

james francis toy iv
homework 9

1. setup equations and find exact integrals

$$\begin{aligned}h_{00} &= 2x^3 - 3x^2 + 1 \\h_{01} &= -2x^3 + 3x^2 \\h_{10} &= x^3 - 2x^2 + x \\h_{11} &= x^3 - x^2\end{aligned}$$

$$\begin{aligned}h'_{00} &= 1/2 \\h'_{01} &= 1/2 \\h'_{10} &= 1/12 \\h'_{11} &= -1/12\end{aligned}$$

2. compute $\text{pk}(x)$ dx integral

$$\int_0^1 y_k h_{00} + h d_k h_{10} + y_{k+1} + y_{k+1} h_{01} + h d_{k+1} h_{11} dt$$

=

$$\left(\frac{y_k}{2} + \frac{h d_k}{12} + \frac{y_{k+1}}{2} - \frac{h d_{k+1}}{12} \right) h$$

3. now we can sum them from $k = 0$ to $k = n - 1$ when doing the first few iterations of this we can see the $\frac{h d_1}{12}$ is cancelling; however, the $\frac{h d_0}{12}$ and $\frac{h d_n}{12}$ terms are not. So we can setup an equation for this by separating the constant from y and $d_0 - d_n$ to arrive at:

$$\left(\frac{1}{2} + 1 + 1 + \dots + 1 + 1 + \frac{1}{2} \right) y + \frac{h}{12} (d_0 - d_n) h$$

4. code

5. code