SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

01.018 Design Integrated Project I

10.002 Physics I

Ethanol-Powered Cable-Guided Rocket System

Group 08 - Cohort 07

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1 Introduction

A model rocket can be constructed from an empty coke bottle, with possible modifications being attached to the external layer of the bottle's body. The rocket is constrained to glide along a cable using a harness. When the ethanol fuel in the bottle is ignited, the high-pressure exhaust from the nozzle can propel the rocket forcefully along the cable. By changing the amount of kinetic friction and air drag, the distance travelled can be controlled.

2 Objective

To determine appropriate values of air drag coefficient and kinetic friction (as defined in 2.1) so as to achieve our target distance of $6 \pm 0.5\,\text{m}$. The original dynamic equation of the rocket is given by

$$m\frac{dv(t)}{dt} = T(t) - \mu_k mgcos\theta - mgcos\theta - \beta v(t). \tag{1}$$

We approximate the original differential equation as

$$T(t_n) - \mu_k mgcos\theta - mgcos\theta - \beta v_n = m\frac{v_{n+1} - v_n}{t_{n+1} - t_n}, \tag{2}$$

with $v_0 = 0$, $t_0 = 0$, $T(t_0) = 0$, $g = 9.80665$ m/s^2, $\theta = 4.8°$, $\Delta t = t_{n+1} - t_n = 0.2$ s and $0 \leq n \leq 33$, where $n \in \mathbb{N}$. The values of μ_k, m and β are to be determined.

Rearranging the equation, we obtain

$$v_{n+1} = \frac{T(t_n) - \mu_k mgcos\theta - mgcos\theta - (\beta - 5m)\, v_n}{5m}. \tag{3}$$

We use this equation, where v_{n+1} is the subject, in Excel to solve the recurrent sequence.

To obtain the distance travelled, we approximate $x = \int_0^{t_T} v(t)dt$ by utilising this useful recurrence relation

$$x_{n+1} = x_n + \frac{(v_{n+1} + v_n)\,(t_{n+1} - t_n)}{2}, \tag{4}$$

where $x_0 = 0$ and $\Delta t = t_{n+1} - t_n = 0.2$ s.

2.1 Definitions

Air drag The force exerted by the air surrounding the rocket, acting opposite to the relative motion of the rocket as it passes through the air.

Kinetic friction The amount of retarding force between two solid surfaces that are moving relative to each other.

2.2 Assumptions

Time interval Since the original dynamic equation is difficult to solve analytically, we will solve it numerically by using an arbitrarily small enough time interval of 0.2 s.

Human reaction time We are assuming that the average human reaction time, which is about 0.2 s, is negligible compared to the rocket's time of flight. This is because we are using video frame analysis to determine the data plot.

Amount of thrust Since we are using the same amount of ethanol to launch the rocket, we will assume that the thrust data is equal to the data provided in the 2D Math Excel Sheet.

Mass of ethanol Since we are using only a very small amount of ethanol to launch the rocket, we will assume that the mass of ethanol is negligible and thus mass of the whole rocket system is constant throughout. We do not need to consider the loss in mass.

Shape of bottle rocket Since the bottle that we are using is a mass-produced commercial and industrially-tested Coca-Cola product, we are assuming that the bottle rocket is axisymmetric.

Flow of $\mathbf{CO_2}$ gas We are assuming that the exhaust flow of the CO_2 produced as a result of the combustion of ethanol through the nozzle is uniform, steady and axisymmetric.

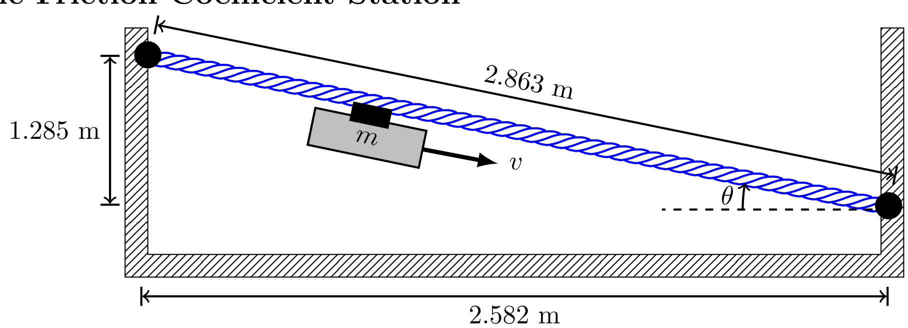
Complete combustion of ethanol We are assuming that the ethanol is completely combusted to form CO_2 and H_2O only through sufficient shaking of the bottle before launch.

Mass of harness We are assuming that the mass of the harness is exactly 22 g.

3 Experimental Data

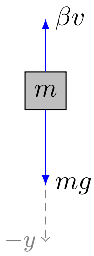
Mass of an empty, unmodified coke bottle	= 25.6495 g
Mass of a rectangular fin	= 2.7460 g
Mass of a clipped delta fin	= 1.6795 g
Volume of ethanol fuel used per flight	= 0.5 mL

3.1 Kinetic Friction Coefficient Station



Distance travelled by rocket	= 2.655 m
Time taken by rocket	= 1.7333 s

3.2 Free-Fall Air Drag Coefficient



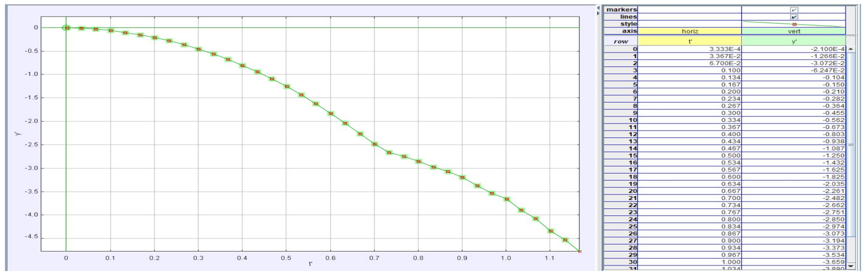


Figure 1: Graph of y against t for the unmodified rocket shape.

Using the Excel Sheet, $\beta \approx 0.0824$ kg/s for the unmodified, empty coke bottle.

3.3 Final Rocket Design Free-Fall Data

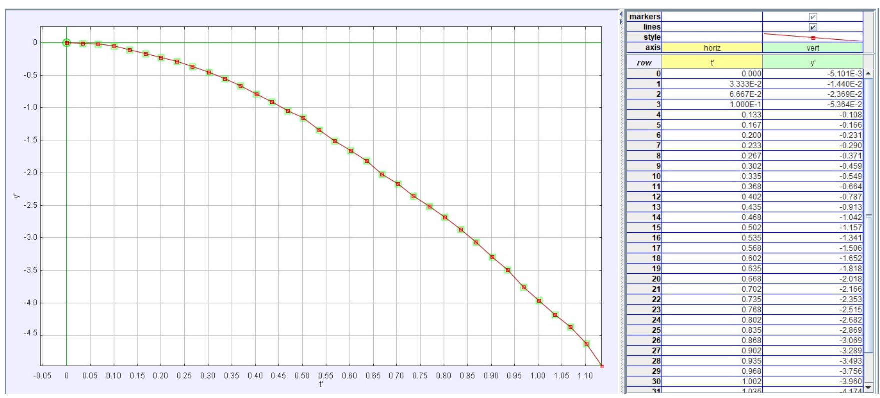


Figure 2: Graph of y against t for the final rocket shape.

Using the Excel Sheet, $\beta \approx 0.1215$ kg/s for the final coke bottle design.

4 Calculations

$$\theta_1 \approx \cos^{-1}\left(\frac{2.582}{2.863}\right) \approx 25.5975^\circ \tag{5}$$

$$\theta_2 \approx \sin^{-1}\left(\frac{1.285}{2.863}\right) \approx 26.6686^\circ \tag{6}$$

$$\text{Average } \theta \approx \frac{25.5975 + 26.6686}{2} \approx 26.1331^\circ. \tag{7}$$

Hence, by using the equation

$$\mu_k = \frac{\sin\theta - \frac{2s}{gt^2}}{\cos\theta}, \tag{8}$$

we can obtain the value of $\mu_k \approx 0.2891$.

Using our experimental data and the Excel Sheet, our unmodified coke bottle will travel a distance of about 10.4215 m. In real-life, it travelled a distance of about 9.2587 m. Thus, to reach our target distance of 6 ± 0.5 m, we need to increase our values of m and β carefully in balance.

5 Results

We modified the rocket by attaching 2 clipped delta fins and 2 rectangular fins as shown in Figure 3a. This would increase the value of m (including the harness) to 56.5005 g and the value of β to 0.1215 kg/s. This resulted in a theoretical value of 6.9053 m (based on the Excel Sheet) and an experimental value of 6.4369 m (Figure 3b). The value of β contributes a larger portion than m in the decrease of total distance travelled by the rocket. The actual value of β needed for this value of m is about 0.1439 kg/s.

(a) Our final rocket design. (b) Our final experimental result.

Figure 3: Relevant data from our third try during the rocket launch session.

6 Discussion

The theoretical value of the distance achieved (based on the Excel Sheet) is 6.9053 m. The percentage discrepancy between the theoretical value and the experimental result obtained here is 7.0213%. Because only a single measurement was made, it is not possible to calculate an estimated standard deviation.

Our choice of attaching 2 clipped delta fins and 2 rectangular fins arises from considering the drag equation and the rocket's aesthetics. We used a stiff cardboard for the material of the fins since we want to avoid fin flutter, which would exponentially increase the drag force and possibly even snap the fins off the rocket.

The drag equation is defined as

$$F_D = \frac{1}{2}\rho u^2 C_D A, \tag{9}$$

where F_D is the component of drag force in the direction of the flow velocity, ρ is the mass density of the air, u is the flow velocity relative to the object, A is the reference area and C_D is the drag coefficient. The value of C_D depends on the Reynolds number.

The Reynolds number is defined as

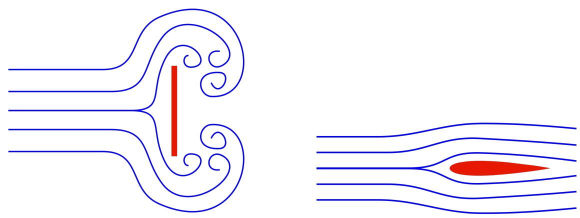
$$Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu}, \tag{10}$$

where ρ is the density of the air, u is the velocity of the fluid with respect to the rocket, L is a characteristic linear dimension, μ is the dynamic viscosity of the air and ν is the kinematic viscosity of the air.

While theoretically, elliptical fins have the lowest induced drag, it may not be necessarily true for small model rockets such as ours. This is because at low Reynolds number with laminar flow dominating, a rectangular fin would result in lower overall drag.

We utilised 2 clipped delta fins and 2 rectangular fins instead of 4 rectangular fins since we do not want too much drag on the rocket. This is because clipped delta fins are more aerodynamic than rectangular fins. Each shape is put opposite each other so as to minimise any unbalanced forces that might destabilise the rocket.

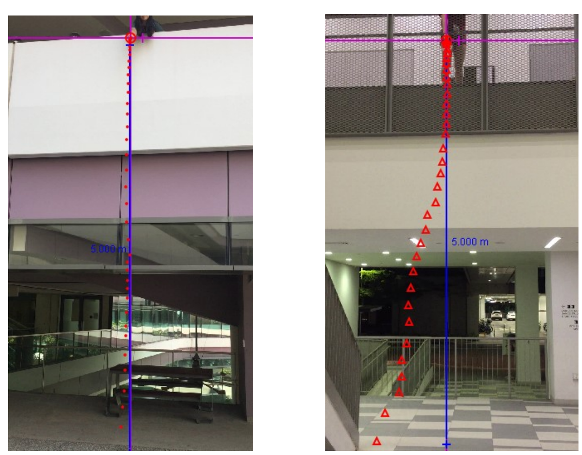
Furthermore, attaching fins instead of, for example, a rectangular piece of cardboard in front of the rocket is more aesthetically pleasing. A rectangular piece of cardboard perpendicularly cutting the airflow would lead to an inaccurate turbulent drag force term of βv^2 instead of βv (Figure 4a), making our calculations inaccurate and thus that is why we actively did not choose to attach a rectangular piece of cardboard in front of the rocket.



(a) A rectangular plate. (b) A typical foil.

Figure 4: Airflow around different shapes.

A possible source of experimental uncertainty is the difficulty of determining the center of mass of the bottle rocket during video frame analysis. This is because the bottle is a rigid body and not a point mass. During free-fall testing, there is also some horizontal force by air drag due to non-uniform distribution of the rocket's mass causing different levels of pressure (Figure 5). This could introduce some errors into our calculations as well.



(a) Unmodified design. (b) Final design.

Figure 5: Plots of the video frame analysis with a calibration stick.

The shaking of the bottle might not be sufficient as well, and thus there could be incomplete combustion. Incomplete combustion could lead to production of CO instead of CO_2, leading to different gas pressure and thus different thrust values than the ones in the Excel Sheet.

Different thrust values could also arise due to different volumes of ethanol pipetted. Some ethanol would inadvertently still be lost as well, even if we tried to keep a finger on the nozzle, especially since ethanol is more volatile than water.

In reality, the ethanol has some mass, which would slow down the rocket during the initial part of the launch and thus the rocket would reach a shorter distance instead.

It is also difficult to make sure that the centre of pressure and the centre of mass is exactly on the central horizontal axis of the bottle rocket. This would result in some unwanted, small rotation (even under the restriction of the harness). Thus, air flow around the rocket could be turbulent instead of laminar. If the air flow is turbulent, the air drag force term would be βv^2 instead of βv. This would yield a larger air resistance and thus shorter distance reached. Turbulent airflow could also be caused by unwanted bending of the fins. As the bottle rockets were used for several testings before the actual launch, the heat produced might deform the bottle and thus cause turbulent airflow as well.

The mass of the harness could differ from one station to another due to various manufacturing processes (even under quality control), and this could introduce more inaccuracies to our measurements.

Because the result obtained is fairly close to the theoretical value (with a percentage discrepancy of below 10%), it is possible that some of these experimental uncertainties have fortuitously cancelled one another.

A possible improvement that could be made is to taper off the thickness of the fins from the root to the tips so as to keep the airfoil constant along the span. This could be done by sanding the fin carefully. Another improvement related to aesthetics is to spray paint the rocket.

7 Conclusion

Overall, we have succeeded in determining appropriate values of β and m so that our bottle rocket could achieve our target distance of $6 \pm 0.5\,\mathrm{m}$. Despite the various assumptions and experimental errors, we still manage to achieve our target distance within the acceptable range.

References

"Problems Reduce Benefits of Elliptical Fins" by Bob Parks. *Journal of the International Spacemodeling Society.* September 1993 (Vol. 1, Number 5). pg 4.