Numerical Methods (MAT 370) - Interpolation

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1 Introduction

The Great Lakes are one of America's most beautiful natural resources. As such, maintaining the water quality is a major concern for the Environmental Protection Agency (EPA), who oversees resource management for the Great Lakes. One of the levels monitored by the EPA is the chlorophyll levels of the lakes, recorded in $\frac{mg}{L}$. These levels correspond with the amount of plant life in the water, which can be used to determine how murky or clear the water is. Additionally, the level of phosphorus, also recorded in $\frac{mg}{L}$, often corresponds to the chlorophyll levels. By using samples of both chlorophyll and phosphorus levels, an interpolating polynomial can be derived that allows for approximations of future chlorophyll levels given a recorded phosphorus level.

2 The Problem

In Spring 2017, 7 samples were taken from the Great Lakes: one from each lake, and three from Lake Erie (Western, Central, and Eastern regional samples). When trying to estimate past chlorophyll levels from 1999, all that is known is that waste treatment attempted to keep the phosphorus concentration of western Lake Eerie to $15\frac{mg}{L}$. However, the chlorophyll concentration was not recorded and so must be approximated from the samples provided.

3 Methodology

Using the 7 samples provided, a Lagrange polynomial was constructed to approximate the concentration of chlorophyll for a given phosphorus concentration. Since there are few data points, there is less of a concern of over-fitting the data and the ease of computing the Lagrange Polynomial makes it a good choice for this approximation. Additionally, because the $15\frac{mg}{L}$ concentration of phosphorus falls between two of the samples given (Lake Michigan and Eastern Lake Erie) and not particularly near either of these points (8 and 17.5), the approximation of the value of interest in 1999 should be reasonably accurate if extreme oscillation does not occur between these points. Using Octave, the Lagrange polynomial was graphed over the domain $[0, 35]$ to ensure that no extreme oscillation occurred near the value of interest that might result in a faulty approximation. After this, the input of $15\frac{mg}{L}$ was given to estimate the chlorophyll level in western Lake Erie in 1999.

4 Results

After graphing the polynomial on the domain $[0, 35]$, seen in Figure 1, no severe oscillation was noticed near 15. This assured that this approximation provided by

the polynomial would not drastically differ from the actual value. Using the value $15\frac{mg}{L}$ as the input phosphorus concentration, the Lagrange Polynomial approximated the chlorophyll concentration to be $3.5839\frac{mg}{L}$ in western Lake Erie in 1999. This is a reasonable approximation, as the two data points closest to $15\frac{mg}{L}$ had corresponding chlorophyll levels 2 and $3.8\frac{mg}{L}$, effectively bounding what the result of the approximation should be. For approximations where the input of phosphorus is greater than $30\frac{mg}{L}$, however, more data should be collected, as only one data point lies in this range and so the sharper curve that occurs after $30\frac{mg}{L}$ may be inaccurate. Additionally, for smaller values the polynomial oscillates below 0 although negative values are impossible for concentrations. However, for phosphorus concentrations between 5 and $25\frac{mg}{L}$, this polynomial should provide a reasonable approximation.

5 Figures

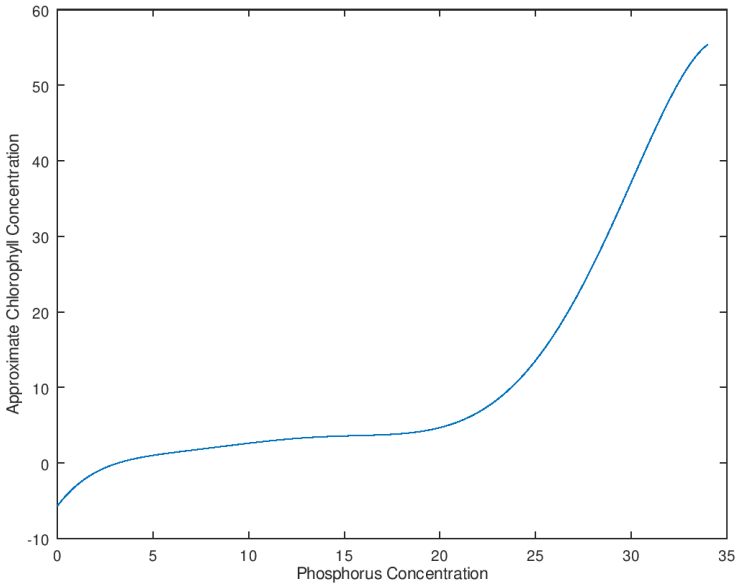

Figure 1: The constructed Lagrange Polynomial on the domain [10,20]