Is Basic Logic Genuinely Paraconsistent and Paracomplete?

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Abstract

We examine whether or not various forms of the law of non-contradiction and the law of excluded middle hold in Sambin's Basic Sequent Logic. We identify at least 128 distinct forms of Non-contradiction and its prime form and a matching number of forms of the Excluded Middle and its prime form.

Contents

Part I
Non-Contradiction and Excluded Middle in Sambin's Basic Logic

$$\vdash ((A \wedge (A \vdash)) \vdash)$$

$$A \wedge (A \vdash) \vdash$$

$$\vdash \top \leftarrow (A \wedge (A \to 0)) \to 0$$

$$\vdash 1 \leftarrow (A \wedge (A \to 0)) \to 0$$

$$\vdash 1 \leftarrow (A \wedge (A \to 0)) \to \bot$$

$$\vdash \top \leftarrow (A \otimes (A \to 0)) \to 0$$

$$\vdash 1 \leftarrow (A \otimes (A \to 0)) \to 0$$

$$\vdash 1 \leftarrow (A \otimes (A \to 0)) \to \bot$$

1 Classical Non-contradiction

$$A \otimes (A \to 0), A \otimes (A \to \bot), A \wedge (A \to 0), A \wedge (A \to \bot) \vdash$$

$$\vdash [(A \otimes (A \to 0)) \circ (A \otimes (A \to \bot)) \circ (A \wedge (A \to 0)) \circ (A \wedge (A \to \bot))] \to 0$$

$$\vdash [(A \otimes (A \to 0)) \circ (A \otimes (A \to \bot)) \circ (A \wedge (A \to 0)) \circ (A \wedge (A \to \bot))] \to \bot$$

2 Classical Excluded Middle

$$\vdash (A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))$$

$$1 \leftarrow [(A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))] \vdash$$

$$\top \leftarrow [(A \wp (1 \leftarrow A)) \circ (A \wp (\top \leftarrow A)) \circ (A \vee (1 \leftarrow A)) \circ (A \vee (\top \leftarrow A))] \vdash$$

3 Paraconsistent and Paracomplete forms of truth

$$\vdash 0 \to A$$

$$\vdash \bot \to A$$

$$\vdash 1 \to A$$

$$\vdash \top \to A$$

$$A \leftarrow 0 \dashv$$

$$A \leftarrow \bot \dashv$$

$$A \leftarrow 1 \dashv$$

$$A \leftarrow \top \dashv$$

3.1 NC in Basic Sequent Logic

3.1.1 Additive NC

$$[A \wedge (1 \leftarrow A)] \vdash$$

$$[A \wedge (\top \leftarrow A)] \vdash$$

$$[A \wedge (A \rightarrow 0)] \vdash$$

$$[A \wedge (A \rightarrow \bot)] \vdash$$

3.1.2 Multiplicative NC

$$[A \otimes (1 \leftarrow A)] \vdash$$

$$[A \otimes (\top \leftarrow A)] \vdash$$

$$[A \otimes (A \rightarrow 0)] \vdash$$

$$[A \otimes (A \rightarrow \bot)] \vdash$$

3.2 NC' in Basic Sequent Logic

3.2.1 Additive NC'

Excluding 1

$$\vdash [A \wedge (1 \leftarrow A)] \rightarrow 0$$

$$\vdash [A \wedge (1 \leftarrow A)] \to \bot$$

$$\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \wedge (\top \leftarrow A)] \to 0$$

$$\vdash [A \wedge (\top \leftarrow A)] \to \bot$$

$$\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]$$

Including 0

$$\vdash [A \wedge (A \to 0)] \to 0$$

$$\vdash [A \wedge (A \to 0)] \to \bot$$

$$\vdash 1 \leftarrow [A \wedge (A \to 0)]$$

$$\vdash \top \leftarrow [A \wedge (A \to 0)]$$

Including Bottom

$$\vdash [A \wedge (A \to \bot)] \to 0$$

$$\vdash [A \wedge (A \to \bot)] \to \bot$$

$$\vdash 1 \leftarrow [A \wedge (A \to \bot)]$$

$$\vdash \top \leftarrow [A \wedge (A \to \bot)]$$

3.2.2 Multiplicative NC'

Excluding 1

$$\vdash [A \otimes (1 \leftarrow A)] \to 0$$

$$\vdash [A \otimes (1 \leftarrow A)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \otimes (\top \leftarrow A)] \to 0$$

$$\vdash [A \otimes (\top \leftarrow A)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (\top \leftarrow A)]$$

Including 0

$$\vdash [A \otimes (A \to 0)] \to 0$$

$$\vdash [A \otimes (A \to 0)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (A \to 0)]$$

$$\vdash \top \leftarrow [A \otimes (A \to 0)]$$

Including Bottom

$$\vdash [A \otimes (A \to \bot)] \to 0$$

$$\vdash [A \otimes (A \to \bot)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (A \to \bot)]$$

$$\vdash \top \leftarrow [A \otimes (A \to \bot)]$$

3.3 NC Proofs in Basic Sequent Logic

3.3.1 Additive NC

$$L \wedge \frac{A \vdash}{[A \wedge (A \to \bot)] \vdash} \qquad L \wedge \frac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{[A \wedge (A \to \bot)] \vdash}$$

$$\frac{A \vdash}{[A \wedge (1 \leftarrow A)] \vdash} \qquad \frac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{[A \wedge (1 \leftarrow A)] \vdash}$$

$$\frac{A \vdash}{[A \wedge (\top \leftarrow A)] \vdash} \qquad \frac{\dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash}}{}$$

$$\frac{A \vdash}{[A \wedge (A \to 0)] \vdash} \qquad \frac{\dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{[A \wedge (A \to 0)] \vdash}}{}$$

3.3.2 Multiplicative NC

$$\frac{A, 1 \leftarrow A \vdash}{[A \otimes (1 \leftarrow A)] \vdash} \otimes L$$

$$\frac{A, \top \leftarrow A \vdash}{[A \otimes (\top \leftarrow A)] \vdash}$$

$$\frac{A, A \to 0 \vdash}{[A \otimes (A \to 0)] \vdash}$$

$$\frac{A, A \to \bot \vdash}{[A \otimes (A \to \bot)] \vdash}$$

3.4 NC' in Basic Sequent Logic

3.4.1 Additive NC'

Excluding 1

$$\dfrac{\dfrac{A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \to 0} \qquad \dfrac{\dfrac{1 \leftarrow A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \to 0}$$

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (1 \leftarrow A) \vdash \bot}}{\vdash [A \wedge (1 \leftarrow A)] \to \bot} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{1 \leftarrow A \vdash \bot}}{[A \wedge (1 \leftarrow A)] \vdash \bot}}{\vdash [A \wedge (1 \leftarrow A)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]} \qquad \dfrac{\vdash 1 \qquad \dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]} \qquad \dfrac{\vdash \top \qquad \dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]}$$

Excluding Top

$$\dfrac{\dfrac{A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \to 0} \qquad \dfrac{\dfrac{\top \leftarrow A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \to 0}$$

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (\top \leftarrow A) \vdash \bot}}{\vdash [A \wedge (\top \leftarrow A)] \to \bot} \qquad \dfrac{\dfrac{\dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash}}{[A \wedge (\top \leftarrow A)] \vdash \bot}}{\vdash [A \wedge (\top \leftarrow A)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]} \qquad \dfrac{\vdash 1 \qquad \dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]} \qquad \dfrac{\vdash \top \qquad \dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}$$

Including 0

$$\dfrac{\dfrac{A \vdash 0}{[A \wedge (A \to 0)] \vdash 0}}{\vdash [A \wedge (A \to 0)] \to 0} \qquad \dfrac{\dfrac{A \to 0 \vdash 0}{[A \wedge (A \to 0)] \vdash 0}}{\vdash [A \wedge (A \to 0)] \to 0}$$

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (A \to 0) \vdash \bot}}{\vdash [A \wedge (A \to 0)] \to \bot} \qquad \dfrac{\dfrac{\dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{[A \wedge (A \to 0)] \vdash}}{[A \wedge (A \to 0)] \vdash \bot}}{\vdash [A \wedge (A \to 0)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (A \to 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to 0)]} \qquad \dfrac{\vdash 1 \qquad \dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{A \wedge (A \to 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to 0)]}$$

$$\frac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (A \to 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to 0)]} \qquad \frac{\vdash \top \qquad \dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{A \wedge (A \to 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to 0)]}$$

Including Bottom

$$\frac{\dfrac{\dfrac{A \vdash 0}{[A \wedge (A \to \bot)] \vdash 0}}{\vdash [A \wedge (A \to \bot)] \to 0} \qquad \dfrac{\dfrac{\dfrac{A \to \bot \vdash 0}{[A \wedge (A \to \bot)] \vdash 0}}{\vdash [A \wedge (A \to \bot)] \to 0}}$$

$$\frac{\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (A \to \bot) \vdash \bot}}{\vdash [A \wedge (A \to \bot)] \to \bot} \qquad \dfrac{L \wedge \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{[A \wedge (A \to \bot)] \vdash}}{\dfrac{[A \wedge (A \to \bot)] \vdash \bot}{\vdash [A \wedge (A \to \bot)] \to \bot}}$$

$$\frac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (A \to \bot) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to \bot)]} \qquad \frac{\vdash 1 \qquad \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{A \wedge (A \to \bot) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to \bot)]}$$

$$\frac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (A \to \bot) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to \bot)]} \qquad \frac{\vdash \top \qquad \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{A \wedge (A \to \bot) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to \bot)]}$$

3.4.2 Multiplicative NC'

All unprovable in B The reason none of these can be proven in B is because they all contain instances of multiplicative NC sequents which are themselves unprovable.

Excluding 1

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (1 \leftarrow A)] \rightarrow \bot$$

$$\vdash 1 \leftarrow [A \otimes (1 \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (1 \leftarrow A)]$$

Excluding Top

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow 0$$

$$\vdash [A \otimes (\top \leftarrow A)] \rightarrow \bot$$

$$\vdash 1 \leftarrow [A \otimes (\top \leftarrow A)]$$

$$\vdash \top \leftarrow [A \otimes (\top \leftarrow A)]$$

Including 0

$$\vdash [A \otimes (A \to 0)] \to 0$$

$$\vdash [A \otimes (A \to 0)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (A \to 0)]$$

$$\vdash \top \leftarrow [A \otimes (A \to 0)]$$

Including Bottom

$$\vdash [A \otimes (A \to \bot)] \to 0$$

$$\vdash [A \otimes (A \to \bot)] \to \bot$$

$$\vdash 1 \leftarrow [A \otimes (A \to \bot)]$$

$$\vdash \top \leftarrow [A \otimes (A \to \bot)]$$

3.5 Partially provable or provable in B

Only additive NC and NC' rules are partially provable or provable in B.

3.5.1 NC

$$L \wedge \frac{A \vdash}{[A \wedge (A \to \bot)] \vdash} \qquad L \wedge \frac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{[A \wedge (A \to \bot)] \vdash}$$

$$\frac{A \vdash}{[A \wedge (1 \leftarrow A)] \vdash} \qquad \frac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{[A \wedge (1 \leftarrow A)] \vdash}$$

$$\frac{A \vdash}{[A \wedge (\top \leftarrow A)] \vdash}$$

$$\frac{A \vdash}{[A \wedge (A \to 0)] \vdash}$$

3.5.2 NC'

Excluding 1

$$\frac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (1 \leftarrow A) \vdash \bot}}{\vdash [A \wedge (1 \leftarrow A)] \to \bot} \qquad \frac{\dfrac{\dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{1 \leftarrow A \vdash \bot}}{[A \wedge (1 \leftarrow A)] \vdash \bot}}{\vdash [A \wedge (1 \leftarrow A)] \to \bot}$$

$$\frac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]} \qquad \frac{\vdash 1 \qquad \dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{A \wedge (1 \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (1 \leftarrow A)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]} \qquad \dfrac{\vdash \top \qquad \dfrac{\dfrac{\dfrac{\vdash A}{1 \vdash A}}{1 \leftarrow A \vdash}}{A \wedge (1 \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (1 \leftarrow A)]}$$

Excluding Top

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (\top \leftarrow A) \vdash \bot}}{\vdash [A \wedge (\top \leftarrow A)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}$$

Including 0

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (A \to 0) \vdash \bot}}{\vdash [A \wedge (A \to 0)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (A \to 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to 0)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (A \to 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to 0)]}$$

Including Bottom

$$\dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \wedge (A \to \bot) \vdash \bot}}{\vdash [A \wedge (A \to \bot)] \to \bot} \qquad \dfrac{L \wedge \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{[A \wedge (A \to \bot)] \vdash}}{[A \wedge (A \to \bot)] \vdash \bot} \Big/ \vdash [A \wedge (A \to \bot)] \to \bot$$

$$\dfrac{\vdash 1 \qquad \dfrac{A \vdash}{A \wedge (A \to \bot) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to \bot)]} \qquad \dfrac{\vdash 1 \qquad \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{A \wedge (A \to \bot) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to \bot)]}$$

$$\dfrac{\vdash \top \qquad \dfrac{A \vdash}{A \wedge (A \to \bot) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to \bot)]} \qquad \dfrac{\vdash \top \qquad \dfrac{\dfrac{\dfrac{A \vdash}{A \vdash \bot}}{A \to \bot \vdash}}{A \wedge (A \to \bot) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to \bot)]}$$

3.6 Unprovable in B

3.6.1 NC

Additive

$$\dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash}$$

$$\dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{[A \wedge (A \to 0)] \vdash}$$

Multiplicative

$$\dfrac{A, 1 \leftarrow A \vdash}{[A \otimes (1 \leftarrow A)] \vdash} \otimes L$$

$$\dfrac{A, \top \leftarrow A \vdash}{[A \otimes (\top \leftarrow A)] \vdash}$$

$$\dfrac{A, A \to 0 \vdash}{[A \otimes (A \to 0)] \vdash}$$

$$\dfrac{A, A \to \bot \vdash}{[A \otimes (A \to \bot)] \vdash}$$

3.6.2 NC'

All multiplicative NC' rules are unprovable because the multiplicative NC rules are unprovable.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{[A \wedge (\top \leftarrow A)] \vdash}}{[A \wedge (\top \leftarrow A)] \vdash \bot}}{\vdash [A \wedge (\top \leftarrow A)] \to \bot}$$

$$\dfrac{\vdash 1 \qquad \dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash 1 \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\frac{\vdash \top \qquad \dfrac{\dfrac{\top \vdash A}{\top \leftarrow A \vdash}}{A \wedge (\top \leftarrow A) \vdash}}{\vdash \top \leftarrow [A \wedge (\top \leftarrow A)]}$$

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{[A \wedge (A \to 0)] \vdash}}{[A \wedge (A \to 0)] \vdash \bot}}{\vdash [A \wedge (A \to 0)] \to \bot}$$

$$\frac{\vdash 1 \qquad \dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{A \wedge (A \to 0) \vdash}}{\vdash 1 \leftarrow [A \wedge (A \to 0)]}$$

$$\frac{\vdash \top \qquad \dfrac{\dfrac{A \vdash 0}{A \to 0 \vdash}}{A \wedge (A \to 0) \vdash}}{\vdash \top \leftarrow [A \wedge (A \to 0)]}$$

$$\frac{\dfrac{A \vdash 0}{[A \wedge (A \to 0)] \vdash 0}}{\vdash [A \wedge (A \to 0)] \to 0} \qquad \frac{\dfrac{A \to 0 \vdash 0}{[A \wedge (A \to 0)] \vdash 0}}{\vdash [A \wedge (A \to 0)] \to 0}$$

$$\frac{\dfrac{A \vdash 0}{[A \wedge (A \to \bot)] \vdash 0}}{\vdash [A \wedge (A \to \bot)] \to 0} \qquad \frac{\dfrac{A \to \bot \vdash 0}{[A \wedge (A \to \bot)] \vdash 0}}{\vdash [A \wedge (A \to \bot)] \to 0}$$

$$\frac{\dfrac{A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \to 0} \qquad \frac{\dfrac{\top \leftarrow A \vdash 0}{[A \wedge (\top \leftarrow A)] \vdash 0}}{\vdash [A \wedge (\top \leftarrow A)] \to 0}$$

$$\dfrac{\dfrac{A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \to 0} \qquad \dfrac{\dfrac{1 \leftarrow A \vdash 0}{[A \wedge (1 \leftarrow A)] \vdash 0}}{\vdash [A \wedge (1 \leftarrow A)] \to 0}$$