# CS2040S – Data Structures and Algorithms

## Lecture 14 – *Trie

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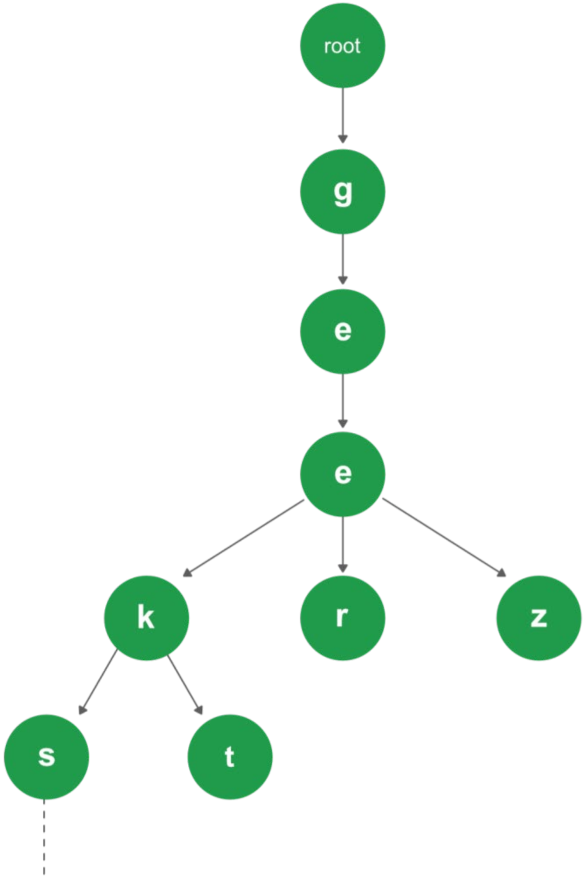
# Trie – A special ordered map for strings



# Motivation for using Trie

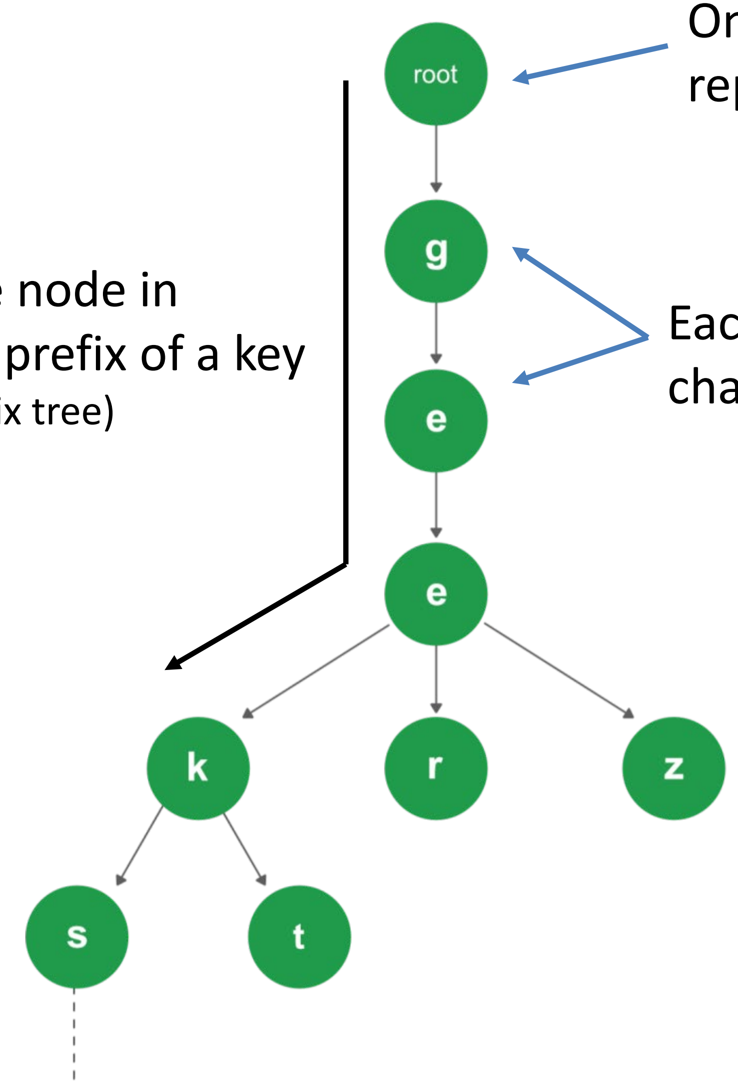
| Cons of using hash table for Strings | Cons of using AVL for Strings |
|---|---|
| High overhead:<br>Requires additional O(K) time to convert string of length K into integer value before hash function can be applied | High overhead:<br>Even though keys are ordered lexicographically, time complexity of insertion/deletion/search operations take O(KlgN) time where K is longest key length. Up to O(lgN) nodes must be compared and each comparision takes O(K) time |
| Does not maintain lexicographical ordering of the String keys | |

- A Trie addresses the weaknesses of both hash table and AVL for storing String keys

# Anatomy of a Trie (1)

Only root does not represent a character

Path from root to some node in the Trie represents the prefix of a key (thus Trie also known as prefix tree)



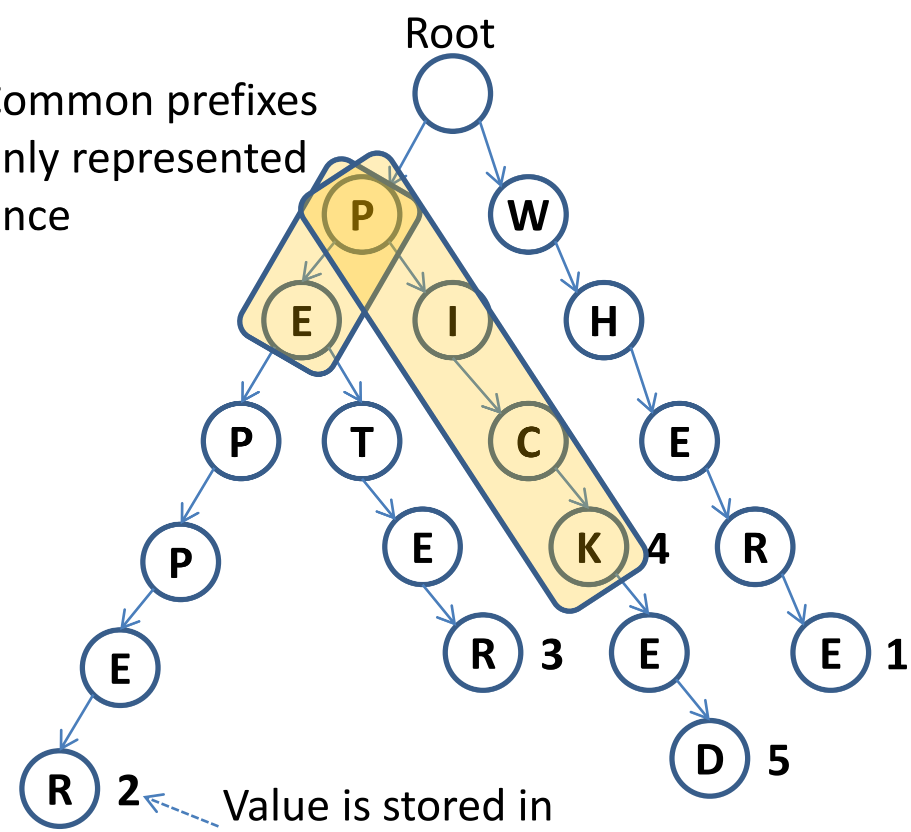

Each node represents a character of a String key

# Anatomy of a Trie (2)

**Key/Value Pairs**

WHERE 1
PEPPER 2
PETER 3
PICK 4
PICKED 5

Common prefixes only represented once




Value is stored in node representing last character of key

# Operations on Trie

- 3 basic operations
  - Retrieval
  - Insertion
  - Removal

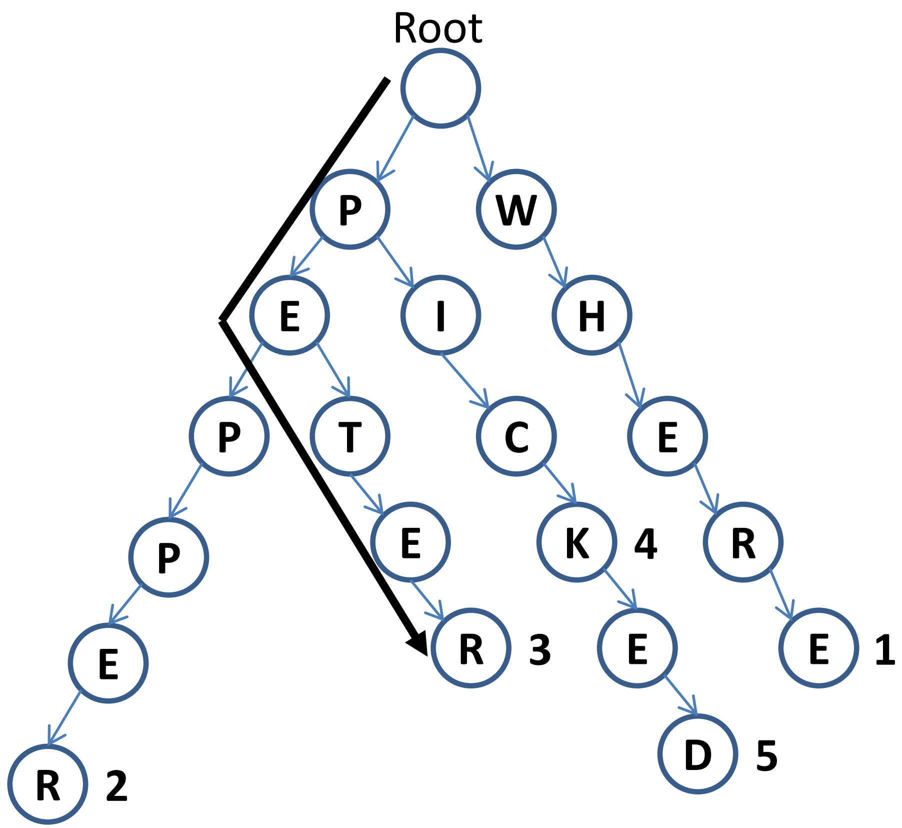
# Retrieval Algorithm

Given a String key S to retrieve

1. Matching process
   i. Start from the root (level 0), check if it has a child node with character matching S[0], if it has move to child node and repeat the matching for S[1] and so on.
   ii. In general for a node at level M, the matching process check if the node has a child matching the character at index M of S
2. Terminating condition
   i. hit a node with no children node matching current character of the key ← return a miss
   ii. hit a node matching last character of key and there is a valid value for that node ← return the value (a hit)
   iii. hit a node matching last character of key and there is no valid value for that node ← return a miss

# Retrieval Examples

- Search for
  - PETER
  - PICK
  - PEP
  - THE



# Insertion Algorithm

- Perform a search for the key similar to retrieval operation until

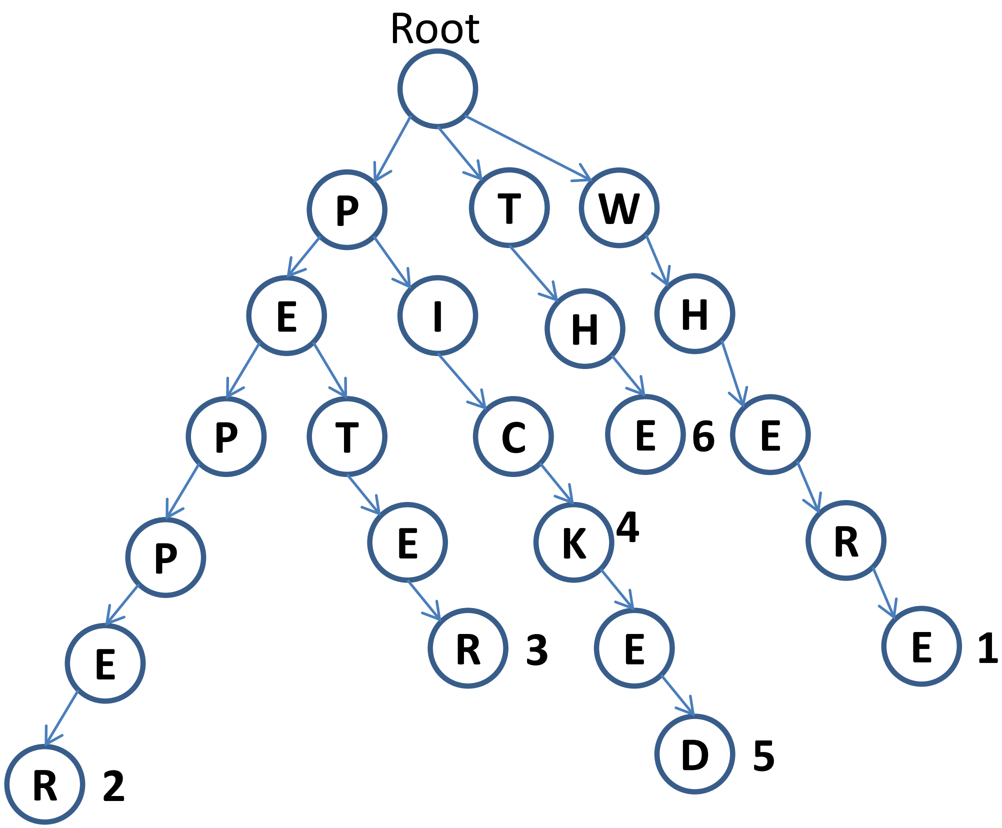
1. Hit a node matching the last character of the key and there is a valid value for the node (existing key/value pair)
   ← update the value with the new value.

2. Hit a node matching the last character of the key and there is no valid value for the node
   ← store the value in this node.

3. We hit a node with no children matching the current character of the string
   ←start inserting the remaining characters as descendent nodes in a linked list like fashion and put the value in the last node.

# Insertion Examples

- Inserting
  - THE 10
  - PICKLED 7
  - PEPP 8



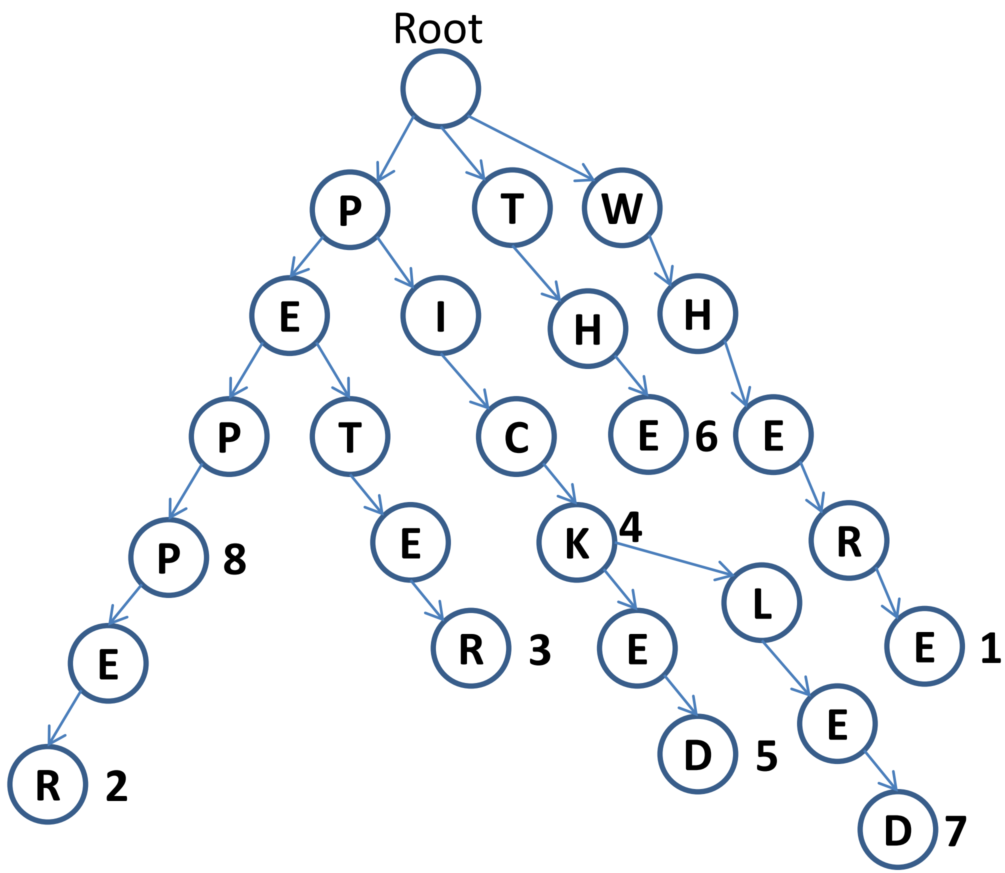
# Deletion Algorithm

- Again perform a search for the key

1. Hit a node with no children matching current character of key ← do nothing
2. Hit a node matching the last character of the key and there is no valid value for the node ← do nothing
3. Hit a node matching the last character of the key and there is a valid value for the node ← remove the value
    i. If it has children nodes do nothing else (it is both a key and also the prefix for other existing keys)
    ii. If it has no children nodes then move back towards the root and start removing nodes. Stop when we hit a node that has at least 1 child (a prefix of some key(s))

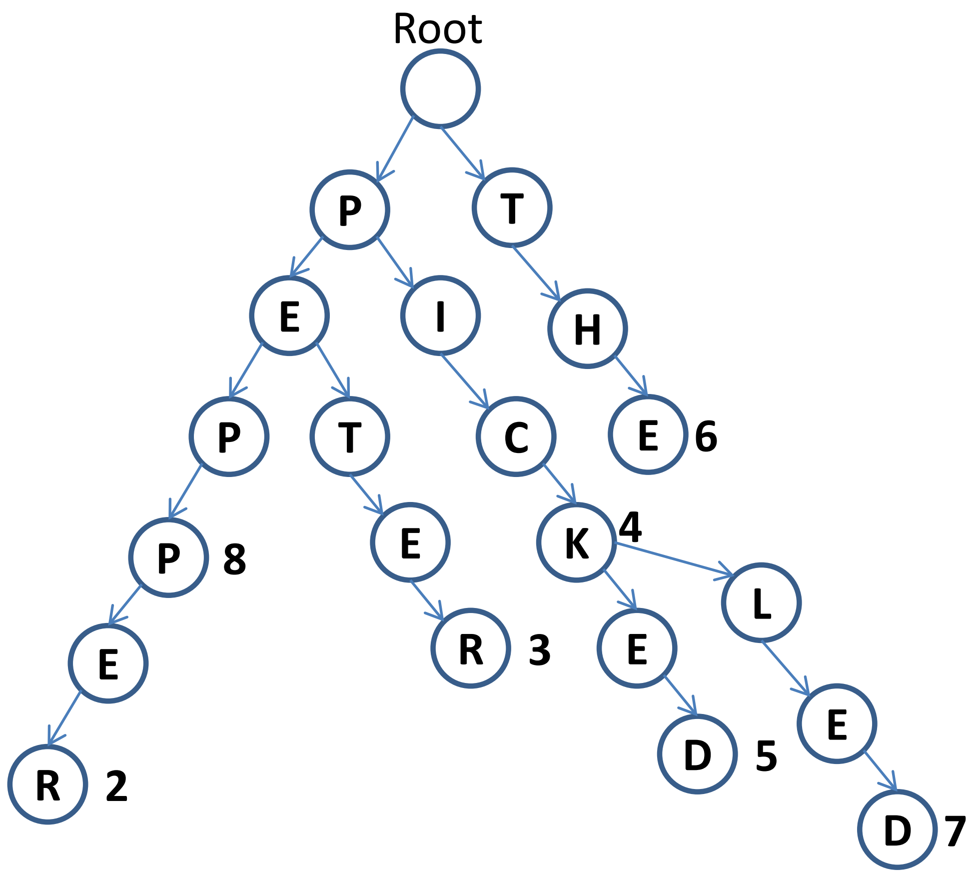
# Deletion Examples (1)

- Deleting
  - WHERE
  - PICKLED
  - TO
  - PEP



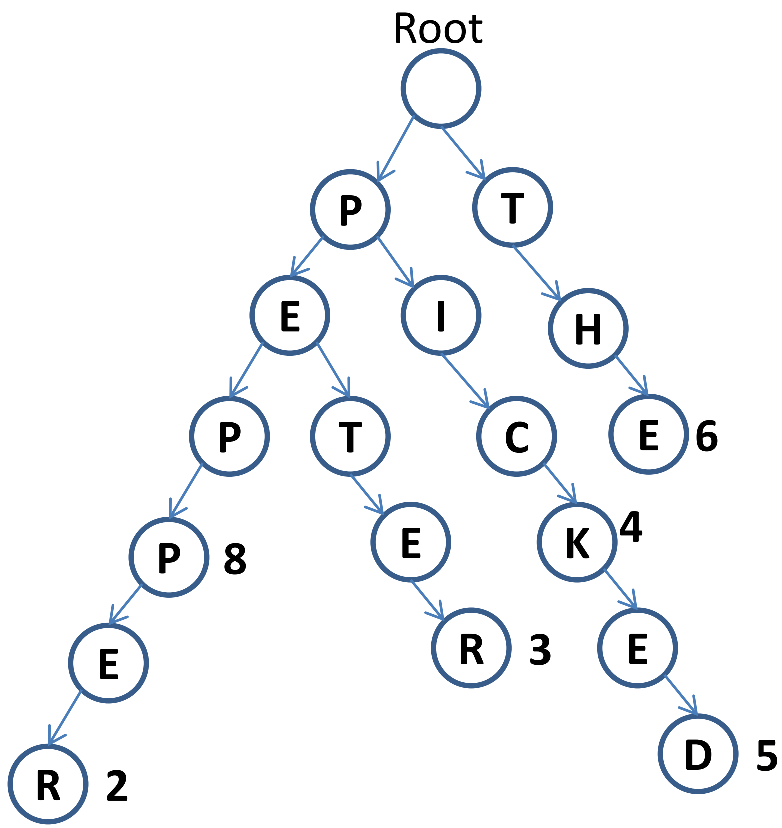
# Deletion Examples (2)

- Deleting
  - WHERE
  - PICKLED
  - TO
  - PEPP



# Deletion Examples (3)

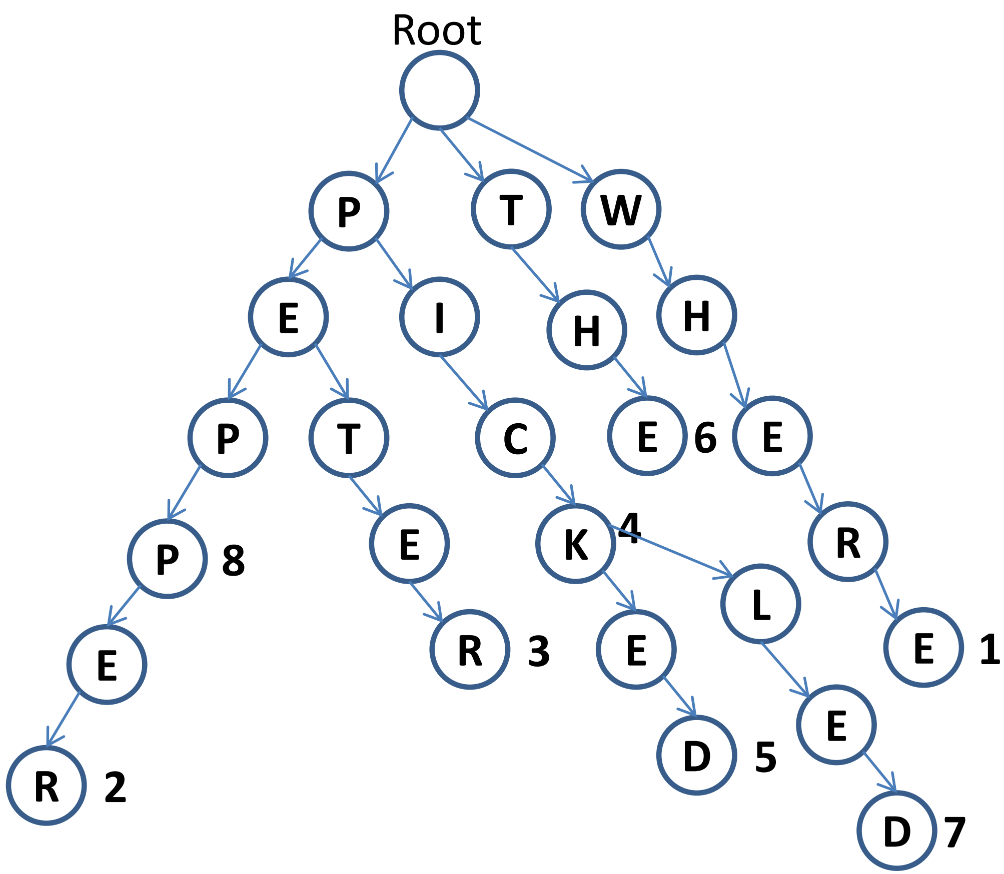
- Deleting
  - WHERE
  - PICKLED
  - TO
  - PEPP



# Sorting Keys in Trie

- Simply perform an pre-order traversal of the Trie (visit children in lexicographical ordering of their characters)
- Keep track of the characters along the path from root to the current node
- If current node has a valid value then output the characters as a string key

# Sorting Example

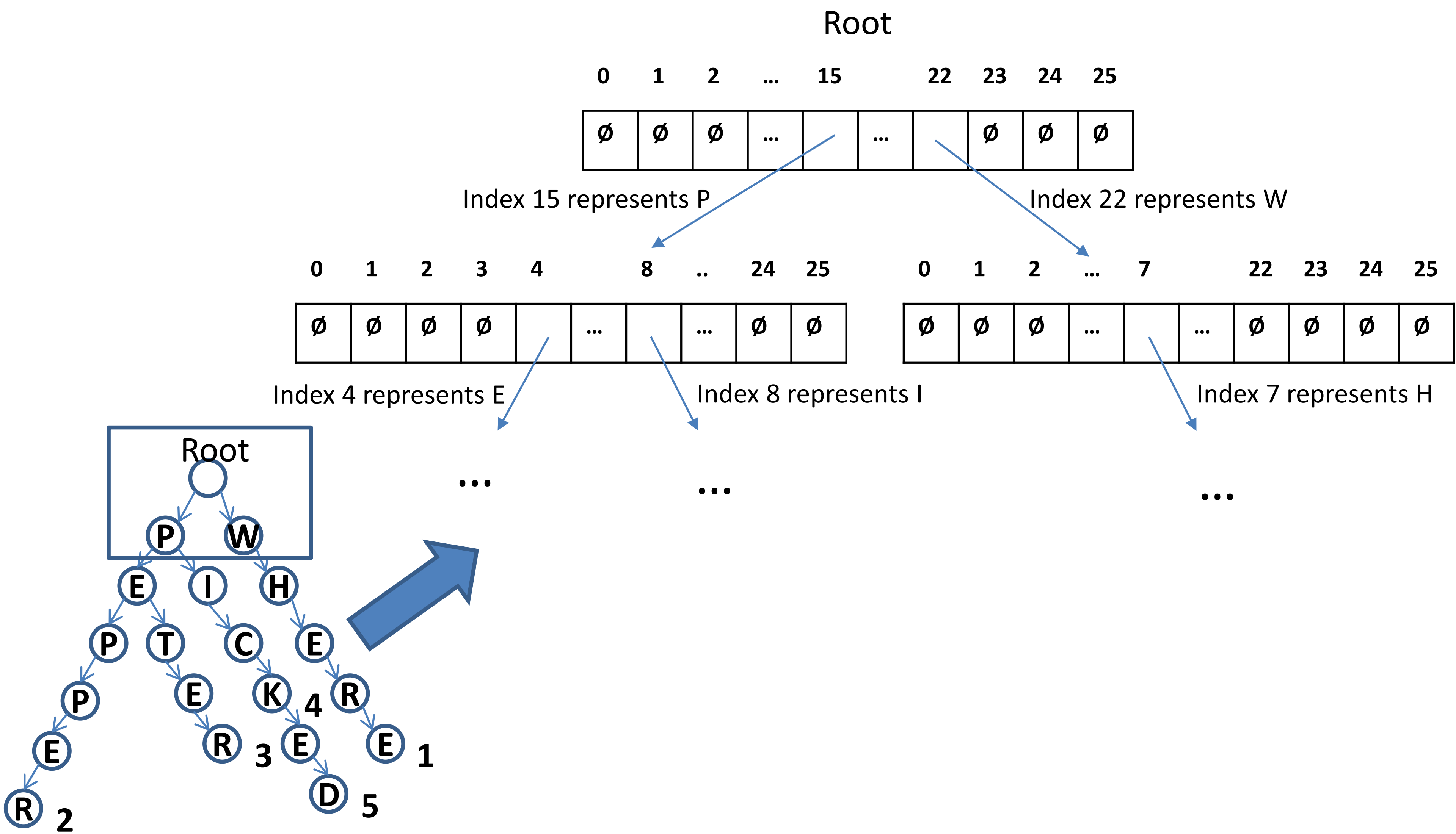



PEPP, PEPPER, PETER, PICK, PICKED, PICKLED, THE, WHERE

# Representing Trie in code (1)

- Use nodes and references like BST/AVL
  - Unlike BST/AVL with at most 2 children per node, there is up to K children per node for a Trie for a language with K alphabets (e.g 26 for English)
  - Instead of using a variable sized array of references for the children, simply use an array of K references where a child node of a particular character is directly mapped to the array index (A == index 0, B == index 1 …)

# Representing Trie in code (2)



# Representing Trie in code (3)

- Additional attributes required in a node
  - **count** ← integer value to keep track of number of children in a node
  - **value** ← attribute to store the value of the key/value pair. Can use a reference type so if it is null means the string represented by the path from the root to this node is not a valid key. If not null means this is a valid key.

# Time complexity analysis of Trie operations

- Worst case for Insertion, Retrieval (successful) and Deletion
  - $O(L)$ where $L$ is the length of the key.

- Average time unsuccessful Retrieval (missing key) in a Trie of $N$ keys
  - Assuming keys are randomly distributed over an alphabet of size $R$. Average time taken if key is not found is $O(log_R N)$ = $O(logN)$ ← independent of key length!

- Worst case for Sorting Keys
  - $O(L' * N)$ where $L'$ is the length of the longest key and $N$ is the number of keys in the Trie

# Space complexity of Trie

- The space required for a Trie containing $N$ keys is at least $O(NR)$ since each node in the Trie has an array of references of size $R$ (alphabet size) and each key is at least length 1

- On average the number of nodes required is then $O(wNR)$ where $w$ is the average key length

- This is more then the space required for hash tableswhich is average $O(wN)$ ← total length of keys

- This is also more then the space required for AVL which is only $O(N)$+$O(wN)$ = $O(wN)$

N nodes for the N keys     Sum of total space required to store the N keys