Analysis of Algorithm

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Here are some equalities that are useful in analyzing algorithms:

Arithmetic series

$$\sum_{i=1}^{n} i = 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \qquad \ldots (1)$$

More generally, if $a_n = a_{n-1} + c$, where c is a constant, then

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n = \frac{n(a_n + a_1)}{2} \qquad \ldots (2)$$

Geometric series

$$\sum_{i=0}^{n} 2^i = 1 + 2 + 4 + \cdots + 2^n = 2^{n+1} - 1 \qquad \ldots (3)$$

More generally, if $a_n = ca_{n-1}$, where $c \neq 1$ is a constant, then

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \cdots + a_n = a_1 \frac{c^n - 1}{c - 1} \qquad \ldots (4)$$

If $0 < c < 1$, then the sum of the infinite geometric series is

$$\sum_{i=1}^{\infty} a_i = \frac{a_1}{1 - c} \qquad \ldots (5)$$

Harmonic series

$$\sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \approx \ln(i + 1) \qquad \ldots (6)$$

Sum of squares

$$\sum_{i=1}^{n} i^2 = 1 + 4 + 9 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} \qquad \ldots (7)$$

Logarithms

$$\log_b a = \frac{1}{\log_a b} \quad \dots (8)$$

$$\log_a x = \frac{\log_b x}{\log_b a} \quad \dots (9)$$

$$\log_2(n!) = n \log_2(n) \quad \dots (10)$$