Numerical Methods (MAT 370) - Integration

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October 6, 2020

1 Introduction

Designing and producing new aircraft requires great attention to detail to ensure the aircraft can safely take flight. One of the important details is the volume of the aircraft's body. Though it is one thing to carefully design the aircraft and know the volume, after a model is created and tested the body may have slightly changed, requiring a new measurement of volume. By finding the area of cross-sections of the aircraft, the total volume can be found after the testing is finished.

2 The Problem

Given a profile of an airfoil as two curves over the domain $[0, 30]$ and data describing the two curves $f(x)$ and $g(x)$, find the area of the airfoil to help find the volume of the aircraft after testing.

3 Methodology

18 data points were provided to describe the two curves f and g. By providing a numerical approximation of the two curves and combining the two approximations, the total area of the airfoil can be found. The Composite Simpson's Rule was used because the error for this approximation is high order. This allows for more data to produce a significantly more accurate approximation using the same code. Because the data points were not evenly spaced out, h was calculated for each point. For the first and final points $h = (b - a)/18 = 30/18 \approx 1.667$ for the 18 data points. For the intermediate points, $h = x_i - x_{i-1}$. Instead of multiplying the sum of $c_i f(x_i)$ by a constant $h/3$ as can be done for evenly spaced data points, each term was multiplied individually by its h_i. Although this introduces the potential for roundoff error, no h is extremely small (with a minimum of 0.375) and so the error introduced should not be significant.

4 Results

In Figure 1, the 18 data points were plotted to show the cross section of the airfoil. The approximated area was 69.116 square meters for the cross section of this airfoil. If more data points were added so that more points could be used with a constant h, a single h could be calculated to reduce potential roundoff error in the calculation. Additionally, additional data points will provide a much more accurate answer because the error term for the Composite Simpson's Rule has order $O(h^4)$.

5 Figures

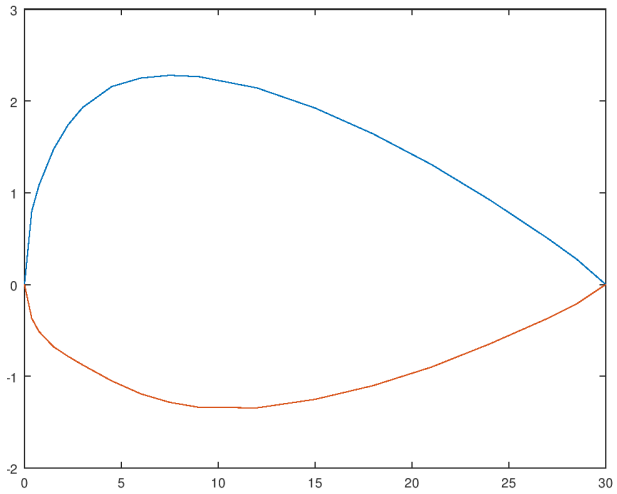

Figure 1: The cross section of the airfoil with the original 18 data points