Numerical Methods (MAT 370) - Linear Systems

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1 Introduction

One of the most popular forms of entertainment for college students is the athletic events held by their schools between their fellow students. Centre College is no different, with a great deal of excitement surrounding the athletic teams each year. One of the most important things about each team is their ranking each year, showing how they compare to the teams from other schools. Centre belongs to the Southern Athletics Association (SAA), made up of nine teams also competing in NCAA Division III athletics. Ranking these teams based on the games they have played gives an idea of how well the respective teams competed during the past season.

2 The Problem

Given the scores of all the SAA football games from 2019, how can the teams best be ranked? Some teams have the same number of wins, which can pose an issue when trying to rank one over the other. Additionally, some games are won by 40 points or more, while other games are only won by 3 points. Finding a way to account for both wins and the total number of points would give a more accurate ranking than the accounting for only one of the two factors.

3 Methodology

To account for both the number of games won and the point differential, an system of equations is necessary. The games played by SAA teams can be represented as a matrix A such that

$$A = \begin{bmatrix} 1 & 0 & -1 & 0 & ... & 0 \\ -1 & 0 & 0 & 1 & ... & 0 \\ ... & ... & ... & ... & ... & ... \\ 0 & 1 & 0 & 0 & ... & -1 \end{bmatrix}$$

where each row represents a game played, each column represents a team, a 1 denotes a win, and a -1 denotes a loss. Additionally, the total point differential for each team can be represented as a vector $\bar{b}$ such that

$$\bar{b} = \begin{bmatrix} d_1 \\ d_2 \\ ... \\ d_n \end{bmatrix}$$

where each d_i represents the total difference in points for each of the teams. Finally, let $\bar{r}$ be a vector containing the teams' ratings such that

$$\bar{r} = \begin{bmatrix} r_1 \\ r_2 \\ \dots \\ r_n \end{bmatrix}$$

Putting the three pieces together, we have the equation

$$A\bar{r} = \bar{b}$$

which can be solved using LU Factorization in MatLab. Additionally, a row of 1's in the matrix and a corresponding 50 in the differential vector are needed to allow a maximum of 50 points to be given when ranking the teams.

4 Results

The 37 games between the 9 teams were translated into the A matrix, and the 37 point differentials were recorded in the $\bar{b}$ vector to give the rating vector

$$\text{Ratings} = \begin{bmatrix} 0.47863 \\ 23.37464 \\ 16.73362 \\ 1.81197 \\ 16.43875 \\ 3.92308 \\ -14.96581 \\ -15.07692 \\ 19.58974 \end{bmatrix}$$

This vector is ordered alphabetically by college name. When rearranged from greatest to least, the rankings of the teams (with ratings) are:

1. Berry (23.37464)
2. Trinity (19.58974)
3. Birmingham Southern (16.73362)
4. Hendrix (16.43875)
5. Millsaps (3.92308)
6. Centre (1.81197)

7. Austin (0.47863)
8. Rhodes (-14.96581)
9. Sewanee (-15.07692)

This ranking, along with the rating of each team, also gives an idea of how much better some teams are than others. For example, the top 4 teams are relatively close together, while the middle 3 and bottom 2 are similarly grouped.

This ranking helps someone hoping to go to the next game know that a matchup between Berry and Sewanee likely will not be a close fought game, while a Birmingham Southern vs. Hendrix game may go down to the final minutes. These examples were indeed true for the games recorded, with Berry blowing out Sewanee 55-0 and Hendrix beating Birmingham Southern in a much more narrow 34-24. This rating also lets Sewanee fans know not to hope for much from their 2019 football team unless they play Rhodes, while Centre fans can hope for good games against Millsaps and Austin and blowout wins against Rhodes and Sewanee.