Linear Algebra (ECE 269)

Face Recognition Using PCA

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December 3, 2019

Part A

According to Turk and Petland, "the associated eigenvalues allow us to rank the eigenvectors according to their usefulness in characterizing the variation among the images." The eigenvalues of the covariance matrix of the training set indicate which eigenvectors encode more information about the variation of the training images. The larger the eigenvalue, the more useful the eigenface is for characterizing the differences in the images.

Figure 1 shows the eigenvalues of the covariance matrix, $C = AA^T$. To select a subset of principal components, M', I would select the eigenvectors corresponding the largest 50 eigenvalues. As seen in Figure 1, the size of the eigenvalues quickly decay and appear to approach a horizontal asymptote after about the top 50. I sorted the eigenvalues (and eigenvectors similarly) in descending order.

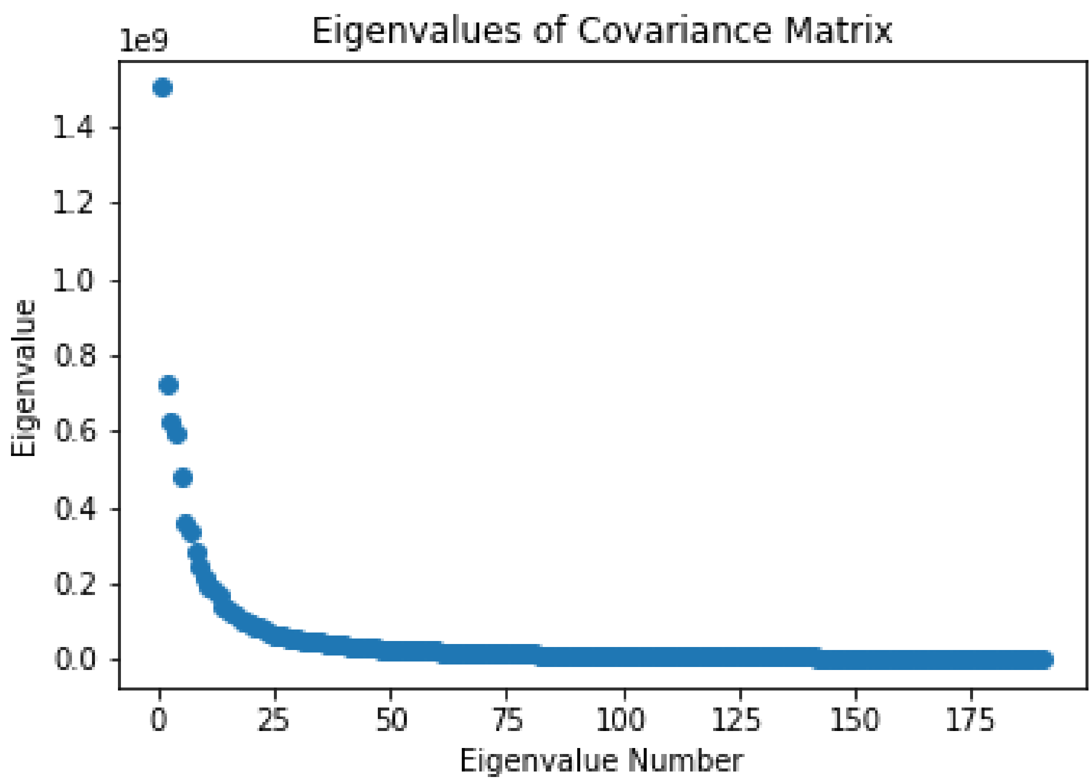


Figure 1: A plot of the sorted list of eigenvalues of the covariance matrix, $C = AA^T$

Part B

In this section the goal is to reconstruct a face from within the training set. Figure 2 shows a plot of the mean squared error with respect to the number of eigenfaces utilized for reconstruction. When using all 190 eigenfaces, the mean squared error is very near zero. In Figure 3, the original image and the reconstructed image are displayed. They are all very similar but the original and reconstruction with 190 eigenfaces are basically indistinguishable by looking at them. The reconstruction with the top 50 eigenfaces is a bit more fuzzy and less detailed which makes sense because it is made of less components.

This result is not surprising since the image being reconstructed not only lies in neutral face space but also belongs to the training set. Principle Component Analysis involves finding a lower dimensional subspace that the training data belongs to. It follows that an image in the training set belongs to that lower dimensional subspace and can therefore be represented as a linear combination of the basis vectors of that subspace.

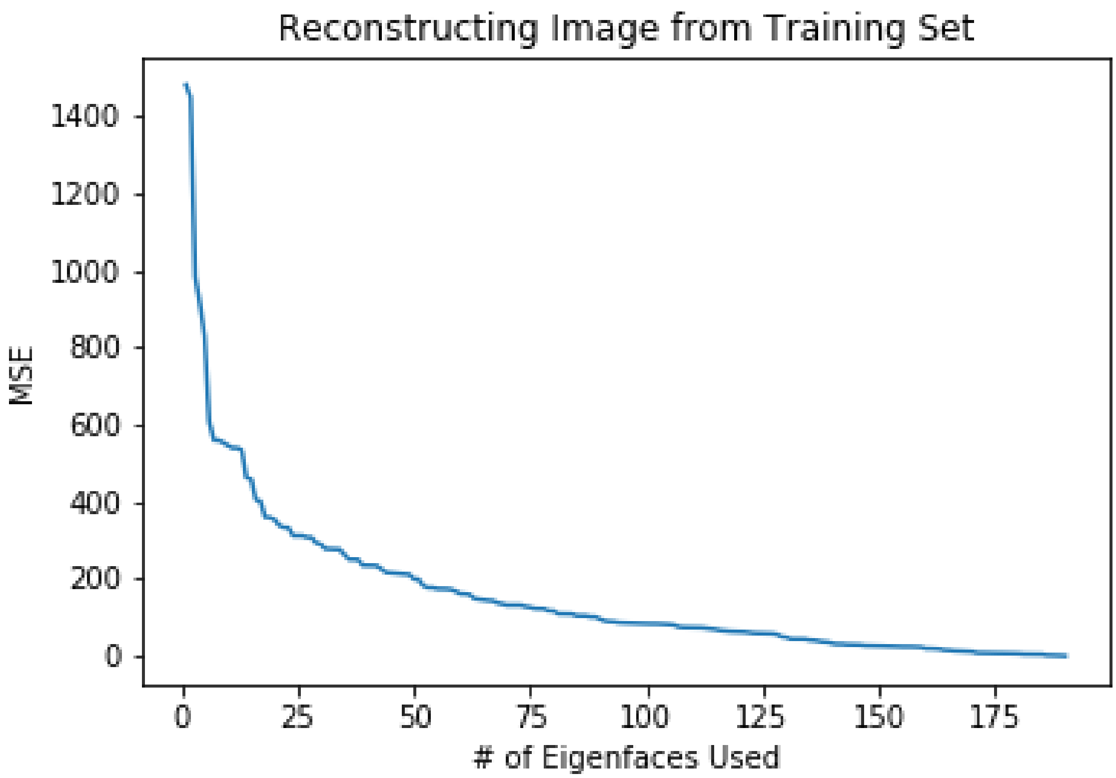
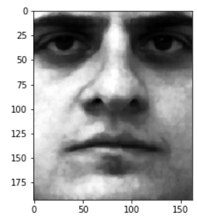
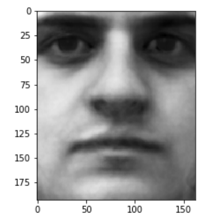


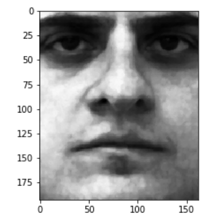
Figure 2: The Mean Squared Error plotted against the number of eigenfaces used for reconstruction. Image taken from set of neutral faces and within training set



(a) Original Image



(b) Top 50 eigenfaces



(c) All 190 eigenfaces

Figure 3: The original image (left), image reconstructed with the top 50 eigenfaces (middle), and image reconstructed with 190 eigenfaces (right)

Part C

In this section, we will be using the same principal components calculated from the set of neutral faces except this time we will be attempting to reconstruct a face from the set of smiling faces. Figure 4 shows a plot of the mean squared error with respect to the number of eigenfaces utilized for reconstruction. When using all 190 eigenfaces, the mean squared error reaches about 200. In Figure 3, the original image and the reconstructed images are displayed. The three images are similar but noticeably different. Unlike with the image in the training set, this image from outside the training set cannot be (nearly) perfectly reconstructed. Again, the reconstruction with the top 50 eigenfaces has less detail but this time the reconstruction with all 190 eigenfaces is also missing some details and is a little fuzzy when compared with the original. This makes sense since the training data had no images of people smiling.

Another observation is that it is more difficult to reconstruct a smiling face if teeth are shown. For example, the plot in Figure 6 shows that the mean squared error for the reconstruction of a face smiling with teeth only reaches about 1000. The image result is also clearly worse as shown in Figure 7. The mouth is simply an undefined blur that looks nothing like the original. This result makes sense since a face with teeth showing would not belong to neutral face space. Because none of the training images have teeth showing (since they are all neutral), teeth cannot be accurately expressed as a linear combination of the eigenfaces.

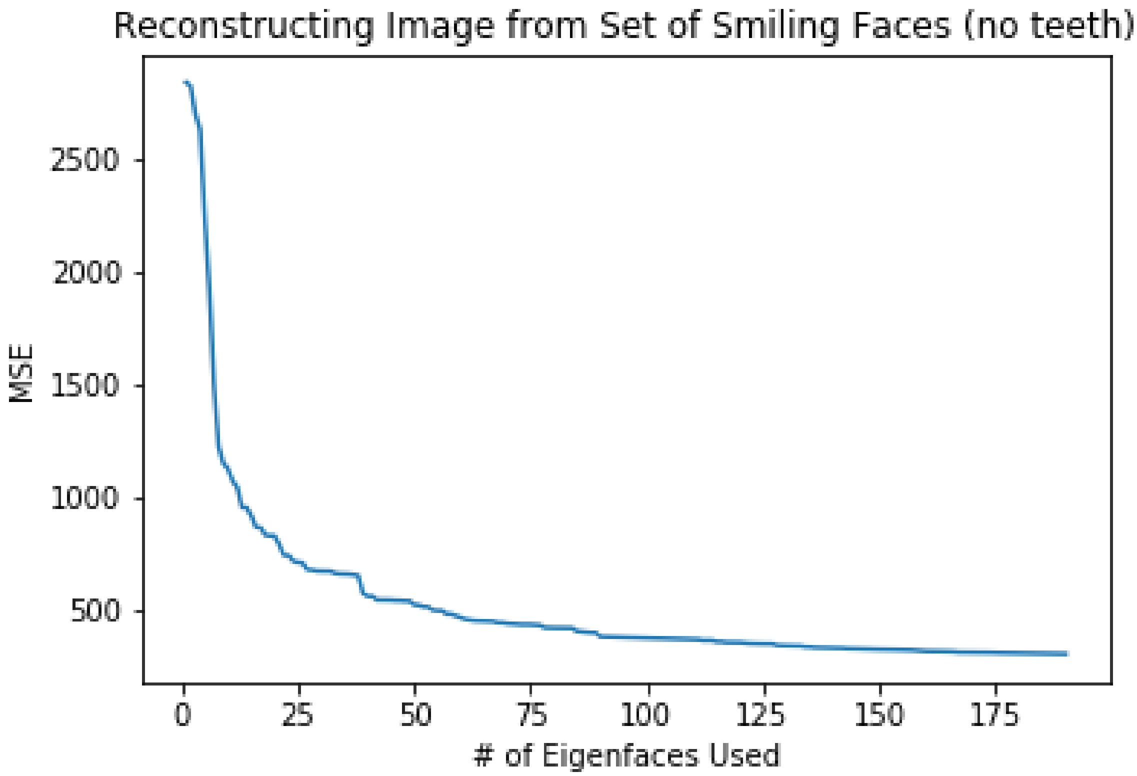
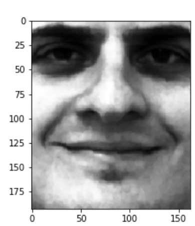
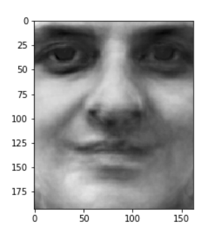


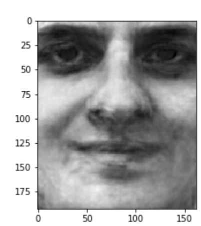
Figure 4: The Mean Squared Error plotted against the number of eigenfaces used for reconstruction. Image taken from set of smiling faces, no teeth showing



(a) Original Image



(b) Top 50 eigenfaces



(c) All 190 eigenfaces

Figure 5: The original image (left), image reconstructed with the top 50 eigenfaces (middle), and image reconstructed with 190 eigenfaces (right)

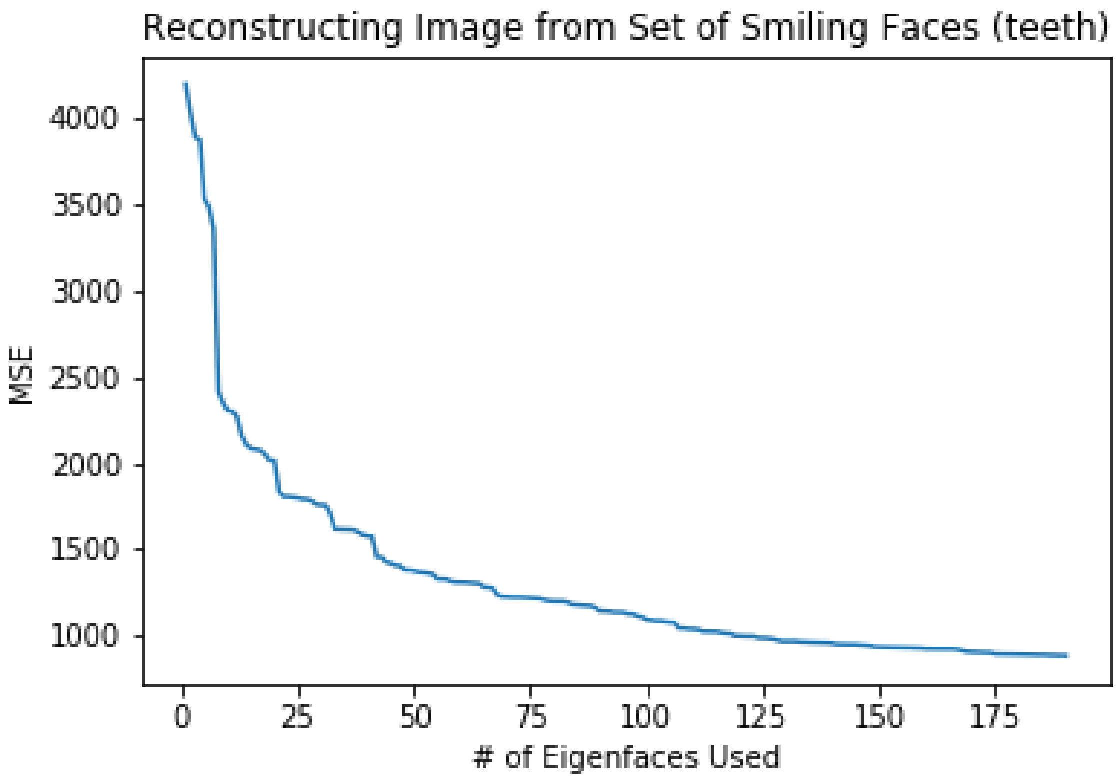
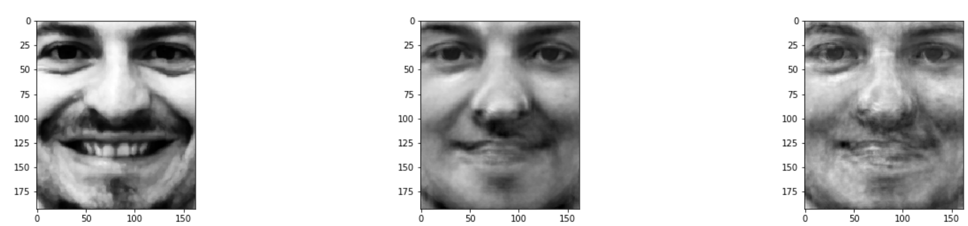


Figure 6: The Mean Squared Error plotted against the number of eigenfaces used for reconstruction. Image taken from set of smiling faces, teeth showing.



(a) Original Image (b) Top 50 eigenfaces (c) All 190 eigenfaces

Figure 7: The original image (left), image reconstructed with the top 50 eigenfaces (middle), and image reconstructed with 190 eigenfaces (right)

Part D

In this section, we will be reconstructing a neutral face that is not part of the training set. The mean squared error reaches about 150 when using all 190 components for reconstruction as shown in Figure 8. As well, the original and reconstructed images look very close as shown in Figure 9.

The reconstruction is very good which makes sense because this neutral image belongs to or at least lies very near neutral face space. It would then be expected that it can be expressed as a linear combination of the eigenfaces. This result shows that this neutral face image is closer to neutral face space than a smiling face even if the person smiling was in the training set (but with a neutral expression).

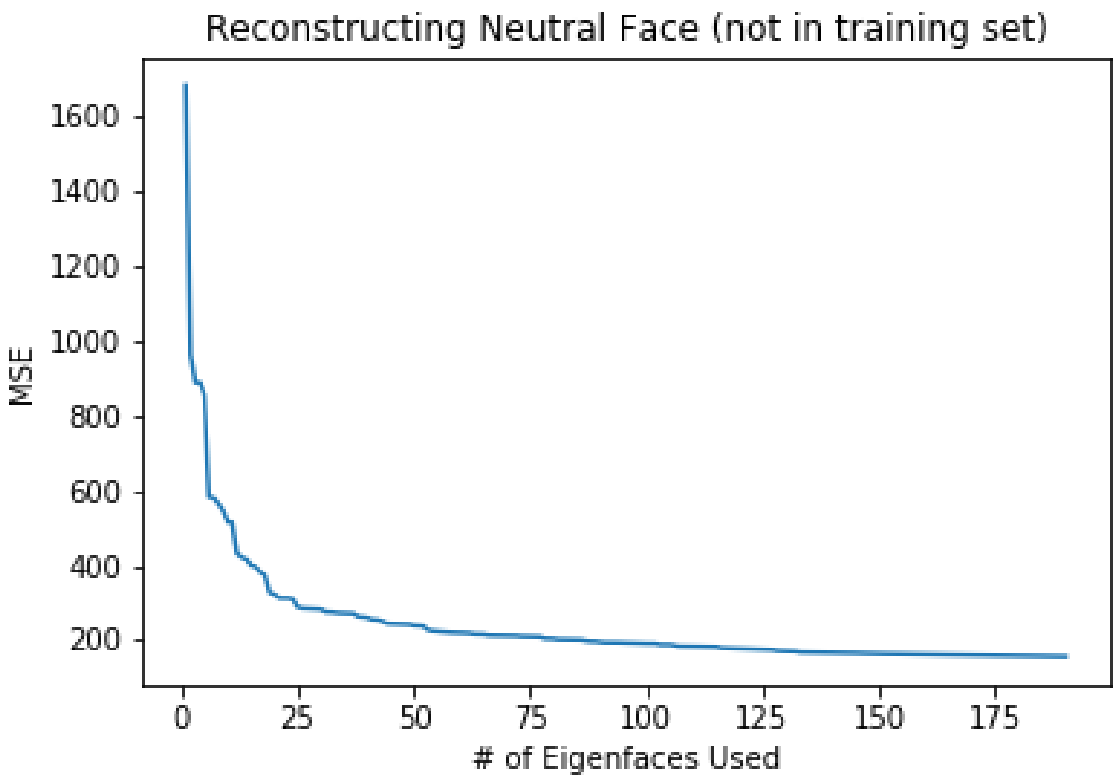
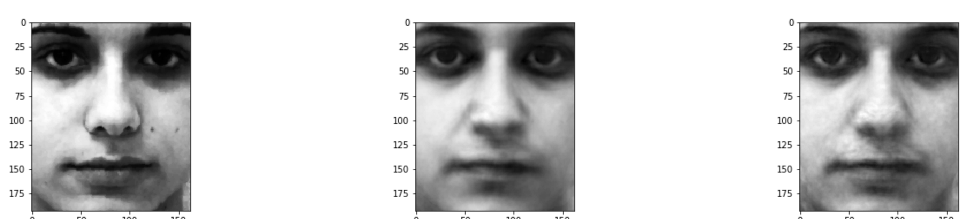


Figure 8: The Mean Squared Error plotted against the number of eigenfaces used for reconstruction. Image taken from set of neutral faces but outside the training set.



(a) Original Image (b) Top 50 eigenfaces (c) All 190 eigenfaces

Figure 9: The original image (left), image reconstructed with the top 50 eigenfaces (middle), and image reconstructed with 190 eigenfaces (right)

Part E

In this section, we attempt to reconstruct an image that is not of a face but of a car. Figure 10 shows that using all 190 components the mean square error is about 2000. This is the highest value seen so far. Figure 11 shows the original image and reconstructed images. The reconstructed image looks nothing like the original image. In fact, a vague outline of a face can be seen in the reconstructed image but no sign of a car can be seen.

This result make sense, since an image of a car would lie no where near face space, let alone neutral face space. Because of its distance from face space, the eigenfaces which span face space can not be used to construct an image of a car.

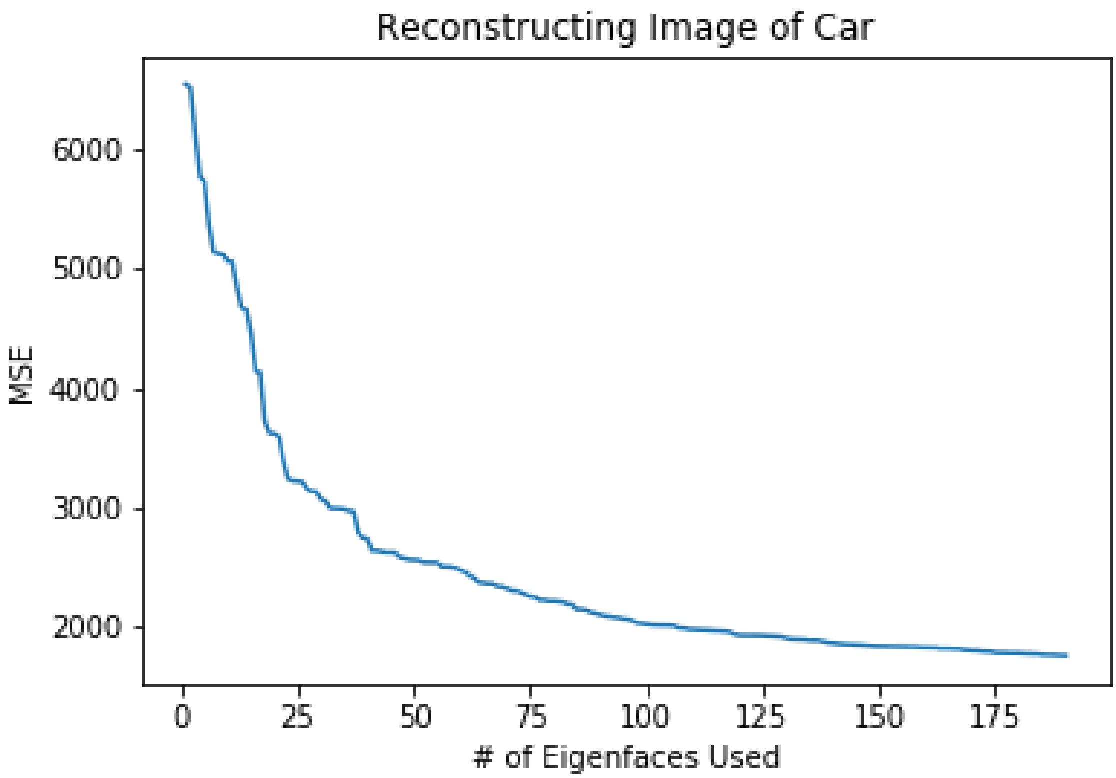
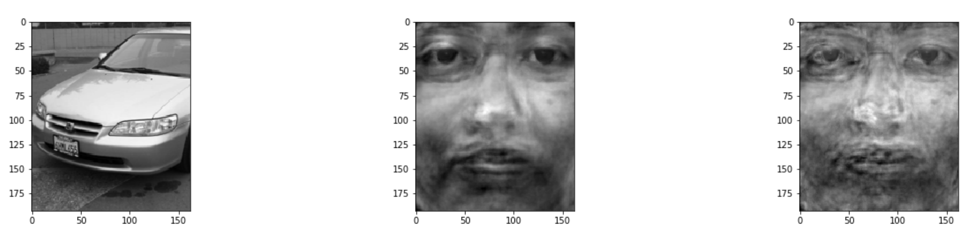


Figure 10: The Mean Squared Error plotted against the number of eigenfaces used for reconstruction. Image of a car.



(a) Original Image (b) Top 50 eigenfaces (c) All 190 eigenfaces

Figure 11: The original image (left), image reconstructed with the top 50 eigenfaces (middle), and image reconstructed with 190 eigenfaces (right)

Part F

Again we will be using an image from the neutral training set except this time the image will be rotated at various angles. Figure 12 shows the mean squared error for the reconstruction (with 190 eigenfaces) for the image at increasing angles of rotation from 0 to 180 degrees. With a rotation angle of about 24 degrees the resulting reconstruction becomes worse than for even the car image with a mean squared error of over 2000. Interestingly, the error is not always increasing but decreases in the range of about 50 to 75 degrees and 150 to 179 degrees.

Figure 13 shows original image (an image from the training set rotated 24 degrees) and the reconstructed image. The two images are not very similar and a vertical face can still be seen in the reconstructed image. Another observation is that the reconstructed image has dark corners like the original image which has black borders cause by the rotation. The reconstruction process attempts to replicate the border.

Figure 14 shows original image (an image from the training set rotated 90 degrees) and the reconstructed image. The two images look nothing alike and in fact a right side up face can be seen faintly in the reconstructed image. The reconstructed image attempted to reproduce the black bars at the top and bottom of the original image.

Figure 15 shows original image (an image from the training set rotated 179 degrees) and the reconstructed image. The two images look nothing alike and in fact a right side up face can be seen faintly in the reconstructed image.

The results of this experiment show that this method of recognition and reconstruction is not rotationally invariant. A recommendation for identifying rotated faces would be to perform a similar rotation transformation on the principal components. If the rotation angle is unknown, one could try various angles to see if they could find the angle that minimizes the squared error.

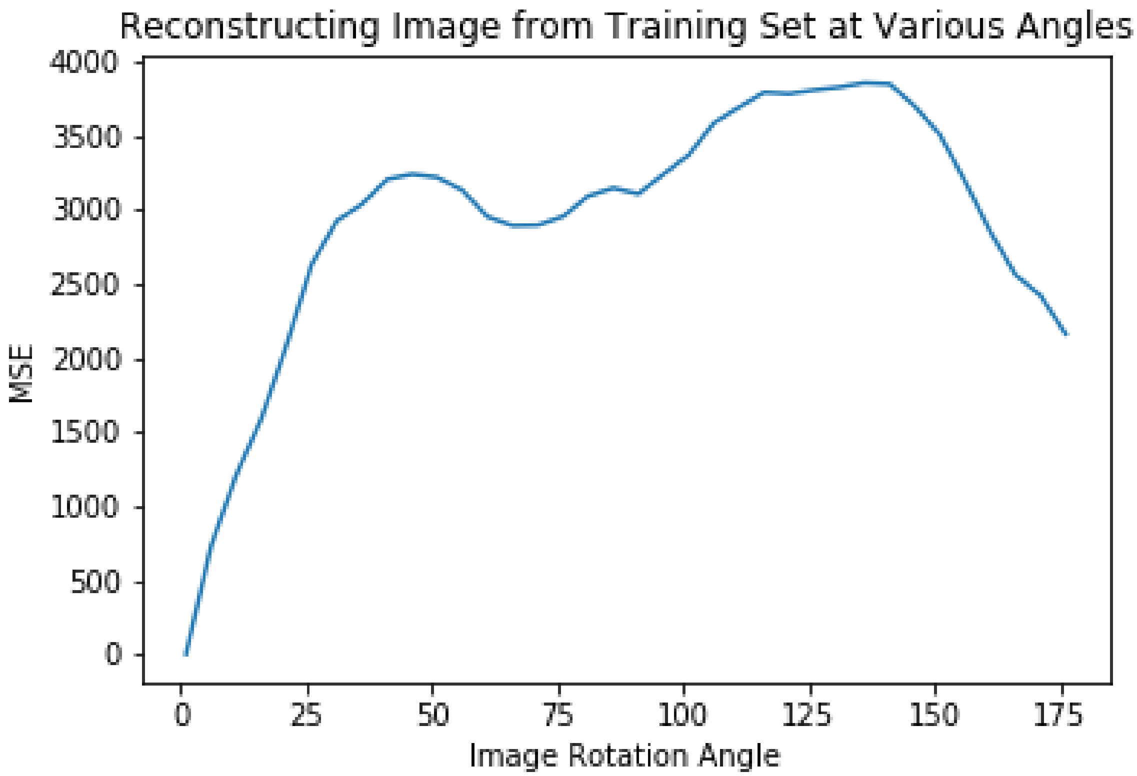
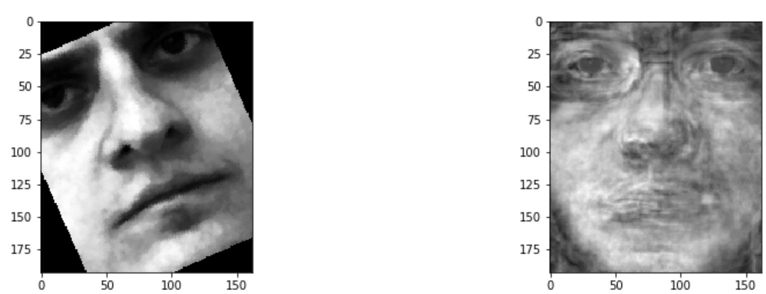


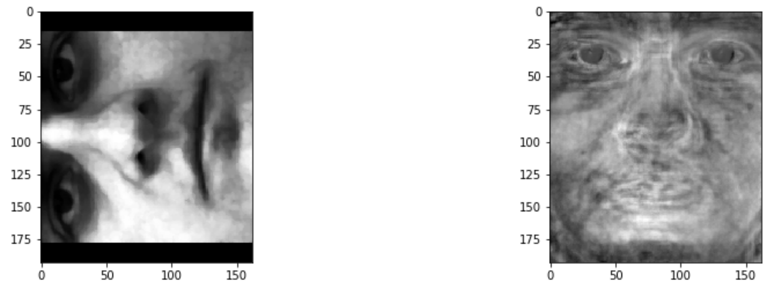
Figure 12: The Mean Squared Error plotted against the image rotation angle for reconstruction using 190 eigenfaces



(a) Original Image

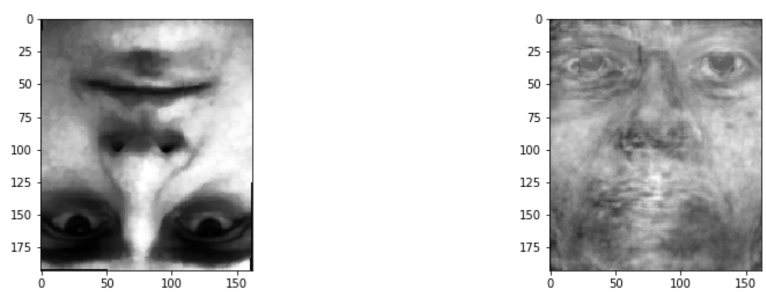
(b) Reconstructed Image

Figure 13: The original image rotated 24 degrees (left) and image reconstructed with 190 eigenfaces (right)



(a) Original Image (b) Reconstructed Image

Figure 14: The original image rotated 90 degrees (left) and image reconstructed with 190 eigenfaces (right)



(a) Original Image (b) Reconstructed Image

Figure 15: The original image rotated 179 degrees (left) and image reconstructed with 190 eigenfaces (right)