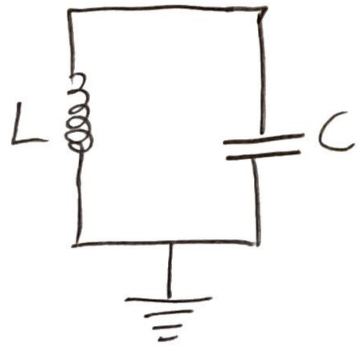
The Transmon Qubit

Previously, we analyzed the LC Tank circuit



Which has a Hamiltonian $\mathcal{H} = 4E_c n^2 + \frac{1}{2}E_L \phi^2$

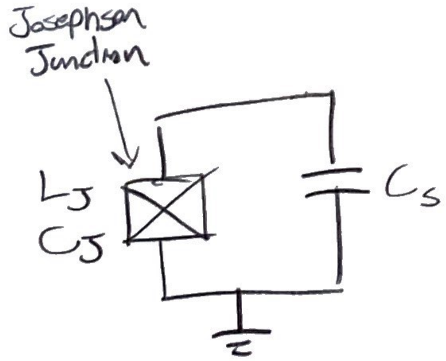
To make a Transmon, all we need to do is swap out the inductor with a quantum circuit element called the "Josephson Junction" (JJ) which is essentially a non-linear inductor. The JJ ~~in conjunction with the capacitor~~ provides the following current & voltage

superconducting critical current

ϕ = reduced flux

$$\begin{cases} I = I_c \sin\phi \\ V = \dfrac{\hbar}{2e}\dfrac{d\phi}{dt} \end{cases}$$

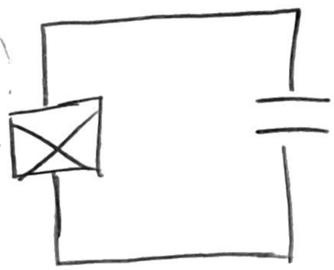
These can be derived from B.C.S. Theory. It would be a good idea to show the derivation of where these come from.



The Transmon

JJ equations

$$\begin{cases} I = I_c \sin\phi \\ V = \dfrac{\hbar}{2e}\dfrac{d\phi}{dt} \end{cases}$$



We already know the energy contribution of the capacitor to be $\underline{\dfrac{Q^2}{2C}} = \underline{4E_c n^2}$

We can use the JJ equations to find out the energy contribution from the Josephson Junction.

$\underline{\text{energy}} = \text{power} \times \text{time}$

$$E = E(t) = \int_{-\infty}^{t} I(t')V(t')\,dt'$$

$$E = \int_{-\infty}^{t} \frac{\hbar I_c}{2e} \sin\phi \frac{d\phi}{dt'}\,dt'$$

$$E = \int \frac{\hbar I_c}{2e} \sin\phi\, d\phi$$

$$E = -\frac{h I_c}{2e \cdot 2\pi} \cos\phi$$

$$\Phi_0 = \frac{h}{2e}$$

$$E = -\frac{\Phi_0 I_c}{2\pi} \cos\phi$$

We define the Josephson Energy to be

$$\boxed{E_J = \frac{\Phi_0 I_c}{2\pi}}$$

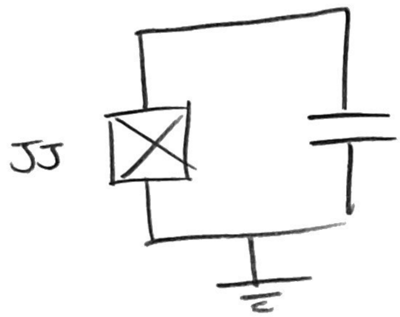
$$\boxed{E = -E_J \cos\phi}$$

Energy contribution of the JJ

We can write the Hamiltonian of the system as the sum of the energy contributions from the capacitor and the JJ.

Hamiltonian of the Transmon Qubit

$$\mathcal{H} = 4E_c n^2 - E_J \cos\phi$$



The Transmon Qubit

Next up on the to-do list...

How can we derive the qubit frequency?

$$\omega_q = \left(\sqrt{8E_J E_c} - E_c\right)/\hbar$$