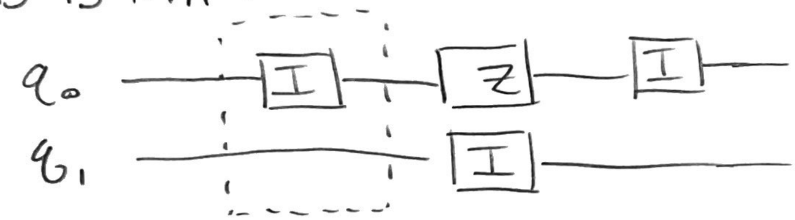
# pm_and_measure.pdf

## Pre-measurement Circuit Implementation & Spin-Z Measurement

To prepare the circuit for a $\langle Z_0 \otimes I_1 \rangle$ measurement, we must decompose $Z_0 \otimes I_1$ in terms of Z gates & other unitaries.

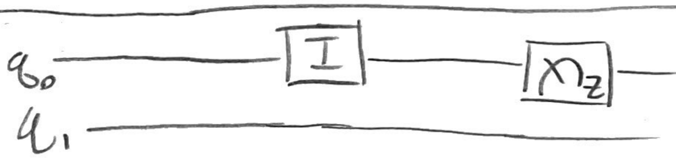
For $Z_0 \otimes I_1$, this is trivial

$Z_0 \otimes I_1$ =




This is the pre-measurement step needed to measure $\langle Z_0 \otimes I_1 \rangle$ with a Z-basis measurement.

PM-circuit for $\langle Z_0 \otimes I_1 \rangle$ + Z-measurement is =

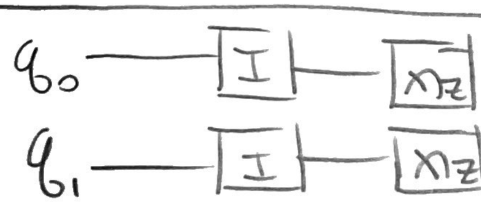



And similiarly, for $\langle I_0 \otimes Z_1 \rangle$

PM-circuit for $\langle I_0 \otimes Z_1 \rangle$ + Z-measurement is =


We also do something similar for $\langle Z_0 Z_1 \rangle$, but we measure both qubits

PM-circuit for $\langle Z_0 \otimes Z_1 \rangle$ + Z-measurement is =

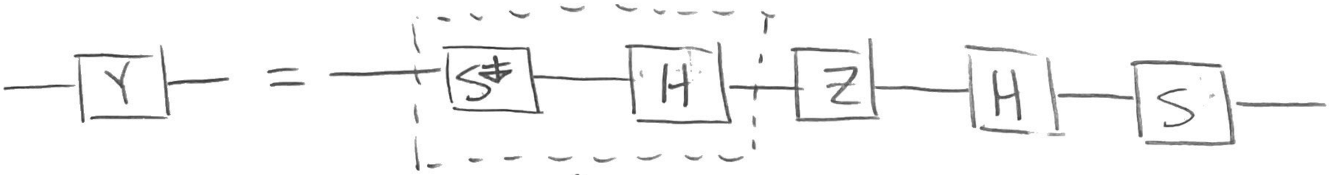


We also need a pre-measurement circuit for $\langle Y_0 \otimes Y_1 \rangle$

We can decompose the Y-gate in terms of a Z-gate & other unitaries.

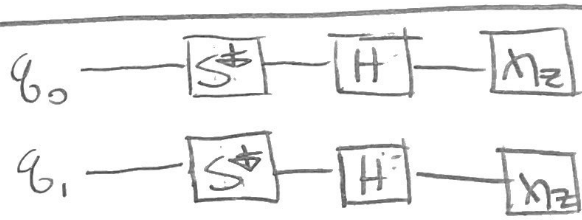
In particular, $Y = (HS^\dagger)^\dagger Z (HS^\dagger)$

In quantum circuit language,




This is the pre-measurement step needed to prepare $q_0$ & $q_1$ in $\langle Y_0 Y_1 \rangle$ for a Z-basis measurement.

PM-circuit for $\langle Y_0 \otimes Y_1 \rangle$ + Z-measurement is =




Lastly, we need a PM circuit for $\langle X_0 \otimes X_1 \rangle$

We can compose the X-gate in terms of a Z-gate & other unitaries.

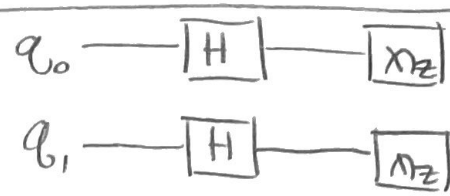
$$X = H^\dagger Z H$$

In quantum circuit language,


pre-measurement step to get everything ready for a Z-basis measurement

Thus,

The PM-circuit for $\langle X_0 \otimes X_1 \rangle$ + Z-measurement is : =




Lastly, I would like to verify the non-trivial operator decompositions using matrix multiplications, because I cited them without proof. In particular, I will verify

1) $X = H^\dagger Z H$

and 2) $Y = (HS^\dagger)^\dagger Z (HS^\dagger)$

1) $$H^\dagger Z H = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = X$$

$\therefore H^\dagger Z H = X$ ✓

2) $$(HS^\dagger)^\dagger Z (HS^\dagger) = S H^\dagger Z H S^\dagger$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

2) $$(HS^\dagger)^\dagger Z (HS^\dagger) = \frac{1}{2}\begin{pmatrix}1 & 1\\ i & -i\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\begin{pmatrix}1 & -i\\ 1 & i\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1\\ i & -i\end{pmatrix}\begin{pmatrix}1 & -i\\ -1 & -i\end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix}0 & -2i\\ 2i & 0\end{pmatrix} = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} = Y$$

$$\therefore (HS^\dagger)^\dagger Z (HS^\dagger) = Y$$