CS2040S: Data Structures and Algorithms

Tutorial Problems for Week 11: Minimum Spanning Tree

For: 25 Oct 2021, Tutorial 9

Problem 1. True or False?

For each of the following, determine if the statement is True or False, justifying your answer with appropriate explanation.

a) The MST is always a connected, undirected graph.

b) The MST will always have $V - 1$ edges.

c) For a graph with unique edge weights, the edge with the largest weight in any cycle of the graph can be included in the MST.

d) For a graph with two disjoint sets of vertices A and B (vertices in A are not in B and vice versa), and another vertex x not inside both the sets, the combined MST of $A \cup x$ and $B \cup x$ is a MST of the original graph.

Problem 2. Lets add some new stuff!

Given a graph G with V vertices and E edges, and the **unique** Minimum Spanning Tree (MST) of G (i.e G has only 1 MST), give an algorithm (including the time complexity) to update the MST if

Problem 2.a. A new edge *(A, B)* is to be inserted into G.

Problem 2.b. A new vertex Y, along with a set of edges connecting Y to the rest of the graph, is inserted to G.

Problem 3. Clearly Has MST Flavor

An ambitious cable company has obtained a contract to wire up the government offices in the city with high speed fiber optics to create a high speed intra-net linking up all the different governmental departments. In the beginning they were confident that the minimum cost of connecting all the offices will be within budget. However, they later found they made a miscalculation, and the minimum cost is in fact too costly. In desperation, they decided to group the government offices into K groups and link up the offices in each group, but not offices between groups to save on the cost. This effectively creates k intra-nets instead of one big intra-net.

Given V government offices, the cost of linking E pairs of government offices, and a budget b, help the company design a program which will tell them what is the smallest value of k (so as to minimize the number of intra-nets). The program also needs to output the government offices in each of the k groups and how they should be linked such that the total cost of all selected links is within budget b.

The program should model the problem as a graph. It should also run in $O(E \log V)$time. Where E and V are the number of vertices (government offices) and edges (possible links between those government offices), respectively.

Problem 4. The MST problem is not obvious

Please download CS2010 Final Exam paper S1, AY2011/2012 from Luminus under "File → Past Year Papers → 2011-12-S1-final.pdf" and solve a problem titled: **Vehicle Monitoring System**.

Problem 5. Is this MST (ICPC Indonesia 2013 question)

Indonesia is the worlds largest archipelago with approximately 17,000 islands scattered for more than 5,000 km from Sabang (west most) in Sumatra island to Merauke (east most) in Papua island. Thus, providing electricity in all cities and towns across all islands is a challenging problem for the government.

Power plants, cities, towns and all other important sites can be represented as a graph where each node represents a site and each edge which connects two different sites represents a cable transferring electricity between the two sites in both directions. You may assume that all sites are connected through some cables and for each pair of sites there is at most one cable connecting them. There is of course a cost to maintain each cable and some of them probably have a very high cost to maintain.

The government has a plan to calculate the minimum total cost to maintain only necessary cables such that all sites are connected to at least one power plant, except for the power plants (which are already connected to themselves). A site is considered connected to a power plant if and only if there is a path which consists of only maintained cables from the site to a power plant.

Consider the following example. There are 9 sites and 3 of them are power plants (A, H and I). The connectivity and the cost of each cable are shown in Figure 1. The minimum total cost to maintain the cables in this example is 22 as shown in Figure 2.

Given an undirected graph (which you may assume is stored in an adjacency list) which represents the connectivity between all sites, determine the minimum total cost needed to maintain cables such that all sites except power plants are connected to at least one power plant.

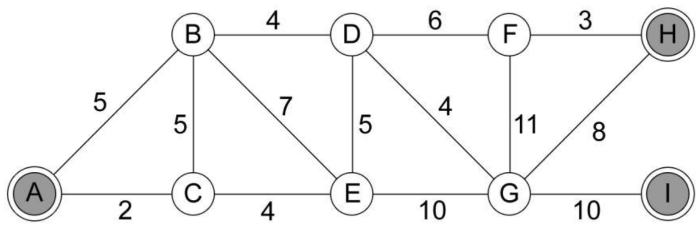

Figure 1: An example of power plants, sites and cables

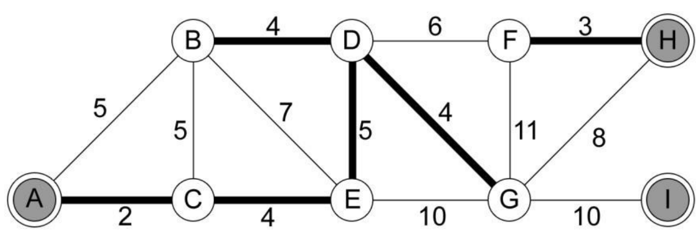

Figure 2: An example of the edges chosen to minimise total cost of cables to build