CS2040S
Data Structures and Algorithms
(e-learning edition)

Graphs!

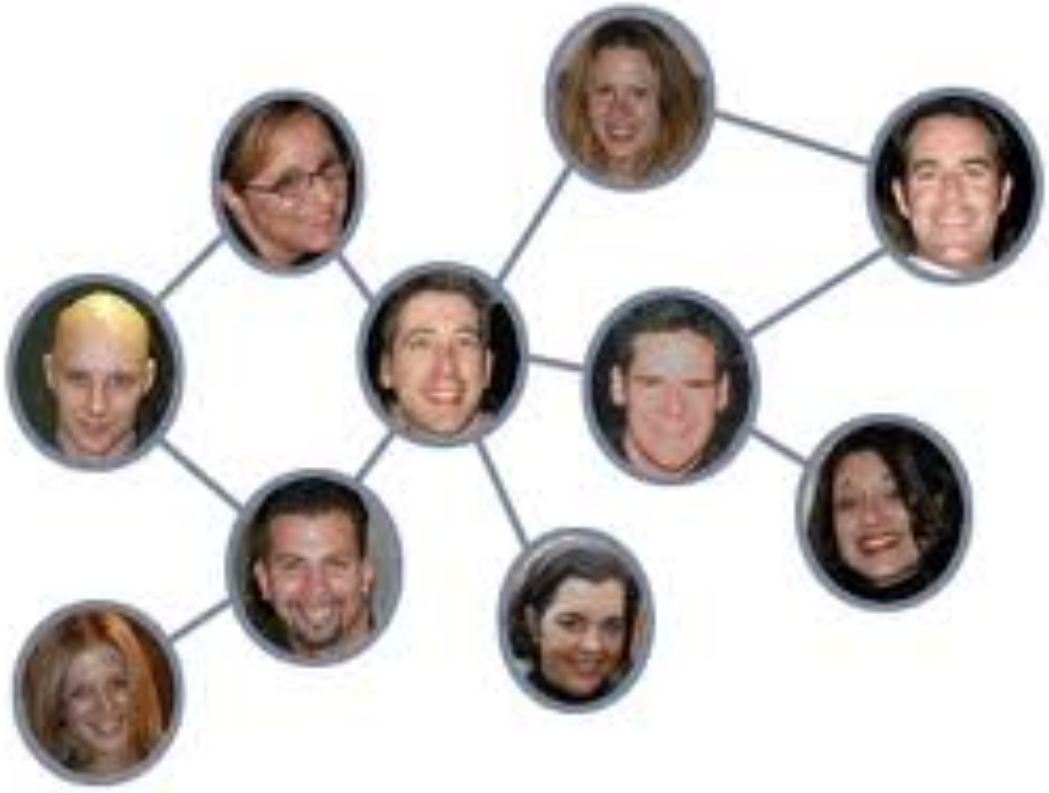
(Part 2)

Where do we find graphs?

Where do we find graphs?

Social network:

- Nodes are people
- Edge = friendship



facebook

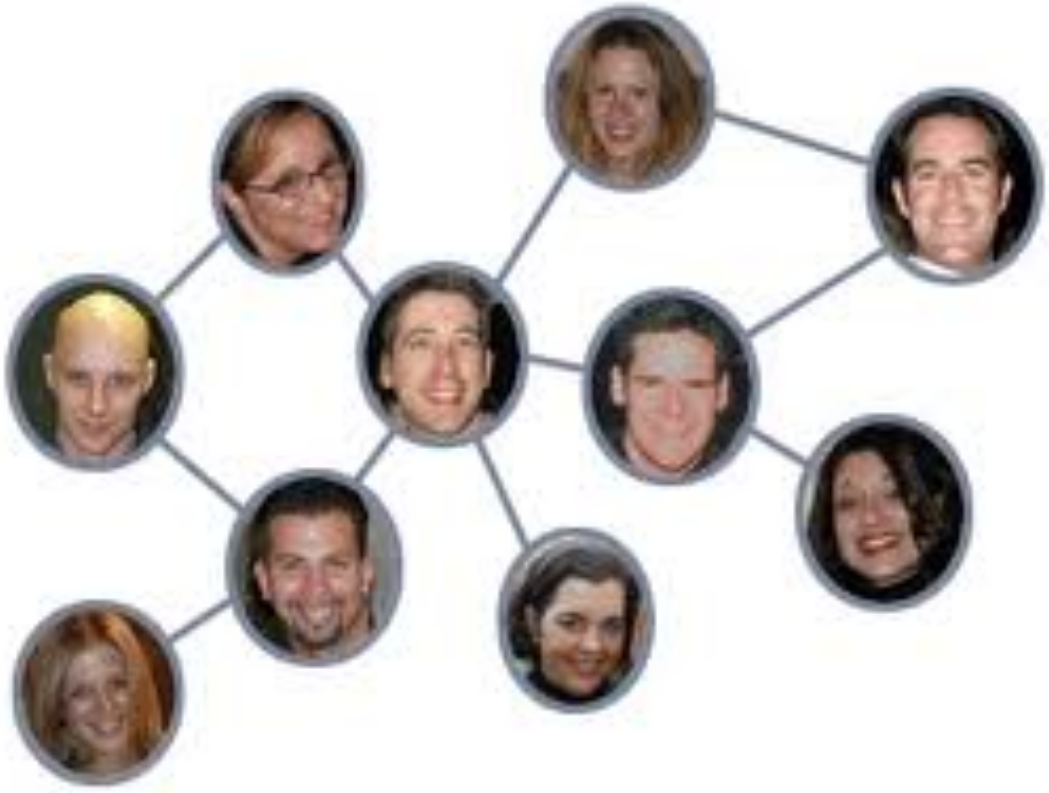
Where do we find graphs?

Social network:

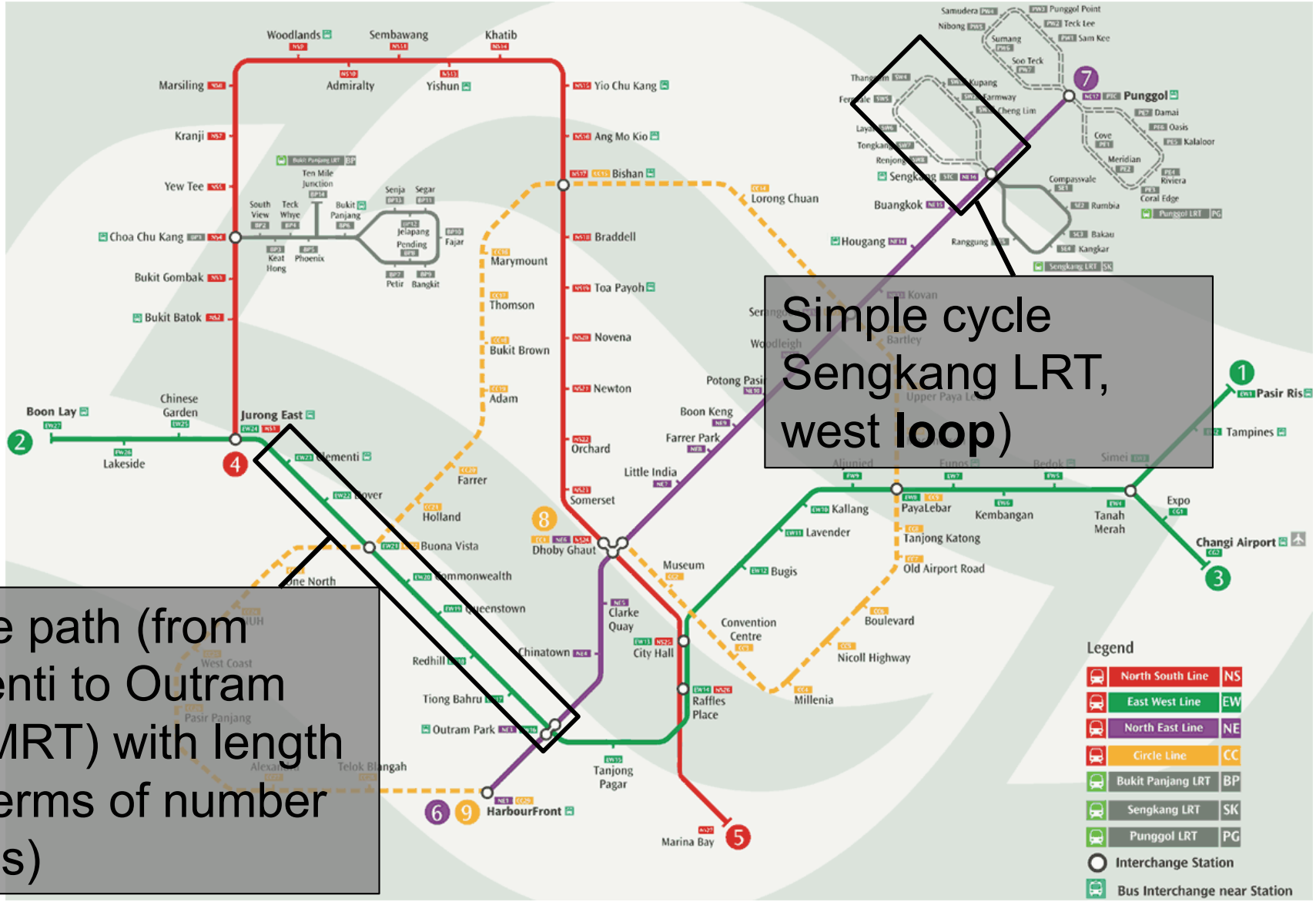
- Nodes are people
- Edge = friendship

Questions:

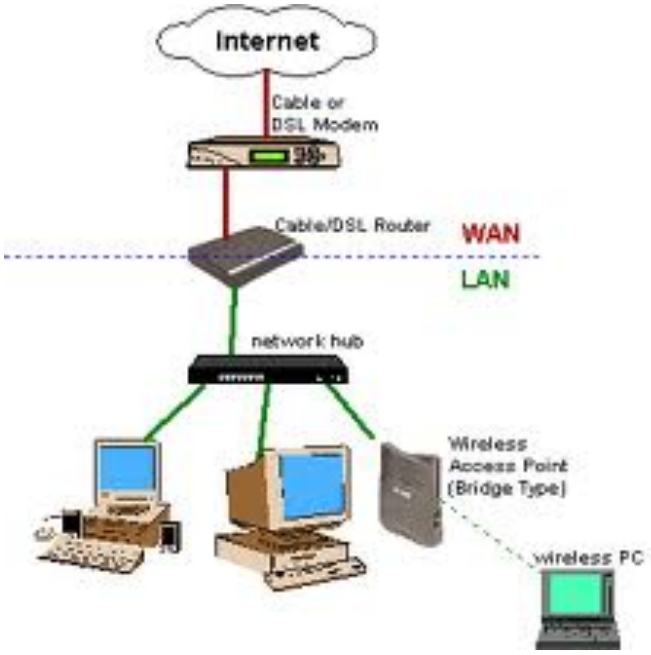
- Connected?
- Diameter?
- Degree?

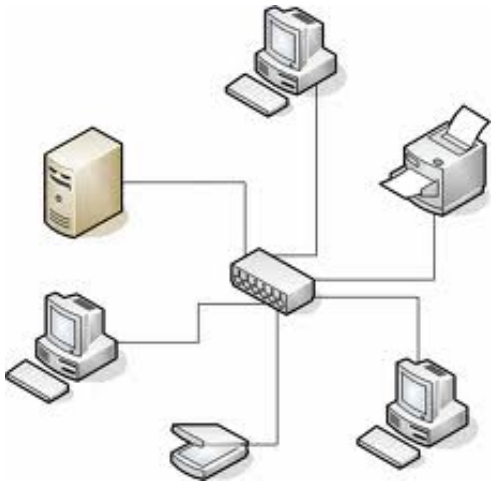


Transportation Network



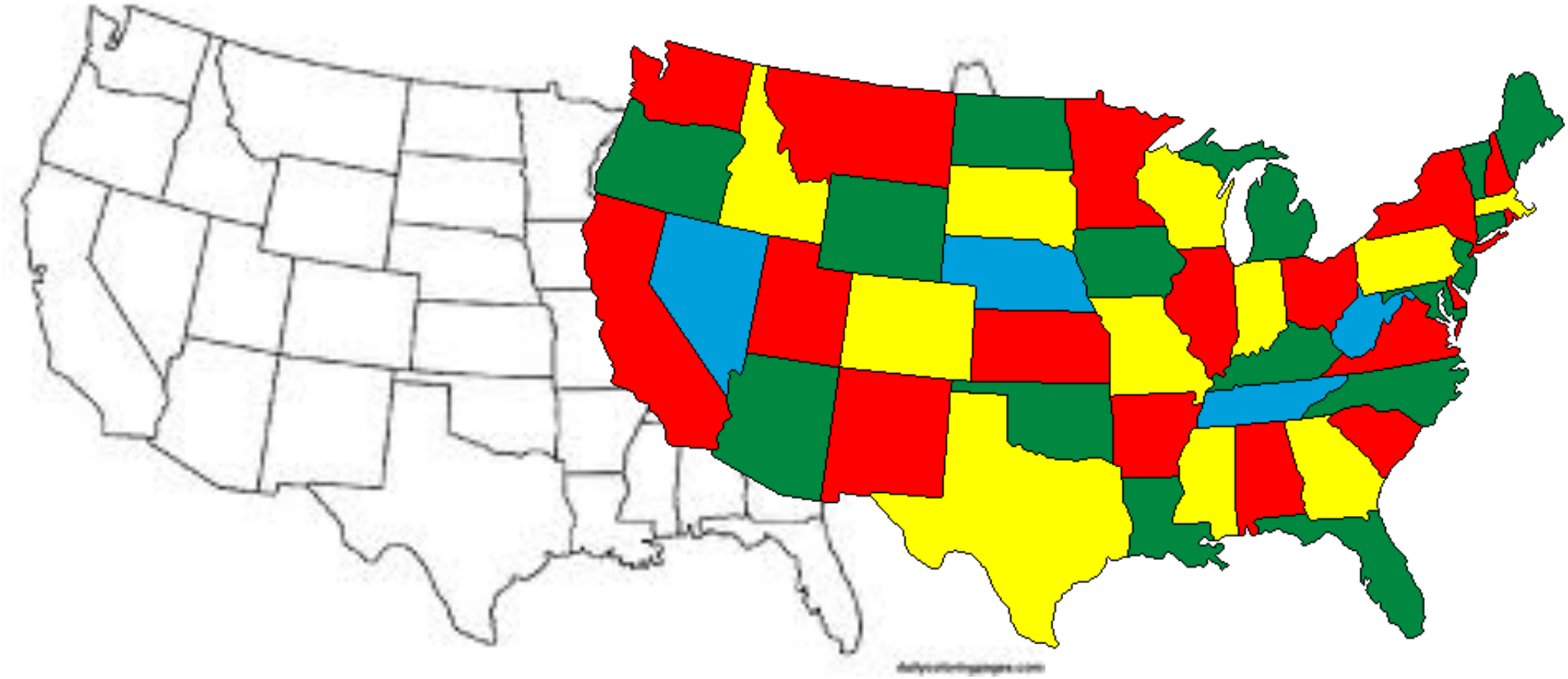
Internet / Computer Networks



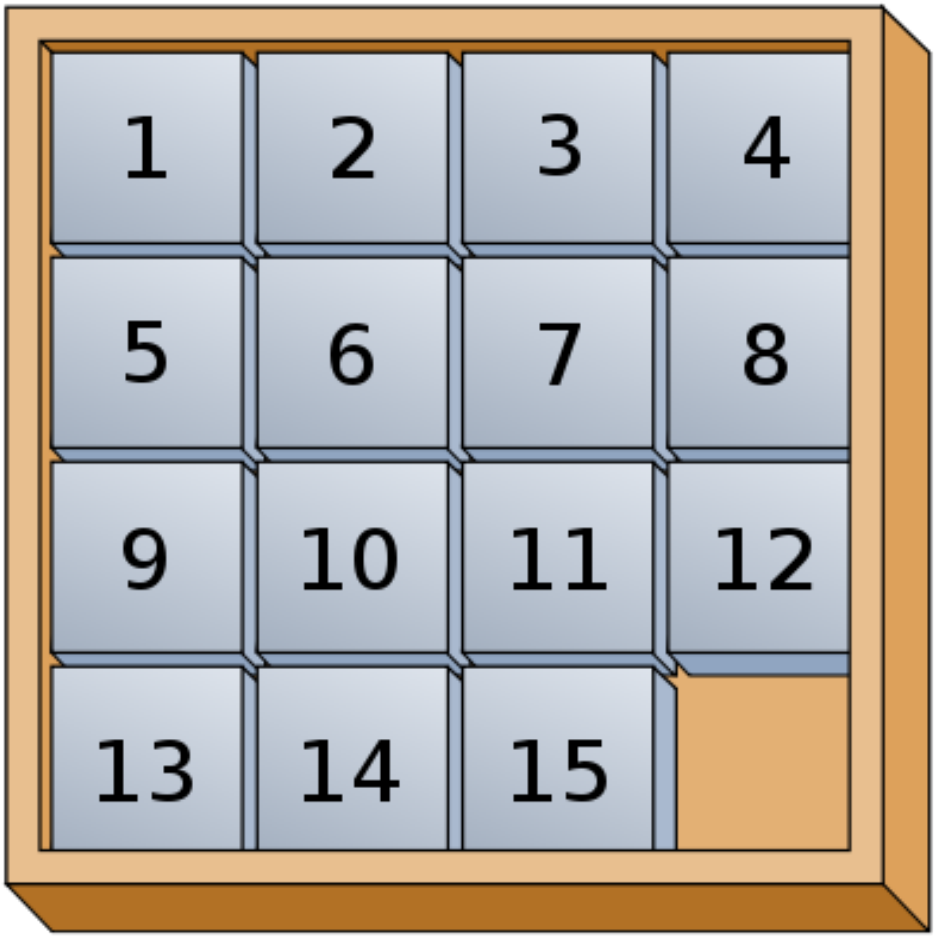


Communication Network

Optimization



Sliding Puzzle



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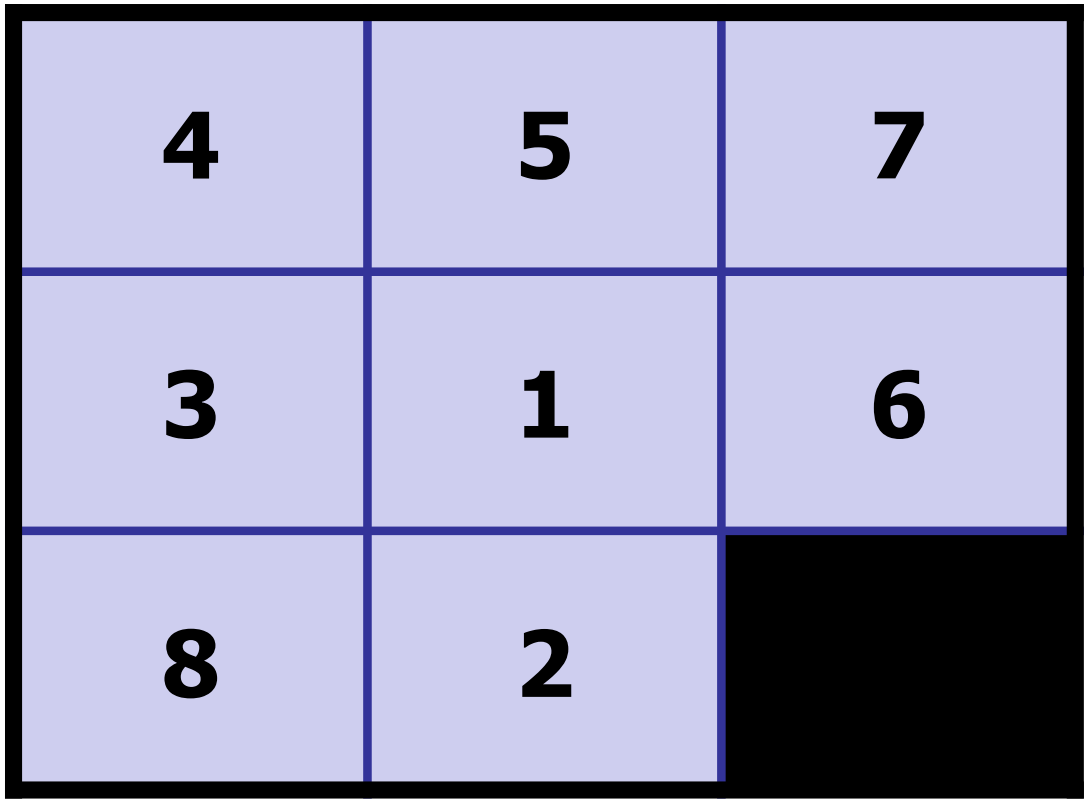

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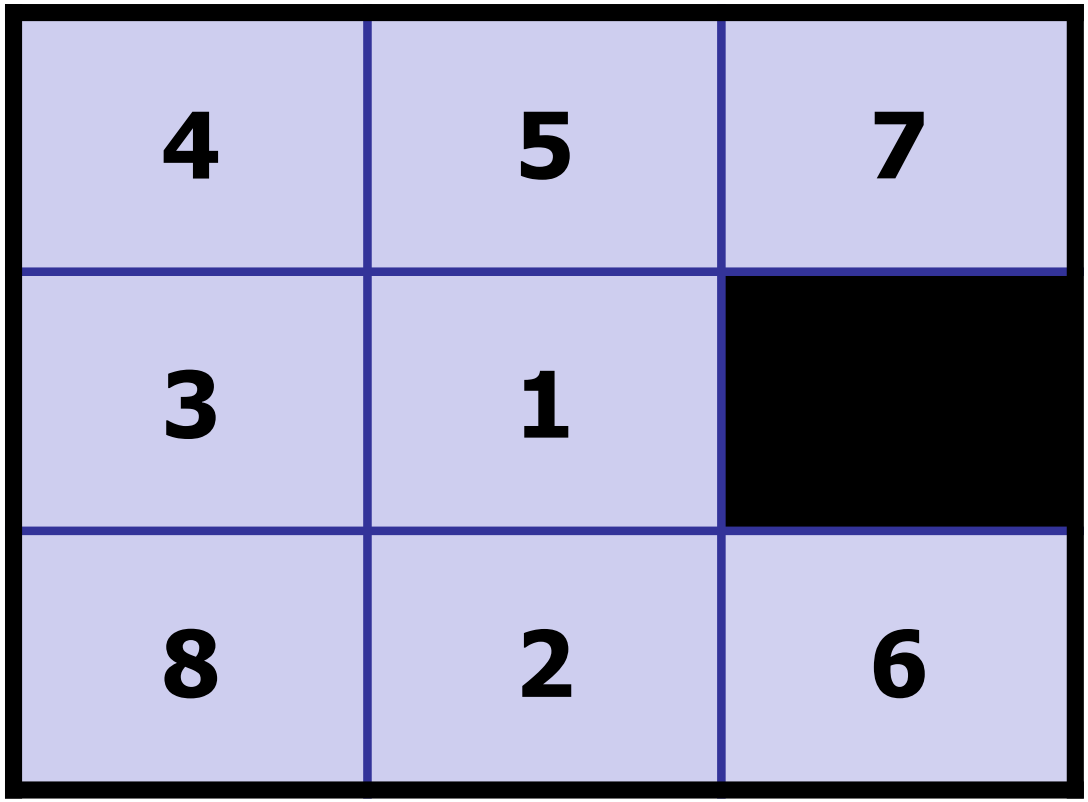
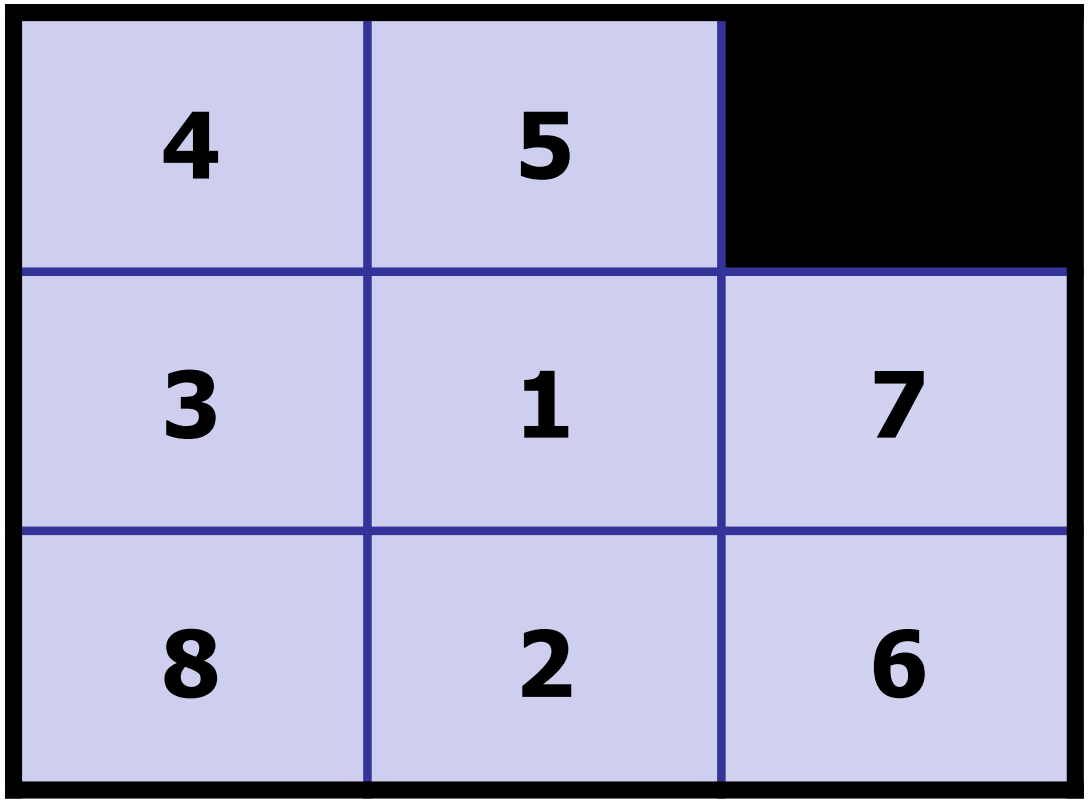


Sliding Puzzle



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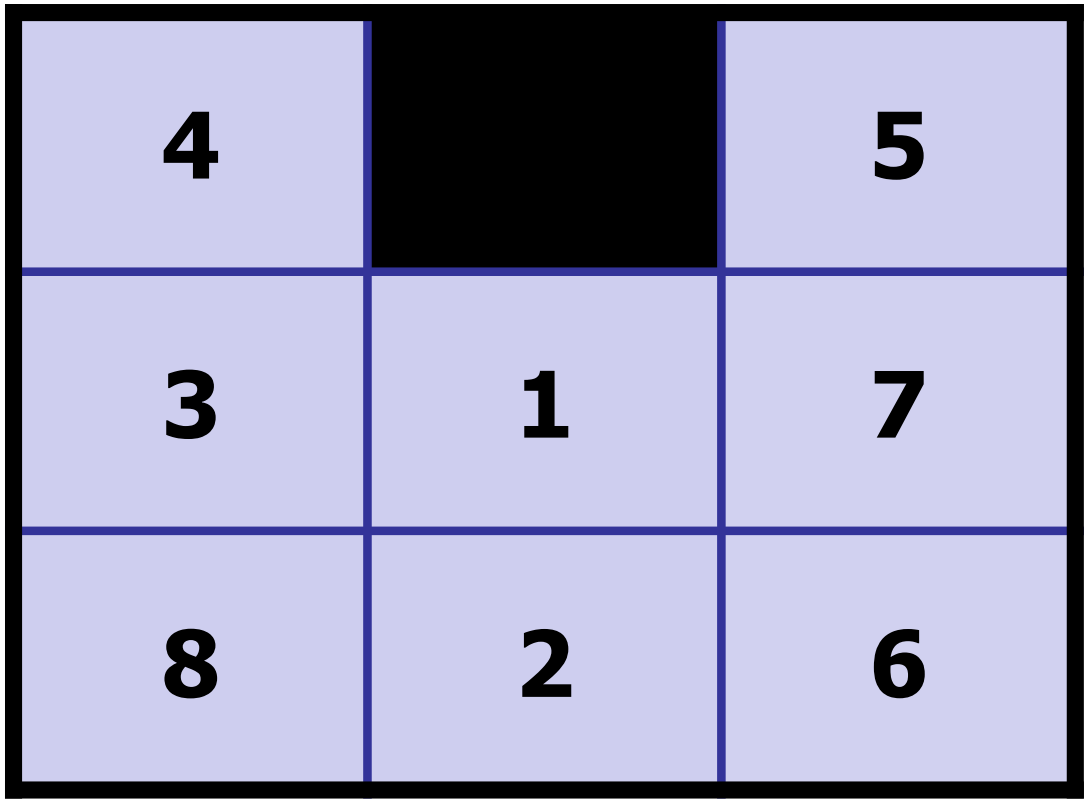

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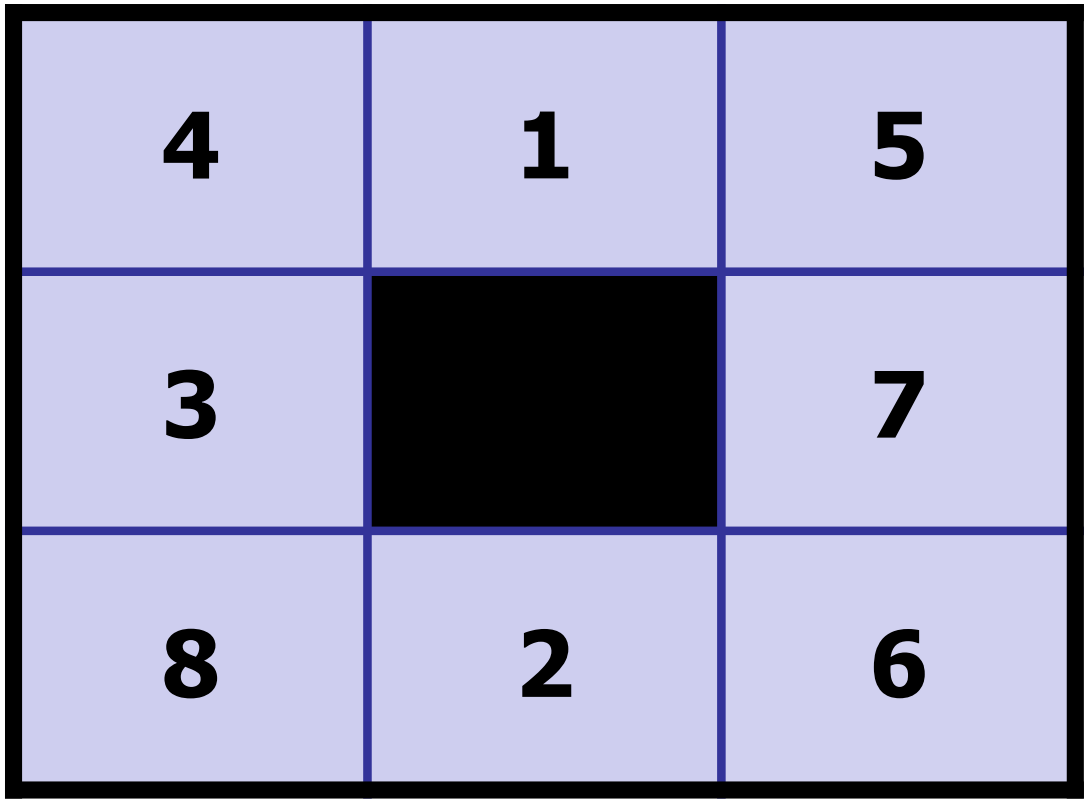
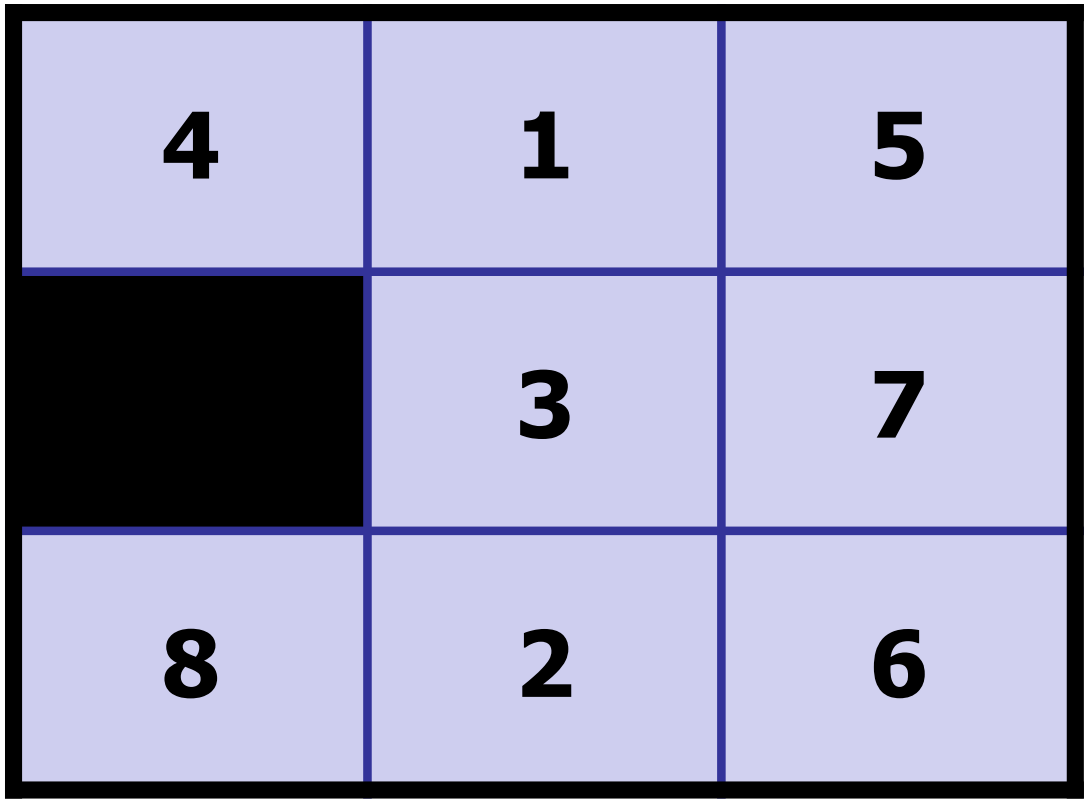
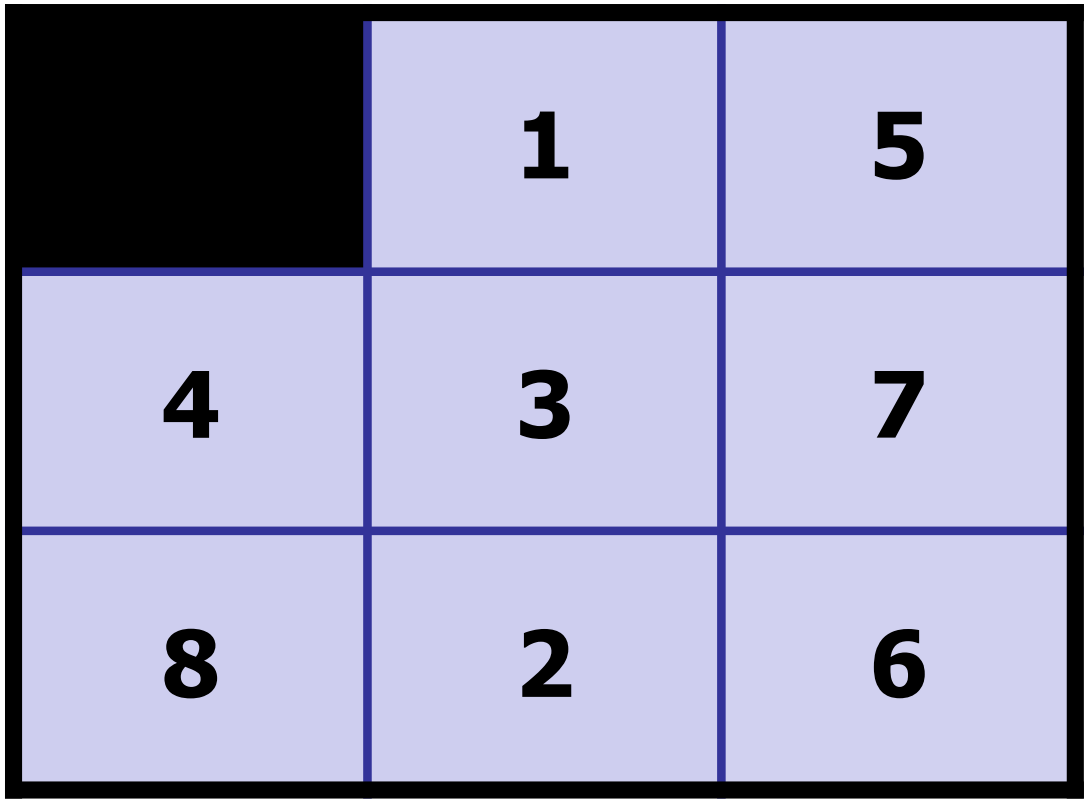


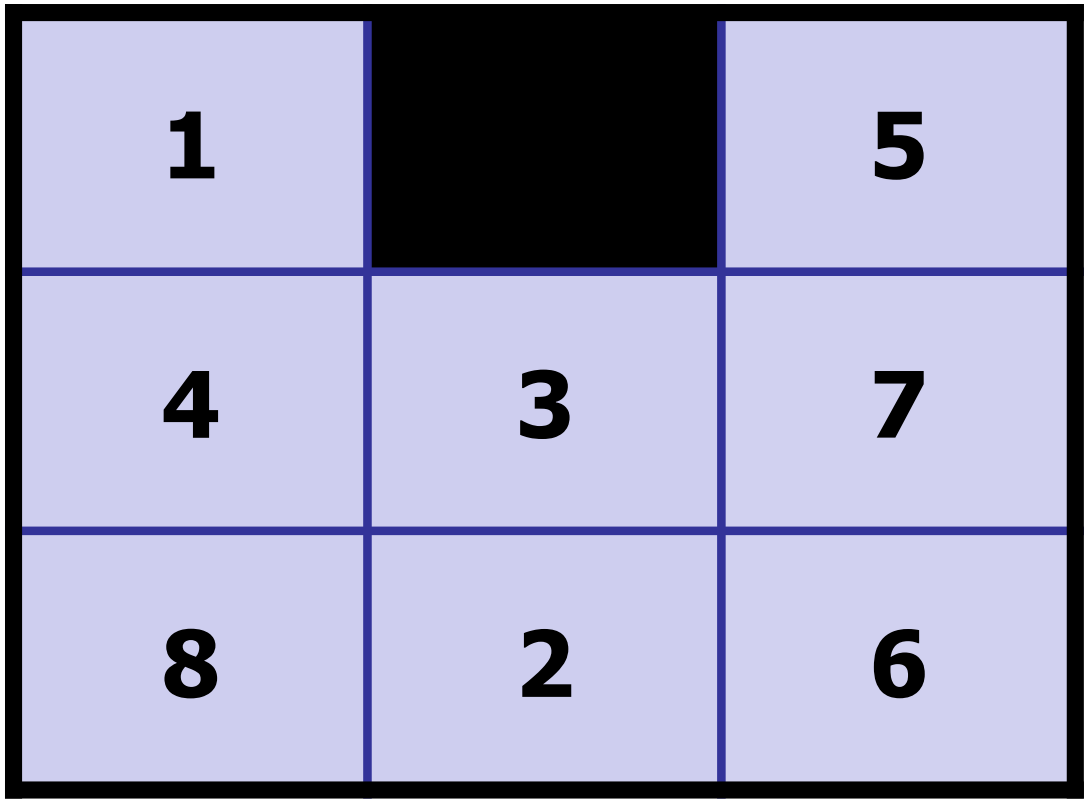
Sliding Puzzle



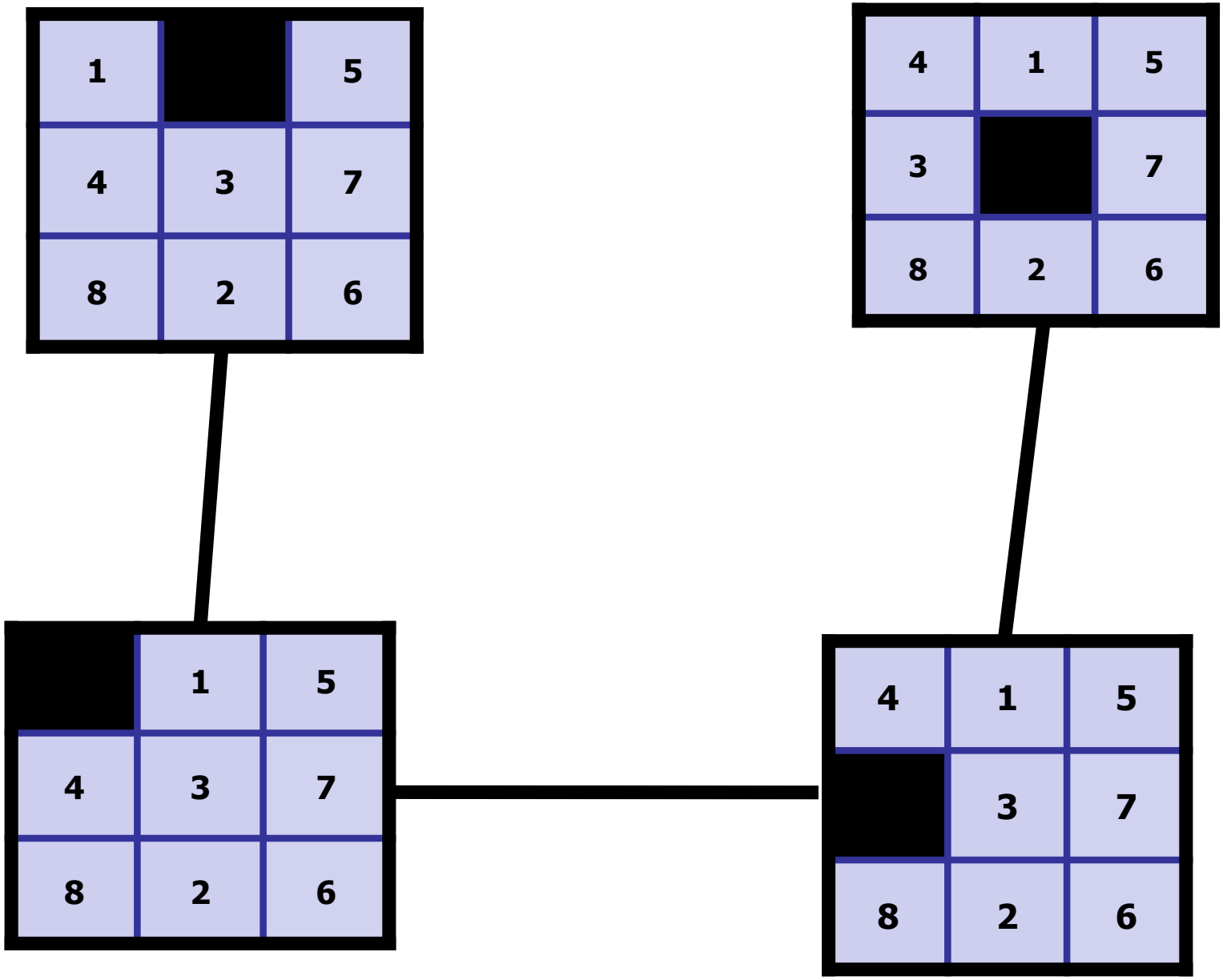
Sliding Puzzle



Sliding Puzzle



Sliding Puzzle is a Graph



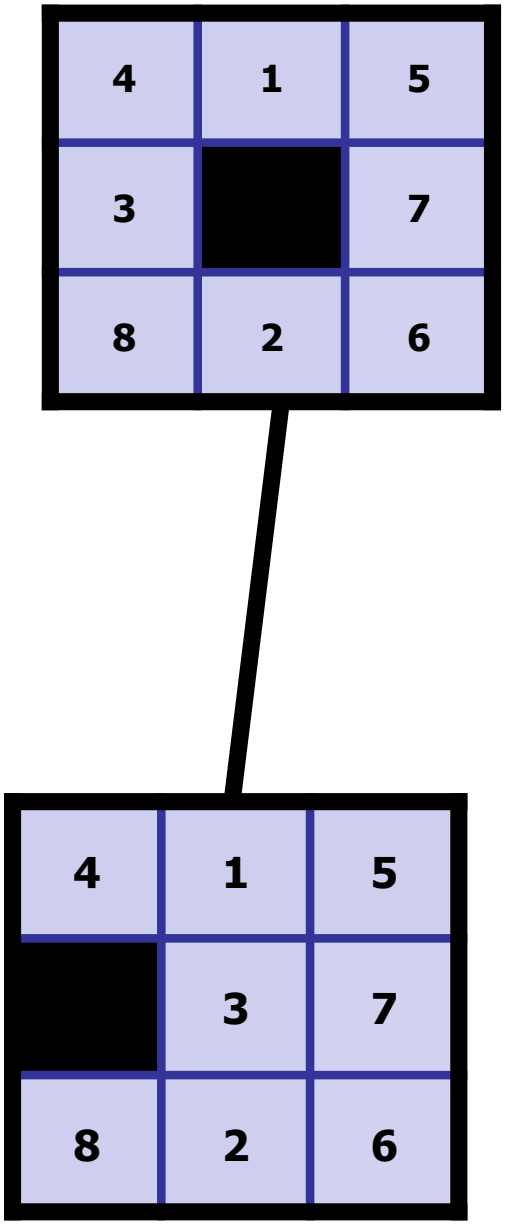
Sliding Puzzle

Nodes:

- State of the puzzle
- Permutation of nine tiles

Edges:

- Two states are edges if they differ by only one move.



What is the maximum degree of the Sliding Puzzle graph?

1. 1
2. 2
3. 3
4. ✓ 4
5. n/2
6. n
7. n!

Sliding Puzzle

Nodes:

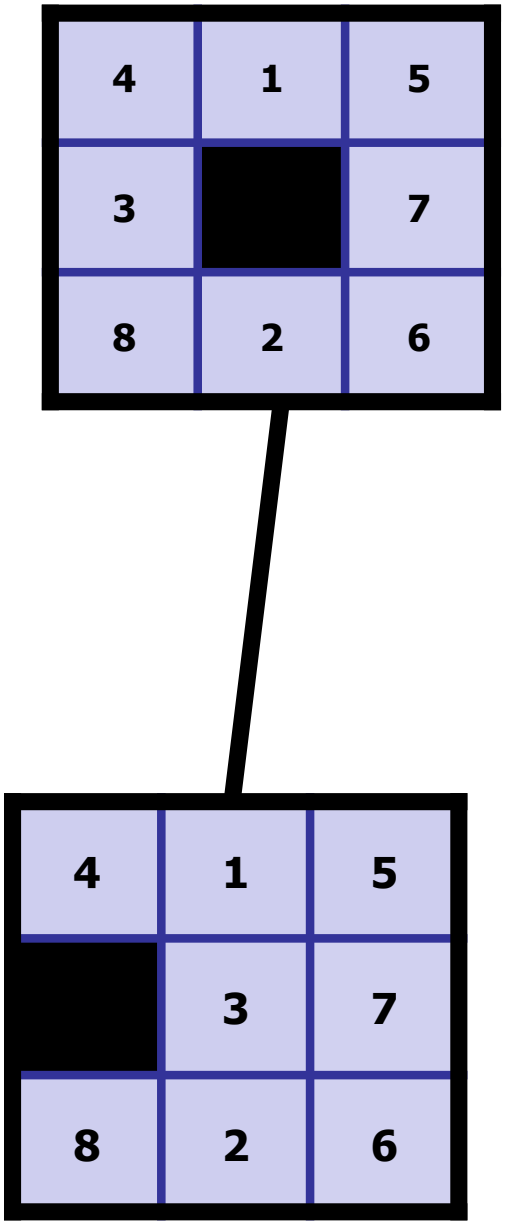
- State of the puzzle
- Permutation of nine tiles

Edges:

- Two states are edges if they differ by only one move.

Nodes = 9! = 362,880

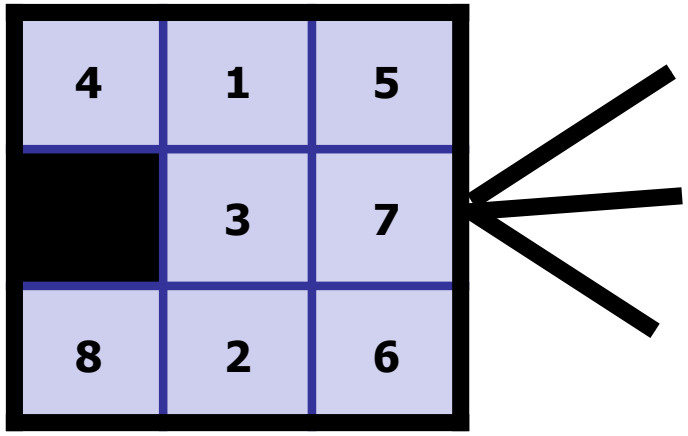
Edges < 4*9! < 1,451,520



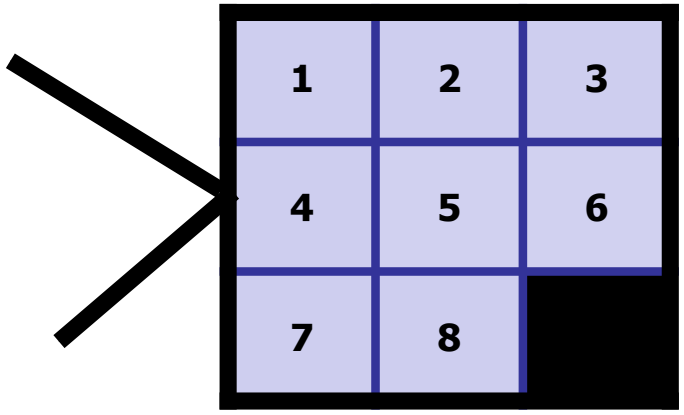
Sliding Puzzle

Number of moves to solve the puzzle?

Initial, scrambled state:



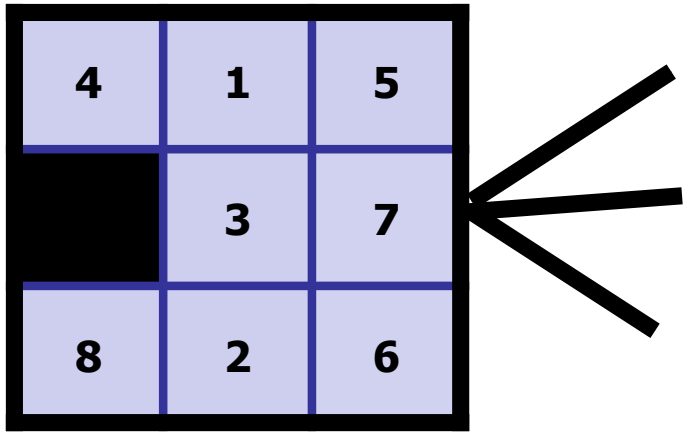
Final, unscrambled state:



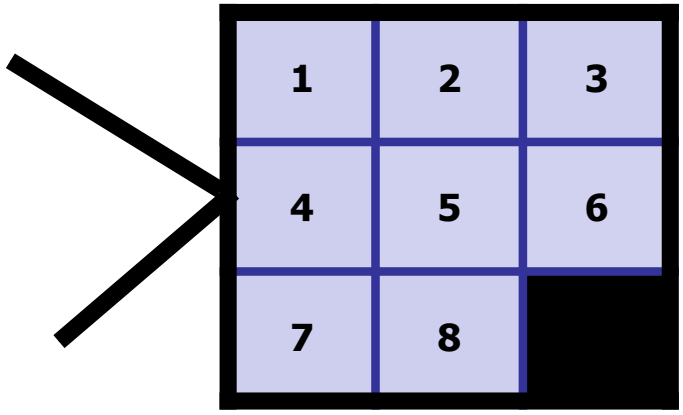
Sliding Puzzle

Number of moves <= Diameter

Initial, scrambled state:



Final, unscrambled state:



2 x 2 x 2 Rubik's Cube

Click me.

2 x 2 x 2 Rubik's Cube

Record solve time: 0.69 seconds

2 x 2 x 2 Rubik's Cube

Configuration Graph

- Vertex for each possible state
- Edge for each basic move
  - 90 degree turn
  - 180 degree turn

Puzzle: given initial state, find a path to the solved state.

2 x 2 x 2 Rubik's Cube

How many vertices?

$$8! \cdot 3^8 = 264{,}539{,}520$$

cubelets

Each cubelet is
in one of 8 positions.

Each of the 8 cubelets
can be in one of three
orientations

2 x 2 x 2 Rubik's Cube

How many vertices?

$$7! \cdot 3^7 = 11{,}022{,}480$$

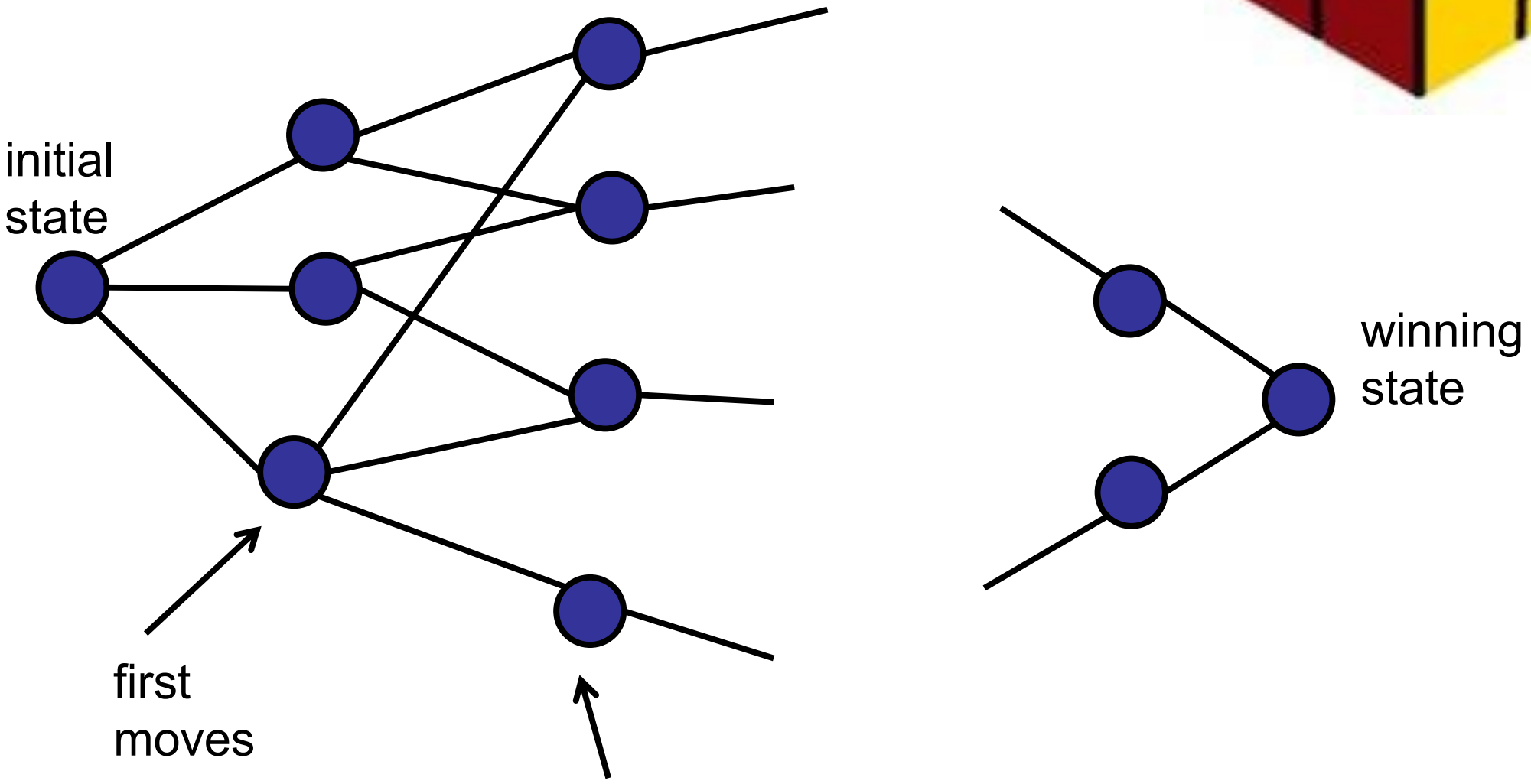
Symmetry:

Fix one cubelet.

Each of the 8 cubelets can be in one of three orientations

2 x 2 x 2 Rubik's Cube

Geography of Rubik's configurations:



Reachable configurations

Distance	90 deg. turns	90/180 deg. turns
0	1	1
1	6	9
2	27	54
3	120	321
4	534	1,847
5	2,256	9,992
6	8,969	50,136
7	33,058	227,536
8	114,149	870,072
9	360,508	1,887,748
0	930,588	623,800
11	1,350,852	2,644
12	782,536	
13	90,280	
14	276	

diameter

Reachable configurations

Distance	90 deg. turns	90/120 deg. turns
0	1	1
1	6	9
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9	360,508	1,887,748
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14	276	

Challenge:
How do you generate this table?

diameter

3 x 3 x 3 Rubik's Cube

Configuration Graph

- 43 quintillion vertices (approximately)
- Diameter: 20
  - 1995: require at least 20 moves.
  - 2008: 20 moves is enough from every position.
  - Using Google server farm.
  - 35 CPU-years of computation.
  - 20 seconds / set of 19.5 billion positions.
  - Lots of mathematical and programming tricks.

3 x 3 x 3 Rubik's Cube

What is the diameter of an (n x n x n) cube?

$\theta(n^2 / \log n)$

Roadmap

Today: Graph Basics

- What is a graph?
- Modeling problems as graphs.
- Graph representations (list vs. matrix)
- Searching graphs (DFS / BFS)