EXERCICE 36
Logique - Quantificateurs

quelle que soit la relation $\mathbf{R}$,
quelle que soit la constante a,
$((\forall x, (\mathbf{R}(x, a) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))) \Rightarrow (\exists x, (\mathbf{R}(x, x) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))))$

Soit la constante a

Montrons $(\forall x, (\mathbf{R}(x, a) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))) \Rightarrow (\exists x, (\mathbf{R}(x, x) \Leftrightarrow (\exists y, \mathbf{R}(y, x))))$ (1)

($\Rightarrow$I)

> *Supposons que* $\forall x, (\mathbf{R}(x, a) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))$ (h1)
>
> *Montrons* $\exists x, (\mathbf{R}(x, x) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))$ (2)
>
> *($\exists$I)*
>
> > *Montrons* $\mathbf{R}(a, a) \Leftrightarrow (\exists y, \mathbf{R}(y, a))$ (3)
> >
> > *($\forall$E)*
> >
> > > *Montrons* $\forall x, (\mathbf{R}(x, a) \Leftrightarrow (\exists y, \mathbf{R}(y, x)))$ (4)
> > >
> > > *d'après* (h1)