NATIONAL UNIVERSITY OF SINGAPORE

SCHOOL OF COMPUTING
WRITTEN QUIZ 2 (6%)
AY2017/18 Semester 1

CS2010 – Data Structures and Algorithms II

8 November 2017 Time allowed: 45 minutes

INSTRUCTIONS TO CANDIDATES

1. Do **NOT** open the question paper until you are told to do so.
2. This question paper contains 5 questions. It comprises EIGHT (8) printed pages, including this page.
3. Answer all questions only on this **Question Paper**. You can use either pen or pencil. Just make sure that you write **legibly**!
4. This is an **Open Book Quiz**. You can check the lecture notes, tutorial files, problem set files, CP3 book, or any other books that you think will be useful.
5. **Pace yourself!** Do not spend too much time on one question.

TUTORIAL GROUP

STUDENT NUMBER: A | | | | | | | | |

For examiners' use only		
Question	***Max***	***Marks***
Q1-3	9	
Q4	16	
Q5	15	
Total	**40**	

Section A – Analysis (9 Marks)

Prove (the statement is correct) or disprove (the statement is wrong) the following statements below. If you want to prove it, provide the proof or at least a convincing argument. If you want to disprove it, provide at least one counter example. 3 marks per each statement below (1 mark for saying correct/wrong, 2 marks for explanation):

1. We can build a connected, directed and positively weighted graph of 4 vertices and 6 edges such that performing bellman ford will result in >= 19 relaxations.

2. Given an undirected and connected graph, if the weight of the shortest path from vertex s to vertex x, i.e $s\sim > x$, is smaller than the weight of the shortest path from s to every other vertex, and the weight of the shortest path from vertex t to vertex x, i.e $t \sim > x$ is smaller than the weight of the shortest path from t to every other vertex, then the shortest path from s to t must be $s \sim > x \sim > t$ in **all cases**.

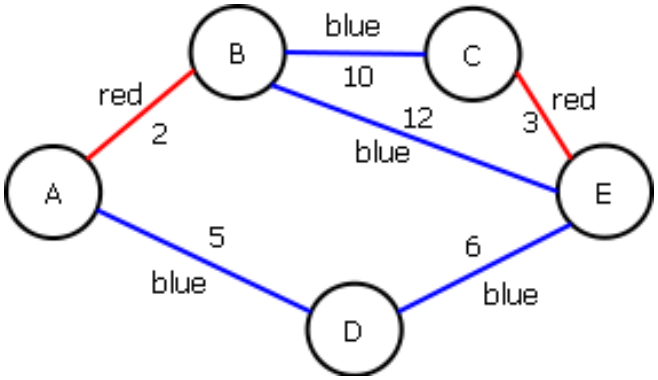
3. There are no graphs where the SSSP spanning tree is also the same as the MST (in terms of the edges used).

For Section B, Partial marks will be awarded for correct answers which do not meet the time complexity required (or if the required time complexity of not given, then any correct answer that is less efficient than the lecturer's answer) . You can write in pseudo-code. Provide enough details to all user defined DSes/algorithms/modifications to taught DSes and algorithms so as to show understanding of the solution, but also ***try be as concise as possible, writing more will take more time and possibly make more mistakes!***

Section B – Application (31 Marks)

4. **Colored paths [16 marks]**

City Z is a very unique city. Its roads are painted either red or blue. Junctions and roads of Z can be represented as an undirected weighted graph, where in addition to the weight (positive value), an edge which represents a road also has an extra parameter color which indicates what color the road is. An example is shown below



a) If a person wants to get from some source vertex X to some destination vertex Y, and only wants to take a path where the roads are only of a certain color K, give the best algorithm you can think of to find the shortest path from X to Y given the above restriction. If there is no such shortest path return infinity. Analyze the time complexity of your algorithm in terms of V and E the number of vertices and edges in the graph respectively. **[8 marks]**

b) Now another person wants to get from some source vertex X to some destination vertex Y, but she only wants to travel a path where the edges **alternate** in color starting from a red edge (i.e red,blue,red,blue…). Now give the best algorithm you can think of to find the shortest path from X to Y given the above restriction. If there is no such shortest path return infinity. Analyze the time complexity of your algorithm in terms of V and E the number of vertices and edges in the graph respectively. **[8 marks]**

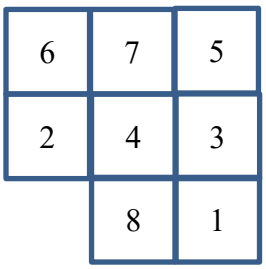
5. **Portal [15 marks]**

Dhell wakes up to find himself trapped in a strange building which consists of only 1 floor. He suddenly realizes that he has knowledge of the layout of the building. The building consists of M rooms where $3 \leq M \leq 100{,}000$. Adjacent rooms are connected to each other by a door, through which Dhell can move between the two room. The time to move from a room r to an adjacent room r' and vice versa is the time for the automatic door connecting r and r' to open (the door will immediately close once Dhell has move through it). There are N doors in all, where $M - 1 \leq N \leq 1{,}000{,}000$. There is also always some way to get from one room to another.

Dhell is stranded in room A and he needs to get to the lobby B (also a room) which contains a portal to exit the building. However with his mysterious knowledge of the layout of the building, he knows there is a sequence of rooms $R = \{A, r_1, \ldots r_k, B\}$, where $1 \leq k \leq M - 2$, which he needs to visit before the portal will open in lobby B, and he can escape.

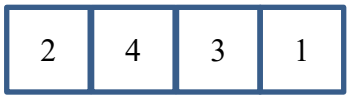
Dhell can go through other rooms as many times as he wants as long as the rooms in R are visted **exactly once** and **in the order they are given**.

In example 1 below, if $R = \{2,3,4,1\}$ Dhell (who will start at 2) can escape by going through the rooms **2**,6,7,5,**3**,**4**,8,**1** in order to escape. However, **2**,**4**,**3**,**4**,8,**1** is not a valid escape path since 4 is first visited before 3 and then it is visited again after 3. Note this is just an example, the rooms might not be square!



Example 1

In example 2 below, if $R = \{2,3,4,1\}$ again, there is no way for Dhell to escape since he has to reach room 4 before room 3.



Example 2

Assume Dhell has knowledge of all the rooms in the building, which rooms are adjacent to each other and the time taken for each of the doors to open. Assume he also has knowledge of the sequence of designated rooms R that must be visited before he can escape.

a) Now model this as a graph problem and give the best algorithm you can think of to compute the **shortest time** for Dhell to escape the building. If there is no way to escape the building, return -1. **[8 marks]**

After escaping the building (so there is way to escape ...), Dhell has put this strange experience behind him. However, one day he suddenly woke up to find himself back in a room of the building! He once again mysteriously has knowledge of the layout of the building, and he realizes the rules to escape has changed.

The layout is still the same as before, but this time, in order to get from his starting room A to the lobby B, he only has to go through one particular room C. Dhell can move through other rooms as many times as he wants but can only move through rooms A,C,B once and in that order. In addition, the longest time taken by any other door he opens while trying to move from A to B must be minimized.

b) Using the same graph modeling as for a) give the best algorithm you can think of to compute a valid path he needs to take to get from A to B such that the portal will open. You may assume there is at least one such path. **[7 marks]**

```
visited[] <- boolean[3];
visited[0] = true;

q<Grid> <- init empty queue
q.offer(add(<A_row, A_col>);

perform BFS on G:
   currV <- q.poll()
   for all outgoing edges <currV, u> from currV:
      if (u !== A
```

== END OF PAPER ==