

Is Basic Logic Genuinely Paraconsistent and Paracomplete?

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Abstract

We examine whether or not various forms of the law of non-contradiction and the law of excluded middle hold in Sambin's Basic Sequent Logic. We identify at least 128 distinct forms of Non-contradiction and its prime form and a matching number of forms of the Excluded Middle and its prime form.

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Part I

Non-Contradiction and Excluded Middle in Sambin's Basic Logic

$$\vdash ((A \wedge (A \vdash)) \vdash)$$

$$A \wedge (A \vdash) \vdash$$

$$\vdash \top \leftarrow (A \wedge (A \rightarrow 0)) \rightarrow 0$$

$$\vdash 1 \leftarrow (A \wedge (A \rightarrow 0)) \rightarrow 0$$

$$\vdash 1 \leftarrow (A \wedge (A \rightarrow 0)) \rightarrow \perp$$

$$\vdash \top \leftarrow (A \otimes (A \rightarrow 0)) \rightarrow 0$$

$$\vdash 1 \leftarrow (A \otimes (A \rightarrow 0)) \rightarrow 0$$

$$\vdash 1 \leftarrow (A \otimes (A \rightarrow 0)) \rightarrow \perp$$

1 Classical Non-contradiction

$$A \otimes (A \rightarrow 0), A \otimes (A \rightarrow \perp), A \wedge (A \rightarrow 0), A \wedge (A \rightarrow \perp) \vdash$$

$$\vdash [(A \otimes (A \rightarrow 0)) \circ (A \otimes (A \rightarrow \perp)) \circ (A \wedge (A \rightarrow 0)) \circ (A \wedge (A \rightarrow \perp))] \rightarrow 0$$

$$\vdash [(A \otimes (A \rightarrow 0)) \circ (A \otimes (A \rightarrow \perp)) \circ (A \wedge (A \rightarrow 0)) \circ (A \wedge (A \rightarrow \perp))] \rightarrow \perp$$

2 Classical Excluded Middle

$$\vdash (A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))$$

$$1 \leftarrow [(A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))] \vdash$$

$$\top \leftarrow [(A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))] \vdash$$

3 Paraconsistent and Paracomplete forms of truth

$$\vdash 0 \rightarrow A$$

$$\vdash \perp \rightarrow A$$

$$\vdash 1 \rightarrow A$$

$$\vdash \top \rightarrow A$$

$$A \leftarrow 0 \dashv$$

$$A \leftarrow \perp \dashv$$

$$A \leftarrow 1 \dashv$$

$$A \leftarrow \top \dashv$$

3.1 NC in Basic Sequent Logic

3.1.1 Additive NC

$$[A \wedge (1 \leftarrow A)] \vdash$$

$$[A \wedge (\top \leftarrow A)] \vdash$$

$$[A \wedge (A \rightarrow 0)] \vdash$$

$$[A \wedge (A \rightarrow \perp)] \vdash$$

3.1.2 Multiplicative NC

$$[A \otimes (1 \leftarrow A)] \vdash$$

$$[A \otimes (\top \leftarrow A)] \vdash$$

$$[A \otimes (A \rightarrow 0)] \vdash$$

$$[A \otimes (A \rightarrow \perp)] \vdash$$

3.2 NC' in Basic Sequent Logic

3.2.1 Additive NC'

Excluding 1

$$\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0$$

$$\vdash [A \wedge (1 \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \wedge (\top \leftarrow A)] \rightarrow 0$$

$$\vdash [A \wedge (\top \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]$$

Including 0

$$\vdash [A \wedge (A \rightarrow 0)] \rightarrow 0$$

$$\vdash [A \wedge (A \rightarrow 0)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \wedge (A \rightarrow 0)]$$

$$\vdash \top \leftarrow [A \wedge (A \rightarrow 0)]$$

Including Bottom

$$\vdash [A \wedge (A \rightarrow \perp)] \rightarrow 0$$

$$\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \wedge (A \rightarrow \perp)]$$

$$\vdash \top \leftarrow [A \wedge (A \rightarrow \perp)]$$

3.2.2 Multiplicative NC'**Excluding 1**

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (\top \leftarrow A)]$$

Including 0

$$\vdash [A \otimes (A \rightarrow 0)] \rightarrow 0$$

$$\vdash [A \otimes (A \rightarrow 0)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (A \rightarrow 0)]$$

$$\vdash \top \leftarrow [A \otimes (A \rightarrow 0)]$$

Including Bottom

$$\vdash [A \otimes (A \rightarrow \perp)] \rightarrow 0$$

$$\vdash [A \otimes (A \rightarrow \perp)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (A \rightarrow \perp)]$$

$$\vdash \top \leftarrow [A \otimes (A \rightarrow \perp)]$$

3.3 NC Proofs in Basic Sequent Logic

3.3.1 Additive NC

$$L \wedge \frac{A \vdash}{[A \wedge (A \rightarrow \perp)] \vdash} \quad L \wedge \frac{\frac{A \vdash}{A \vdash \perp}}{A \rightarrow \perp \vdash} \frac{}{[A \wedge (A \rightarrow \perp)] \vdash}$$

$$\begin{array}{c}
\frac{A \vdash}{[A \wedge (1 \leftarrow A)] \vdash} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{[A \wedge (1 \leftarrow A)] \vdash} \\
\frac{A \vdash}{[A \wedge (\top \leftarrow A)] \vdash} \quad \frac{\frac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash} \\
\frac{A \vdash}{[A \wedge (A \rightarrow 0)] \vdash} \quad \frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{[A \wedge (A \rightarrow 0)] \vdash}
\end{array}$$

3.3.2 Multiplicative NC

$$\begin{array}{c}
\frac{A, 1 \leftarrow A \vdash}{[A \otimes (1 \leftarrow A)] \vdash} \otimes L \\
\frac{A, \top \leftarrow A \vdash}{[A \otimes (\top \leftarrow A)] \vdash} \\
\frac{A, A \rightarrow 0 \vdash}{[A \otimes (A \rightarrow 0)] \vdash} \\
\frac{A, A \rightarrow \perp \vdash}{[A \otimes (A \rightarrow \perp)] \vdash}
\end{array}$$

3.4 NC' in Basic Sequent Logic

3.4.1 Additive NC'

Excluding 1

$$\frac{\frac{A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0} \quad \frac{\frac{1 \leftarrow A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0}$$

$$\frac{\frac{\frac{A \vdash}{A \vdash \perp}}{A \wedge (1 \leftarrow A) \vdash \perp}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow \perp} \quad \frac{\frac{\frac{\frac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{1 \leftarrow A \vdash \perp}}{[A \wedge (1 \leftarrow A)] \vdash \perp}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow \perp}$$

$$\frac{\frac{A \vdash}{\vdash 1} \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash} \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]}$$

$$\frac{\frac{A \vdash}{\vdash \top} \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash} \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]}$$

Excluding Top

$$\frac{\frac{A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow 0} \quad \frac{\frac{\top \leftarrow A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow 0}$$

$$\begin{array}{c}
\frac{A \vdash}{A \vdash \perp} \\
\frac{A \vdash \perp}{A \wedge (\top \leftarrow A) \vdash \perp} \\
\frac{A \wedge (\top \leftarrow A) \vdash \perp}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow \perp}
\end{array}
\qquad
\begin{array}{c}
\frac{\top \vdash A}{\top \leftarrow A \vdash} \\
\frac{\top \leftarrow A \vdash}{[A \wedge (\top \leftarrow A)] \vdash} \\
\frac{[A \wedge (\top \leftarrow A)] \vdash}{[A \wedge (\top \leftarrow A)] \vdash \perp} \\
\frac{[A \wedge (\top \leftarrow A)] \vdash \perp}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow \perp}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \wedge (\top \leftarrow A) \vdash} \\
\frac{\vdash 1 \quad A \wedge (\top \leftarrow A) \vdash}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}
\end{array}
\qquad
\begin{array}{c}
\frac{\top \vdash A}{\top \leftarrow A \vdash} \\
\frac{\top \leftarrow A \vdash}{A \wedge (\top \leftarrow A) \vdash} \\
\frac{A \wedge (\top \leftarrow A) \vdash}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \wedge (\top \leftarrow A) \vdash} \\
\frac{A \wedge (\top \leftarrow A) \vdash}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}
\end{array}
\qquad
\begin{array}{c}
\frac{\top \vdash A}{\top \leftarrow A \vdash} \\
\frac{\top \leftarrow A \vdash}{A \wedge (\top \leftarrow A) \vdash} \\
\frac{A \wedge (\top \leftarrow A) \vdash}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}
\end{array}$$

Including 0

$$\begin{array}{c}
\frac{A \vdash 0}{[A \wedge (A \rightarrow 0)] \vdash 0} \\
\frac{[A \wedge (A \rightarrow 0)] \vdash 0}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow 0}
\end{array}
\qquad
\begin{array}{c}
\frac{A \rightarrow 0 \vdash 0}{[A \wedge (A \rightarrow 0)] \vdash 0} \\
\frac{[A \wedge (A \rightarrow 0)] \vdash 0}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow 0}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \vdash \perp} \\
\frac{A \vdash \perp}{A \wedge (A \rightarrow 0) \vdash \perp} \\
\frac{A \wedge (A \rightarrow 0) \vdash \perp}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow \perp}
\end{array}
\qquad
\begin{array}{c}
\frac{A \vdash 0}{A \rightarrow 0 \vdash} \\
\frac{A \rightarrow 0 \vdash}{[A \wedge (A \rightarrow 0)] \vdash} \\
\frac{[A \wedge (A \rightarrow 0)] \vdash}{[A \wedge (A \rightarrow 0)] \vdash \perp} \\
\frac{[A \wedge (A \rightarrow 0)] \vdash \perp}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow \perp}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \wedge (A \rightarrow 0) \vdash} \\
\frac{\vdash 1 \quad A \wedge (A \rightarrow 0) \vdash}{\vdash 1 \leftarrow [A \wedge (A \rightarrow 0)]}
\end{array}
\qquad
\begin{array}{c}
\frac{A \vdash 0}{A \rightarrow 0 \vdash} \\
\frac{A \rightarrow 0 \vdash}{A \wedge (A \rightarrow 0) \vdash} \\
\frac{A \wedge (A \rightarrow 0) \vdash}{\vdash 1 \leftarrow [A \wedge (A \rightarrow 0)]}
\end{array}$$

$$\begin{array}{c}
\frac{\frac{A \vdash}{A \wedge (A \rightarrow 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \rightarrow 0)]} \quad \frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{\frac{A \wedge (A \rightarrow 0) \vdash}{\vdash \top \leftarrow [A \wedge (A \rightarrow 0)]}}
\end{array}$$

Including Bottom

$$\begin{array}{c}
\frac{A \vdash 0}{[A \wedge (A \rightarrow \perp)] \vdash 0} \quad \frac{A \rightarrow \perp \vdash 0}{[A \wedge (A \rightarrow \perp)] \vdash 0} \\
\hline
\vdash [A \wedge (A \rightarrow \perp)] \rightarrow 0 \quad \vdash [A \wedge (A \rightarrow \perp)] \rightarrow 0
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \vdash \perp} \quad \frac{A \vdash}{A \vdash \perp} \\
\hline
\frac{A \wedge (A \rightarrow \perp) \vdash \perp}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp} \quad \frac{L \wedge \frac{A \rightarrow \perp \vdash}{[A \wedge (A \rightarrow \perp)] \vdash}}{\frac{[A \wedge (A \rightarrow \perp)] \vdash \perp}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp}}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \vdash \perp} \quad \frac{A \vdash}{A \vdash \perp} \\
\hline
\frac{A \rightarrow \perp \vdash}{A \wedge (A \rightarrow \perp) \vdash} \quad \frac{A \rightarrow \perp \vdash}{A \wedge (A \rightarrow \perp) \vdash} \\
\hline
\frac{\vdash 1 \quad A \wedge (A \rightarrow \perp) \vdash}{\vdash 1 \leftarrow [A \wedge (A \rightarrow \perp)]} \quad \frac{\vdash 1 \quad A \wedge (A \rightarrow \perp) \vdash}{\vdash 1 \leftarrow [A \wedge (A \rightarrow \perp)]}
\end{array}$$

$$\begin{array}{c}
\frac{A \vdash}{A \vdash \perp} \quad \frac{A \vdash}{A \vdash \perp} \\
\hline
\frac{A \wedge (A \rightarrow \perp) \vdash}{\vdash \top \leftarrow [A \wedge (A \rightarrow \perp)]} \quad \frac{A \wedge (A \rightarrow \perp) \vdash}{\vdash \top \leftarrow [A \wedge (A \rightarrow \perp)]}
\end{array}$$

3.4.2 Multiplicative NC'

All unprovable in B The reason none of these can be proven in B is because they all contain instances of multiplicative NC sequents which are themselves unprovable.

Excluding 1

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (\top \leftarrow A)]$$

Including 0

$$\vdash [A \otimes (A \rightarrow 0)] \rightarrow 0$$

$$\vdash [A \otimes (A \rightarrow 0)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (A \rightarrow 0)]$$

$$\vdash \top \leftarrow [A \otimes (A \rightarrow 0)]$$

Including Bottom

$$\vdash [A \otimes (A \rightarrow \perp)] \rightarrow 0$$

$$\vdash [A \otimes (A \rightarrow \perp)] \rightarrow \perp$$

$$\vdash 1 \leftarrow [A \otimes (A \rightarrow \perp)]$$

$$\vdash \top \leftarrow [A \otimes (A \rightarrow \perp)]$$

3.5 Partially provable or provable in B

Only additive NC and NC' rules are partially provable or provable in B.

3.5.1 NC

$$\begin{array}{c}
\frac{A \vdash}{[A \wedge (A \rightarrow \perp)] \vdash} \quad L \wedge \quad \frac{\frac{A \vdash}{A \vdash \perp} \quad A \rightarrow \perp \vdash}{[A \wedge (A \rightarrow \perp)] \vdash} \quad L \wedge \\
\frac{A \vdash}{[A \wedge (1 \leftarrow A)] \vdash} \quad \frac{\frac{\vdash A}{1 \vdash A} \quad 1 \leftarrow A \vdash}{[A \wedge (1 \leftarrow A)] \vdash} \\
\frac{A \vdash}{[A \wedge (\top \leftarrow A)] \vdash} \\
\frac{A \vdash}{[A \wedge (A \rightarrow 0)] \vdash}
\end{array}$$

3.5.2 NC'

Excluding 1

$$\begin{array}{c}
\frac{\frac{A \vdash}{A \vdash \perp} \quad A \wedge (1 \leftarrow A) \vdash \perp}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow \perp} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A} \quad 1 \leftarrow A \vdash \perp}{[A \wedge (1 \leftarrow A)] \vdash \perp}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow \perp} \\
\frac{\vdash 1 \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A} \quad 1 \leftarrow A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]}
\end{array}$$

$$\begin{array}{c}
\frac{\vdash \top \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]} \quad \frac{\frac{\frac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash} \quad \frac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]}
\end{array}$$

Excluding Top

$$\begin{array}{c}
\frac{\frac{A \vdash}{A \vdash \perp}}{A \wedge (\top \leftarrow A) \vdash \perp} \\
\vdash [A \wedge (\top \leftarrow A)] \rightarrow \perp \\
\\
\frac{\vdash 1 \quad \frac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]} \\
\\
\frac{\vdash \top \quad \frac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}
\end{array}$$

Including 0

$$\begin{array}{c}
\frac{\frac{A \vdash}{A \vdash \perp}}{A \wedge (A \rightarrow 0) \vdash \perp} \\
\vdash [A \wedge (A \rightarrow 0)] \rightarrow \perp \\
\\
\frac{\vdash 1 \quad \frac{A \vdash}{A \wedge (A \rightarrow 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \rightarrow 0)]}
\end{array}$$

$$\frac{\vdash \top \quad \frac{A \vdash}{A \wedge (A \rightarrow 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \rightarrow 0)]}$$

Including Bottom

$$\frac{\frac{A \vdash}{A \vdash \perp} \quad \frac{A \wedge (A \rightarrow \perp) \vdash \perp}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp}}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp}$$

$$\frac{L \wedge \frac{\frac{A \vdash}{A \vdash \perp} \quad \frac{A \rightarrow \perp \vdash}{[A \wedge (A \rightarrow \perp)] \vdash}}{[A \wedge (A \rightarrow \perp)] \vdash \perp}}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow \perp}$$

$$\frac{\vdash 1 \quad \frac{A \vdash}{A \wedge (A \rightarrow \perp) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \rightarrow \perp)]}$$

$$\frac{\vdash 1 \quad \frac{\frac{A \vdash}{A \vdash \perp} \quad \frac{A \rightarrow \perp \vdash}{A \wedge (A \rightarrow \perp) \vdash}}{A \wedge (A \rightarrow \perp) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \rightarrow \perp)]}$$

$$\frac{\vdash \top \quad \frac{A \vdash}{A \wedge (A \rightarrow \perp) \vdash}}{\vdash \top \leftarrow [A \wedge (A \rightarrow \perp)]}$$

$$\frac{\vdash \top \quad \frac{\frac{A \vdash}{A \vdash \perp} \quad \frac{A \rightarrow \perp \vdash}{A \wedge (A \rightarrow \perp) \vdash}}{A \wedge (A \rightarrow \perp) \vdash}}{\vdash \top \leftarrow [A \wedge (A \rightarrow \perp)]}$$

3.6 Unprovable in B

3.6.1 NC

Additive

$$\frac{\top \vdash A}{\frac{\top \leftarrow A \vdash}{[A \wedge (\top \leftarrow A)] \vdash}}$$

$$\frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{[A \wedge (A \rightarrow 0)] \vdash}$$

Multiplicative

$$\frac{A, 1 \leftarrow A \vdash}{[A \otimes (1 \leftarrow A)] \vdash} \otimes L$$

$$\frac{A, \top \leftarrow A \vdash}{[A \otimes (\top \leftarrow A)] \vdash}$$

$$\frac{A, A \rightarrow 0 \vdash}{[A \otimes (A \rightarrow 0)] \vdash}$$

$$\frac{A, A \rightarrow \perp \vdash}{[A \otimes (A \rightarrow \perp)] \vdash}$$

3.6.2 NC'

All multiplicative NC' rules are unprovable because the multiplicative NC rules are unprovable.

$$\frac{\frac{\frac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash}}{[A \wedge (\top \leftarrow A)] \vdash \perp} \rightarrow \perp$$

$$\frac{\vdash 1 \quad \frac{\frac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\frac{\frac{\frac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\frac{\frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{[A \wedge (A \rightarrow 0)] \vdash}}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow \perp}$$

$$\frac{\frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{A \wedge (A \rightarrow 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \rightarrow 0)]}$$

$$\frac{\frac{\frac{A \vdash 0}{A \rightarrow 0 \vdash}}{A \wedge (A \rightarrow 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \rightarrow 0)]}$$

$$\frac{\frac{A \vdash 0}{[A \wedge (A \rightarrow 0)] \vdash 0}}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow 0} \quad \frac{\frac{A \rightarrow 0 \vdash 0}{[A \wedge (A \rightarrow 0)] \vdash 0}}{\vdash [A \wedge (A \rightarrow 0)] \rightarrow 0}$$

$$\frac{\frac{A \vdash 0}{[A \wedge (A \rightarrow \perp)] \vdash 0}}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow 0} \quad \frac{\frac{A \rightarrow \perp \vdash 0}{[A \wedge (A \rightarrow \perp)] \vdash 0}}{\vdash [A \wedge (A \rightarrow \perp)] \rightarrow 0}$$

$$\frac{\frac{A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow 0} \quad \frac{\frac{\top \leftarrow A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \rightarrow 0}$$

$$\frac{\frac{A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0} \qquad \frac{\frac{1 \leftarrow A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0}$$