Swing Option

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Introduction to Financial Engineering

Outline

1. BACKGROUND AND SIGNIFICANCE
2. SWING OPTION
3. REAL-WORLD APPLICATION
4. ASSET MODEL
5. PRICING ALGORITHM
6. SENSITIVITY ANALYSIS

Swing Option

1

Swing Option คือ สัญญาประเภทหนึ่งที่นักลงทุนใช้ในตลาดพลังงานซึ่งอนุญาตให้ผู้ถือ Option ซื้อพลังงานในปริมาณที่กำหนดไว้ล่วงหน้าในราคาที่กำหนดไว้

2

Swing Option นั้นอาจถูกรู้จักในชื่อ **Take-or-Pay** หรือ **Take-and-Pay** Contract โดย Swing Option มีการซื้อขายกันนอกตลาดหลักทรัพย์ **(Over-the-counter)**

Swing Option

How to use

1. ผู้ถือ Option จะสามารถใช้สิทธิ์เพื่อซื้อสินทรัพย์เพิ่มเติมตอนราคาสูงกว่าราคาใช้สิทธิ์
2. ผู้ถือ Option จะสามารถใช้สิทธิ์เพื่อซื้อสินทรัพย์น้อยลงตอนราคาน้อยกว่าราคาใช้สิทธิ์

Payoff

Payoff = max(0, n X (S_T - K))

Restriction

1. จำนวนสิทธิที่สามารถ exercise ได้ (n_right)
2. ปริมาณสินทรัพย์อ้างอิงที่สามารถซื้อเพิ่มหรือลดได้ในแต่ละงวด (local constraint)
3. ปริมาณสินทรัพย์อ้างอิงรวมที่สามารถซื้อเพิ่มหรือลดได้ตลอดทั้งสัญญา (global constraint)

Real-World Application

Oil Pipeline Company

Pipeline Congestion

Swing Down

ASSET MODEL

1. Mean-reverting Ornstein-Uhlenbeck Process

2. Censured Mean Reversion Binomial Model

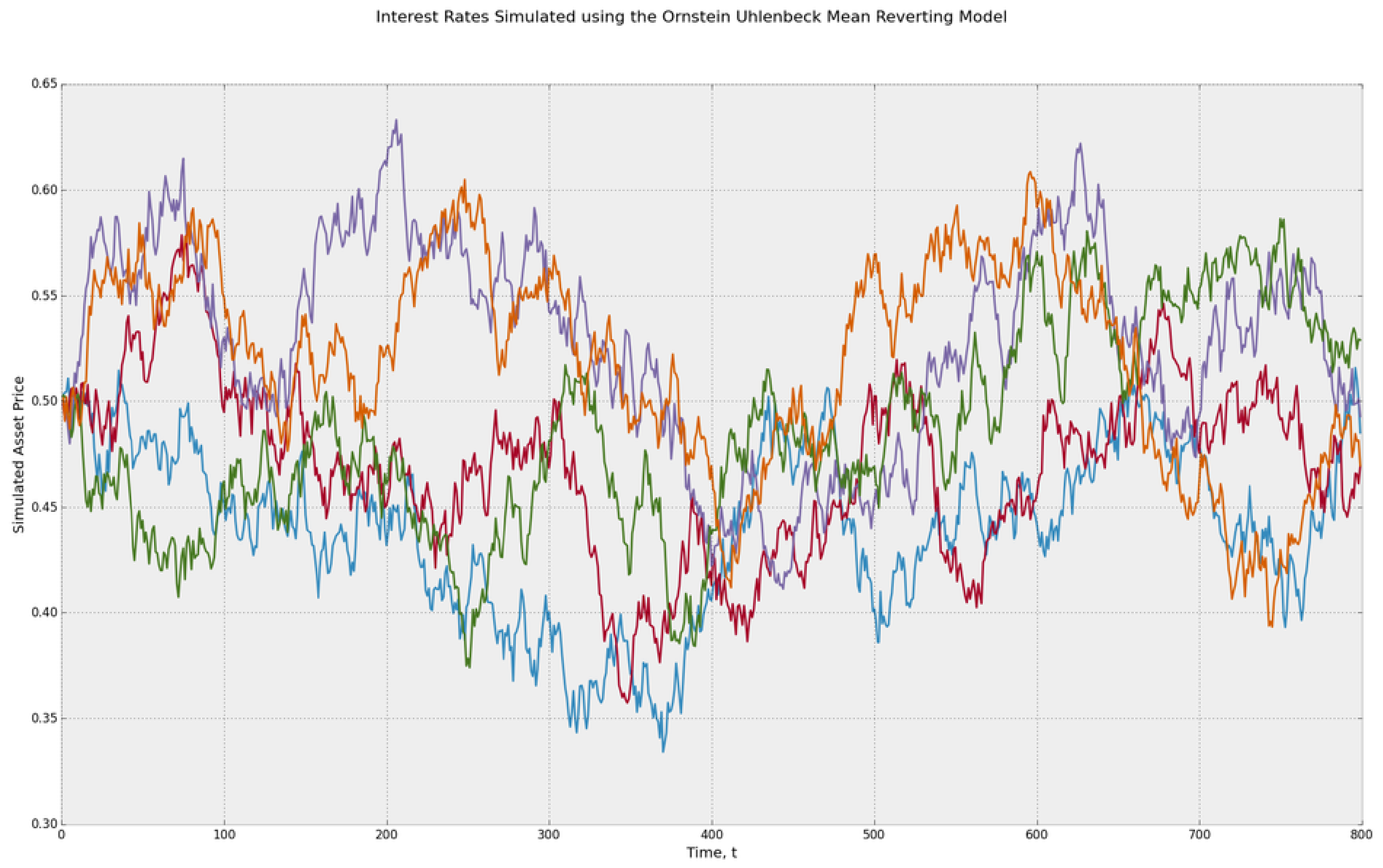
Mean-reverting Ornstein-Uhlenbeck Process

Speed of Mean Reversion > 0 Long-run mean Volatility

$$dx_t = \kappa(\bar{x} - x_t) + \sigma dW_t$$

$$\bar{x} = \ln(\bar{S}) - \frac{\sigma^2}{2\kappa}$$

$$x_t = \ln(S_t)$$



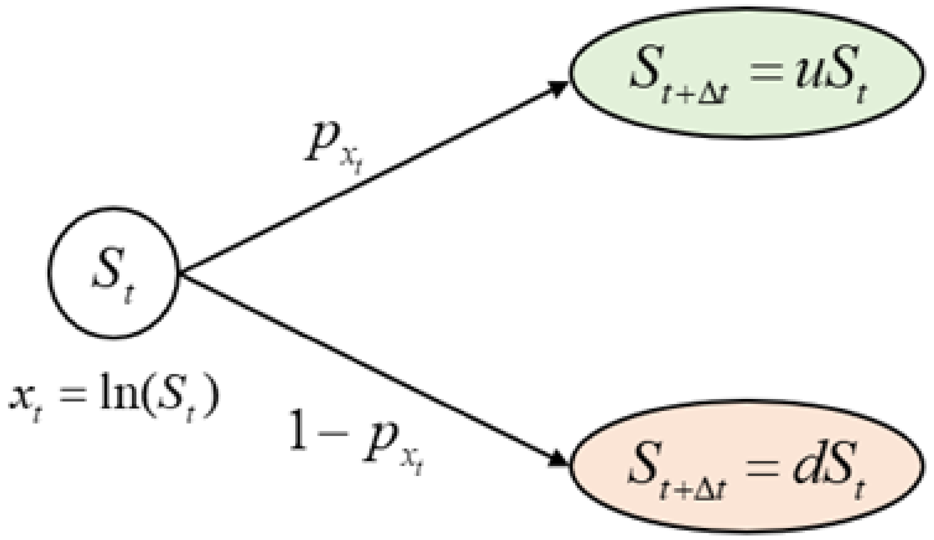
http://www.turingfinance.com/random-walks-down-wall-street-stochastic-processes-in-python/
https://www.researchgate.net/figure/Trajectory-Ornstein-Uhlenbeck-process-observe-the-reversion-rate-b-3-towards-the_fig1_281965169

Censured Mean Reversion Binomial Model

Nelson and Ramaswamy (1990)

By using the **moment matching technique** of

1. Expection value of natural log of the asset price
2. Variance of the natural log of the asset price



$$x_{t+\Delta t} = \ln(uS_t) = \ln(u) + \ln(S_t) = U + x_t$$

$$x_{t+\Delta t} = \ln(dS_t) = \ln(d) + \ln(S_t) = D + x_t$$

$$u = \exp(\sigma\sqrt{\Delta t})$$

$$d = \exp(-\sigma\sqrt{\Delta t})$$

$$p_{x_t} = \max\left(0,\ \min\left(1,\ \frac{1}{2} + \frac{1}{2}\,\frac{\kappa(\bar{x} - x_t)\sqrt{\Delta t}}{\sigma}\right)\right)$$

transform to risk-neutral probability

Normalized Risk Premium

$$\tilde{p}_{x_t} = \max\left(0,\ \min\left(1,\ \frac{1}{2} + \frac{1}{2}\,\frac{\kappa\left(\left(\bar{x} - \frac{\lambda_x}{\kappa}\right) - x_t\right)\sqrt{\Delta t}}{\sigma}\right)\right)$$

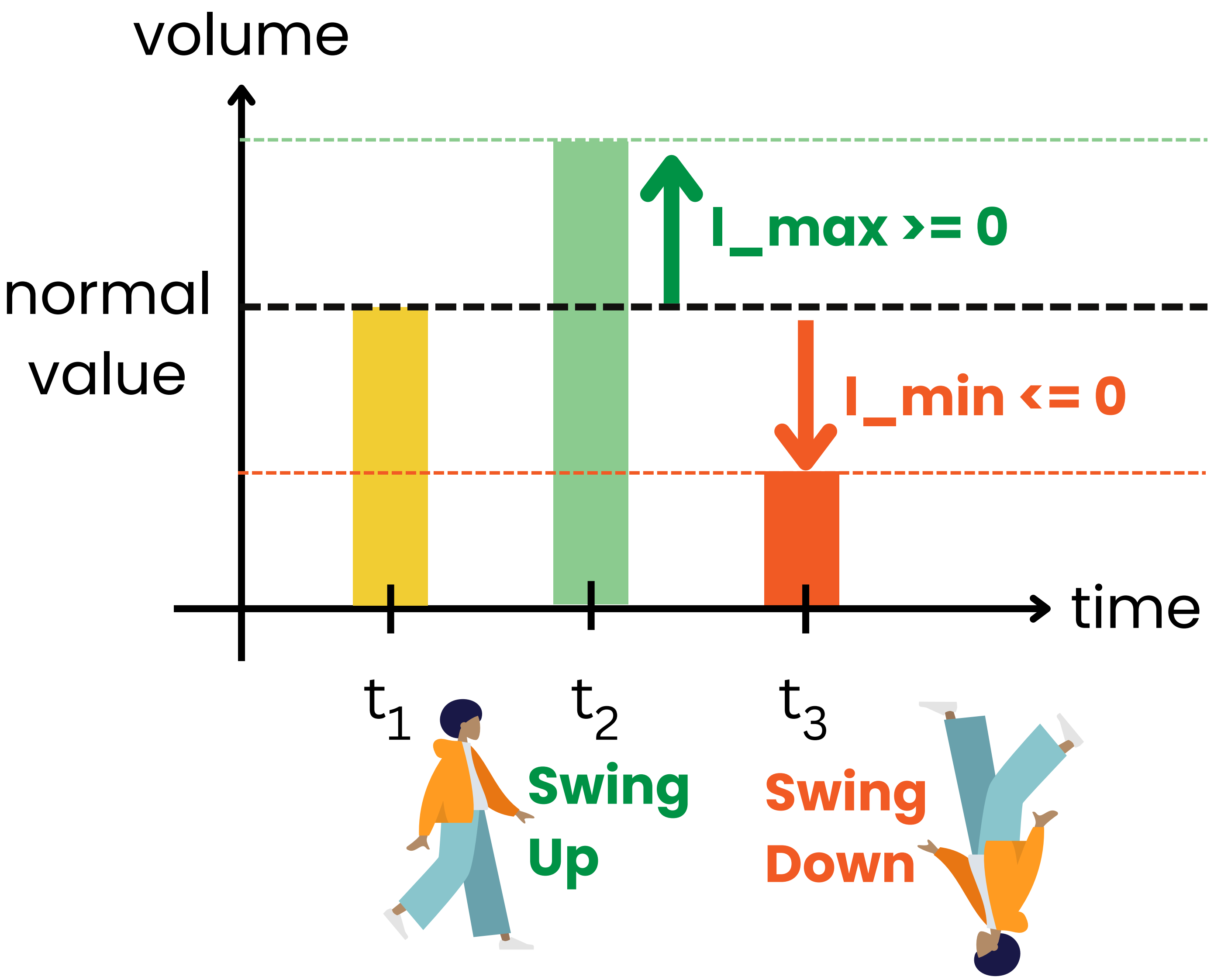
PRICING ALGORITHM

1. Swing Option with only Local Constraints

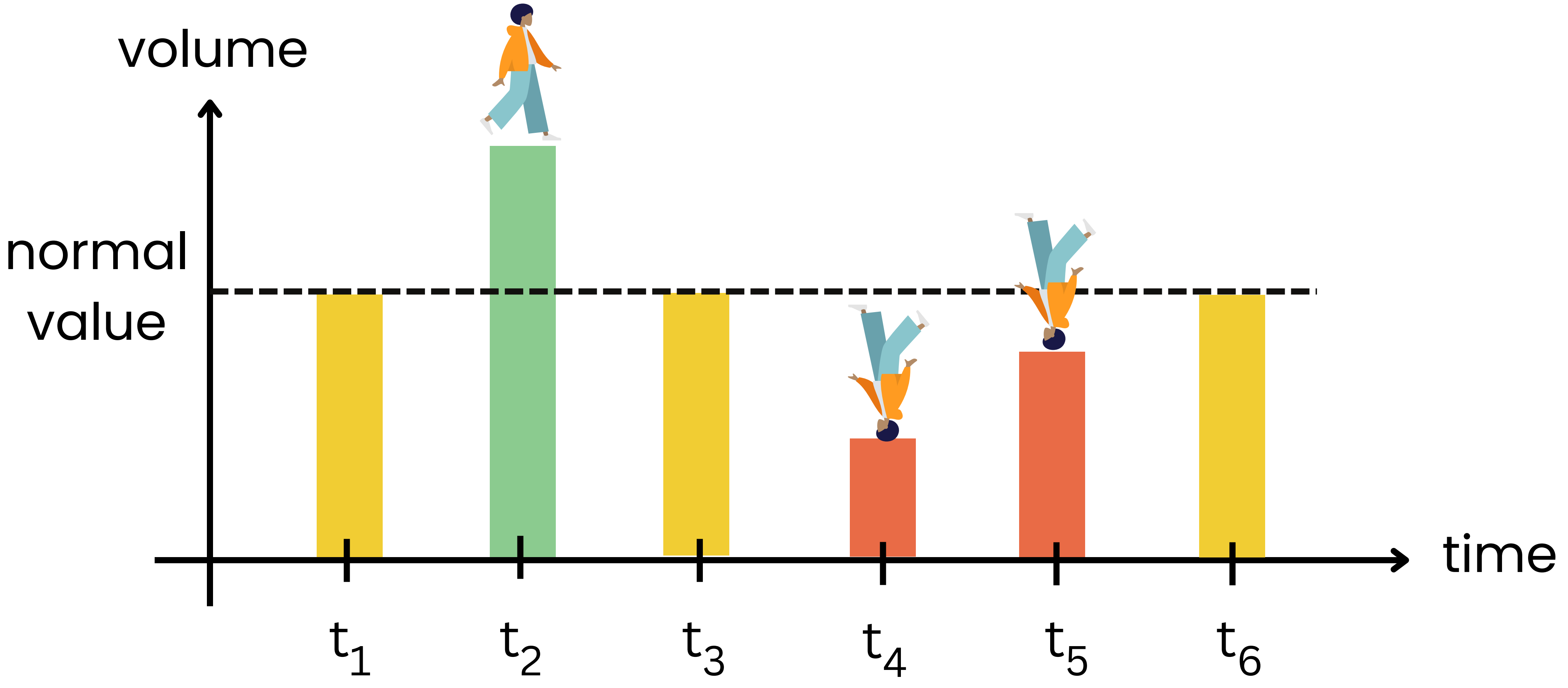
2. Swing Option with both Local and Global constraints

Swing Option with only Local Constraint

I_min and **I_max**



n_right



Example: n_right = 4, n_time = 6

Swing Option with only Local Constraint

Bang-bang Consumption

Since there is no limit in overall swing value

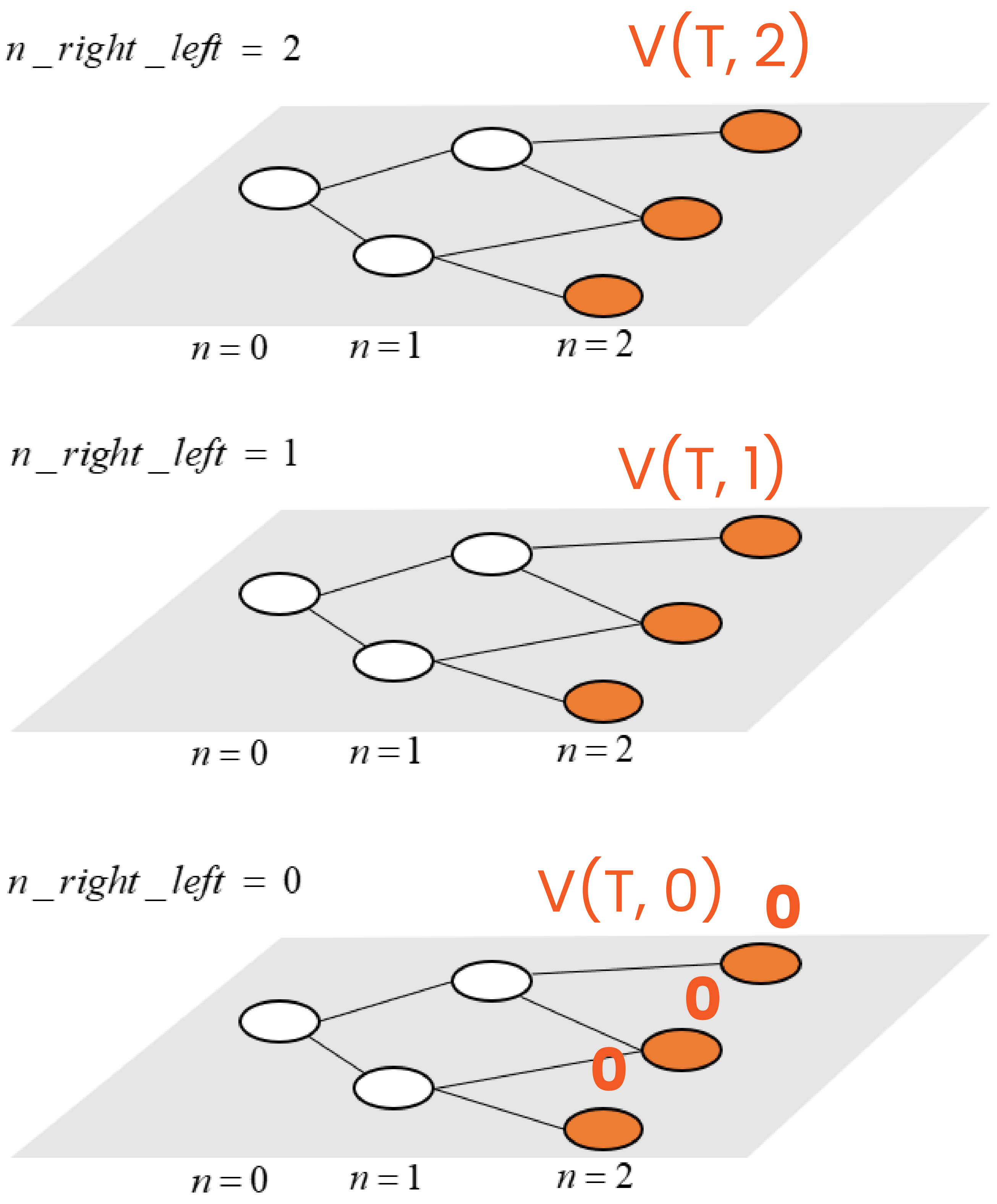
In order to maximize the payoff, when exercises, always swings at the maximum possible value. (I_min, I_max)

Local Constraint Payoff Function

When exercises,

$$payoff_1(t) = \max \begin{cases} I\_max * (S_t - K) \\ I\_min * (S_t - K) \\ 0 \end{cases}$$

Swing Option with only Local Constraint



Option's Value V(t, n_right_left)

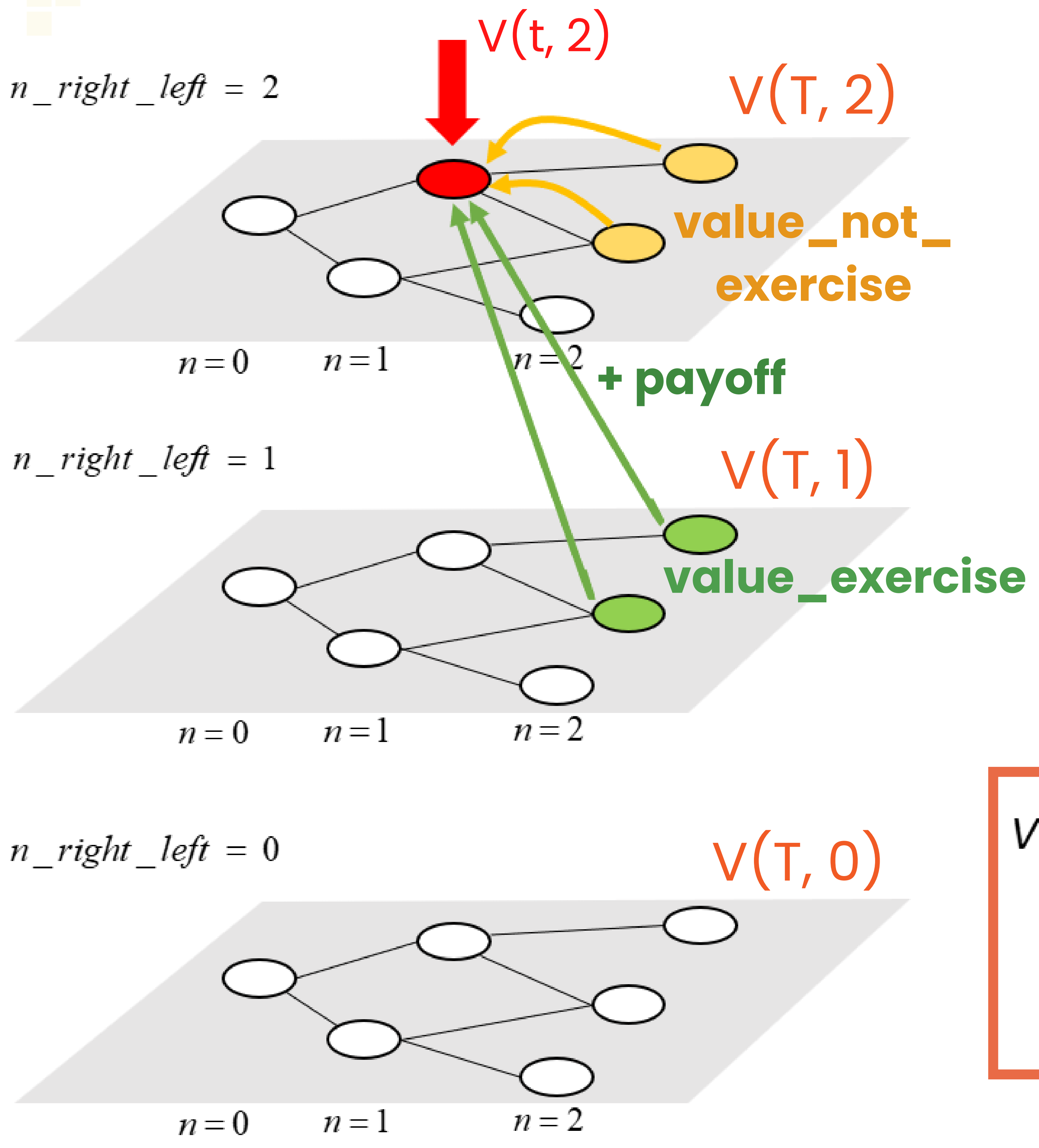
time จำนวนสิทธิที่เหลืออยู่

1. **Compute Payoff at time T for all n_right_left (n = 0 to n_right)**

$$V(T,n) = \begin{cases} payoff_1(T) & \text{if } n \geq 1 \\ 0 & \text{if } n = 0 \end{cases}$$

$$payoff_1(t) = \max \begin{cases} l\_max*(S_t - K) \\ l\_min*(S_t - K) \\ 0 \end{cases}$$

Swing Option with only Local Constraint



2. Backward Calculation

$$value\_not\_exercise = e^{-r*dt}\left(p*V^u(t+1,\ n)+(1-p)*V^d(t+1,\ n)\right) \quad : n \geq 1$$

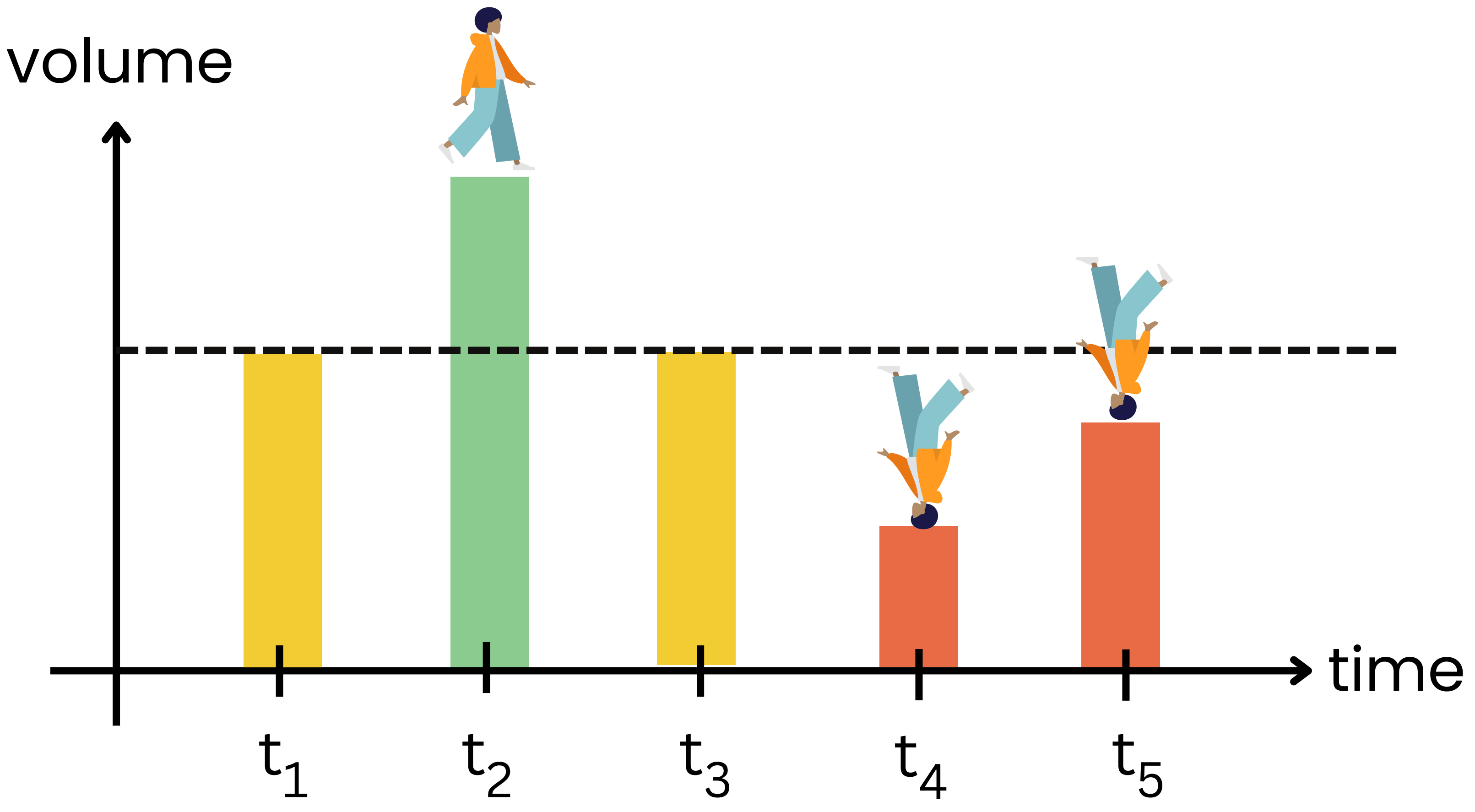
$$value\_exercise = \begin{cases} e^{-r*dt}\left(p*V^u(t+1,\ n-1)+(1-p)*V^d(t+1,\ n-1)\right) & if\ \ n>1 \\ 0 & if\ \ n=1 \end{cases}$$

$$V(t,n) = \begin{cases} \max(value\_not\_exercise,\ value\_exercise + payoff_1(t)) & if\ \ n \geq 1 \\ 0 & if\ \ n = 0 \end{cases}$$

Price at time 0 = V(0, n_right)

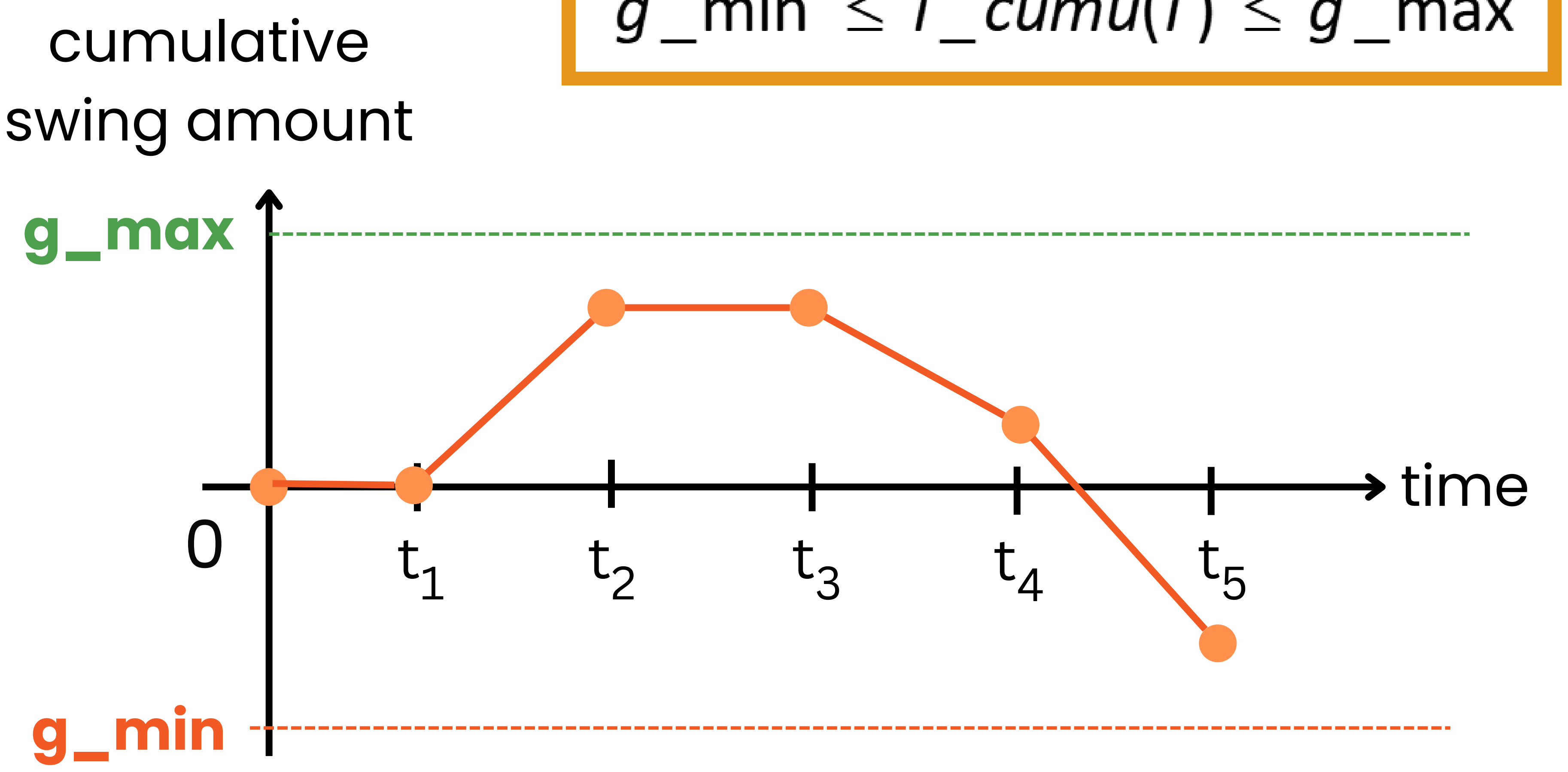
Swing Option with only Global Constraint

g_min and **g_max**



Example: n_right = 3, n_time = 5

$$g\_min \le l\_cumu(T) \le g\_max$$



$$l\_cumu(t) = \sum_{i=1}^{j} l_{t_i}; \quad t \in [t_j, t_{j+1})$$

Remark In this case, the swing amount in each exercise, l_min, l_max, g_min, g_max **must be** multiple of some real number in order to obtain finite number of option trees.

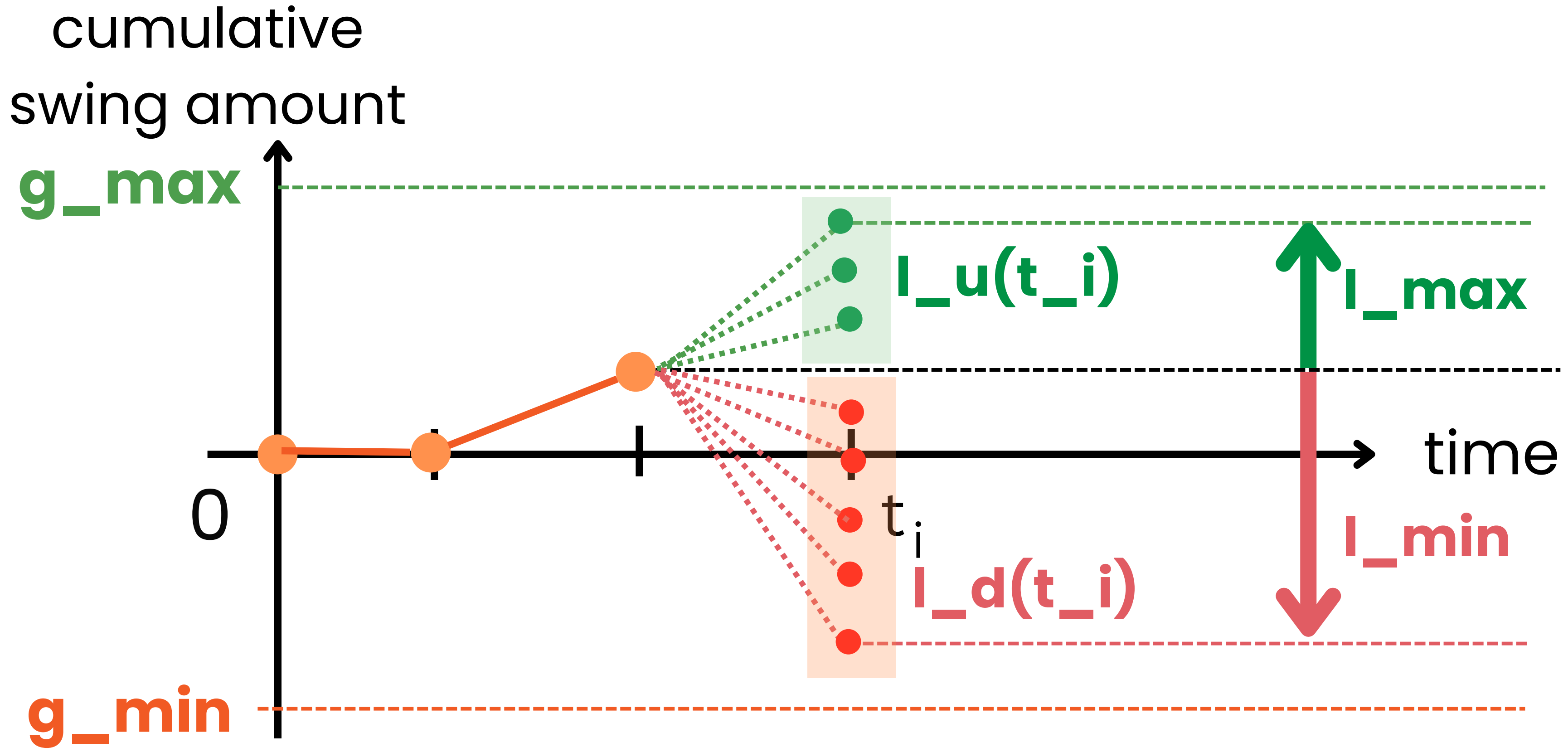
Swing Option with only Global Constraint

Bang-bang Consumption

Global Constraint Payoff Function

swing amount

$$payoff_2(t,\ l) = \max\begin{cases} l * (S_t - K) \\ 0 \end{cases}$$



Local Constraint Payoff Function

$$payoff_1(t) = \max\begin{cases} l\_max * (S_t - K) \\ l\_min * (S_t - K) \\ 0 \end{cases}$$

$$l \in l\_u(t) \cup l\_d(t)$$

Swing Option with only Global Constraint

Option's Value V(**t**, **n_right_left**, **l_cumu**)

time จำนวนสิทธิที่เหลืออยู่ cumulative swing amount

1. **Compute Payoff at time T for all n_right_left, l_cumu**

$$V(T, n, l\_cumu) = \begin{cases} \max \begin{cases} payoff_2(T, \max(l\_u(T))) \\ payoff_2(T, \min(l\_d(T))) \end{cases} & if\ n \geq 1 \\ 0 & if\ n = 0 \end{cases}$$

still bangable at maturity date

Swing Option with only Global Constraint

2. Backward Calculation

$$value\_not\_exercise = e^{-r*dt}\begin{pmatrix} p*V^u(t+1,\ n,\ l\_cumu) \\ +(1-p)*V^d(t+1,\ n,\ l\_cumu) \end{pmatrix} \qquad : n \geq 1$$

$$value\_exercise(l) = \begin{cases} e^{-r*dt}\begin{pmatrix} p*V^u(t+1,\ n-1,\ l\_cumu + l) \\ +(1-p)*V^d(t+1,\ n,\ l\_cumu + l) \end{pmatrix} & if\ \ n > 1 \\ 0 & if\ \ n = 1 \end{cases}$$

$$V(t,n,l\_cumu) = \begin{cases} \max\begin{cases} value\_not\_exercise \\ \max\limits_{l_t \in l\_u(t) \cup l\_d(t)} (value\_exercise(l_t) + payoff_2(t,\ l_t)) \end{cases} & if\ n \geq 1 \\ 0 & if\ \ n = 0 \end{cases}$$

Price at time 0 = V(t = 0, n_right_left = n_right, l_cumu = 0)

Sensitivity Analysis

1. Number of exercise right
2. Strike Price
3. Long-run mean & Speed of mean reversion
4. Volatility

Default Constraint

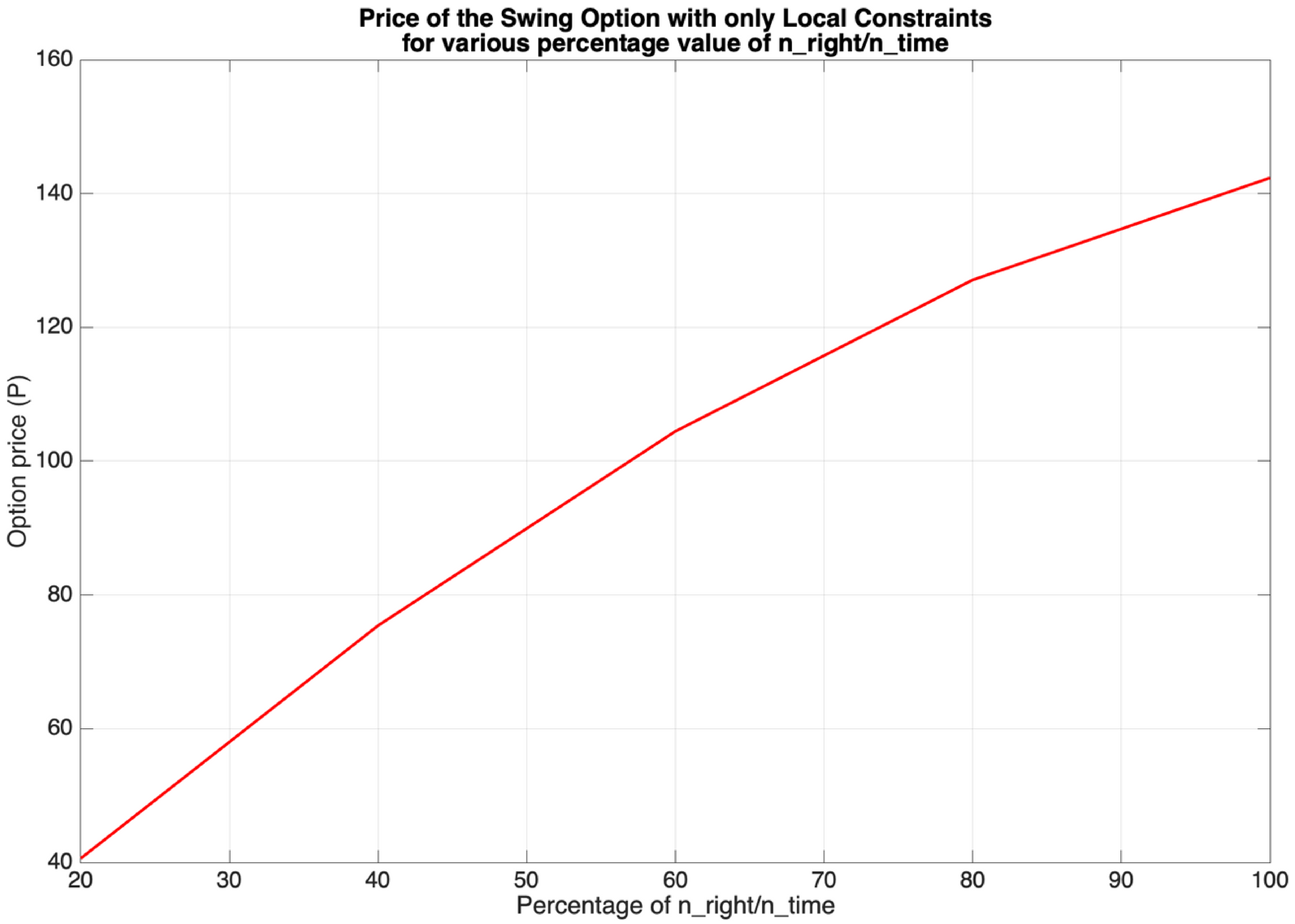
Local Constraint

- Number of exercise right = 2
- Number of Installment = 5
- Local Constraint Min = -1
- Local Constraint Max = 1
- Stock Price at the beginning = 100
- Strike Price = 100
- Long-run Mean = 100
- Sigma = 0.7
- Speed of mean reversion = 1
- Risk Premium = 0.1
- Risk-free Rate = 0.1
- Time to maturity = 1
- Number of timestep = 1000

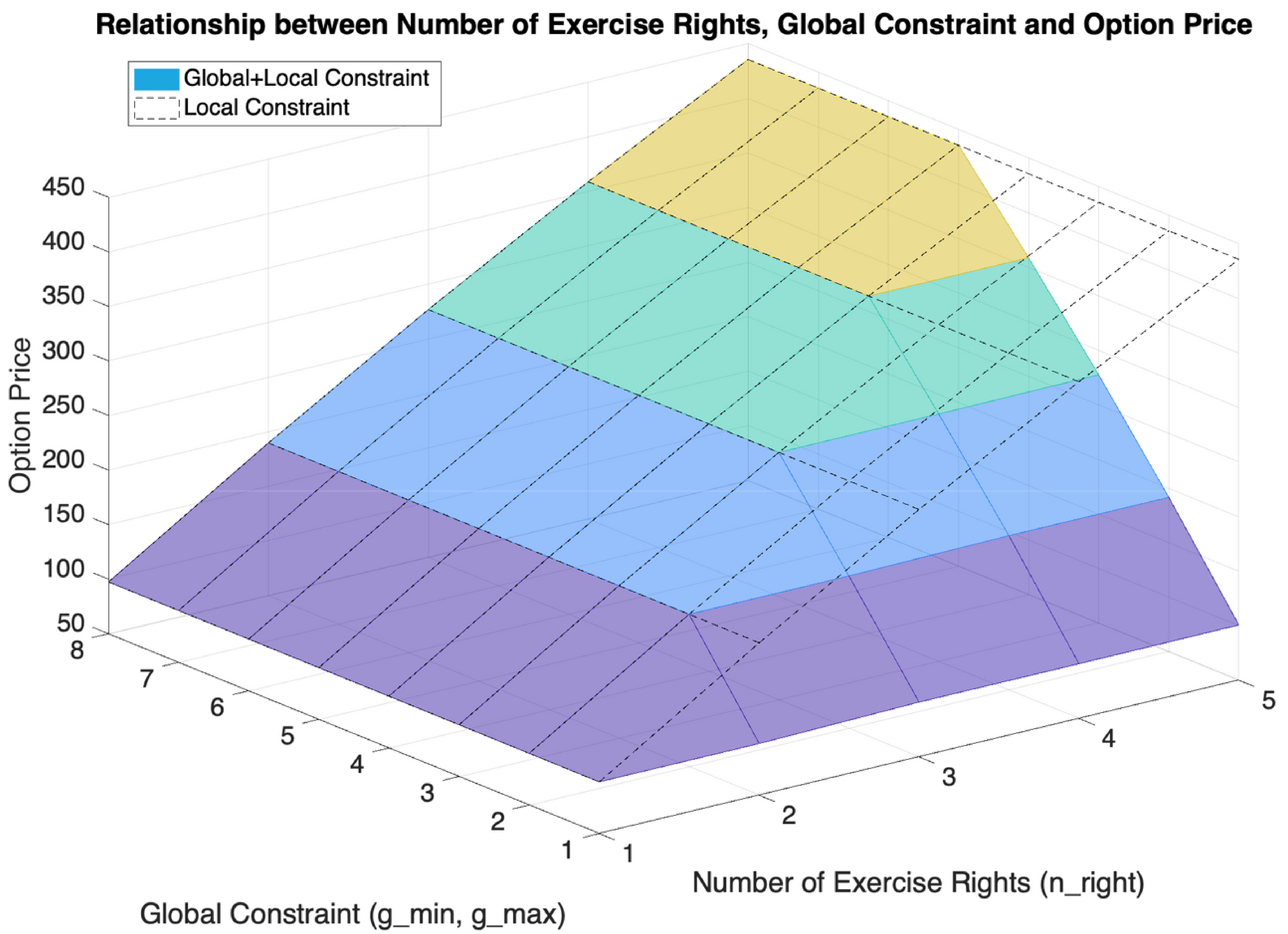
Global Constraint

- Multiplier = 1
- Global Constraint Min = -3
- Global Constraint Max = 3

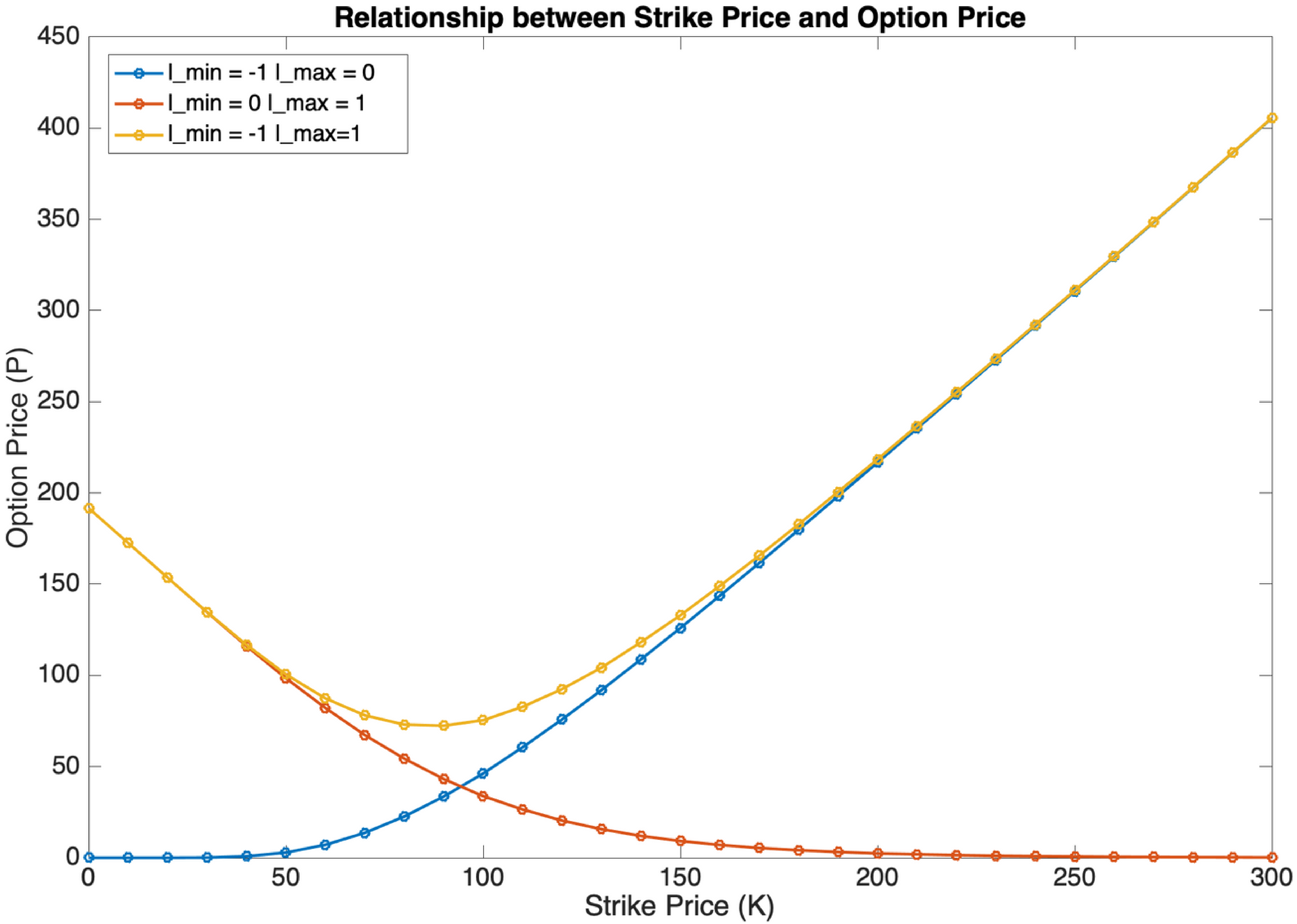
Local Swing Option and Number of exercise rights



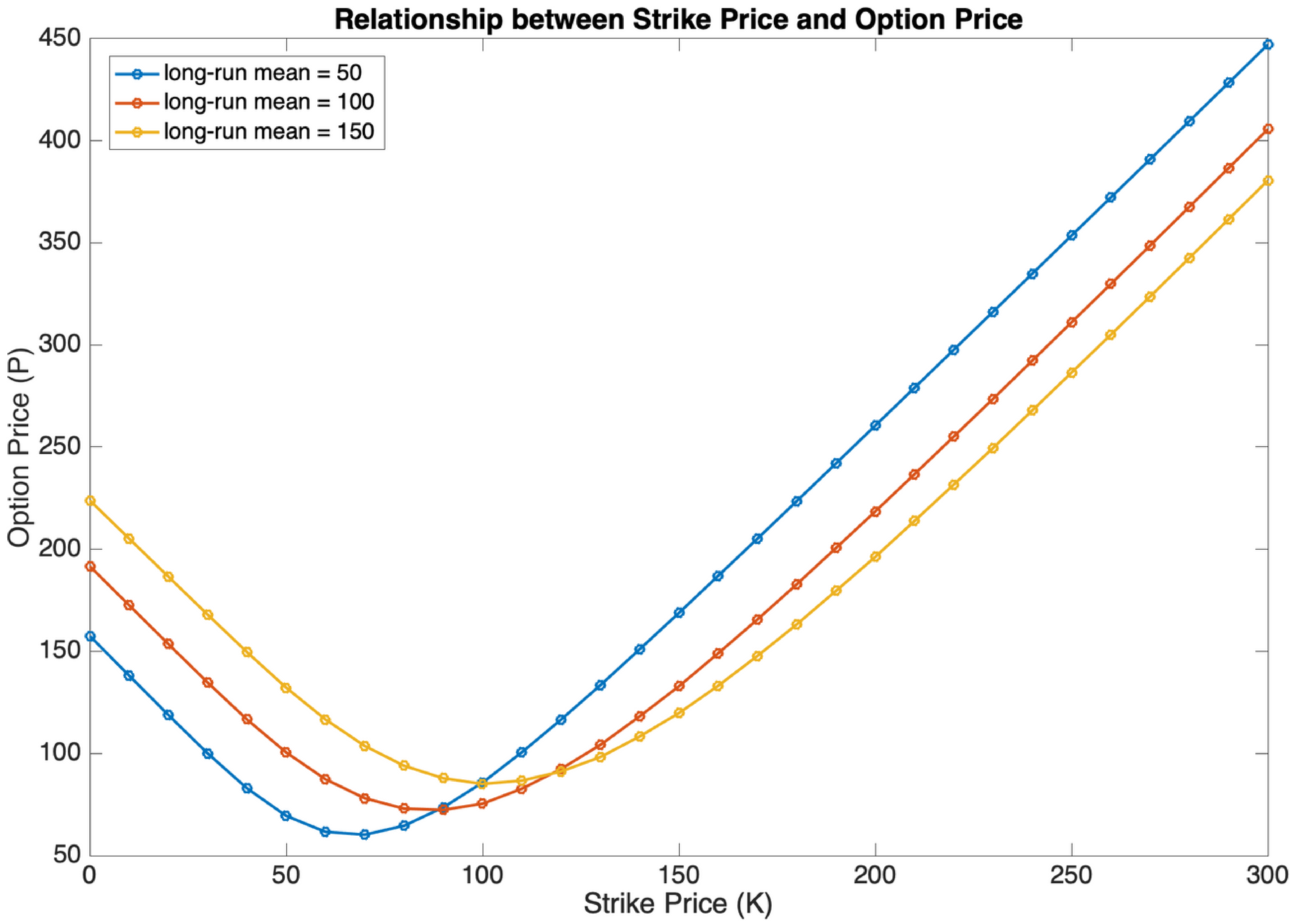
Global Swing Option and Number of exercise rights



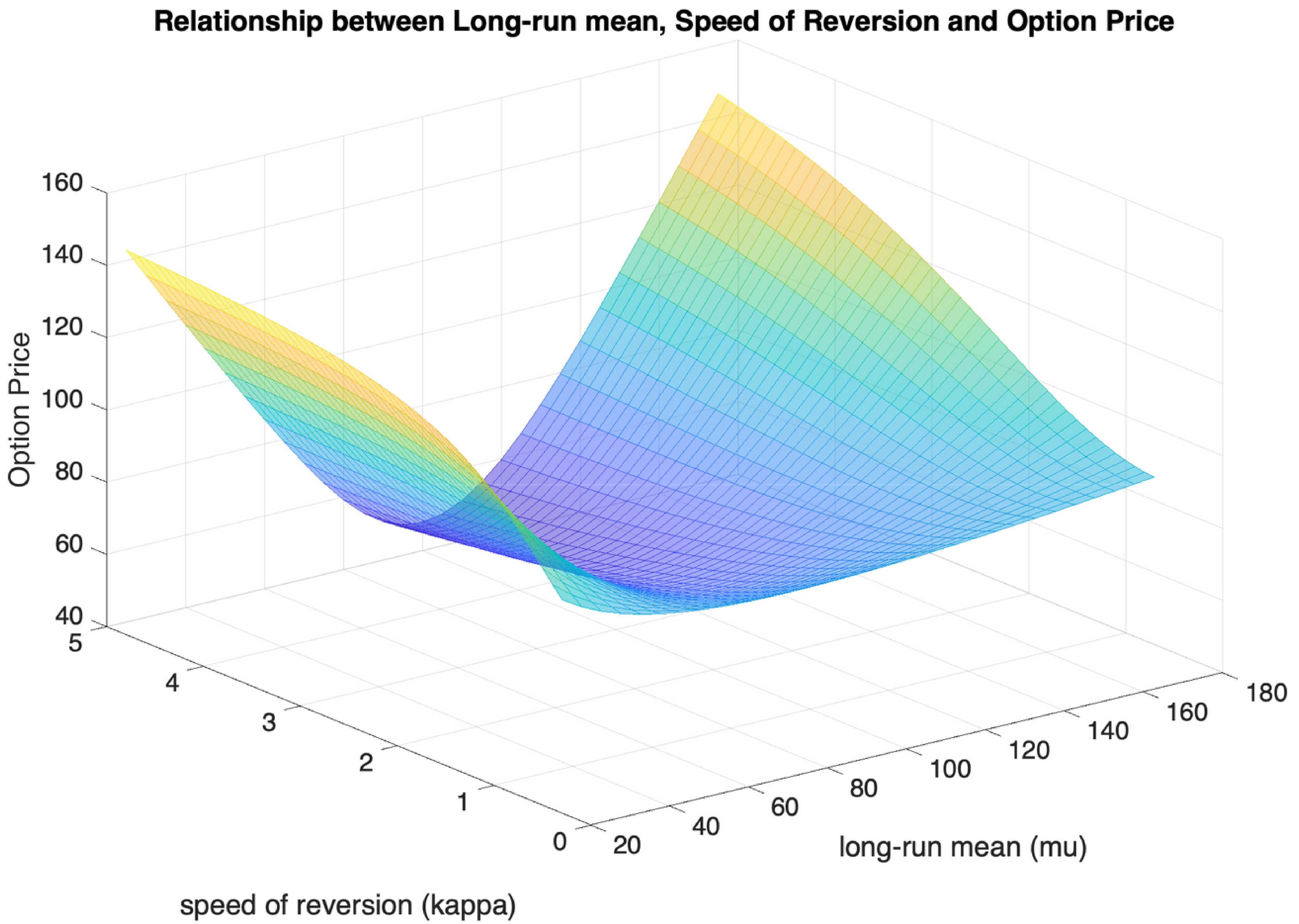
Local Swing Option and Strike Price



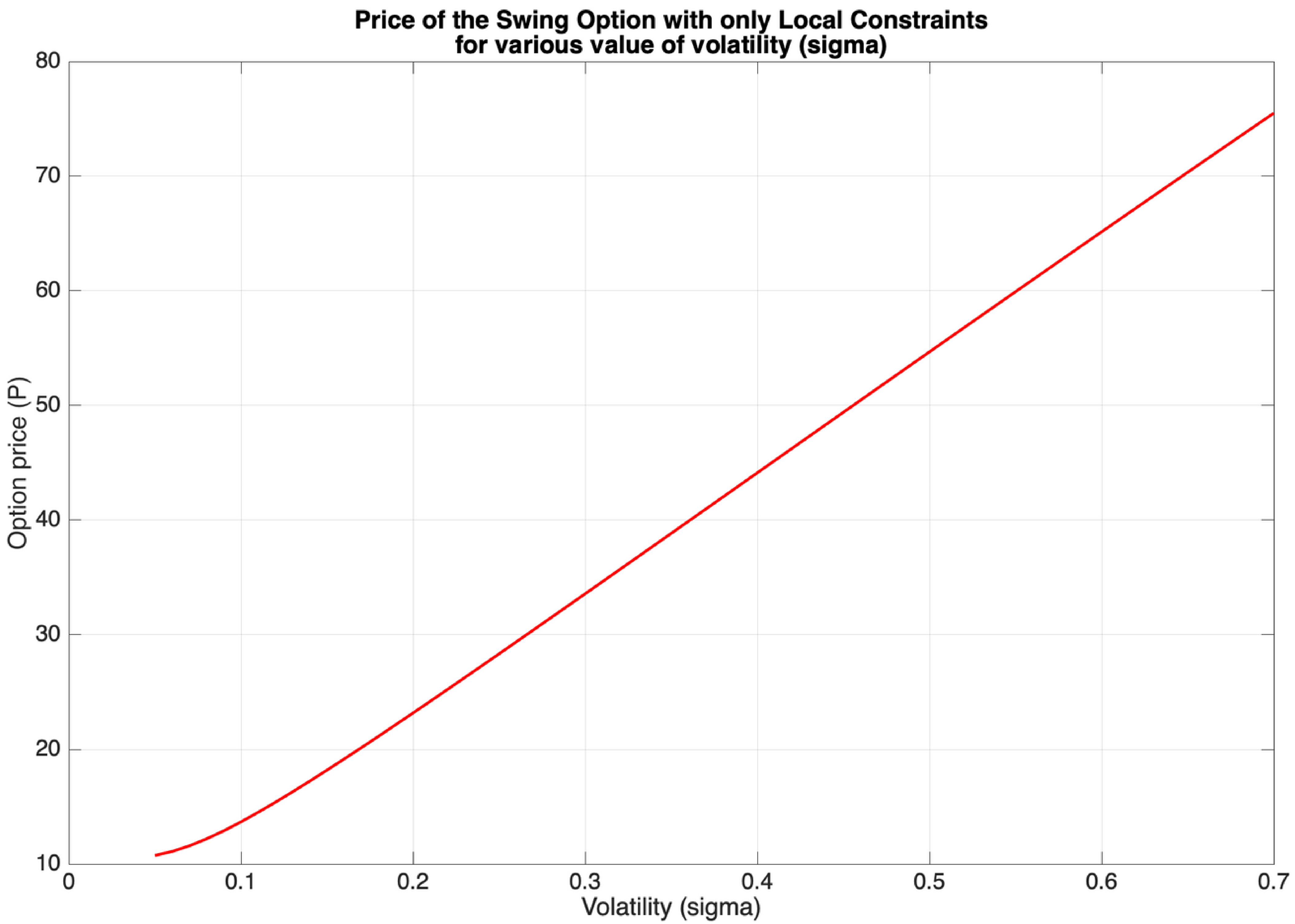
Local Swing Option and Strike Price



Local Swing Option, Speed of mean reversion and Long-run mean



Local Swing Option and Volatility



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Thank You!

Q&A