Prove convexity:

Zero-order definition of convexity

Let +bij = -aij

Proof 1: Generalized proof

Introduce

Group into

Expand

Factorize

Group into

Use triangle inequality

Use a variant of the reverse triangle inequality

Proof 2: Decompose function into kii (tighter bound on inequality) and kij

Let +bij = -aij

Step 1: Focus on kii and bii, or i = j

Re-arrange terms into 2-norms

Re-arrange as

Use triangle inequality:

Expand

Cancel out

Remove absolute-value operator: 2-norms are always non-negative

Expand

Re-arrange into

Group and

Factorize into

Isolate common factor

Factorize

Step 2: Focus on kij and bij, where i ≠ j. Same as proof 1, except we focus on i ≠ j

Step 3: Putting Step 1 and Step 2 together

Introduce

Introduce

Re-arrange sum into matrix form