ECE536 Computer Vision Assignment 1

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Answer:

2.1

a) No answer, because $v = [1\ 2\ 3]$ is a vector while 1 is a scalar, they cannot add up.

b) No answer, because v is of size 1×3 while v^T is of size 3×1, the dimension doesn't match.

c) $v \cdot v^T = [1 \quad 2 \quad 3] \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 14$

d) If we consider $\boldsymbol{B}$ as three columns of vector $[b_1 \quad b_2 \quad b_3]$ and calculate $v \times b_1$, $v \times b_2$, $v \times b_3$ separately:

$$v \times b_1 = \begin{bmatrix} 2 \cdot 5 - 3 \cdot 3 \\ 3 \cdot 1 - 1 \cdot 5 \\ 1 \cdot 3 - 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$v \times b_2 = \begin{bmatrix} 2 \cdot 6 - 3 \cdot 4 \\ 3 \cdot 2 - 1 \cdot 6 \\ 1 \cdot 4 - 2 \cdot 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$v \times b_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Then $\quad v \times \boldsymbol{B} = [1 \quad 2 \quad 3] \times \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

e) $v \cdot \boldsymbol{B} = [1 \quad 2 \quad 3] \cdot \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 0 \end{bmatrix} = [22 \quad 28 \quad 0]$

f) $eig(A) = \det(A - \lambda \cdot I)) = \det\left(\begin{pmatrix} 3 & 2 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right) = \det \begin{pmatrix} 3-\lambda & 2 & 0 \\ 1 & 2-\lambda & 1 \\ -1 & 0 & 1-\lambda \end{pmatrix}$

$$= -\lambda^3 + 6\lambda^2 - 9\lambda + 2$$

Let $-\lambda^3 + 6\lambda^2 - 9\lambda + 2 = 0 \qquad \lambda = 2, 2+\sqrt{3}, 2-\sqrt{3}$

g) Property of eigenvalue/eigenvectors:

One of the functions of calculating eigenvalue/eigenvectors is to reduce the dimension of the original matrix, therefore is useful for data compression.

The eigenvalue/eigenvectors can be used to represent original matrix A in the eigenspace. Matrix A can be recovered from eigenvalue/eigenvectors: $A = P\Lambda P^{-1}$, where P is the matrix composed of eigenvectors $v_1, v_2, \ldots, v_n$ and $\Lambda = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_n \end{pmatrix}$. The larger the eigenvalue λ_i is, the more important its eigenvector is to the original matrix A. By deleting the smaller eigenvalues and keep the larger ones, most information of the matrix A is preserved and the data is compressed (dimension reduction).