Paraconsistent and Paracomplete Systems

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April 13 2022

Abstract

Every paraconsistent or paracomplete system invalidates one of the laws of indiscernibility or one of the laws of logical explosion.

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Part I
General Paraqualifications

1 Paracomplete Systems

A system is generally paracomplete if it invalidates one or both of the laws of indiscernibility.

1.1 Indiscernibility of Identicals

$$\dfrac{\dfrac{\exists x \exists y \left[x = y \land \exists F \left(Fx \oplus Fy\right)\right] \vdash \bot}{\exists x \exists y \left[x = y \nrightarrow \forall F \left(Fx \leftrightarrow Fy\right)\right] \vdash \bot}}{\vdash \forall x \forall y \left[x = y \rightarrow \forall F \left(Fx \leftrightarrow Fy\right)\right]}$$

1.2 Identity of Indiscernibles

$$\dfrac{\dfrac{\exists x \exists y \left[\exists F \left(Fx \leftrightarrow Fy\right) \land x \neq y\right] \vdash \bot}{\exists x \exists y \left[\exists F \left(Fx \leftrightarrow Fy\right) \nrightarrow x = y\right] \vdash \bot}}{\vdash \forall x \forall y \left[\forall F \left(Fx \leftrightarrow Fy\right) \rightarrow x = y\right]}$$

2 Paraconsistent Systems

A system is generally paraconsistent if it invalidates some number of the laws of inconsistency.

2.1 Logical Explosion

$$\dfrac{\exists x \exists y \left[\left(x \land \neg x\right) \land \neg y\right] \vdash \bot}{\vdash \forall x \forall y \left[\left(x \land \neg x\right) \rightarrow y\right]}$$

2.2 Logical Trivialization

$$\frac{\exists x \exists y \, [y \wedge (\neg x \vee x)] \vdash \bot}{\vdash \forall x \forall y \, [y \rightarrow (x \wedge \neg x)]}$$

$$\cfrac{\cfrac{\exists x \exists y \, [y \wedge (\neg x \vee x)] \vdash \bot}{\vdash \forall x \forall y \, [(\neg x \vee x) \rightarrow \neg y]}}{\vdash \forall x \forall y \, [\neg (x \wedge \neg x) \rightarrow \neg y]}$$