due to Sand and Air

Run Number	Orifice Differential Pressure In. of H <sub>2</sub> O	Orifice Static Pressure In. of H <sub>2</sub> O	Pressure along Pipe at Taps					Time per Run Sec.	Weight of Sack and Sand Gms.	
			1	2	3	4	5			
1	2.00	3.05	1.35	1.30	1.25	1.20	1.30	1.00	0.80	5598
2	6.95	9.25	3.20	2.95	2.85	2.80	2.75	2.35	1.90	3679
3	10.00	12.90	4.25	4.20	3.95	3.75	3.65	3.10	2.50	3911
4	12.45	15.55	4.85	4.80	4.45	4.30	4.25	3.50	2.90	3122
5	14.15	17.60	6.00	5.45	5.00	4.90	4.85	4.00	3.30	3052
6	14.95	18.65	5.55	5.75	5.20	5.05	5.05	4.70	4.10	3116
7	16.80	21.00	6.60	6.50	6.00	5.80	5.80	5.45	4.75	3984
8	14.60	14.60	2.65	2.55	2.40	2.40	2.25	2.20	1.70	4537
9	3.20	4.90	1.90	1.90	1.75	1.80	1.70	1.75	1.45	4261
10	2.45	3.60	1.20	1.20	1.10	1.15	1.05	1.10	0.90	2104
11	4.70	6.05	1.95	1.90	1.75	1.75	1.70	1.60	1.45	1746
12	6.65	8.30	2.50	2.50	2.30	2.25	2.20	2.10	1.85	1592
13	10.65	13.05	3.90	3.85	3.50	3.50	3.45	3.25	2.90	1658
14	12.55	15.20	4.45	4.45	4.05	4.00	3.90	3.65	3.25	1626
15	14.90	18.00	5.25	5.25	4.75	4.75	4.60	4.35	3.80	1100
16	3.65	4.95	1.70	1.70	1.55	1.60	1.55	1.35	1.10	2163
17	1.30	2.15	0.75	0.80	0.75	0.75	0.70	0.60	0.50	3380
18	Run Omitted	3.45	1.40	1.20	1.10	1.05	1.00	1.05	1.00	1026
19	2.60	5.90	1.75	1.60	1.55	1.60	1.55	1.60	1.50	1130
20	3.60	8.30	2.50	2.45	2.20	2.10	2.15	2.00	1.80	137.8
21	6.75	10.60	3.20	3.15	2.90	2.85	2.75	2.60	2.30	951
22	8.65	13.05	3.85	3.85	3.50	3.45	3.40	3.35	3.15	1109
23	10.75	15.55	4.50	4.45	4.05	4.00	3.90	3.70	3.25	747
24	13.00	18.50	4.95	4.95	4.55	4.50	4.40	4.25	4.10	736
25	14.80	2.00	0.75	0.75	0.70	0.65	0.70	0.70	3.60	483
26	1.50	2.00	0.75	0.75	0.70	0.65	0.70	0.70	0.55	1453
27	Run Omitted	1.75	0.55	0.65	0.55	0.60	0.50	0.60	0.50	136.0
28	1.05	3.30	1.05	1.05	1.00	0.95	0.90	0.95	0.80	217.7
29	2.55	5.70	1.70	1.75	1.55	1.60	1.55	1.60	1.35	981
30	4.60	10.25	2.40	2.40	2.20	2.20	2.15	2.00	1.70	736
31	6.65	8.15	3.00	3.00	2.70	2.65	2.60	2.40	2.15	558
32	8.50	12.80	3.65	3.70	3.35	3.30	3.25	3.20	3.00	547
33	No Flow <sup>10</sup>	12.80	3.65	3.70	3.35	3.30	3.25	3.20	3.00	474
34	10.40	12.55	2.20	0.75	0.80	0.70	0.75	0.70	0.60	203.1
35	1.60	4.45	1.75	1.75	1.30	1.20	1.20	1.15	1.00	217
36	3.55	3.55	2.40	2.40	2.05	1.75	1.75	1.75	1.50	854
										568
37-40	1.55	Minimum Transport Velocity by Visual Method								

## Calculated Data

Table I

## Orifice Calibration

Run Number	Linear Velocity of Air, $U$ , Feet / Sec.	Orifice Coefficient $c$	Reynolds Number $N_{Re}$
1a	16.70	0.880	17,640
2a	17.35	0.816	18,300
3a	21.10	0.716	22,000
4a	26.75	0.642	28,550
5a	21.40	0.603	22,700
6a	20.10	0.707	21,250
7a	18.65	0.717	19,720
8a	31.00	0.610*	32,800
9a	49.00	0.610*	51,800
10a	69.30	0.610*	73,250

\*Assumed values of Orifice coefficient,  $c$ .

## Calculated Data

Table II

Run Number	Orifice Velocity Ft. / Sec.	Pipe Velocity Ft. / Sec.	Air Weight Rate Lbs. / Min.
1	58	27.40	6.03
2	109	51.50	11.32
3	130	61.30	13.50
4	145	68.50	15.07
5	152	71.80	15.80
6	158	74.70	16.40
7	168	79.40	17.46
8	88	41.50	9.14
9	73	34.40	7.57
10	64	30.20	6.65
11	88	41.50	9.14
12	105	49.50	10.90
13	132	62.30	13.70
14	144	68.00	14.96
15	158	74.50	16.40
16	77	36.30	8.00
17	46	21.70	4.78
18	Run Omitted		
19	65	30.70	6.75
20	77	36.40	8.01
21	104	49.00	10.79
22	120	56.70	12.50
23	134	63.30	13.93
24	148	70.00	15.40
25	158	74.70	16.45
26	50	23.60	5.20
27	Run Omitted		
28	40	19.35	4.26
29	65	30.70	6.76
30	84	41.50	9.14
31	105	56.60	12.45
32	119	57.10	12.80
33	Run Omitted		
34	130	61.40	13.50
35	52	24.60	5.41
36	77	36.40	8.00

## Calculated Data

Table III

Run Number	Loading Ratio R	$\frac{\Delta P_f}{\Delta P_a}$	Reynolds Number $N_{Re}$
1	0.7540		42,200
2	0.2604	1.182	79,250
3	0.1935	1.198	94,300
4	0.1510	1.114	105,400
5	0.1475	1.384	110,500
6	0.1434	1.050	115,000
7	0.1517	1.087	122,000
8	0.4740	1.280	63,900
9	0.5630	1.311	52,900
10	0.2830	1.390	46,500
11	0.1585	1.176	63,900
12	0.1004	1.053	76,100
13	0.0716	1.000	95,900
14	0.0648	1.028	104,600
15	0.3810	----	115,000
16	0.1843	1.132	55,800
17	0.4160	1.250	33,400
18	Run Omitted		
19	0.0133	1.820	47,200
20	0.0993	1.225	56,000
21	0.0757	1.025	75,300
22	0.0356	1.083	87,200
23	0.0365	1.060	97,300
24	0.0269	----	107,800
25	0.1415	----	115,000
26	0.1988	1.305	36,300
27	Run Omitted		
28	0.1258		29,800
29	0.0690		47,250
30	0.0329		63,800
31	0.0260		87,100
32	0.0195		86,300
33	Run Omitted		
34	0.0043		94,500
35	0.0678		37,800
36	0.0265		56,000

Minimum Transport Velocity: Orifice Diff. Press. = 0.23

(In. of H<sub>2</sub>O)Pipe Velocity, = 10.15  
(Ft. / Sec.) $N_{Re}$  = 15,610

## Calculated Data

Table IV

Run Number	$\alpha - 1$	$\frac{\alpha - 1}{r}$	$Re \left( \frac{d}{D} \right)^2 \left( \frac{w}{\rho} \right)$
1	-	-	-
2	0.182	0.698	17,320
3	0.198	1.022	20,600
4	0.114	0.755	23,050
5	0.384	2.600	24,150
6	0.050	0.349	25,150
7	0.087	0.573	26,650
8	0.280	0.591	43,960
9	0.311	0.552	11,570
10	0.390	1.375	10,190
11	0.176	1.113	14,000
12	0.053	0.528	16,650
13	0.000	0.000	21,000
14	0.028	0.432	22,900
16	0.132	0.716	12,050
17	0.250	0.601	7,316

### Calculated Data

Distance from the particle feed funnel to the pressure taps  
in pipe diameters.

From particle feed funnel to tap #1 = 1.63 PD  
From particle feed funnel to tap #2 = 3.67 PD  
From particle feed funnel to tap #3 = 7.00 PD  
From particle feed funnel to tap #4 = 10.32 PD  
From particle feed funnel to tap #5 = 13.65 PD  
From particle feed funnel to tap #6 = 17.00 PD  
From particle feed funnel to tap #7 = 23.65 PD  
From particle feed funnel to tap #8 = 37.00 PD  
From particle feed funnel to tap #9 = 56.70 PD

### Particle Screen Analysis

Particle screen using Taylor standard sieves and a Ro-Tap Agitator. A one hundred gram sample was analyzed.

Screen No.	Screen opening, inches	Weight, grams	Percentage by weight
16 mesh	0.0469	0.0	0.0
35 mesh	0.0164	188.5	88.5
48 mesh	0.0116	7.5	7.5
65 mesh	0.0082	3.5	3.5
100 mesh	0.0059	0.5	0.5

Particle Density

Weight sample grams	Volume of H <sub>2</sub> O displaced, cc	Density grams/cc
100	36.7	2.72
100	37.2	2.69
50	18.5	2.70

Average density: 2.703 grams/cubic centimeter  
 $2.703 \times 62.4 = 168.5$  pounds/cubic foot

Miscellaneous Dimensions

Diameter of spouting	3.000 inches
Diameter of orifice	2.0625 inches
Orifice pressure taps (throat)	
upstream	3.2500 inches
downstream	1.5000 inches
Average Particle Diameter	.0295 inches
Constants	
g =	32.17 ft. <sup>2</sup> /sec.
Centipoises to ft.-lb./sec.	.000672

## Discussion of Results

The first consideration of this investigation was the study of the pressure drop as the distance below the point of admittance of the solid was increased.

Pressure readings from 1.835 to 53.7 pipe diameters below the feed funnel gave good correlation with Wood and Bailey's hypothesis (1) of a constant drop beginning 30 to 40 diameters below the admittance of the solid. In the plots of the various runs (Figures 6 to 7b) the constant loss becomes apparent at approximately 25 diameters. As mentioned earlier the type of feed apparatus will determine, to some degree, the location of the point of constancy. In this case the gravity feed dropped the sand in so that there was not as great a transfer of energy to the sand from the air and therefore the pressure losses became constant more quickly. Also the fact that the particles were smaller and lighter would facilitate the transfer. The change in slope of the plots of these runs is much more noticeable at the lower velocities.

The calculations of the total pressure drop using the equations presented earlier (Equations 1-8) gave poor correlation with the experimental work and therefore must be presented with some doubt as to their validity in this case. The calculations have been omitted from this report.

Using the equations of Gasterstadt (9) good agreement was found for the medium loadings. As can be seen in Figures 8 and 9 the curves are mean plots, the best that can be made

from the data. The equation is satisfied by the conditions and can be held as valid for sand in addition to wheat. A more complete and accurate check must be made using other materials before the generalization of this equation can be accepted.

The optimum transport velocity for this system was found by plotting the amount of material transported per unit time against the amount of air per unit time and the velocity. It was found to be approximately 20 ft./sec. The amount transported falls off sharply as the velocity increases. This can be due to pressure being exerted against the sand in the feed apparatus. It is indicated to be so because the decrease continues until a pulsating flow is encountered at about 70-75 ft./sec. The pulsating flow can be attributed to "vapor-lock" or the fact that the pressure exerted by the air is equal to or greater than the pressure forcing the sand into the pipe. This problem would not be present in other types of feed mechanisms, i.e. venturi or pressure feed. A similar plot of loading,  $R$ , vs. Reynolds numbers,  $N_{Re}$ , was made and a similar result obtained. (Figures 10 and 11)

Minimum transport velocities varied considerably. Dalle Vaille's equation gave a value higher than the experimental value. In the first case, the calculated value, a velocity of 17.95 feet per second was found while the second case, the experimental values, velocities of 10.51 and 23.3 feet per second were found. The difference can be accounted for in the fact that the particles were not of uniform size and weight. The observed movement favored the light, smaller particles

and therefore gave a velocity between that of the heavy particles and that of the light particles. An average of the two experimental values gives better agreement: 16.87 to 17.95 (ft./sec.). Dalle Valle's equation does not take into account the variation in size but uses the specific gravity of the mixture.

The plot of the horizontal conveying of wheat and oats (7) compared to a similar plot of sand (Fig. 12) indicates similar behavior to the grains but it is accomplished at much smaller pressures and much slower velocities. The plots both show a large amount of generalization in their curves and therefore should be investigated further.

The lack of correlation between the work done here and previous work can be attributed to many things, the chief source of variance being in the experimental work. The manometers used were not sensitive enough to register small changes of pressure and the scale used could not be read with accuracy. Conversion to inclined type gages would permit a larger scale to be used and necessitate greater movement of the indicating liquid for a small change in pressure. Positive reading, static, gages would permit more rapid and accurate readings which, of necessity, must be done during the short time of the run.

The photographs made of various loadings show a definite variation in the flow density from the top to the bottom of the tube. At low velocities the bottom section of the tube carries the greater portion of the flow but as the velocity

increases the distribution becomes better and the density tends to equalize throughout the tube. These photographs indicate a rapid method of determining loadings and optimum transport velocities. Picture records kept of flows at various known loadings could be used as a nephelometer and compared with unknown flows giving an easy, rapid, and fairly accurate determination of the air rate necessary for greatest transport. Sampling traverses made in conjunction with photographs could be used to determine the density of flow for a given loading. These pictures in turn can be compared with an unknown flow and the loading determined by matching the proper photograph to the flow.

Better photographs will be possible with certain changes in the apparatus. The use of polaroid filters on the lights, in addition to the lens, will cut the reflection to a minimum. Replacement of the glass tubing was necessary after approximately twenty-five runs because the glass surface was pitted to such a degree by the sand that it was impossible to obtain useful photographs. The installation of hard glass or a hard transparent plastic would overcome this difficulty. By constructing the entire unit of a transparent material, from the feed funnel to the last pressure tap, it would be possible to make a photographic study of the entire flow pattern. Any false assumptions in the present flow theories could be found and a more profound study of flow patterns made.

Enlargement of the scope of this investigation to include solids of various physical properties and using various feed mechanisms would give results of greater value to industry as a whole.

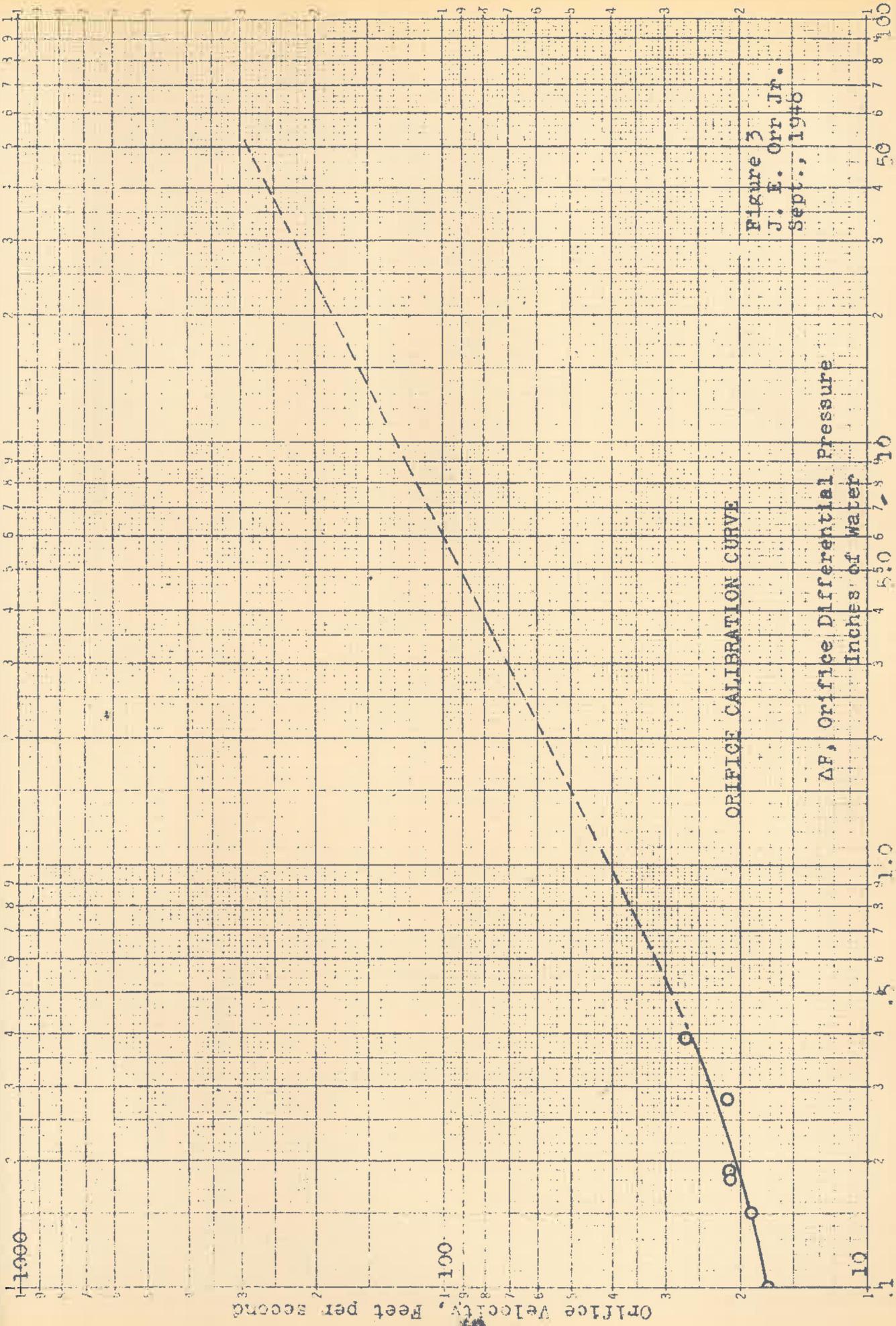


Figure 5  
J. E. Orr Jr.  
Sept., 1946

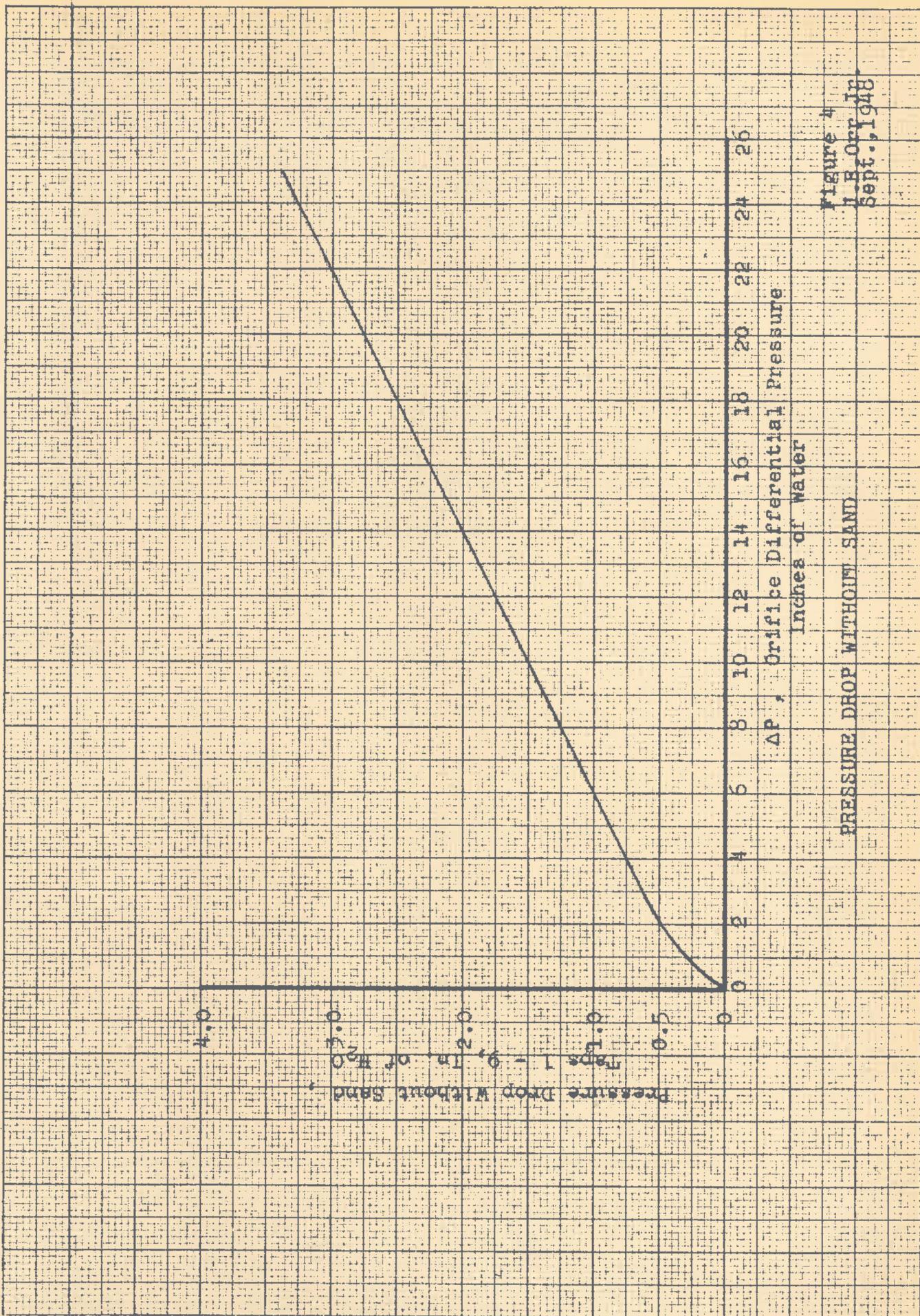


Figure 4  
Sept., 1948

PRESSURE DROP WITHOUT SAND

$\Delta P$ , Orifice Differential Pressure  
Inches of Water

0 2 4 6 8 10 12 14 16 18 20 22 24 26

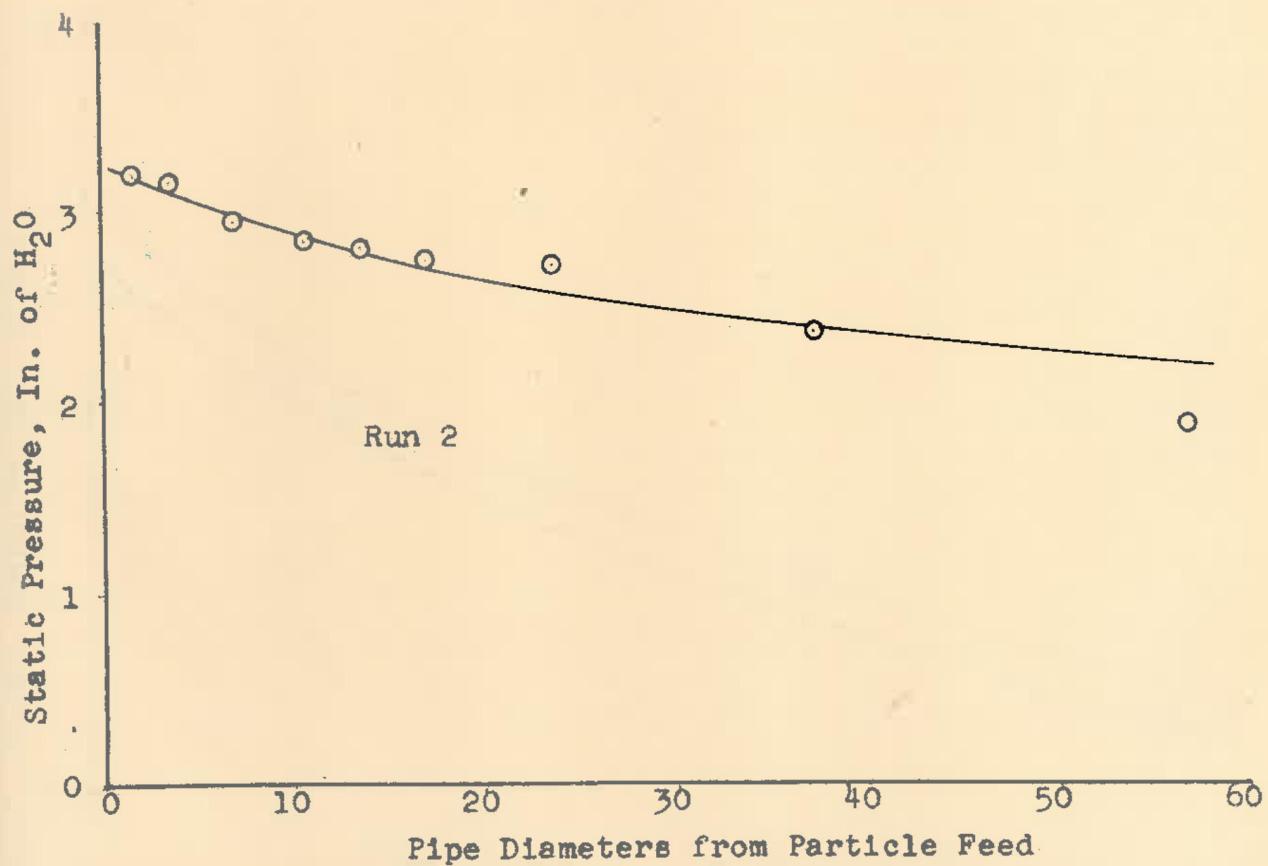
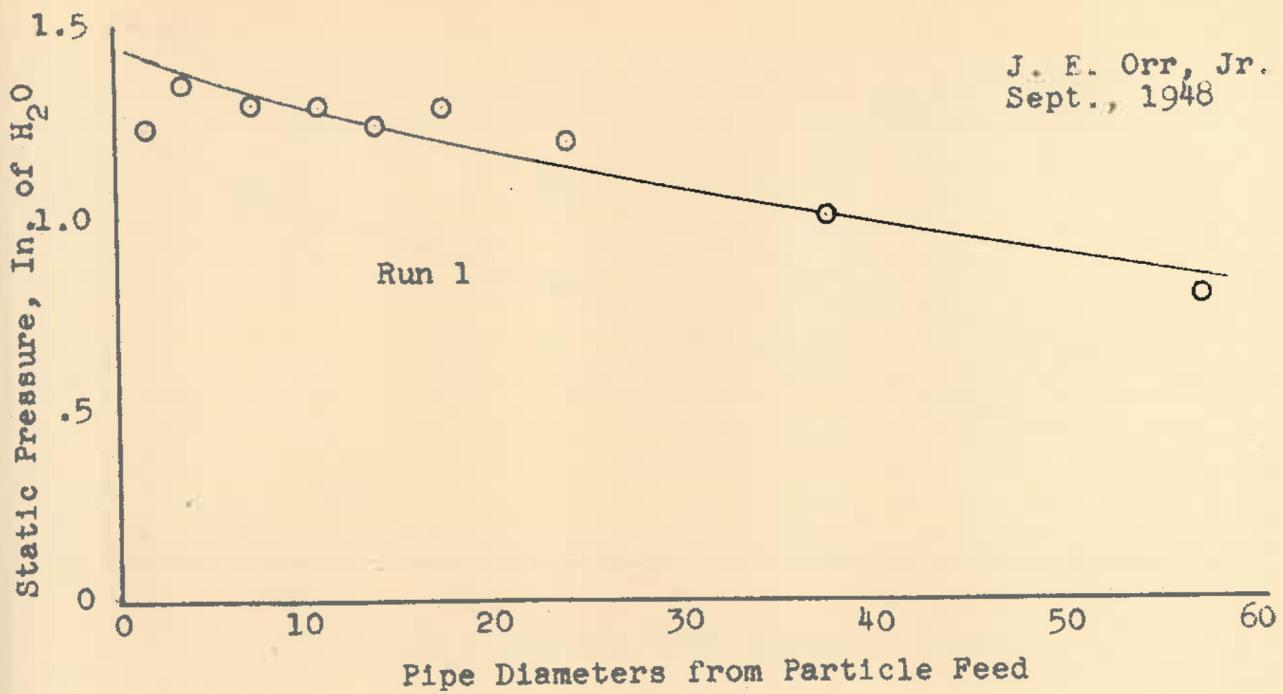


Figure 5a

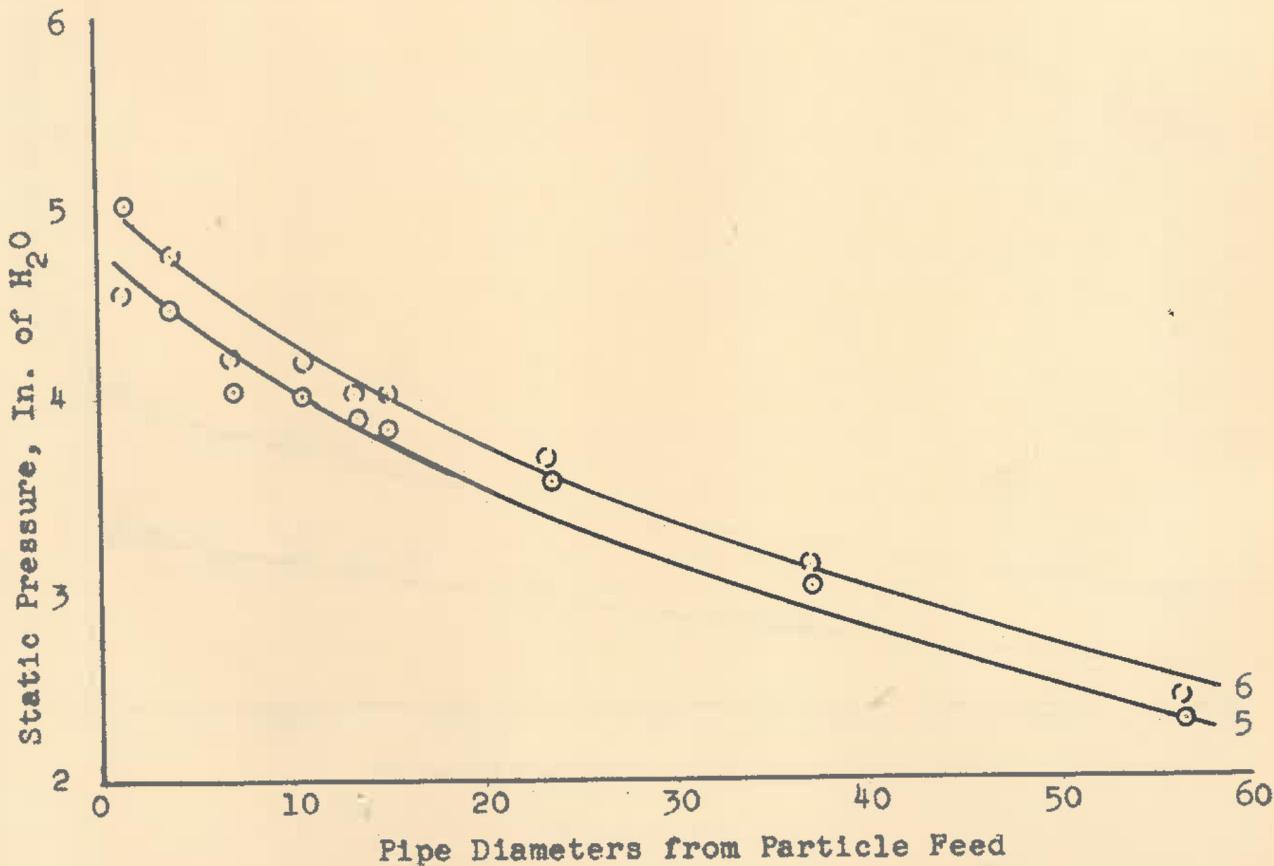
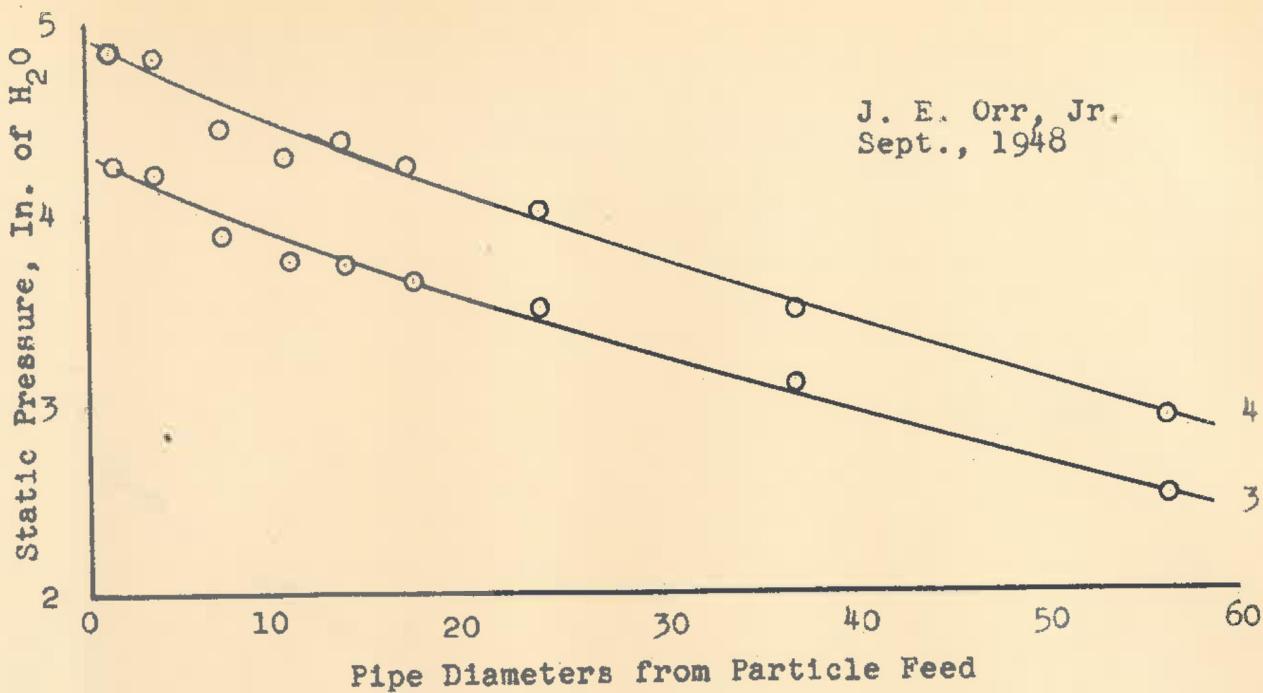


Figure 5b

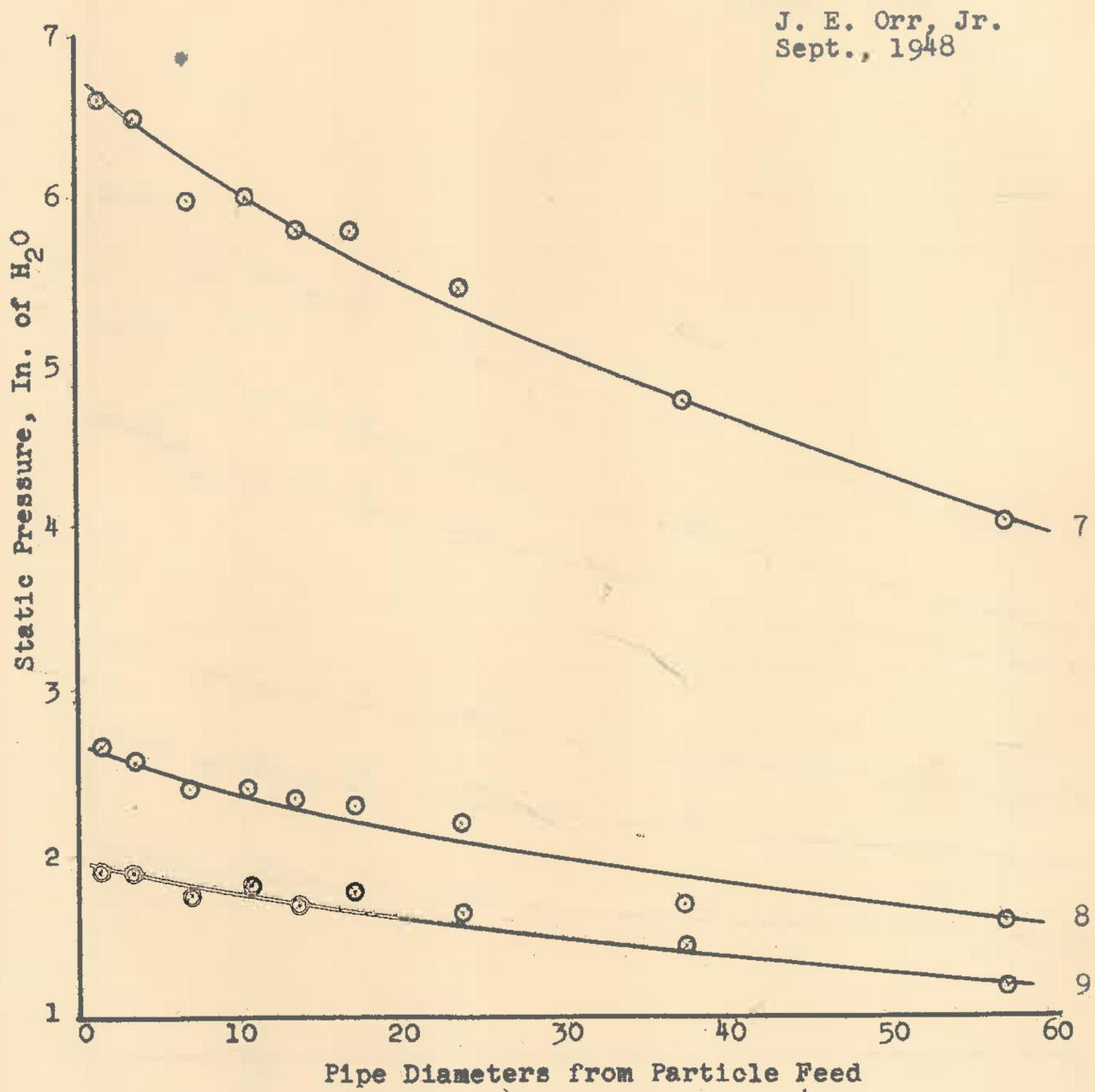


Figure 6

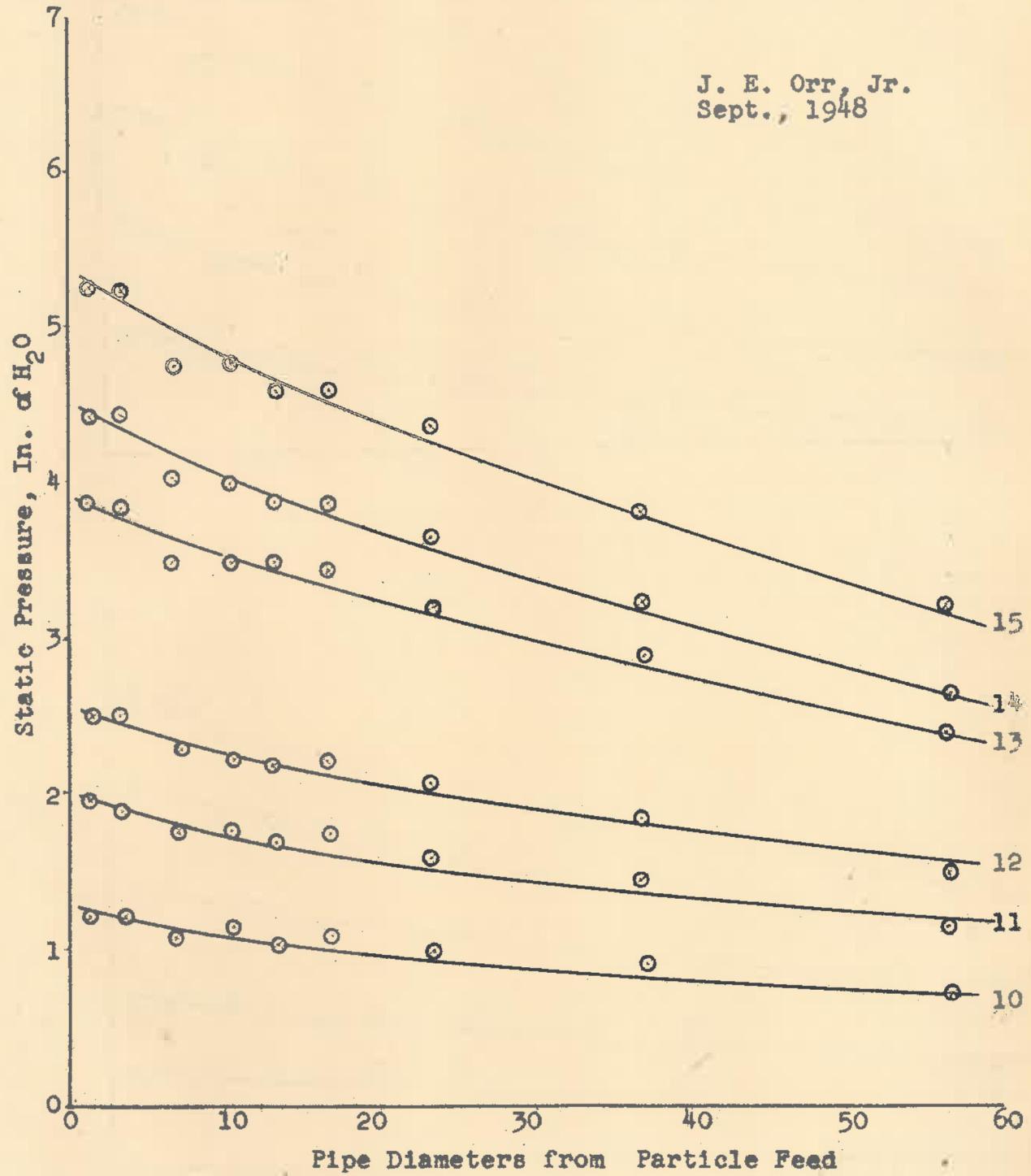
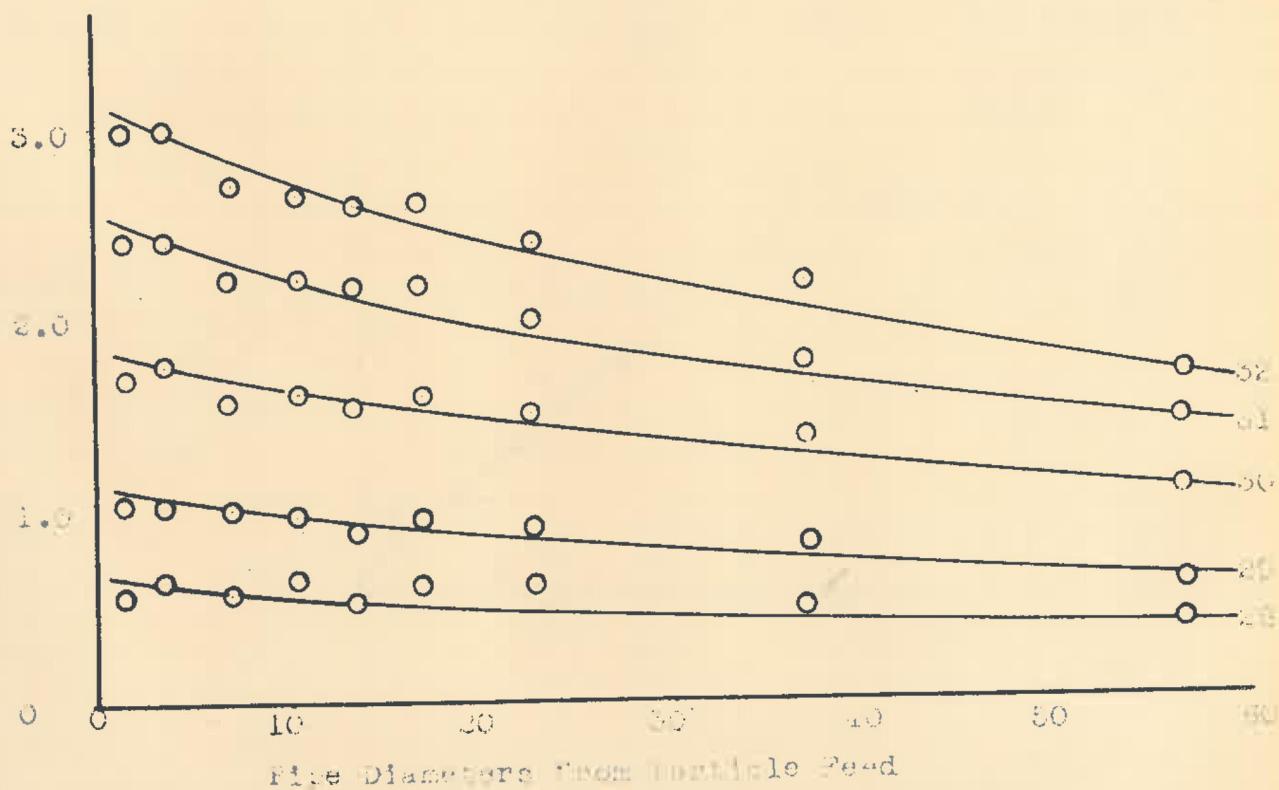
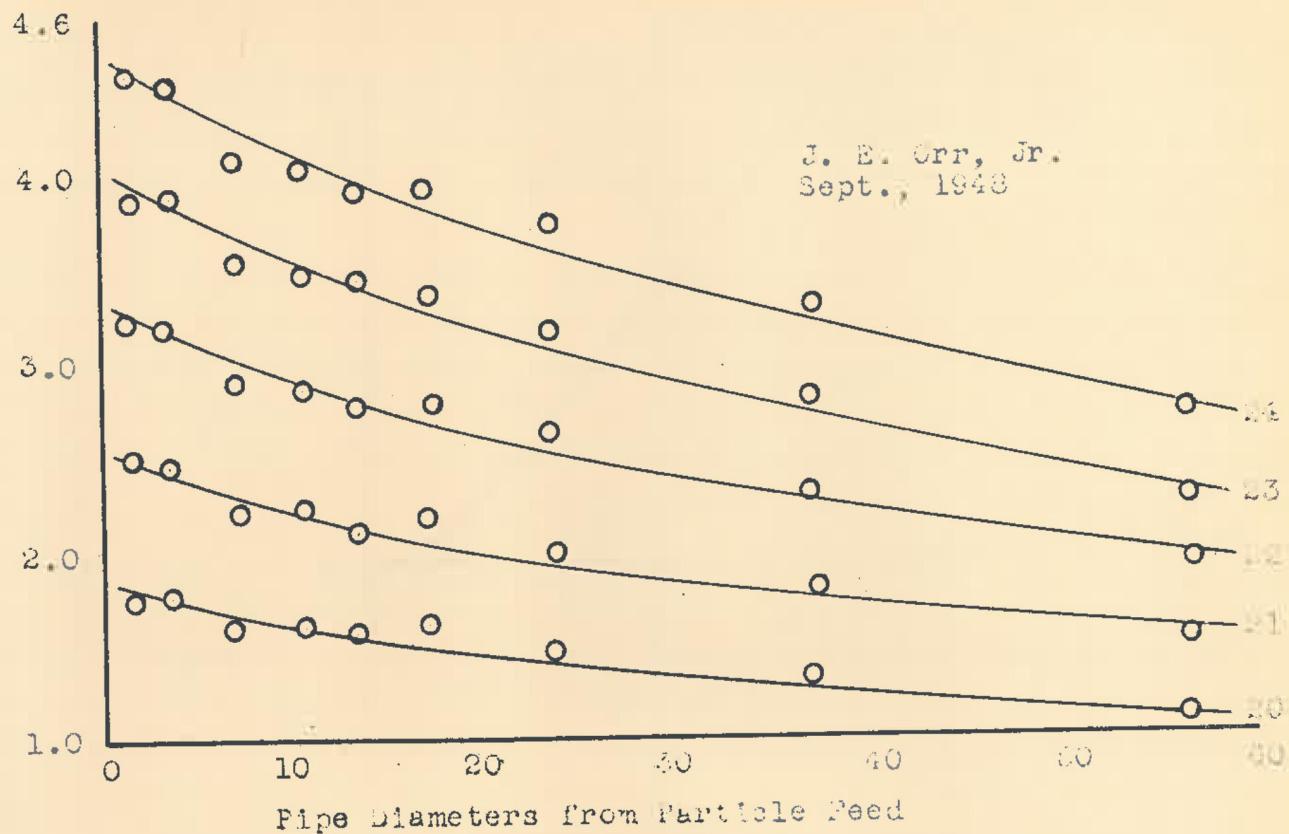


Figure 7a



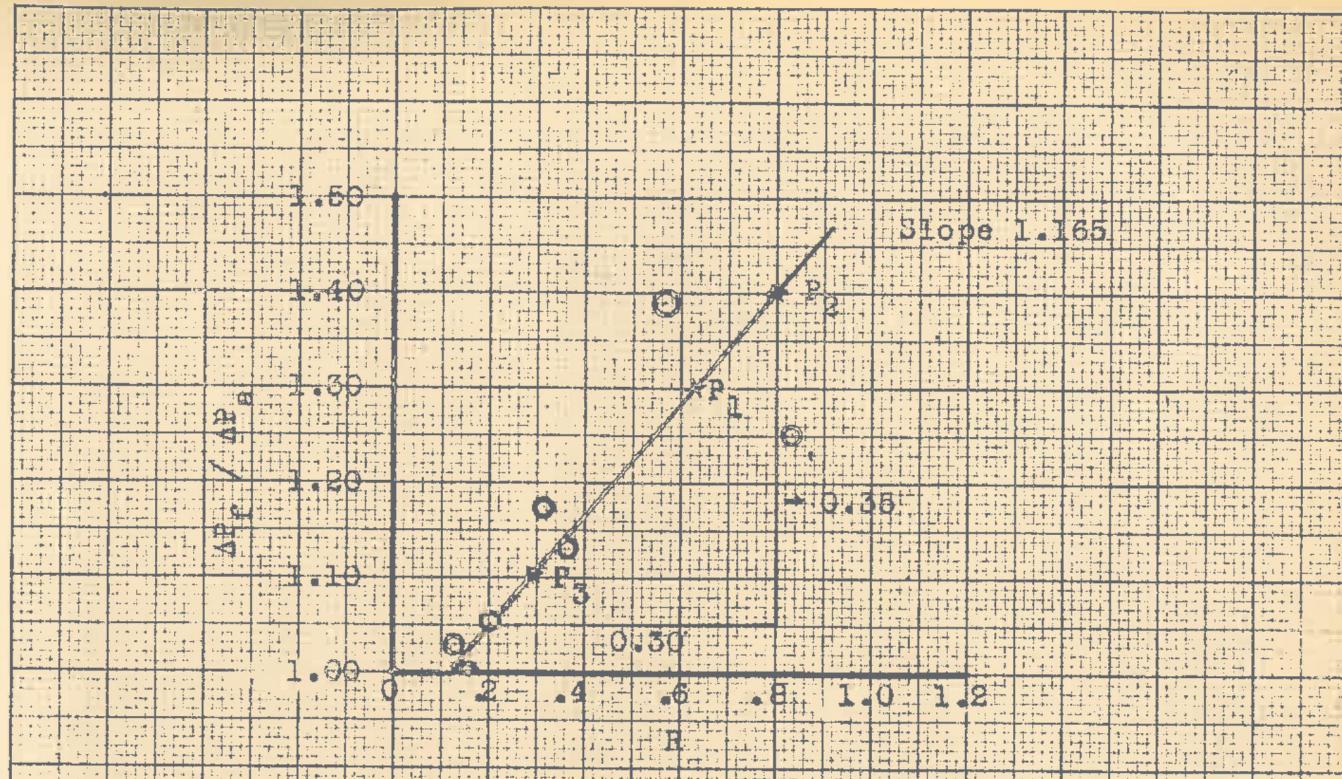


Figure 8

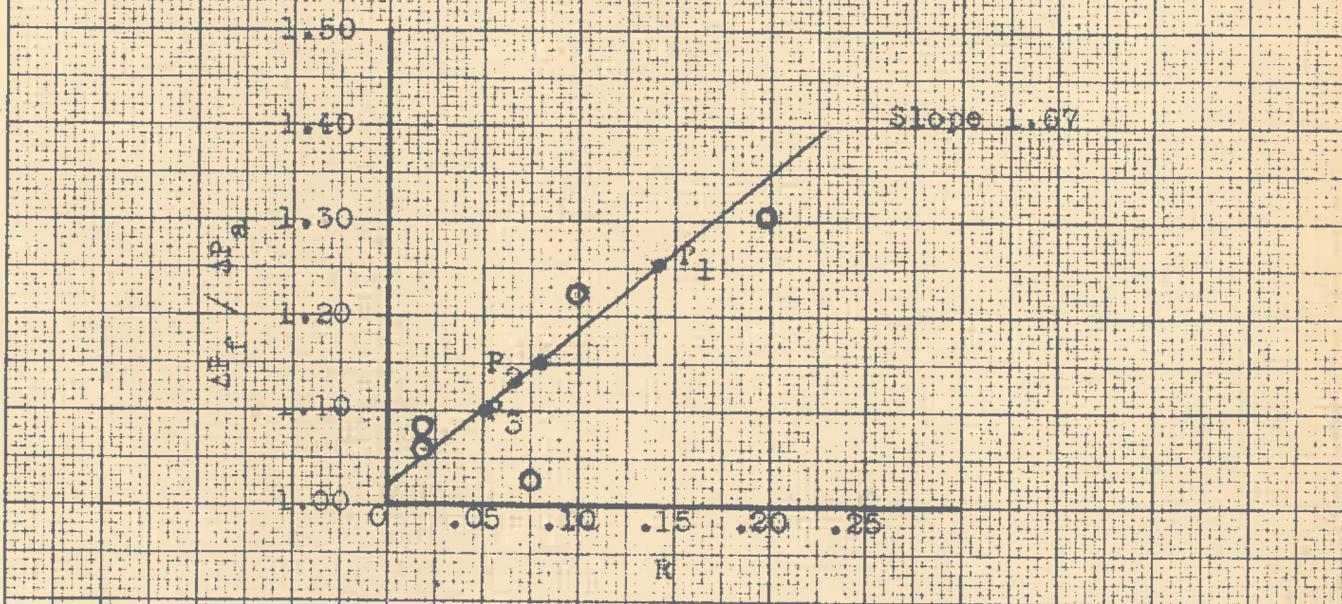


Figure 9

J. H. Orr, Jr.  
Sept., 1948

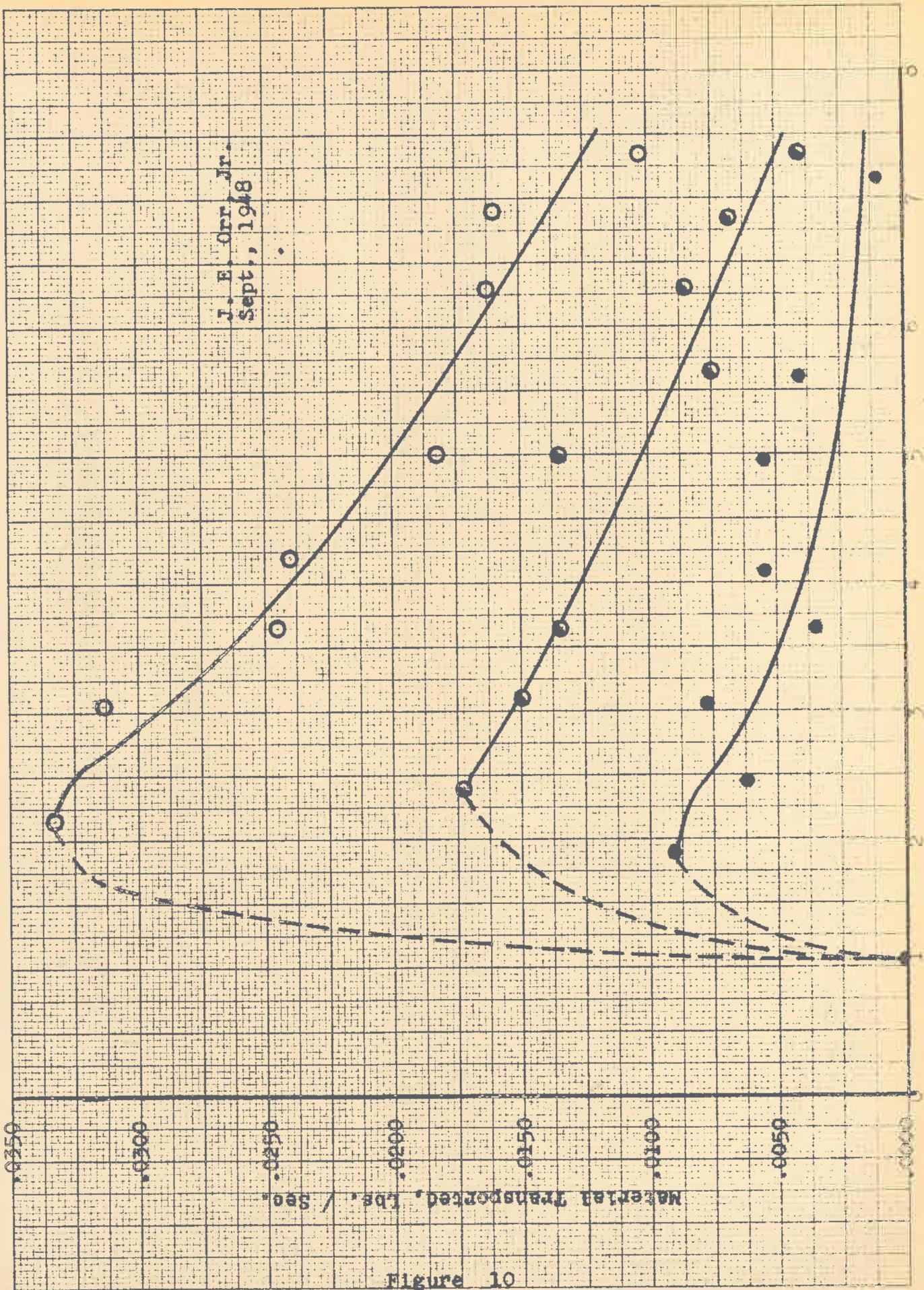
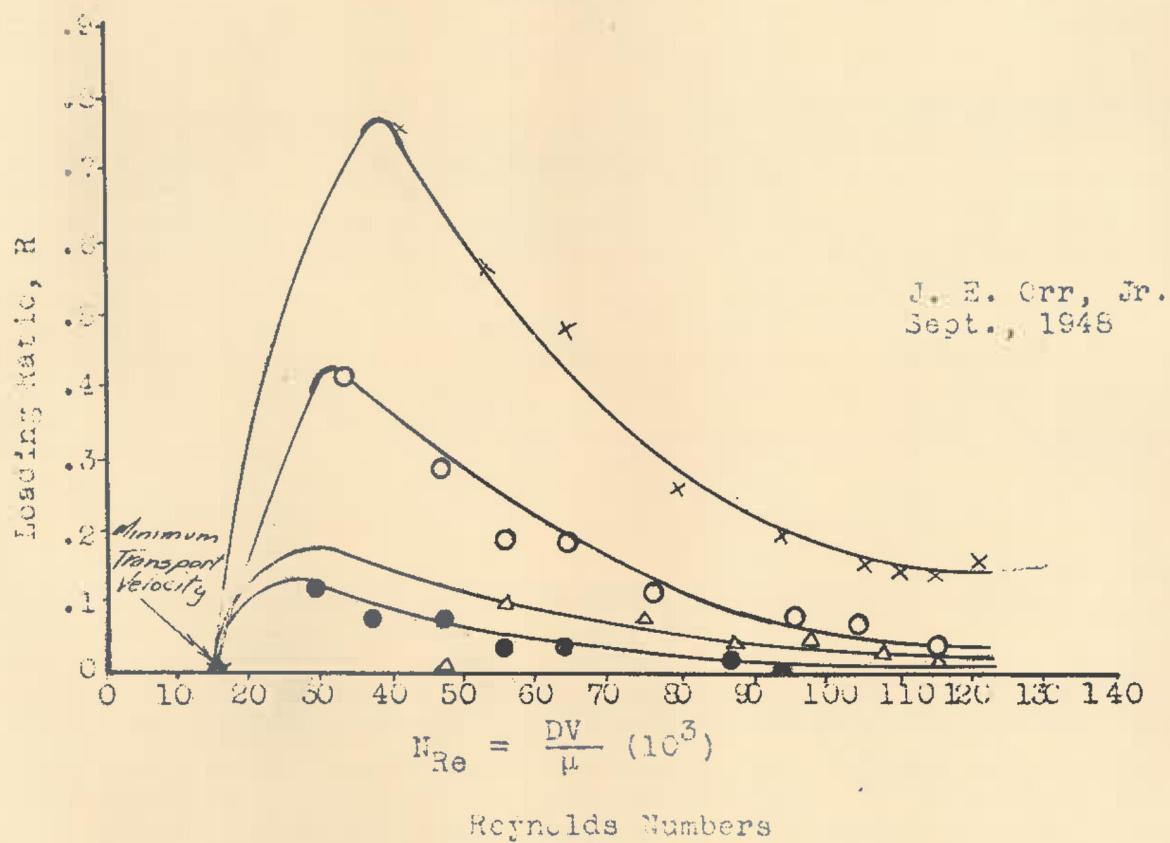


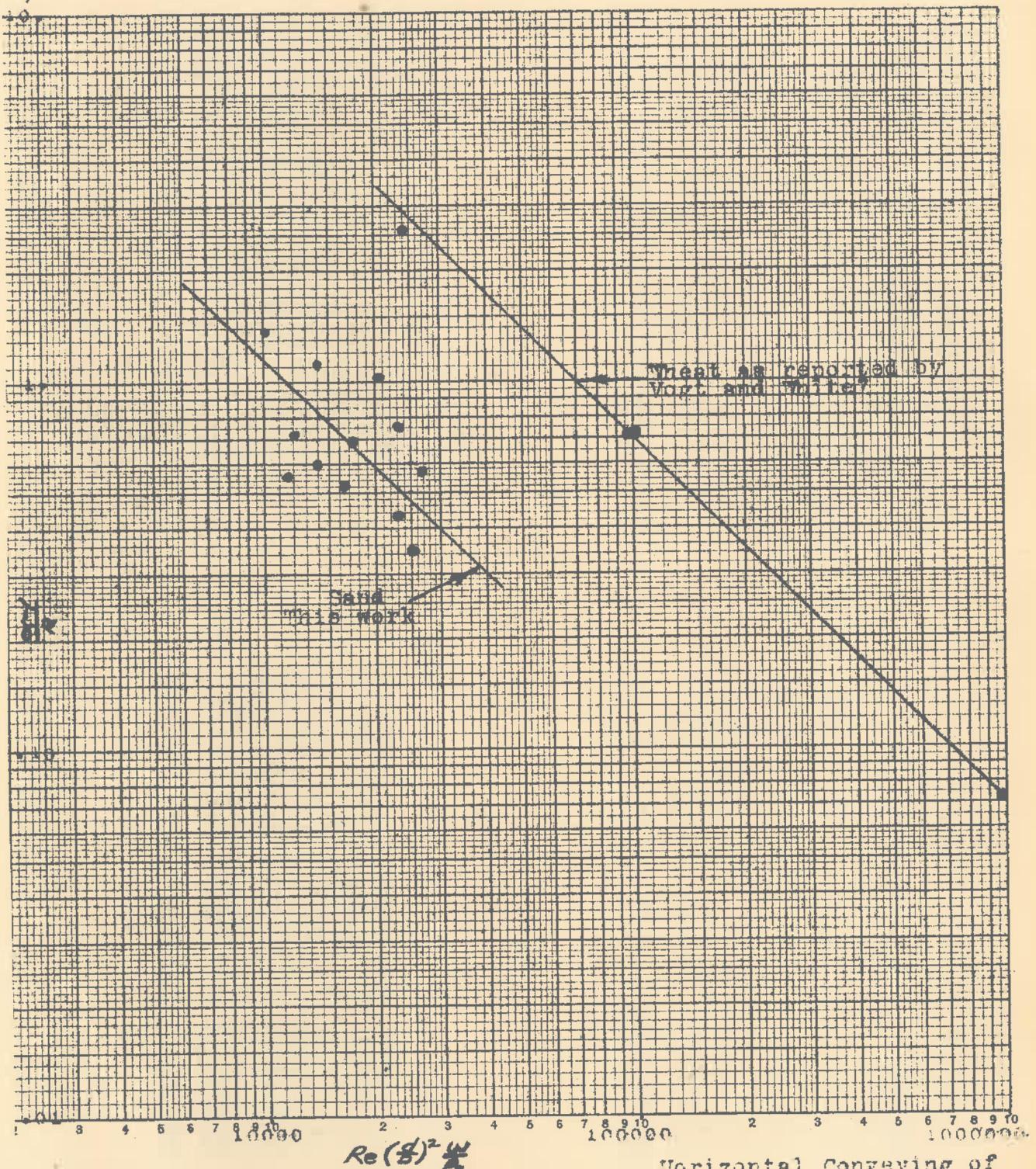
Figure 10

Legend: Runs 1 - 9 ( x )  
 Runs 10 - 17 ( o )  
 Runs 19 - 26 ( △ )  
 Runs 28 - 36 ( ● )



Favorable Transport Velocities

Figure 11



Horizontal Conveying of  
Wheat + Cats with Air  
J. E. Orr Jr. Fig. 12

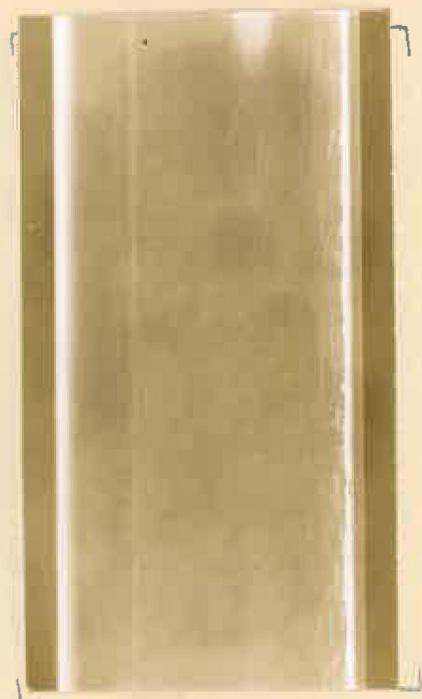


Illustration 13: Visual Determination of Minimum Transport Velocity



Illustration 14: Loading,  $R = \frac{103}{70}$   
Air Velocity,  $V = 70$  ft./sec.



Illustration 15: Loading,  $R = \frac{1475}{71.8}$   
Air Velocity,  $V = 71.8$  ft./sec.



Illustration 16: Loading,  $R = \frac{1434}{74.5}$   
Air Velocity,  $V = 74.5$  ft./sec.

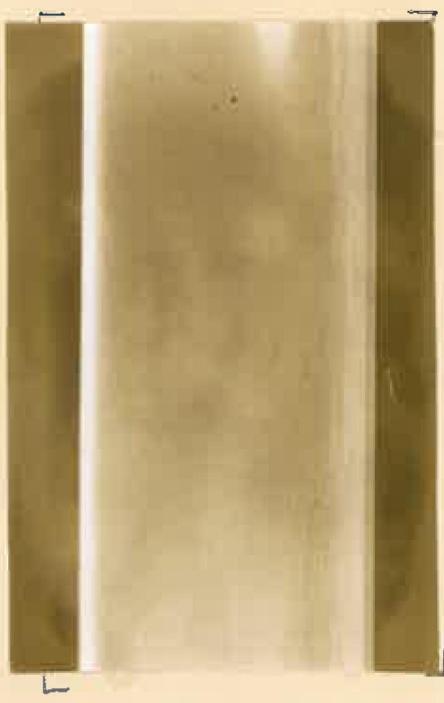


Illustration 17: Loading, R = .0648  
Air Velocity, V = 68 ft./sec.

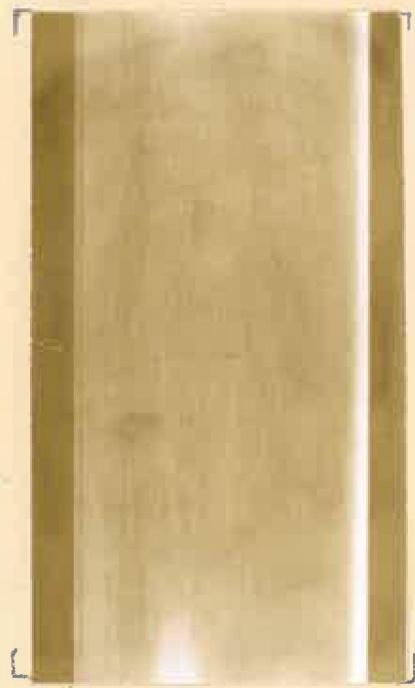


Illustration 19: Loading, R = .1842  
Air Velocity, V = 36.3 ft./sec.



Illustration 18: Loading, R = .0381  
Air Velocity, V = 74.5 ft./sec.

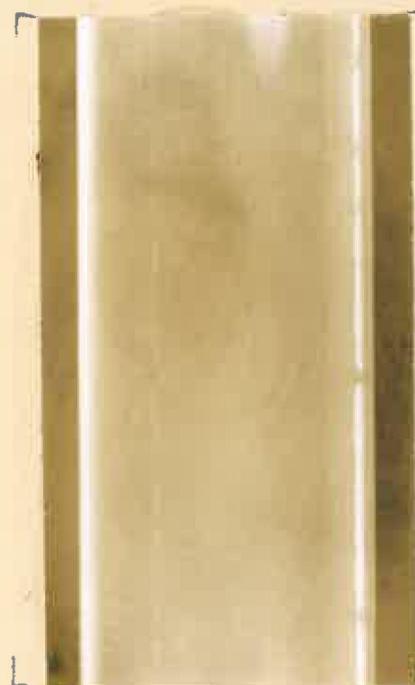


Illustration 20: Loading, R = .1988  
Air Velocity, V = 56.7 ft./sec.

## Sample Calculations

### Orifice Calibration

#### Tank conditions to orifice conditions

$$U_o = \frac{P_t V_t}{P_o} ; U_o = \left( \frac{408.9}{410.6} \right) (4) = 3.98 \text{ cubic feet}$$

#### Volumetric rate of flow through orifice

$$q_o = \frac{U_o}{t} ; q_o = \frac{3.98}{10.3} = .387 \text{ cubic feet/sec.}$$

#### Area of orifice

$$S_o = \frac{\pi D^2}{(144)(4)} ; S_o = \frac{(\pi)(2.062)^2}{(144)(4)} = .0232 \text{ square feet}$$

#### Linear velocity through orifice, U

$$U = \frac{q_o}{S_o} ; U = \frac{.387}{.0232} = 16.7 \text{ feet/second}$$

#### Average density of air

$$P = P - \frac{1}{2}dP ; P = 410.6 - \frac{1}{2} (.08) = 410.56 \text{ inches of water}$$

$$\rho_a = (.0808) \left( \frac{492}{540} \right) \left( \frac{410.56}{407.2} \right) = .0743 \text{ lb./cubic foot}$$

#### Fluid Head

$$H = \frac{dp}{12} \frac{\rho_{H_2O}}{\rho_{air}} ; H = \left( \frac{.08}{12} \right) \left( \frac{62.3}{.0743} \right) = 5.59 \text{ feet of Air}$$

#### Orifice coefficient, C,

$$C = \frac{U}{\sqrt{2gH}} \quad C = \frac{16.7}{\sqrt{(64.4)(5.59)}} = .820$$

Reynolds Numbers, N<sub>Re</sub>

$$N_{Re} = \frac{DV\rho}{\mu} ; N_{Re} = \frac{(2.0625)(16.7)(.0743)}{(12)(.000672)(.018)}$$

$$N_{Re} = 17,640$$

Particle feed rate, lbs./min.

$$W = \frac{(5500 \text{ gms})(60 \text{ sec.})(\text{lbs.})}{(159.4 \text{ sec.})(\text{min.}) (453.6 \text{ gms.})}$$

$$W = 4.55 \text{ lbs./min.}$$

Weight Rate of Air Flow, lbs./min.

$$W_a = (27.36 \frac{\text{ft.}}{\text{sec.}})(60 \frac{\text{sec.}}{\text{min.}})(0.491 \frac{\text{ft.}^2}{\text{min.}})(.0745 \frac{\text{lb.}}{\text{ft.}^3})$$

$$W_a = 6 \text{ lb./min.}$$

Mass Velocity of Air, G<sub>a</sub>, lbs./sec. ft.<sup>2</sup>

$$G_a = \frac{(6 \text{ lbs.}) (\text{min.})}{(\text{min.}) \frac{(60 \text{ sec.}) (.0491 \text{ ft.}^2)}{}}$$

$$G_a = 2.03 \text{ lbs./sec. ft.}^2$$

Loading Ratio, R

$$R = \frac{\text{Wt. material transported, lbs./min.}}{\text{Wt. air moved, lbs./min.}}$$

$$R = \frac{4.55 \text{ lbs./min.}}{6 \text{ lbs./min.}} = .758$$

Reynolds Number

$$N_{Re} = \frac{DVF}{\mu}$$

$$N_{Re} = (3/12)(27.4/.000672)(.0745/.018)$$

$$N_{Re} = 42,200$$

$$\frac{\Delta P_f / \Delta P_a}{\Delta T_f / \Delta T_a} \text{ For use in Gasterstadt Plots}$$
$$\frac{\Delta P_f / \Delta P_a}{\Delta T_f / \Delta T_a} = 1.370/1.10 = 1.18$$

Gasterstadt Equation :  $\frac{\Delta Pf}{\Delta Pa} = 1 \neq RS$

Runs 10 - 17 From fig.  $S = 1.165$

$$(P_1) 1.3 \approx 1 \neq (.31)(1.165) = 1.35$$

$$(P_2) 1.4 \approx 1 \neq (.4)(1.165) = 1.46$$

$$(P_3) 1.1 \approx 1 \neq (.15)(1.165) = 1.175$$

Runs 19-26 From fig.  $S = 1.67$

$$(P_1) 1.25 \approx 1 \neq (.14)(1.67) = 1.234$$

$$(P_2) 1.15 \approx 1 \neq (.08)(1.67) = 1.134$$

$$(P_3) 1.1 \approx 1 \neq (.05)(1.67) = 1.084$$

#### Average Particle Size

$$D_{35} : \frac{.0469 \neq .0164}{2} = .03165$$

$$D_{48} : \frac{.0164 \neq .0116}{2} = .0140$$

$$D_{65} : \frac{.0116 \neq .0082}{2} = .0099$$

$$D_{100} : \frac{.0082 \neq .0058}{2} = .0070$$

$$D = \frac{(.03165)(88.5) \neq (.0140)(.075) \neq (.0099)(.035)}{(.0070)(.005)} \text{ inches}$$

$$D = .02953 \text{ inches}$$

#### Minimum transport velocity

$$(a) 1.55 \text{ in H}_2\text{O} \quad 23.6 \text{ ft./sec.}$$

$$.23 \text{ in H}_2\text{O} \quad 10.15 \text{ ft./sec.}$$

$$\text{Average} \quad 16.87 \text{ ft./sec.}$$

(b) Dalle Valle Equation

$$v = 6000 \left( \frac{2.7}{2.7 + 1} \right)^{0.348} d$$

$$v = 6000 \left( \frac{2.7}{3.7} \right) (.0295)^{0.398}$$

$$v = 17.95 \text{ ft./sec.}$$

### Nomenclature

- $\alpha$  = relative pressure drop, dimensionless  
A = cross-sectional area of pipe, sq. ft.  
c = orifice coefficient, dimensionless  
D = diameter of pipe, ft.  
d = particle diameter, inches  
f = friction factor, dimensionless  
G = mass velocity of air in pipe, lbs./sec.(sq. ft.)  
 $g_c$  = conversion factor,  $\frac{\text{lbs.}}{\text{lb. force}} \frac{\text{ft.}}{\text{sec.}^2}$   
H = fluid head, ft. of fluid flowing  
k = empirical constants  
L = length of pipe, including equivalent length for bends, ft.  
 $N_{Re}$  = Reynolds Number, dimensionless  
P = pressure, in. H<sub>2</sub>O  
 $\Delta p$  = pressure drop, lbs. force/sq. ft.  
 $\Delta p_a$  = pressure drop for air alone, lbs. force/sq. ft.  
 $\Delta p_{accel}$  = pressure drop for acceleration, lbs. force/sq. ft.  
 $\Delta p_f$  = pressure drop due to friction for the mixture, lbs. force/sq.ft.  
 $\Delta p_v$  = pressure drop to support air and solids, lbs. force/sq.ft.  
n = loading  
 $R = N_s/N_a$   
s = slope, dimensionless  
 $\gamma$  = specific gravity  
t = time, sec.  
 $V_a$  = superficial velocity of air, ft./sec.  
 $V_t$  = minimum transport velocity, ft./sec.

- $v_s$  = velocity of solids, ft./sec.  
 $v$  = volume, cu. ft.  
 $w_a$  = weight of air, lbs.  
 $w_s$  = weight of solids, lbs.  
 $w_a$  = weight rate flow of air, lbs./sec.  
 $w_s$  = weight rate flow of solids, lbs./sec.  
 $\mu_a$  = absolute viscosity of air, lb./(ft.)(sec.)  
 $\rho_a$  = density of air, lbs./cu. ft.  
 $\rho_m$  = density of mixture, lbs./cu.ft.  
 $\rho_s$  = true density (not bulk density) of solids, lb./cu.ft.

### References

- (1) Alves, G. E., "Solid-Gas Flow In Pipes", University of Delaware, Division of Chemical Engineering, Newark, Delaware, 1948, pg. 25-30.
- (2) Dalle Valle, J. M., "Determining Minimum Air Velocities for Exhaust Systems", Heating, Piping and Air Conditioning, 4 639-41 (1932) September.
- (3) Gasterstadt, "The Experimentelle untersuchung des Pneumatischen Fördervorganges", Z. V. D. I. 68, 617-624 (1924).
- (4) Kiddoo, R. C., "The Flow of Solid Vapor Suspensions Through Circular Pipes", Thesis, University of Delaware, Newark, Delaware, 1948.
- (5) Perry, J. H. "Chemical Engineer's Handbook", McGraw-Hill Book Co., New York, New York, 1938.
- (6) Walker, Lewis, McAdams, and Guilliland, "Principles of Chemical Engineering", McGraw-Hill Book Co., New York, Third Edition, 1938.
- (7) Vogt, S. G., White, R. H., "Friction in the Flow of Suspensions", Ind. Engn. Chem. 40, 1731-1738, 1948, (Sept.).