Skipper

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I. INTRODUCTION

Nearly everyone has tried to throw a stone on a body of water and count the number of skips the stone was able to make. Of course the more, the better. Intuitively, there are some empirical rules for the best throw – one should throw circular and flat stones with a high speed, a small angle with the water surface and with a flick of the wrist to give the stone a spin. These rules can also be understood using the laws of physics. The collision between the stone and the water surface is quite complex because it involves the fluid dynamics around the immersed stone.[1,2] Energy dissipation and angular destabilisation contribute to the sinking of the stone after a few rebounds.

This paper will consider a very simplified description of the motion of a stone skipping over a water surface using forces, momentum and energy. This paper will also propose the creation of a device which could help us determine the best conditions for a significant amount of skips.

II. MOTIVATION AND CREATIVITIES

We were inspired by Skippa, a rock-skipping device built by Mark Rober.[3] Several customisations that we implemented include:

1. We used cheaper alternatives as our budget limit was \$60. This implied that we could not purchase the clay pigeon thrower used by Mark Rober, especially since the clay pigeon thrower exceeds \$100 and that it could only be obtained from the United States of America with a long shipping duration. Thus, we built the whole device from scratch.

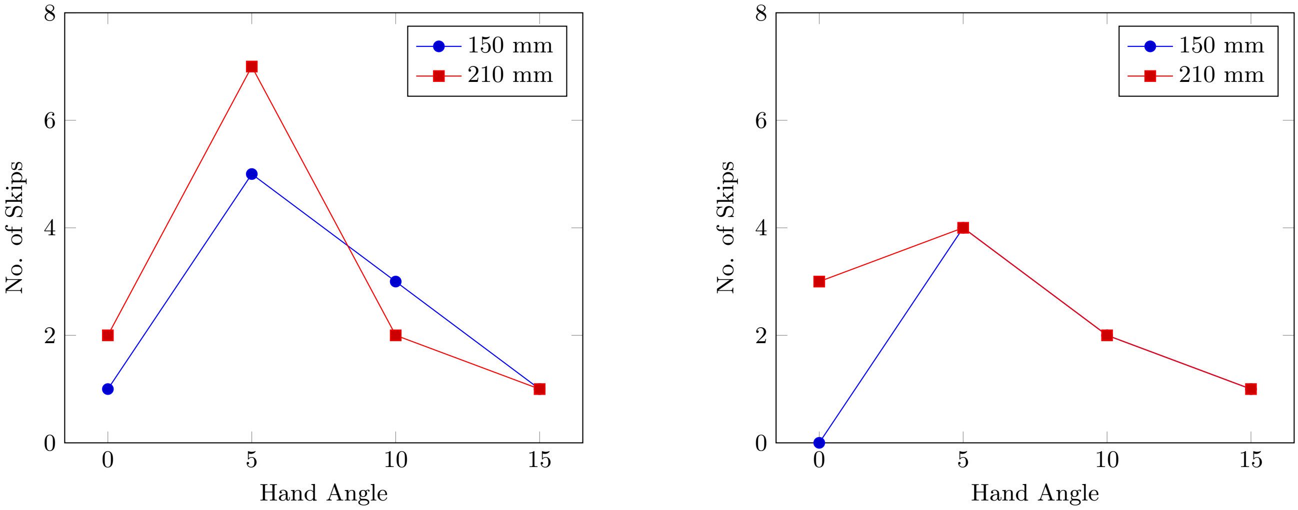
2. We used wood for the thrower, PVC pipes for the base and polymer clay as the material for the modelled circular stones.

III. RESULT AND ANALYSIS

Stone No.	Diameter / mm	Thickness / mm	Hand Angle / °	Spring Extension / mm	No. of Skips
1	40.5	10.5	0	150	1
2	40.5	10.5	5	150	5
3	40.5	10.5	10	150	3
4	40.5	10.5	15	150	1
5	40.5	10.5	0	210	2
6	40.5	10.5	5	210	7
7	40.5	10.5	10	210	2
8	40.5	10.5	15	210	1
9	45.0	15.0	0	150	0
10	45.0	15.0	5	150	4
11	45.0	15.0	10	150	2
12	45.0	15.0	15	150	1
13	45.0	15.0	0	210	3
14	45.0	15.0	5	210	4
15	45.0	15.0	10	210	2
16	45.0	15.0	15	210	1

P. S. We were also able to skip the stones on the sand.

We used a constant distance of the stone from the pivot of 155 mm and a constant arm angle of 0°.



(a) Stone with 40.5 mm diameter and 10.5 mm thickness.

(b) Stone with 45.0 mm diameter and 15.0 mm thickness.

FIG. 1: Plots for the two different stones.

A. Force Analysis

(a) Stone at the launcher part of the device (top-view).

(b) Stone when it touches the water surface (side-view).

FIG. 2: Free body diagrams for the stone at different times.

The long runway of the thrower's 'hand' allows static friction in the direction parallel to the thrower's long axis to act on the stone for a longer time (as shown in Figure 2a) and thus the stone would achieve more angular momentum at the end, achieving more angular stability. Meanwhile, the component of static friction in the direction perpendicular to the thrower's long axis pushes the stone forward, increasing its v_{x0}.

When the stone collides with the water surface, the equations of motion for the stone's center of mass are

$$M\frac{dv_x}{dt} = -\frac{1}{2}\rho_w v^2 A_{im}\left(C_l \sin\theta + C_f \cos\theta\right) \tag{1a}$$

$$M\frac{dv_z}{dt} = -Mg + \frac{1}{2}\rho_w v^2 A_{im}\left(C_l \cos\theta - C_f \sin\theta\right), \tag{1b}$$

where M is the mass of the stone, v is the velocity of the stone (with $v^2 = v_x^2 + v_z^2$), g is acceleration due to gravity, A_{im} is the immersed area of the stone in contact with the water surface, C_l and C_f are the lift and friction coefficients respectively, ρ_w is the mass density of water and θ is the tilt angle of the stone with the water surface. In our case, θ is the same as the hand angle as our arm angle is constantly $0°$. Note that we do not take into consideration the value of v_y as it approximately does not contribute to the number of skips achieved by the stone. The value of v_y will only determine how much the stone's path will bend from its initial straight path when it was launched from the device.

For a circular stone, the immersed area A_{im} is given by

$$A_{im}(s) = R^2 \left[\cos^{-1}\left(1 - \frac{s}{R}\right) - \left(1 - \frac{s}{R}\right)\sqrt{1 - \left(1 - \frac{s}{R}\right)^2} \right], \tag{2}$$

where $s = \frac{|z|}{\sin\theta}$ (the maximum immersed height) and R is the stone's radius.

The condition for the stone to skip is that a maximum depth must be reached before the stone is fully immersed. By using the constant energy condition of the system and some dimensionless variables (derived by Lydéric Bocquet),[4] we could explicitly solve the condition for skimming to be

$$v_{x0} > v_c = \sqrt{\frac{4Mg\sin\theta}{\pi R^2 C\rho_w \sin\theta - M\tan^2\beta}}, \tag{3}$$

where v_c is the minimum critical velocity, $\tan\beta = \frac{v_{z0}}{v_{x0}}$ and $C = C_l\cos\theta - C_f\sin\theta \approx C_l$.

More skips could be achieved if v_c is lower. This would mean a smaller v_{z0}, a larger v_{x0}, a smaller M and a larger R. This confirms our intuition of throwing a light and flat stone with a high linear speed with an initial height as close to the water surface as possible. In our case, since we are using polymer clay with a uniform density to craft the stones, an increase in R would inevitably lead to an increase in M. Thus, based on our data, the bigger stone achieves a lower number of skips due to this effect. Furthermore, both the numerator and the denominator contain $\sin\theta$, which makes the determination of the relationship of the number of skips with θ difficult. We will attempt to find this relationship using other methods.

B. Momentum Analysis

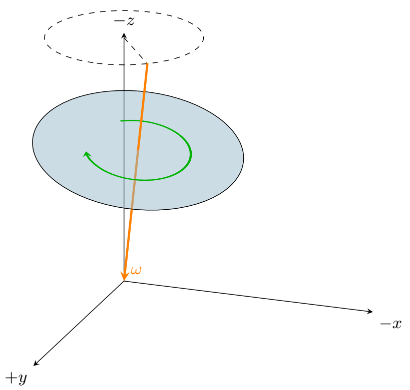


FIG. 3: Graph of the stone's rotation in 3D. Stone is depicted as a 2D circle for simplification purposes.

By using Euler equations, we could derive the equation for the tilt angle, θ, of the stone with respect to the surface of water to be

$$\ddot{\theta} + \left[\dot{\phi}_0\left(\frac{J_0 - J_1}{J_1}\right)\right]^2 (\theta - \theta_0) = \frac{M_\theta}{J_1}, \tag{4}$$

where $\dot{\phi}_0$ is the initial spin angular velocity, θ_0 is the initial tilt angle of the stone before its first skip, J_0 and J_1 are moments of inertia of the stone in the directions normal and along the stone respectively and M_θ is the projection of the torque due to water flow in the y-direction. Thus, for a successful skip, θ needs to remain approximately constant. The condition for this is described as

$$\dot{\phi}_0 \approx \omega \gg \sqrt{\frac{g}{R}}. \tag{5}$$

This is supported by our data, which shows how the stone with a larger radius would achieve a lower initial angular velocity, and thus would skip less due to it possessing less angular stability.

From Lydéric Bocquetś research paper,[4] the number of skips achieved by the stone, N_c, can be non-prescriptively approximated by the equation

$$N_c \approx \frac{R\dot{\phi_0}^2}{g}. \tag{6}$$

This is obtained by considering angular destabilisation of the stone. This is supported by our experimental data, which shows that as the spring extension is increased, the maximum angular speed achieved would be larger and thus $\dot{\phi_0}$ would be higher. Unsurprisingly, the stone with a smaller radius achieved more number of skips. This could be due to the fact that it requires more time for the larger stone to reach the same $\dot{\phi_0}$. As $\dot{\phi_0}$ contributes quadratically, compared to the linear contribution of R, $\dot{\phi_0}$ has a more significant impact on the value of N_c. Thus, considered together, this would correspond to a higher N_c.

C. Energy Analysis

By only considering the x-component of the stone's velocity and neglecting wind and air friction, after each collision with the water surface, the loss in kinetic energy of the stone is approximately given by

$$W = -\int_0^t F_x(t)v_x(t)\,\mathrm{d}t \approx -\mu M g \ell, \tag{7}$$

where t is the collision time, $\mu = \frac{C_l \sin\theta + C_f \cos\theta}{C}$ and ℓ is defined as

$$\ell = v_{x0}\frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{M\sin\theta}{C\rho_w R}}. \tag{8}$$

Thus, after N collisions, the velocity of the stone obeys the relation

$$\frac{1}{2}M{v_{xf}}^2 - \frac{1}{2}M{v_{x0}}^2 = -N\mu M g \ell, \tag{9}$$

where the number of skips N_c will be when the total energy loss is larger than the initial kinetic energy. Hence, N_c is given accordingly by

$$N_c = \frac{{v_{x0}}^2}{2g\mu\ell}. \tag{10}$$

This is supported by our data, which indicates that a higher v_{x0} due to a larger spring extension would lead to a larger N_c. The relationship of N_c with θ is still yet unclear since both ℓ and μ are dependent on the value of θ. Considering ℓ only, we would like the value of θ to be as small as possible to achieve a larger N_c. However, our data shows that a value of $\theta = 5°$ would lead to a higher number of skips than $\theta = 0°$. This might be due to some alteration to the final value of N_c by μ. Thus, our 'sweet spot' is the value of $\theta = 5°$.

IV. CHALLENGES AND SOLUTIONS

1. The pivot between the thrower and the base column was too weak since it was made from acrylic. This causes the thrower to oscillate too much as it is not so stable. This could lead to the further end of the thrower stealing some energy from the stone and disrupts the rotation of the stone if the stone collides with the thrower. An improvement would be to cut metal with waterjet to replace the whole pivot connection. This would strengthen the connection and prevents unnecessary oscillations.

2. The spring that we obtained is not strong enough and thus the stones were only able to skip a maximum of 7 times. A possible improvement would be to utilise a stronger spring with a larger spring constant.

3. The polymer clay is of a low density and the stones actually float in seawater. This causes the reaction buoyant force to be lower than expected and thus the stone would skip lower than expected.

4. We conducted our data collection at Siloso Beach, Sentosa Island, Singapore. At the beach, the surface of water oscillates with the ocean waves and tides. This alters the shape of the water surface and thus, the direction of forces would be different compared to a flat body of water. At the beach, due to the temperature and pressure gradient between the land area and the sea area, winds are frequent. This also hinders us from achieving accurate readings as the winds would exert additional forces on the stone that was being skipped. An improvement that could be done would be to conduct the testing at a water reservoir or a swimming pool with no or less winds and a flat surface of water.

5. The PVC pipes were not very strong. As such, the PVC pipes actually bend towards its insides as we tried to tighten the bolt. Thus, a possible improvement would be to use metal for the whole pivot column connected to the base.

6. The plane of the beach's surface had a non-zero angle with respect to the surface of the water. This would lead to a systematic error of the angle measured.

7. Our mechanism assumes that the spring solely extends linearly with no bending. In reality, with the structure that we have built, the spring would bend a little as it is bent through the angle when the thrower is turned.

8. It is assumed that:

   (a) The mass of the polymer clay stays constant throughout its motion.

   (b) The shape of the polymer clay stays constant throughout its motion, i.e. the stones are rigid.

   (c) The angle of attack, α, remains constant along each impact with the water.

   If any of these assumptions are not true, the theoretical calculations that we performed would be less accurate.

V. CONCLUSION AND FUTURE WORK

We succeeded in determining the important factors that affect the maximum number of skips of the stone. Based on our experiments, an arm angle of 5° with a fairly strong initial force is optimal to achieve the most number of skips. This differs with the empirical result of a research paper[5] that discovered 20° as the 'magic' angle. Also, we were only able to achieve single-digit numbers of skips. This might be due to the accumulation of all the different inaccuracies. For further improvement, we could construct a properly structured base with a low centre of gravity to further stabilise the device, instead of just weighing down the base with sand of half the base's height. With a stronger base, we would not require to fix the hand and thus the hand angle would also be able to be changed. We could also follow Mark Rober's mechanism instead of our current mechanism to prevent the bending of the spring. Finally, a high-speed camera mounted on a drone could be used in order for us to be able to experimentally determine the value of the stone's angular velocity at any point in time during the stone's collisional process.

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