Numerical Methods (MAT 370) - Finding Conditions for Cyanobacteria Dominance

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September 6, 2020

1 Introduction

Aquatic ecosystems are composed of many different levels, from the large organisms such as fish to the micro-organisms like phytoplankton. Each level is important for the overall health and stability of the ecosystem. Phytoplankton are the foundation for many river ecosystems, and maintaining healthy phytoplankton communities is an important aspect of river management. One of the possible dangers is a low diversity community, which is compounded when the dominant specie(s) is less nutritious or even harmful for the larger organisms that rely on phytoplankton as a food source. Cyanobacteria are a genus of phytoplankton that can sometimes produce toxins as well as out-compete more nutritious phytoplankton species. A large bloom of cyanobacteria that dominates the community can have a strong negative effect on the ecosystem as a whole.

One issue in managing rivers to prevent these kinds of blooms is a lack of research. In the Upper Mississippi River (UMR), for example, very little research currently exists on the phytoplankton communities, with even less focusing on the impact of cyanobacteria and what might cause harmful blooms. Over the summer, I worked with the University of Wisconsin La-Crosse and the USGS to better understand the phytoplankton of the UMR using 181 samples collected by the USGS from 1996 to 2012 spanning from Minnesota to Missouri. This led to the creation of several models that helped to understand what can lead to a rise in cyanobacteria in the communities. These models helped connect the most important environmental factors to changes in phytoplankton genera, which can then be used to estimate future changes in the community.

2 The Problem

When cyanobacteria become dominant in a community, serious harm is done to the ecosystem. Typically, this is set at 80% of the total phytoplankton abundance. The first model used to find the conditions that could lead to a bloom in cyanobacteria measures the impact other phytoplankton genera have on cyanobacteria relative abundance.

$$\text{Cyanobacteria } \% = -.77x_1 - .69x_2 - .724x_3 - .555x_4 - .944x_5 - .919x_6 + 77.391$$

such that each x_i represents the relative abundance of another phytoplankton genera. In order to combine with the second model, this model has been simplified into terms of the relative abundance of bacillariophyta, the phytoplankton genera that typically composes 70% or more of phytoplankton communities in the UMR. The simplified model for relative abundance of cyanobacteria is:

$$\text{Cyanobacteria } \% = -.408x_1 + 38.282$$

The second model measures the impact of changes in the nitrogen to phosphorus ratio on bacillariophyta. This model was selected because nitrogen to phosphorus ratio was connected to phytoplankton community health during my summer research with moderate to strong correlations with changes to phytoplankton genera abundance.

$$\text{Bacillariophyta } \% = 75.008 + 20.181x - 14.33x^2$$

When combined with the first model, we have an equation that can minimize the most abundant phytoplankton population while also maximizing the relative abundance of cyanobacteria:

$$\text{Cyanobacteria } \% = 7.679 - 8.234x + 5.847x^2$$

Setting this equal to 80, the level at which cyanobacteria would dominate the community, we have an equation with a root at the nitrogen to phosphorus ratio that could lead to a harmful cyanobacteria bloom:

$$g(x) = -72.321 - 8.234x + 5.847x^2$$

Finding the root of this equation provides an important level for organizations such as the Upper Mississippi River Restoration Program (UMRR) to use when monitoring the health of the river and phytoplankton communities.

3 Methodology

To find the root of this equation, the Secant Method was used to search for the root of the equation $g(x)$. This method was selected because of the fast convergence and ease of setting up the equation. Initial approximations were identified graphically using Octave. After plotting the function, I identified a root between 4 and 5. Using $p_0 = 4, p_1 = 4.1, TOL = 10^{-5},$ and $N_0 = 10^2$, I was then able to estimate the root of the equation.

4 Results

After five iterations, an approximation for the root was found at $x \approx 4.2909$. This represents the ratio between available nitrogen and phosphorus in the water, the two most important nutrients for phytoplankton. Research from the summer confirms this generally in that most phytoplankton prefer much higher nitrogen levels than phosphorus, while cyanobacteria thrive in environments with a smaller nitrogen to phosphorus ratio. This number allows for members of the UMRR and other interested groups to monitor the river to ensure that nitrogen and phosphorus levels do not approach this ratio, and so helping to prevent the large blooms of cyanobacteria that can be so detrimental to the environment.