NELSON MANDELA UNIVERSITY

JULY 2021 EMA

MODULE DESCRIPTION	**Artificial Intelligence**
MODULE CODE	**WHAI401**
FACULTY	**Science**
QUALIFICATION	**BSc Hons / BCom Hons**
EXAMINATION DATE	**16 July 2021**
DURATION (IN MINUTES)	**150**
TOTAL MARKS	**80**
PAGES	**5 (including this page)**
ADDENDUMS	**n/a**
EXAMINER	**Dr M.C. du Plessis**
MODERATOR	**Prof S. Viriri (UKZN)**
INSTRUCTIONS	**Answer all questions. Submit a single .pdf document.**
REQUIREMENTS	**n/a**

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1 AI through Search

Bidirectional Search (searching simultaneously from the start and the goal state) is used to improve the time and space complexity of search algorithms. Although Bidirectional Search normally employs Iterative Deepening from the one end and a Breadth First Search from the other, it is also possible to used it in conjunction with other search techniques.

You have to adapt you solution to the Uniform Cost Search which you implemented in Assignment 1 to a Bidirectional Search which performs two Uniform Cost searches from the start and the goal node simultaneously until they meet halfway. You can determine the correctness of your solution by seeing if the same path cost is found as was found by the normal Uniform Cost Search. Do not attempt a multi-threaded solution.

1. What do you notice regarding the number of cells visited on each of the three mazes?
2. Submit your solution by pasting all your code in your submission pdf.

[20]

2 Formal Logic

1. Consider the following statements:

   If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it has horns. The unicorn is magical if it has horns.

   Use the following propositions:
   p : *The unicorn is mythical*;
   q : *The unicorn is immortal*;
   r : *The unicorn is a mammal*;
   s : *The unicorn has horns*;
   t : *The unicorn is magical*;

   Prove using using Proof by Contradiction that:
   The unicorn has horns.
   Use the theorems given in the appendix to perform the proof. Clearly state which theorem you are using at each step. [17]

2. Use Wang's algorithm to prove that:

$$\neg p,\ \neg p \Rightarrow r,\ q \Rightarrow \neg r,\ \neg q \Rightarrow s\ \models\ p \Leftrightarrow q$$

   [14]

3. Translate the following into Clausal Form:

$$\forall x(\ \exists y(P(x,y) \wedge R(y)) \Rightarrow\ \exists y(Q(f(x),y) \wedge R(g(x),y)))$$

   [12]

4. Consider these formulas that are in Clausal Form:
   C_1: $M(x) \vee Q(x)$
   C_2: $\neg M(Clive)$
   B: $Q(y)$

   Make use of the Resolution Procedure and appropriate substitutions to prove that: $C_1, C_2\ \models\ B$.
   [7]

3 Prolog

1. Consider the following extract of a Prolog knowledge-base which represents the family tree of the Zulu and Swazi royal houses (for example, King Mpande is the father of King Cetshwayo). Note that only male family members are listed:

```
parent(mpande,cetshwayo).
parent(ngwane,sobhuza).
parent(dinuzulu,solomon).
parent(mswatiII,ludvonga).
parent(senzangakhona,mpande).
parent(nyangayezizwe,goodwill).
parent(mbandzeni,ngwane).
parent(sobhuza,mswatiIII).
parent(senzangakhona,dingane).
parent(solomon,nyangayezizwe).
parent(mswatiII,mbandzeni).
parent(dinuzulu,athur).
parent(sobhuza,thumbumuzi).
parent(senzangakhona,ushaka).
parent(cetshwayo,dinuzulu).
parent(solomon,mcwayizeni).
```

Create rules in the knowledge-base to support the following (in each case give all the rules that you add to the knowledge-base and any queries you write):

(a) A query to determine whether two rulers are brothers (note that a ruler cannot be his own brother). Who are the brothers of King Ushaka? [3]

(b) A query to determine whether a ruler is the uncle of another. Who are the rulers who had uncles that where also rulers? [2]

(c) A query to determine whether two rulers are related. Is King Ushaka related to King Goodwill? Is King Senzangakhona related to King Dingane? Is King Dinuzulu related to King Mbandzeni? [5]

Final Total: 80 marks

4 Appendix

Theorem		Name
Logical Equivalences		
$p \vee p \equiv p$	$p \wedge p \equiv p$	Idempotent Law
$x \vee y \equiv y \vee x$	$x \wedge y \equiv y \wedge x$	Commutative law
$p \vee (q \vee r) \equiv (p \vee q) \vee r$	$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$	Associative law
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive law
$\neg(p \vee q) \equiv \neg p \wedge \neg q$	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's law
$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$	Absorption law
If t is a tautology and f is a contradiction		
$\neg t \equiv f$	$\neg f \equiv t$	$t - f$ rule 1
$\neg p \vee p \equiv t$	$\neg p \wedge p \equiv f$	$t - f$ rule 2
$t \vee p \equiv t$	$t \wedge p \equiv p$	$t - f$ rule 3
$f \vee p \equiv p$	$f \wedge p \equiv f$	$t - f$ rule 4
$\neg(\neg p) \equiv p$		Involution Law
$p \Rightarrow q \equiv \neg p \vee q$		Conditional rule 1
$p \Rightarrow q \equiv \neg q \Rightarrow \neg p$		Conditional rule 2
$p \Leftrightarrow q \equiv (p \Rightarrow q) \wedge (q \Rightarrow p)$		Equivalence rule 1
$p \Leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$		Equivalence rule 2
$\neg(p \Leftrightarrow q) \equiv (p \Leftrightarrow \neg q) \equiv (\neg p \Leftrightarrow q)$		Equivalence rule 3
Logical Consequences		
$(p \Rightarrow q),\ p \models q$		Detachment or modus ponens
$(p \Rightarrow q),\ \neg q \models \neg p$		Contrapositive inference
$(p \Rightarrow q),\ (q \Rightarrow r) \models (p \Rightarrow r)$		Chain rule
$p \models p \vee q$		$\vee$-introduction
$p,\ q \models p \wedge q$		$\wedge$-introduction
$p \wedge q \models p$		$\wedge$-elimination
$p \Rightarrow q \models (p \vee r) \Rightarrow (q \vee r)$	$p \Rightarrow q \models (p \wedge r) \Rightarrow (q \wedge r)$	Implication rule 1
$p \Leftrightarrow q \models p \Rightarrow q$	$p \Leftrightarrow q \models q \Rightarrow p$	$\Leftrightarrow$-elimination

Table 1: Theorems