Magnetic Levitation System

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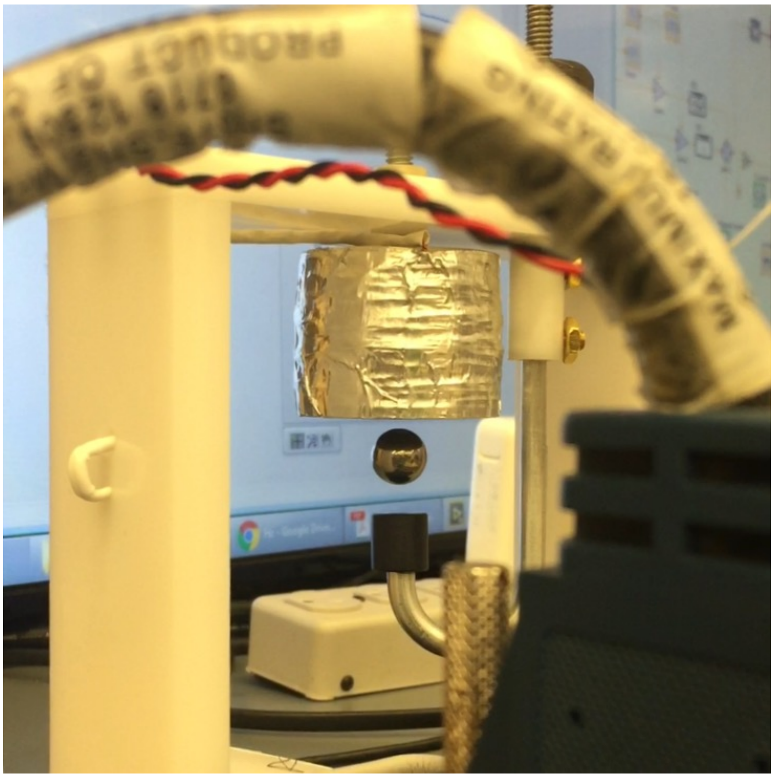


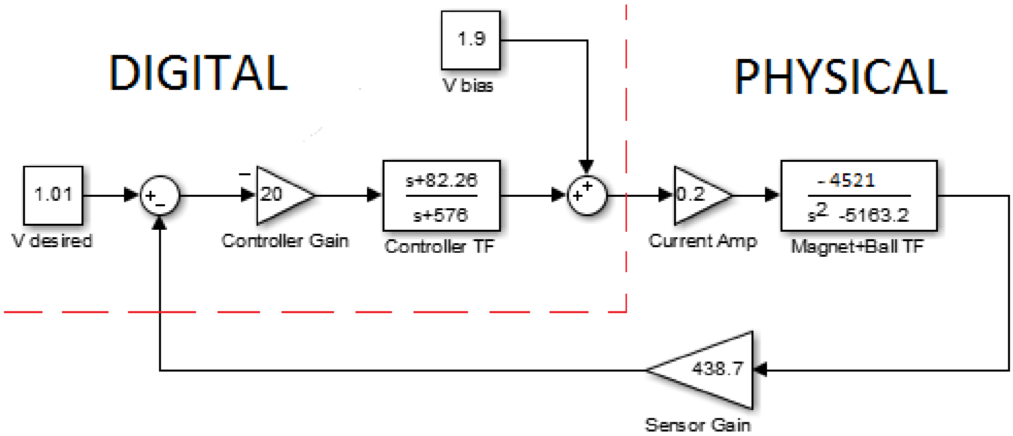
The **magnetic levitation system** maintains the commanded position of a paramagnetic ball using negative feedback from position sensors to control the driver circuit of an electromagnet.

The following summarizes the principle design mechanism of the mechatronic system, followed by a detailed outline of the project and the underlying math, physics, and engineering concepts.

Summary

A light emitting and detecting sensor scheme measures the ball-magnet's displacement, compares the signal with the commanded signal through negative feedback, and a controlling transfer function–designed to compensate for the physical system–outputs a voltage to drive the electromagnetic force on the ball-magnet. The physical system is made up of a voltage to current electromagnetic driver (operation amplifier), the electromagnet (plant), and the sensing scheme. The transfer function of the plant is modeled using the golden rules of operation amplifiers, linearization of the electromagnetic force about an operating position, and a linear fit to empirical data from the balls position and sensor output voltage.

Physical System and Model

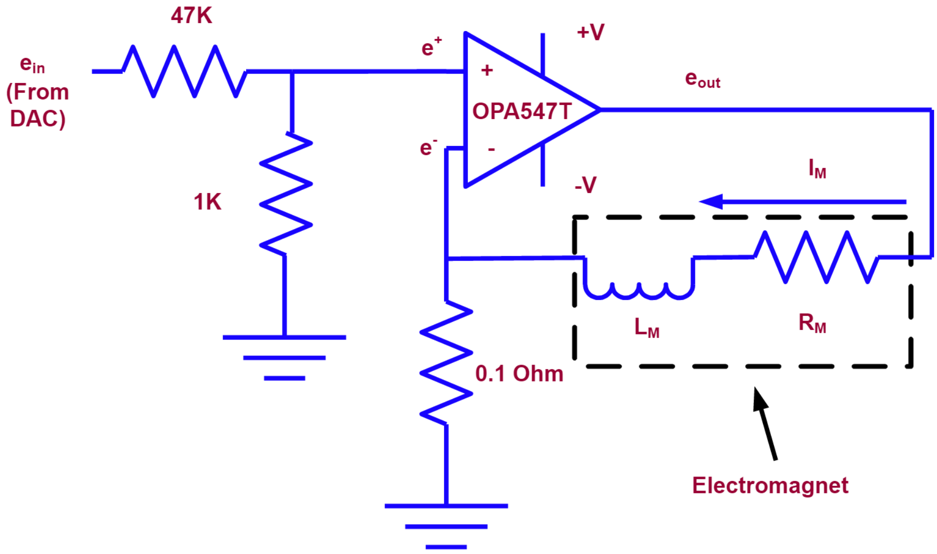


Important subsystems are identified and included in the model of the physical system. The block diagram above shows the important components of the control system, with the digital and physical subsystems separated by the red dotted line. The PHYSICAL system is comprised of three units: the current driver, the electromagnet (plant), and the position sensing scheme. These three subsystems make up the open-loop system, which will be mathematically modeled, combined into one transfer function, and used to design a compensating transfer function in Simulink. An NI DAQ is used to carry a signal to and from the DIGITAL system, which comprises digital logic in the form of a LabVIEW VI.

Mathematical Model

1. Current Driver (Amp)

In this subsystem, the DAQ delivers a voltage to the driver circuit which, using a high-current power op-amp (OPA547T) as a voltage-to-current amplifier, outputs a driving current for the plant according to the schematic displayed below.



The mathematical model of this driver circuit serves as a transfer function from voltage of the driver circuit to the driving current sent into the electromagnet. We assume accurate/constant resistor values and ideal op-amp behavior. The voltage e^+ is attained from a voltage divider, $e^+ = e^-$ assumes the golden rules of op-amps, and then voltage over resistance gives the driving current I_M.

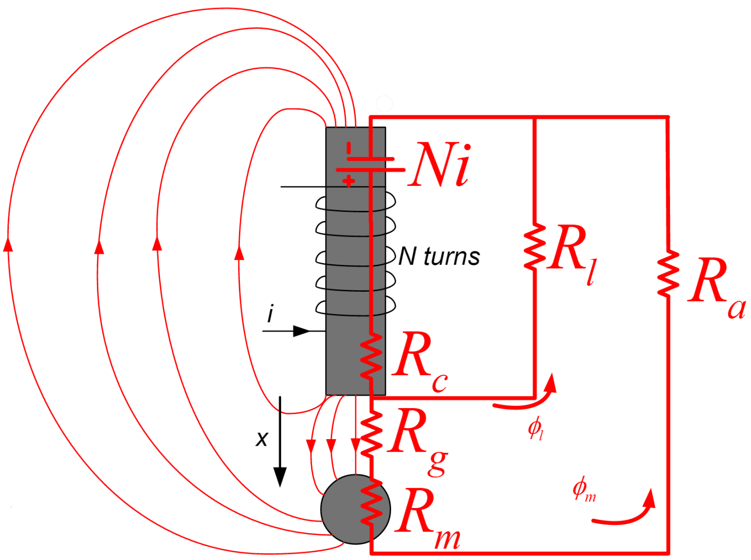
$$I_M = (\frac{1k\Omega}{47k\Omega + 1k\Omega})(\frac{1}{0.1\Omega})e_{in} = 0.2083 * e_{in}$$

$$D(s) = \frac{I_M}{e_{in}} = 0.2083$$

The schematic is made up of resistors, an OPA547 (MUST ATTACH HEAT SINK due to high currents), and our electromagnetic driver which is represented by an inductor and resistor in series. By supplying a 15kΩ current limiting resistor to the op-amp, the current is limited to 500mA. This corresponds to a voltage range of 0V to 2.5V, which is within the 0V to 5V range supplied by the DAQ.

2. Electromagnet & Ball (Plant)

The operation is simple: supply a current and receive a magnetic force. In order to develop a transfer function for this subsystem, a mathematical model of magnetic force must be linearized.



The magnetic flux lines flow through the electromagnetic core, across an air gap, into the paramagnetic ball, and loop back around. With a few assumptions, a lumped parameter model resembles an electrical circuit, characterizing the magnetism as a magnetomotive force (Ni) driving a magnetic flux (ϕ) through magnetic reluctance (R), when N is the number of turns in the coil.

Assuming linear magnetic material, $B = \mu H$ (H is magnetic field intensity, μ is magnetic permeability), and uniform flux density, $B * A = \phi$, each loop of wire is characterized by $H * L = i$, where L is length of magnetic flux travel and A is the cross sectional area they travel through. For a coil, this means $Ni = \frac{BL}{\mu} = \frac{\phi L}{A\mu} = \phi(\frac{L}{\mu A})$, since $H = \frac{B}{\mu}$ and $B = \frac{\phi}{A}$. Writing $R = \frac{L}{\mu A}$ provides the following equation:

$$Ni = \phi\Sigma R = (\phi_m + \phi_l)(R_c) + \phi_m R_g + \phi_m R_m + \phi_l R_l + \phi_m R_a$$

A few simplifying assumptions:

- The magnetic permeability of iron is much greater than that of air, so $R_g,\ R_l,\ R_a >> R_m,\ R_c$, so R_m and R_c are insignificant.
- Assume very little leakage flux, most flows straight through gap to the magnetic ball, $\phi_m + \phi_l \approx \phi_m = \phi$
- The cross sectional area at R_a is very large so the term is insignificant

- Assume no fringing flux lines from electromagnet to magnetic ball

This simplifies the above equation of Ni down to one term of the lumped parameter model:

$$Ni = \phi(\frac{L_{gap}}{\mu_{air} * A_{gap}}) = \phi(\frac{x}{\mu A}) \implies \phi = N\mu A(\frac{i}{x})$$

The linkage flux is equal to both $N\phi$ and Li so $L = \frac{N\phi}{i} = \frac{N^2\mu A}{x}$

The potential energy, U, in an inductor is $U = Li^2 = \frac{N^2\mu A}{x}i^2$ The force of the electromagnet is the derivative of potential energy, U, with respect to displacement, x:

$$F = N^2\mu A i^2(-\frac{1}{x^2}) = -N^2\mu A(\frac{i^2}{x^2}) = -C(\frac{i^2}{x^2})$$

C is a parameter that represents all of the constants, which we solve for with steady state equations as some of the constants are practically unattainable. The differential equation of motion is now derived from the forces acting on the ball: $m\ddot{x} = mg - C(\frac{i^2}{x^2})$. To linearize this 2nd order ODE, the magnetic force undergoes a first order Taylor expansion about an equilibrium operating position ($\bar{x}$) and current ($\bar{i}$) such that $mg = C(\frac{\bar{i}^2}{\bar{x}^2})$:

$$C(\frac{i^2}{x^2}) \approx C(\frac{\bar{i}^2}{\bar{x}^2}) - (\frac{2\bar{i}^2}{\bar{x}^3})\hat{x} + C(\frac{2\bar{i}}{\bar{x}^2})\hat{i}$$

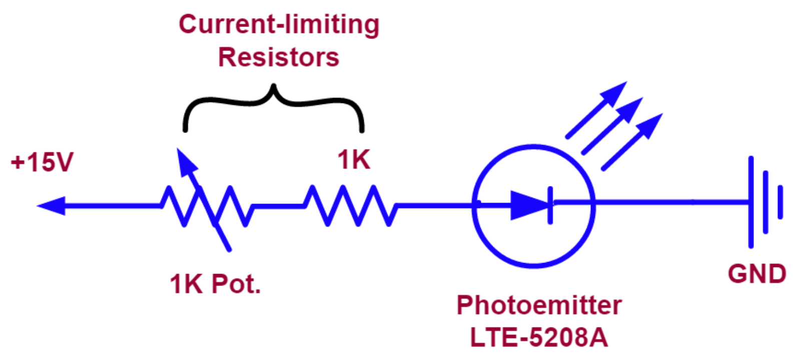
substituting into the equation of motion gives $m\hat{\ddot{x}} = mg - C(\frac{\bar{i}^2}{\bar{x}^2}) - (\frac{2\bar{i}^2}{\bar{x}^3})\hat{x} + C(\frac{2\bar{i}}{\bar{x}^2})\hat{i}$. Finally, applying the equilibrium condition to get rid of mg gives $m\hat{\ddot{x}} = C(\frac{2\bar{i}^2}{\bar{x}^3})\hat{x} - C(\frac{2\bar{i}}{\bar{x}^2})\hat{i}$. Solving for the transfer function between position and current gives:

$$P(s) = \frac{\hat{x}}{\hat{i}} = \frac{-\frac{C}{m}\frac{2\bar{i}}{\bar{x}^2}}{s^2 - \frac{C}{m}\frac{2\bar{i}^2}{\bar{x}^3}}$$

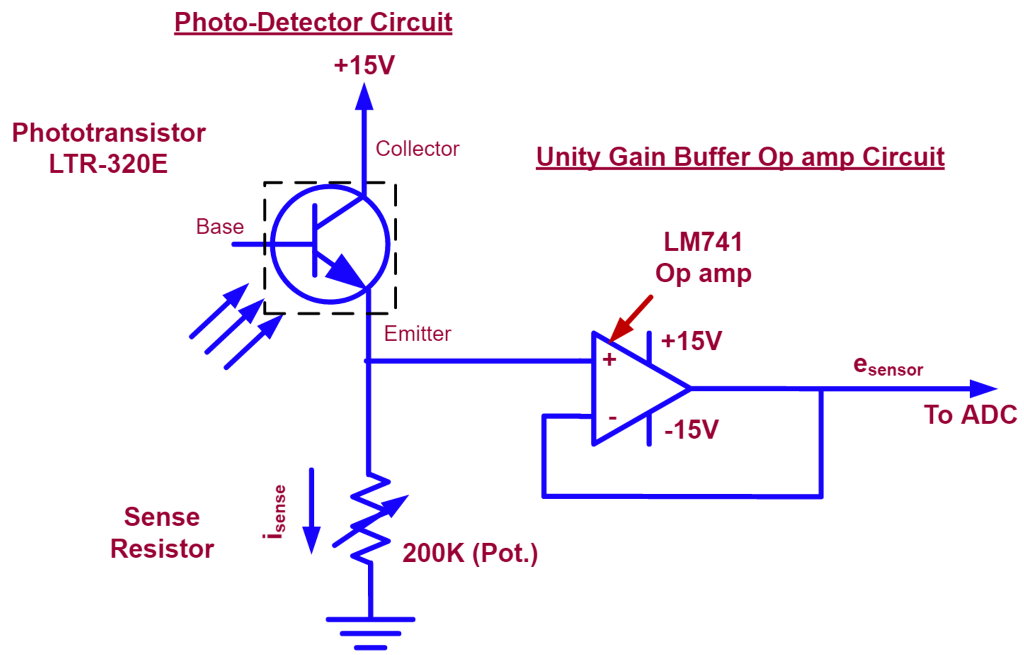
3. Position Sensor

Photo-emitter and photo-detector circuits are used to measure the position of the magnetic ball. The specific devices are spectrally matched with regards to intensity and sensitivity. As the ball rises closer to the electromagnet, it blocks emitted light, and the voltage in the detector drops accordingly. While the sensors are mounted into the system frame, they require the following wiring:

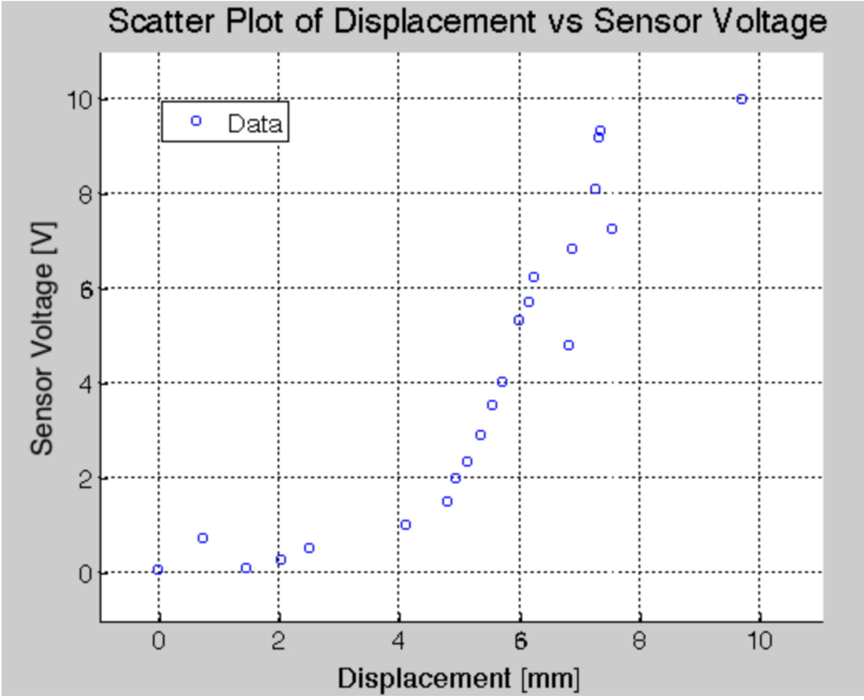
The **photo-emitter** leads are fed to a breadboard, and connected in series with current-limiting resistors, one of which is a 1K potentiometer which was set to 50 ohms. NOTE: Use at least 138 ohms with a 15V supply in order keep the current below the maximum forward current for the emitter LED.



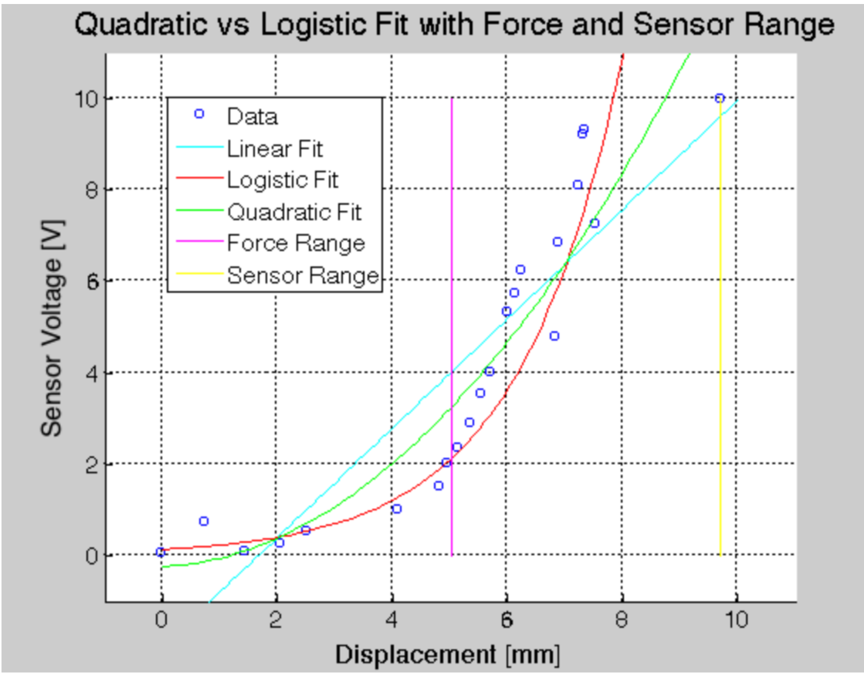
The **photo-detector** consists of a phototransistor in series with a 200K potentiometer, which adjusts the magnitude of the output voltage (e_{sensor}). The voltage at the emitter end of the transistor is sensed using a unity gain buffer (LM741) before it is read by the DAQ. The buffer serves to protect the sensing circuit from loading effects, however it is unnecessary in this case as the input impedance to the DAQ is sufficiently large.



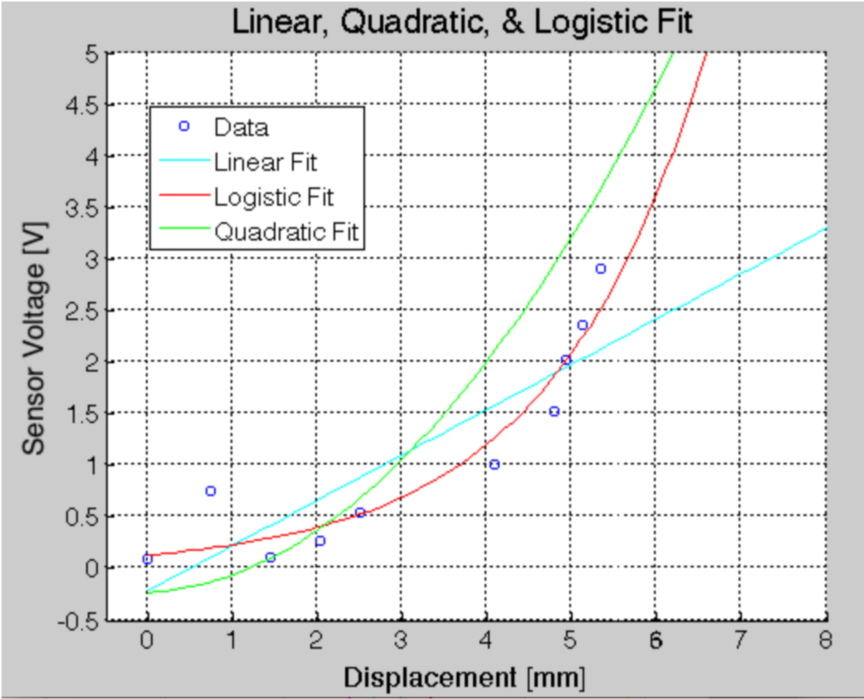
The sensor mounting and environment is subject to variability, so empirical data serves as the mathematical model of the sensing scheme. The sensor outputs between 0V and 10V, which corresponds to approximately 0mm to 10mm displacement of the ball from the bottom of the electromagnet. Below is a scatter plot of 20 data points, about 0.5mm apart, pf voltage vs. displacement.



Below, a linear, exponential, and quadratic fit to the data along with two bounds on the displacement: the yellow line represents the range of the sensor (when it outputs its maximum 10V) and the purple line represents the range of the electromagnetic force (above which it cannot lift the weight of the ball) according to the maximum 5V we can supply to the driver circuit.



In order to operate within the range of the sensor AND the range of force needed to lift the magnet, so a linear, exponential, and quadratic fit is placed on the more relevant 10 data points shown below. The open loop transfer function is taken from the linear fit. The exponential fit is more accurate and used as a look-up table for our control system using a position command (instead of a voltage command). More on this later.



Since the sensor gain must be $\frac{V}{X}$, the slope of the linear fit reasonably represents the transfer function as the y-intercept is insignificantly small:

$$S(s) = \frac{V}{X} = 0.4387$$

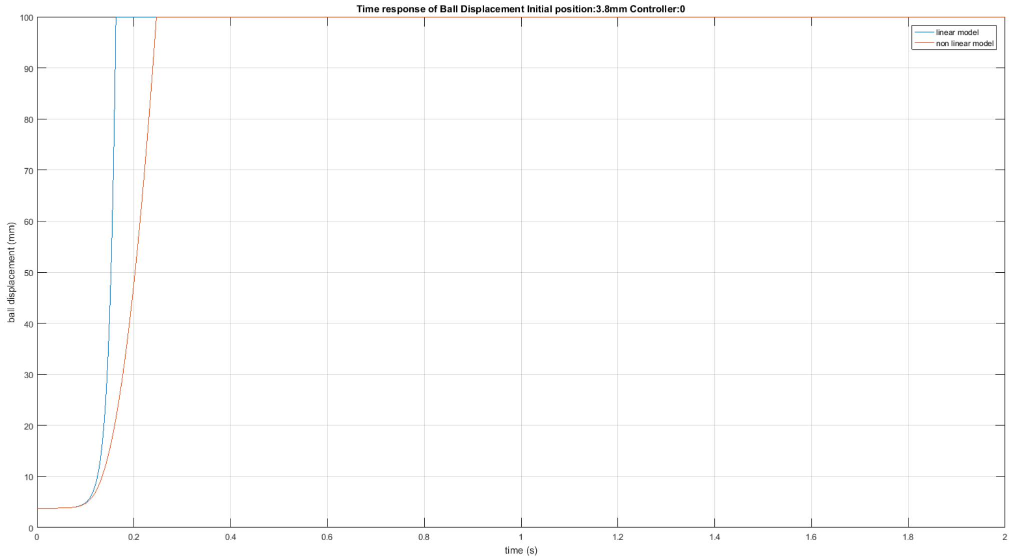
The open-loop system has an approximate transfer function (mathematically modeled):

$$OL(s) = D(s)P(s)S(s) = 0.2083\left(\frac{-\frac{C}{m}\frac{2\bar{i}}{\bar{x}^2}}{s^2 - \frac{C}{m}\frac{2\bar{i}^2}{\bar{x}^3}}\right)0.4387 = -0.09138\frac{\frac{C}{m}\frac{2\bar{i}}{\bar{x}^2}}{s^2 - \frac{C}{m}\frac{2\bar{i}^2}{\bar{x}^3}}$$

Simulation

In order to assess the fidelity of a linear model, simulations are run for a linear and non-linear model of the system using simulink. The non-linearities include the second order equations of motion of the magnet and ball system, the exponential fit to the sensor dynamics, as well as saturation blocks.

Many simulations were run. The one shown below begins with the ball at 3.8mm from the electromagnet–well below the equilibrium position. As expected, the ball falls to the ground as shown below by the displacement increasing with the acceleration of gravity.



As expected, the linear model (shown in blue) underestimates the force of the electromagnet, simulating a ball that falls at a faster rate. The non-linear model (shown in red) more accurately represents the magnets force and manages to affect the ball's trajectory with greater magnitude.

Parameter Identification

1. Equilibrium Position ($\bar{x}$)

Deciding an equilibrium or operating position is arbitrary to a certain degree. The operating position must be away from the boundaries set by the sensor reading and the electromagnetic force. These system outputs are integral to the control system, so avoiding boundaries is necessary for a robust system due to perturbations, disturbance, and noise. With that caveat, 3.88mm was chosen as it is well within the 5mm boundary, while providing enough distance to make levitation apparent.

$$\bar{x} = 3.8mm$$

2. Bias Voltage and Equilibrium Current ($\bar{i}$)

In implementing controls about an operating position, a bias voltage drives enough force to hold the ball at the equilibrium position. This bias voltage drives an electromagnetic force to balance the weight of the ball at the operating position. This was experimentally found to be 1.9V by ramping the voltage until the ball wobbled and a small perturbation lifted the ball. This voltage, along with the driver circuit transfer function, give the current at the equilibrium position.

$$\bar{i} = 0.2083\frac{A}{V} * 1.9V = 396mA$$

3. Constant C

The constant relating position and current to the force on the ball is determined using the steady state of the equation $m\ddot{x} = mg - C(\frac{i^2}{x^2})$, which gives $C = mg(\bar{x}^2/\bar{i}^2)$. The mass of the ball is measured at 8.5 grams, which in SI units solves for:

$$C = 7.6783 * 10^{-6}$$

4. Desired Voltage

Having linearized the differential equation of motion about a specific position and current, the desired voltage for commanding the control system must be determined. The desired voltage corresponds to the sensor output with the ball at its equilibrium position, since our feedback is read from the sensor output. This desired voltage effectively serves as the commanded position, and was measured at 1.01V.

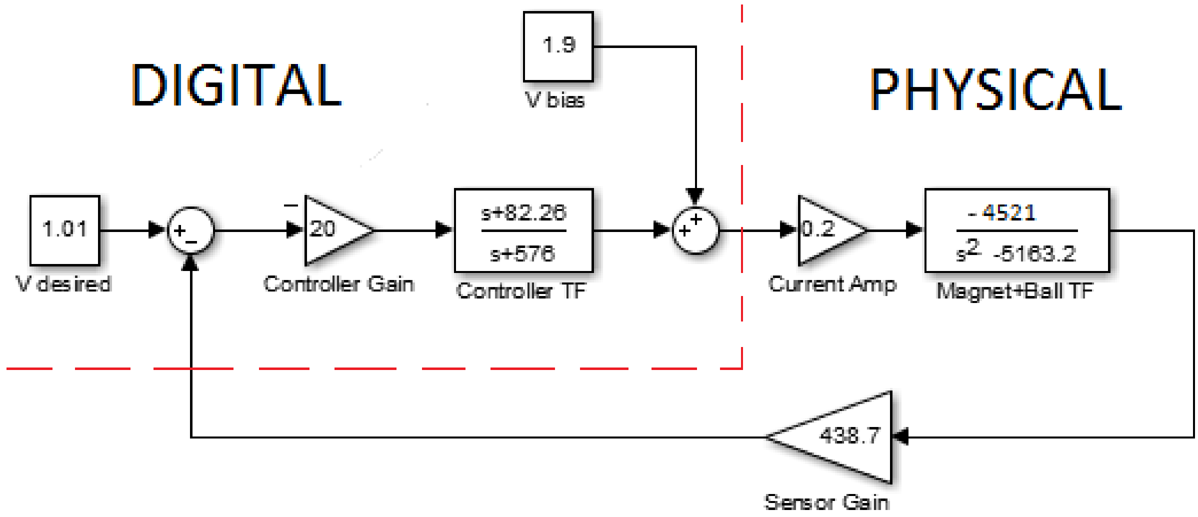
Controller Design and Hardware

1. Power

The sensors and the operational amplifiers are powered (+15V, -15V, and ground) by a DC power supply that provides GND and ±15V. The DAQ takes in a signal from the photo-detecting circuit and outputs a voltage to the driver circuit.

2. System Block Diagram

Revisiting the digital and physical system model below, the user input desired voltage and bias voltage are shown in the block diagram. The open loop transfer function is the product of the three transfer function in the physical system below, labeled Current Amp, Magnet+Ball TF, and Sensor Gain. A controller is designed to compensate for the system, labeled Controller Gain and Controller TF.



3. Negative Transfer Function

It is worth noting the negative sign placed outside of the Controller Gain. Our system requires negative feedback, yet our physical system is modeled with a negative transfer function. Therefore, we "fix" the controller with a negative sign to create overall negative feedback.

The negative transfer function of the plant is simply a consequence of how the input (current) and output (displacement) are defined. Since displacement is distance between the ball and the magnet, increasing current causes this displacement to decrease, resulting in a negative relationship (although increasing the current increases the "height" of the ball).

A simple example demonstrates the need for the added negative sign. Suppose the ball is dropping. The displacement is too large, and the measured voltage from the sensor is above the desired voltage. The error (Desired Voltage - Measured Voltage) is negative, and therefore exits the controller transfer function as a negative error as well. The voltage sent into the current amp will be below the bias voltage and the plant will receive a current lower than the equilibrium current – clearly the ball will drop as it requires a higher current to raise the ball back to equilibrium. Designing a controller does not take into account the negative transfer function of the plant, and therefore the controller must be fixed with a negative sign.

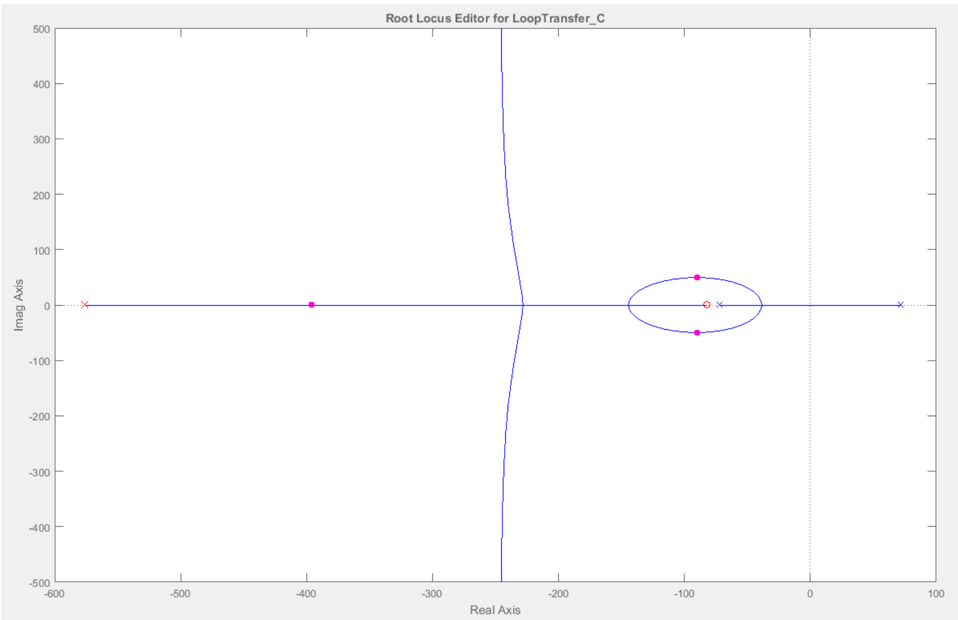
4. MATLAB Controller Design

MATLAB's Sisotool was used to design the controller in order to quickly create a stable controller with proper phase margin and gain margin. Gain margin and phase margin are inversely related and cause a trade-off in controller design. Generally, a gain margin of 3+ and phase margin between 30 and 60 degrees provides reasonably robust stability and sufficient bandwidth. However, these rules depend on the system, and in our case a gain margin of -7.67 is sufficient.

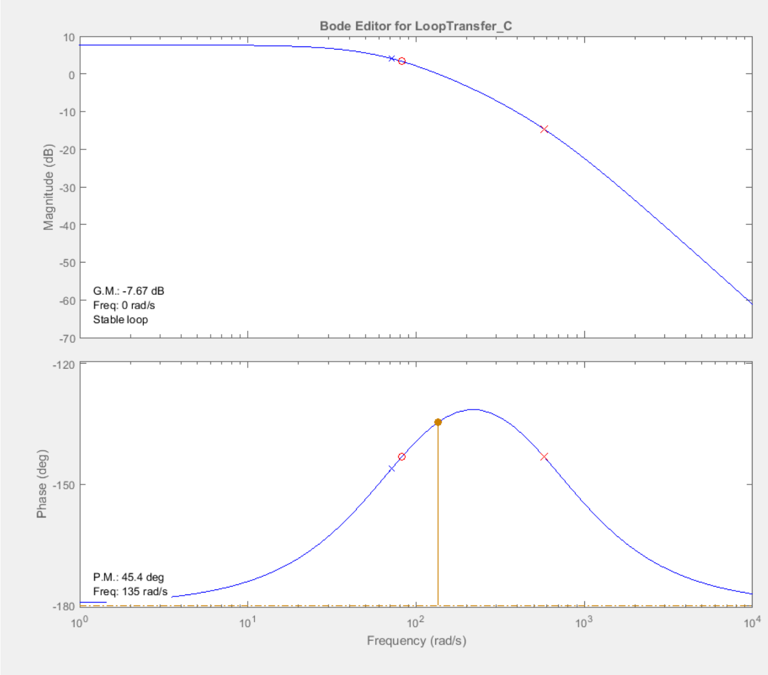
The root locus plot for the closed loop system is shown below. As all the poles are on the left hand plane, the overall system is stable. The closed loop transfer function resulted in:

$$T = \frac{CP}{1 - CP}$$

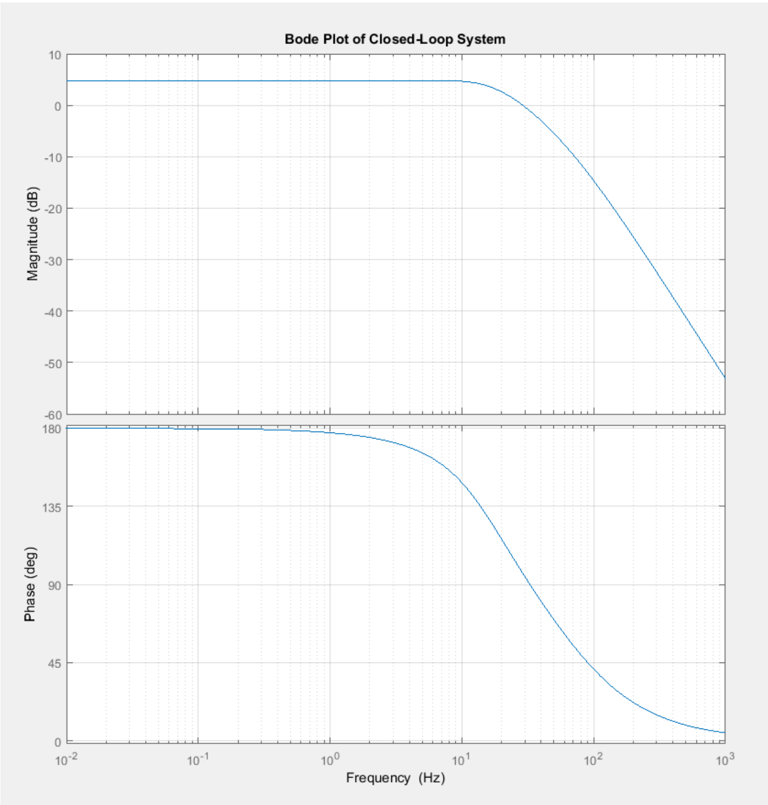
$$\frac{-86940s^4 - 5.723 \cdot 10^7 s^3 - 3.67 \cdot 10^9 s^2 + 2.955 \cdot 10^{11} s + 2.127 \cdot 10^{13}}{s^6 + 1152s^5 + 408390s^4 + 4.533 \cdot 10^7 s^3 + 2.707 \cdot 10^8 s^2 - 2.648 \cdot 10^{11} s - 1.242 \cdot 10^{13}}$$



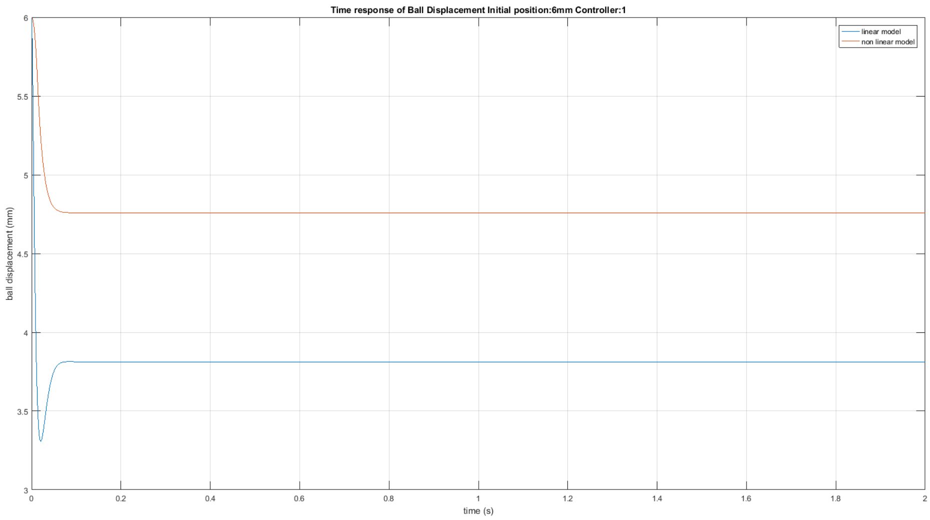
The open loop bode plot (the controller and plant) displays a gain margin of -7.67 dB and a phase margin of 45.4 degrees.



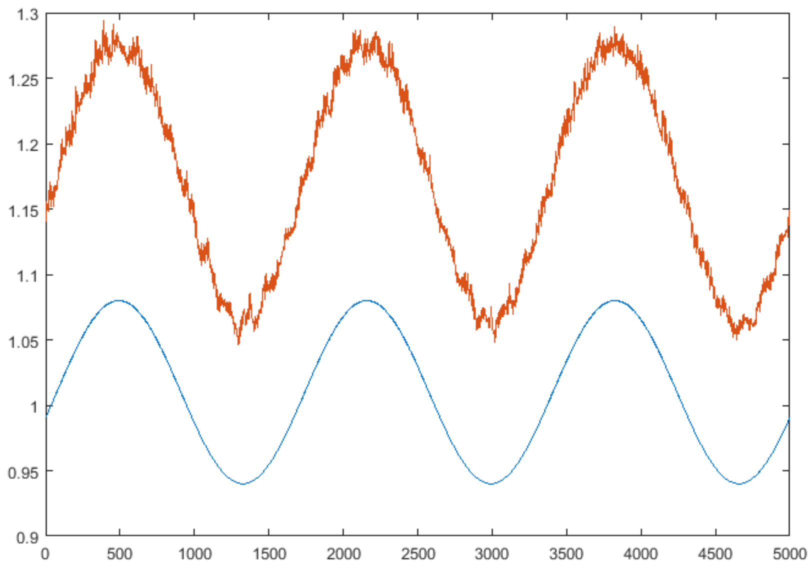
The closed loop bode plot is shown below and the bandwidth is 23.3Hz.



A simulation was ran to test the efficacy of our controller. Below is the displacement of the ball over time using a linear model (in blue) and a non-linear model (in red) for the plant. Both reach stability, however, the linear model reaches stability at a different and less ideal equilibrium displacement since the linear model does not accurately model the highly non-linear system.



In order to test the robustness of the controller, the bandwidth and stability was put to test by replacing the constant command (desired voltage) with a sinusoidal command. The results are shown below with the commanded signal in blue and the displacement (voltage from the photo-detector) in red.



The controller allowed tracking the sinusoidal command over a large range from 0.003Hz to 30Hz. A phase difference of zero was calculated from experimental data at all frequencies in this range. However, the experimental bode plot of the controlled system matches the theoretical bode plot up to about 20Hz, close to the bandwidth or the system. After this our system is not stable and the magnitude ratios no longer match theoretical values. Above 10Hz, the output from the sensor gets increasingly noisy and the calculated phase difference may not be entirely accurate.