Let p: Capital investment increases

q: Inflation decreases

r: Unemployment decreases

s: Taxes are raised

Thus p => q ^ r, ¬q => s, ¬s => p

Prove p => q ^ r, ¬q => s, ¬s => p |= ¬r => s

|  | | | | **A** | **B** | **C** |  | |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **p** | **q** | **r** | **s** | **p=>q^r** | **¬q=>s** | **¬s=>p** | **A^B^C** | **¬r=>s** |
| T | T | T | T | T | T | T | T | T |
| T | T | T | F | T | T | T | T | T |
| T | T | F | T | F | T | T | F | T |
| T | T | F | F | F | T | T | F | F |
| T | F | T | T | F | T | T | F | T |
| T | F | T | F | F | F | T | F | T |
| T | F | F | T | F | T | T | F | T |
| T | F | F | F | F | F | T | F | F |
| F | T | T | T | T | T | T | T | T |
| F | T | T | F | T | T | F | F | T |
| F | T | F | T | T | T | T | T | T |
| F | T | F | F | T | T | F | F | F |
| F | F | T | T | T | T | T | T | T |
| F | F | T | F | T | F | F | F | T |
| F | F | F | T | T | T | T | T | T |
| F | F | F | F | T | F | F | F | F |

Therefore A^B^C |= ¬r=>s

∎

1. A1: p => q ^ r Premise 1

A2: ¬q => s Premise 2

A3: ¬s => p Premise 3

A4: p => r ^-elimination using A1

A5: ¬r => ¬p Conditional rule 2 using A5

A6: ¬p => s Conditional rule 2 using A3

A7: ¬r => s Chain rule using A6 and A7

∎

1. A1: p => q ^ r Premise 1

A2: ¬q => s Premise 2

A3: ¬s => p Premise 3

A4: ¬(¬r=>s) Negation of the consequence

A5: ¬(r V s) Conditional rule 1 using A5

A6: ¬r ^ ¬s De Morgan’s Law using A6

A7: ¬s ^-elimination using A7

A8: p => r ^-elimination using A1

A9: ¬s => r Chain rule using A3 and A9

A10: r Detachment using A10 and A7

A11: ¬r ^-elimination using A7

∎

1. (p => q ^ r) ≡ (¬p V (q ^ r)) ≡ (¬p V q) ^ (¬p V r)

(¬q => s) ≡ (q V s)

(¬s => p) ≡ (s V p)

¬(¬r => s) ≡ ¬(r V s) ≡ (¬r ^ ¬s)

S1: ¬p V q

S2: ¬q V r

S3: q V s

S4: s V p

S5: ¬r

S6: ¬s

S7: ¬q Resolvent of S2 and S5

S8: q Resolvent of S3 and S6

S9: ∎ Resolvent of S7 and S8

1. Step 1 - Convert to Standard Form

(¬pVq)^(¬pVr), qVs, sVp |= rVs

Step 2 - AND/OR Removal

¬pVq, ¬pVr, qVs, sVp |= r, s

Step 3 - Theorem Splitting

* 1. ¬p, ¬pVr, qVs, sVp |= r, s

Step 1 - Negation removal

¬pVr, qVs, sVp |= r, s, p

Step 2 - Theorem Splitting

* + 1. ¬p, qVs, sVp |= r, s, p

Step 1 - Negation removal

qVs, sVp |= r, s, p

Step 2 - Theorem Splitting

* + - 1. q, sVp |= r, s, p

Step 1 - Theorem Splitting

q, s |= r, s, p and q, p |= r, s, p

Both are true

* + - 1. s, sVp |= r, s, p

True since s appears on both sides

* + 1. r, qVs, sVp |= r, s, p

True since r appears on both sides

* 1. q, ¬pVr, qVs, sVp |= r, s

Step 1 - Theorem Splitting

* + 1. q, ¬p, qVs, sVp |= r, s

Step 1 - Negation Removal

q, qVs, sVp |= r, s, p

Step 2 - Theorem Splitting

* + - 1. q, sVp |= r, s, p

Step 1 - Theorem Splitting

q, s |= r, s, p and q, p |= r, s, p

Both are true

* + - 1. q, s, sVp |= r, s, p

True since s appears on both sides

* + 1. q, r, qVs, sVp |= r, s

True since r appears on both sides

∎