STATS 509 HW2

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Q1

a

$\mathbb{P}(R < -V\tilde{a}R) = q$

$F(-V\tilde{a}R) = q < 0.5$

Therefore, $-V\tilde{a}R < \mu$ since double exponential distribution is symmetric on μ

$\frac{1}{2}exp(\lambda(-V\tilde{a}R - \mu)) = q$

$V\tilde{a}R = -\mu - \frac{ln(2q)}{\lambda}$

$VaR = V\tilde{a}R \times Price = -\mu - \frac{ln(2q)}{\lambda} \times Price$

b

Since $V\tilde{a}R = -\mu - \frac{ln(2q)}{\lambda}$,and q < 0; $\lambda > 0$ and $V\tilde{a}R$ is strictly bigger than -μ.

Loss X ~ DExp($-\mu, \lambda$), Relative ES = $\frac{1}{q}\int_{V\tilde{a}R_q}^{\infty} x\frac{1}{2}\lambda exp(-\lambda(x+\mu))dx$

$ES = \frac{1}{2q}exp(-\lambda(V\tilde{a}R+\mu))(V\tilde{a}R+\frac{1}{\lambda})$

From a) we know $V\tilde{a}R = -\mu - \frac{ln(2q)}{\lambda}$, plug this is

Gives us $ES = V\tilde{a}R + \frac{1}{\lambda} = -\mu - \frac{ln(2q)}{\lambda} + \frac{1}{\lambda}$

c

In [1]:

```
source("startup.R")
df = read.csv("Nasdaq_daily_Jan1_2019-Dec31_2021.csv")
head(df)
```

A data.frame: 6 × 7

	Date	Open	High	Low	Close	Adj.Close	Volume
	<chr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1	2019-01-02	6506.91	6693.71	6506.88	6665.94	6665.94	2261800000
2	2019-01-03	6584.77	6600.21	6457.13	6463.50	6463.50	2607290000
3	2019-01-04	6567.14	6760.69	6554.24	6738.86	6738.86	2579550000
4	2019-01-07	6757.53	6855.60	6741.40	6823.47	6823.47	2507550000
5	2019-01-08	6893.44	6909.58	6795.86	6897.00	6897.00	2380290000
6	2019-01-09	6923.06	6985.22	6899.56	6957.08	6957.08	2422590000

Note the the MLE of location parameter μ is the sample median and scale parameter $1/\lambda$ is the sample mean deviation from the median

Source: https://stats.stackexchange.com/questions/281682/how-to-fit-a-data-against-a-laplace-double-exponential-distribution-and-check

In [2]:

```
n = length(df$Adj.Close)
R = df$Adj.Close[-1]/df$Adj.Close[-n]-1
m = median(R)
data.frame(m = m, lambda = 1/mean(abs(R - m)))
l = 1/mean(abs(R- m))
```

A data.frame: 1 × 2

m	lambda
<dbl>	<dbl>
0.001913407	99.42223

$\hat{\mu} = 0.001913407$, and $\hat{\lambda} = 99.4222$

In [3]:

```
var_rl = -1 * qdexp(p = 0.01, mu = m, lambda = l)
print(var_rl)
var = 10^7 * var_rl
print(c('Relative Var is:',var_rl))
print(c('Var is:',round(var,2)))
p = 0.01
ES = 1e7*(var_rl-1/l)
print(c("Estimated Shortfall is:",round(ES,2)))
```

```
[1] 0.03743416
[1] "Relative Var is:"    "0.0374341607656987"
[1] "Var is:"    "374341.61"
[1] "Estimated Shortfall is:" "273760.48"
```

The probability of loss less than 374341.61 usd is 0.99.

Given that a loss is occuring at or greater than $ 374341.61, the mean loss of this porfolio is $ 273760.48.

d

using MLE estimator of μ, σ

$\mu = samplemean, \hat{\sigma}^2 = \frac{n}{n-1} samplevariance$

In [5]:

```
m = mean(R)
variance = var(R) * n / (n-1)
sd = sqrt(variance)
print(c('mu and sigma:',m,sd))
var_rl = qnorm(p, m, sd) * -1
print(c("Relative VaR is :",var_rl))
var = 10^7 * var_rl
print(c("VaR is :",round(var,2)))
ES = -m + sd / p *(dnorm((qnorm(p = p, mean = 0, sd = 1)),0,1))
print(ES*1e7)
```

```
[1] "mu and sigma:"       "0.00126080759968301" "0.0155792444898162"
[1] "Relative VaR is :"  "0.0349819346983633"
[1] "VaR is :"  "349819.35"
[1] 402612.2
```

$\hat{\mu} = 0.0012608$, and $\hat{\sigma} = 0.015579$

The value at risk is 349819.35 usd. It means the probability of loss less than $ 349819.35 is 0.99.

The Expected shortfall is 402612.2 usd , which means given a loss at or below 349819.35, the mean loss of this portfolio is 402612.2 usd.

Try to compare the kurtosis and skewness

In [6]:

```
library("moments")
sk = skewness(R)
kt = kurtosis(R)-3 # to excessive kurtosis
print(paste("True Return Skewness is ",sk))
print(paste("True Return Kurtosis is ",kt))
sk1 = 0
kt1 = 3
sk2 = 0
kt2 = 6
data.frame(skewness = c(sk, sk1, sk2),kurtosis = c(kt,kt1,kt2),row.names =
```

```
[1] "True Return Skewness is  -0.743521849028927"
[1] "True Return Kurtosis is  11.1955040430353"
```

A data.frame: 3 × 2

	skewness	kurtosis
	<dbl>	<dbl>
True	-0.7435218	11.1955
Normal	0.0000000	3.0000
Double Exponential	0.0000000	6.0000

Based on the Skewness and kurtosis, Double exponential distribution is closer to the True distribution, which should provide more accurate result upon prediction.

Q2

a

From the prompt, we know that $F(\mu) = 0.9$

Expand the conditional probability by joint dividing marginal probability.

$\mathbb{P}(X \leq x | X \geq \mu) = \frac{\mathbb{P}(\mu \leq X \leq x)}{\mathbb{P}(X \geq \mu)} = \frac{F(x) - F(\mu)}{1 - F(\mu)}$, where F is the c.d.f of r.v. X.

This gives us

$\frac{F(x)-0.9}{0.1} = 1 - (1 + \frac{\xi(x-\mu)}{\sigma})^{-1/\xi}, x \geq \mu$

Solve for F(x) will give us the c.d.f of X

F(x) =

$$\begin{cases} 1 - 0.1(1 + \frac{\xi(x-\mu)}{\sigma})^{-1/\xi} & \text{if } x \geq \mu \\ 0.9 & \text{if } x < \mu \end{cases}$$

By definition X is loss

$\mathbb{P}(X > V\tilde{a}R) = q = 0.01$

$V\tilde{a}R = F^{-1}(0.01) = Q(0.01)$, where Q denotes the quantile function of F(x)

$V\tilde{a}R = \mu + \frac{\sigma(10^{\xi}-1)}{\xi}$

b

$ES = \mathbb{P}(X < x|X > V\tilde{a}R) = \frac{F(x)-(1-q)}{q} = 1 - 10(1 + \frac{\xi(x-\mu)}{\sigma})^{-1/\xi}, x \geq V\tilde{a}R$

This is equivalent to

= $1 - (1 + \frac{\xi(x-\mu+\frac{\sigma}{\xi}-\frac{10^\xi}{\xi})}{10^\xi\sigma})^{-1/\xi}$

If we let $\sigma' = 10^\xi\sigma, \mu' = \mu + \frac{10^\xi\sigma-\sigma}{\xi}$, then we can rewrite the shortfall distribution into a Generalized Pareto distribution.

$F_{\mathbf{ES}}(x) = 1 - (1 + \frac{\xi(x-\mu')}{\sigma'})^{-1/\xi}, if x > \mu'$

c

Because ES ~ GPD(μ', σ', ξ),

$\mathbb{E}(ES) = \mu' + \frac{\sigma'}{1-\xi} = \mu + \frac{10^\xi\sigma-\sigma}{\xi} + \frac{10^\xi\sigma}{1-\xi}$

In [ ]:

```

```