Numerical Methods (MAT 370) - Root Finding Methods

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September 5, 2020

1 Introduction

As Olin Hall is being refurnished, an important issue to consider is the arrangement of the new hallways. The way the halls are angled affects the dimensions of any furniture that can be purchased. While most classroom items are relatively small and may not be affected by turns in the hallway, the chalkboards that will be installed in each room are the most problematic. Because the chalkboards ideally would fill nearly an entire wall, it is important to determine the maximum length that could be maneuvered through the halls into each classroom. This can be estimated using the widths of the hallways, the angle at which they meet, and a few trigonometric functions.

2 The Problem

One of the more problematic intersections of hallways comes when two halls of differing widths intersect. One such hallway changes from a wider 9 feet to a narrower 7 feet of room. These halls intersect to form a 125°, or 2.1817 radians, angle. The relationship between the length of the board and the angles of this intersection can be expressed by the following equation:

$$\ell(\gamma) = 9csc(.95989 - \gamma) + 7csc(\gamma)$$

where γ is the angle between the board and the wall of the smaller hallway. To find the maximum length that would be able to travel through this intersection, the above equation must be minimized with respect to γ. The first derivative of the equation is:

$$\ell'(\gamma) = 9csc(.95989 - \gamma)cot(.95989 - \gamma) - 7csc(\gamma)cot(\gamma)$$

The next step is determining the roots of this derivative, which in turn informs the minimum value of γ and so the maximum value of ℓ, the length of the chalkboard.

3 Methodology

In order to estimate the roots of $\ell'(\gamma)$, I chose the Secant Method. This method allows for a fast convergence without requiring the second derivative Newton's Method did. Using Octave, I was able to plot the function on the domain $[0, \pi - 2.1817] = [0, 0.95989]$. This domain was chosen because γ cannot be any larger than 0.95989 radians, and so any other maximums are not valid. After plotting the function on this domain, seen in Figure 1, I was able to identify that the root was near 0.4 but less than 0.5. I then used the Secant Method with $p_0 = 0.4$, $p_1 = 0.41$, a tolerance of 10^{-5}, and with $N_0 = 25$.

4 Results

After 5 iterations, the root was estimated to be $\gamma \approx 0.45118$, or $25.851°$. Returning to the original equation $\ell(\gamma)$, we find the maximum length of a chalkboard that can be maneuvered through this intersection to be:

$$\ell(0.45118) = 9csc(0.95989 - 0.45118) + 7csc(0.45118) = 34.533\text{feet}$$

Because the root was only an approximation and the width of the board was ignored, as well as the fact that it would be difficult to order a chalkboard 34.533 feet long, I would recommend that a board no larger than 34 feet be ordered to maximize the length of the board while still being able to deliver it to the appropriate classroom.

5 Figures

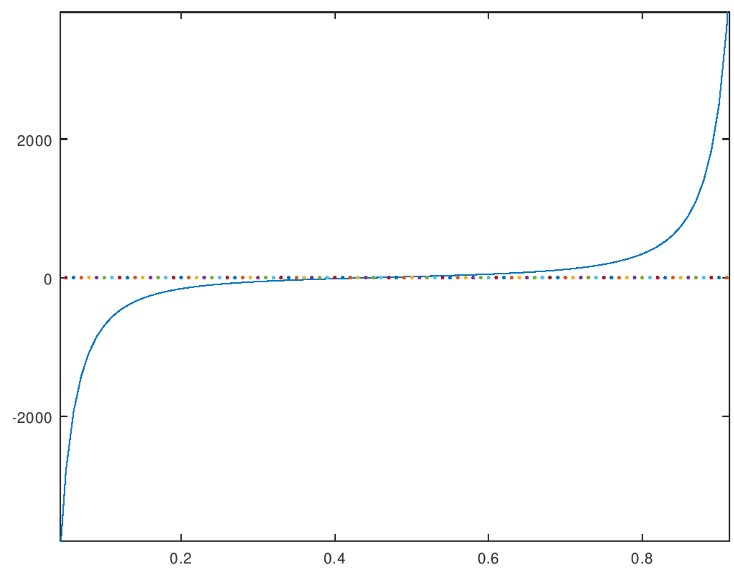

Figure 1: $\ell'(\gamma)$ on $[0, 0.95989]$