Paraconsistent and Paracomplete Systems

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Abstract

Every paraconsistent or paracomplete system invalidates one of the laws of indiscernibility or one of the laws of logical explosion.

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Part I

General Paraqualifications

1 Paracomplete Systems

A system is generally paracomplete if it invalidates one or both of the laws of indiscernibility.

1.1 Indiscernibility of Identicals

$$\frac{\neg\forall F\left(Fx \leftrightarrow Fy\right) \vdash \neg\left(x = y\right)}{x = y \vdash \forall F\left(Fx \leftrightarrow Fy\right)}$$

$$\frac{\exists F\left(Fx \not\leftrightarrow Fy\right) \vdash x \neq y}{x = y \vdash \forall F\left(Fx \leftrightarrow Fy\right)}$$

1.2 Identity of Indiscernibles

$$\frac{\neg\left(x = y\right) \vdash \neg\forall F\left(Fx \leftrightarrow Fy\right)}{\forall F\left(Fx \leftrightarrow Fy\right) \vdash x = y}$$

$$\frac{x \neq y \vdash \exists F\left(Fx \not\leftrightarrow Fy\right)}{\forall F\left(Fx \leftrightarrow Fy\right) \vdash x = y}$$

1.3 Excluded Middle

$$\frac{\neg\left(x \vee \neg x\right) \vdash \neg\top}{\bot \vdash x \vee \neg x}$$

2 Paraconsistent Systems

A system is generally paraconsistent if it invalidates some number of the laws of inconsistency.

2.1 Logical Explosion

$$\frac{\neg y \vdash \neg (x \wedge \neg x)}{x \wedge \neg x \vdash y}$$

2.2 Contradictory Consequence

$$\frac{\neg (x \wedge \neg x) \vdash \neg y}{y \vdash x \wedge \neg x}$$

2.3 Non-Contradiction

$$\frac{\neg \bot \vdash \neg (x \wedge \neg x)}{x \wedge \neg x \vdash \bot}$$