

CS2040S

Data Structures and Algorithms

(e-learning edition)

Graphs!

(Part 1)

Roadmap

Today: Graph Basics

- What is a graph?
- Modeling problems as graphs.
- Graph representations (list vs. matrix)
- Searching graphs (DFS / BFS)

Roadmap

Next: Searching Graphs

- Searching graphs
- Shortest path problem
- Bellman-Ford Algorithm
- Dijkstra's Algorithm

Roadmap

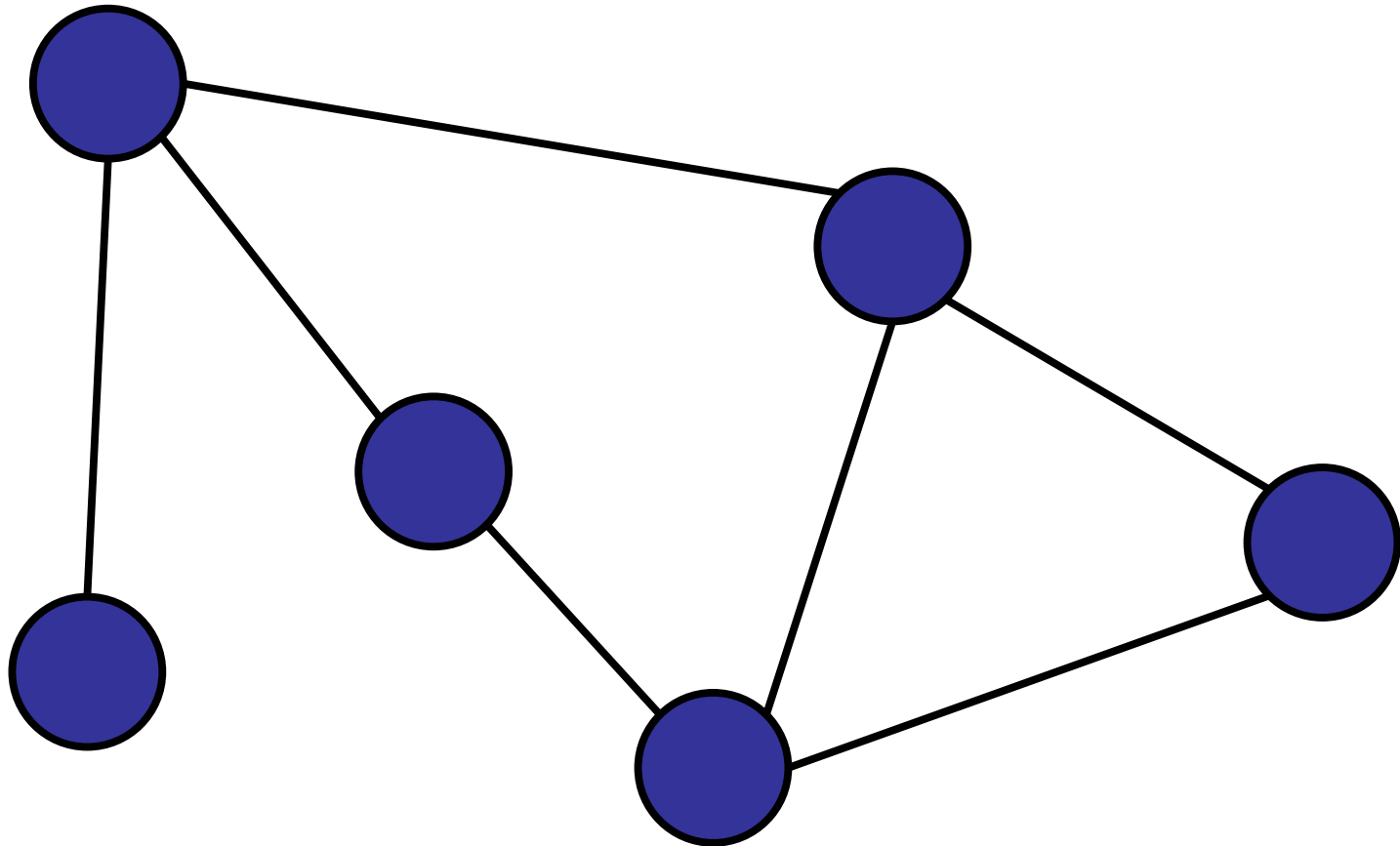
Next next:

- Connected component problem
 - Union-Find data structure
- The Minimum Spanning Tree Problem
 - Kruskal's Algorithm
 - Prim's Algorithm

What is a graph?

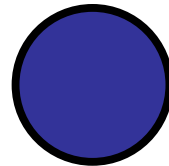
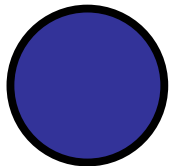
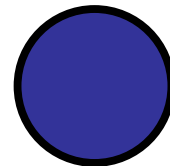
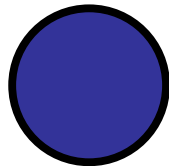
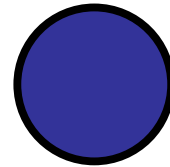
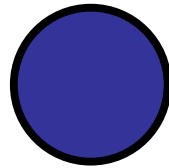
Is it a graph?

- ✓ 1. Yes
- 2. No.



Is it a graph?

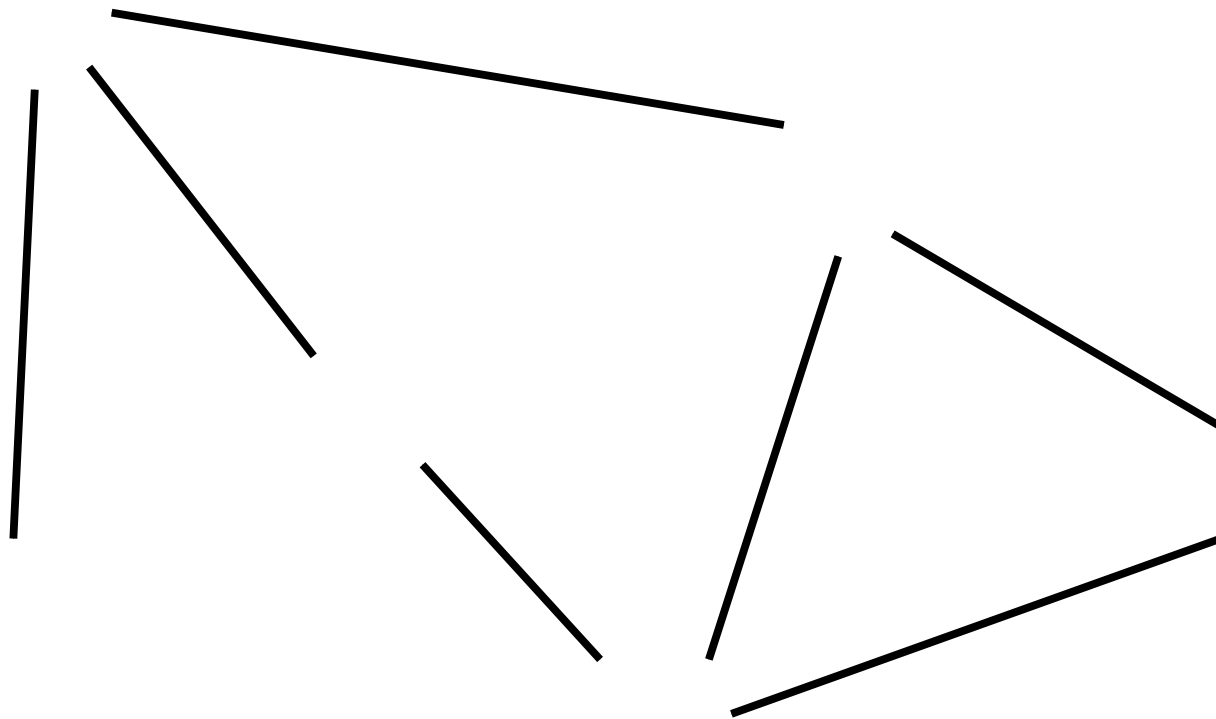
- ✓ 1. Yes
- 2. No.



Is it a graph?

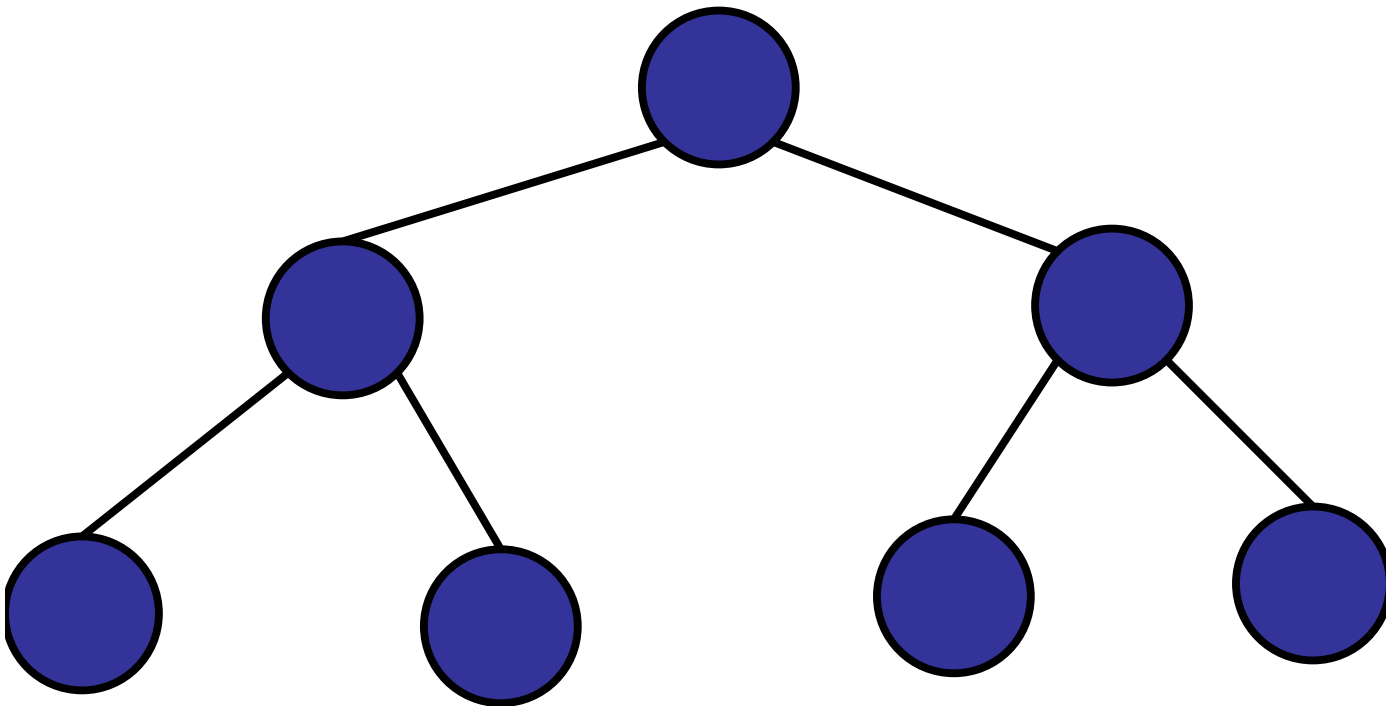
1. Yes

✓ 2. No.



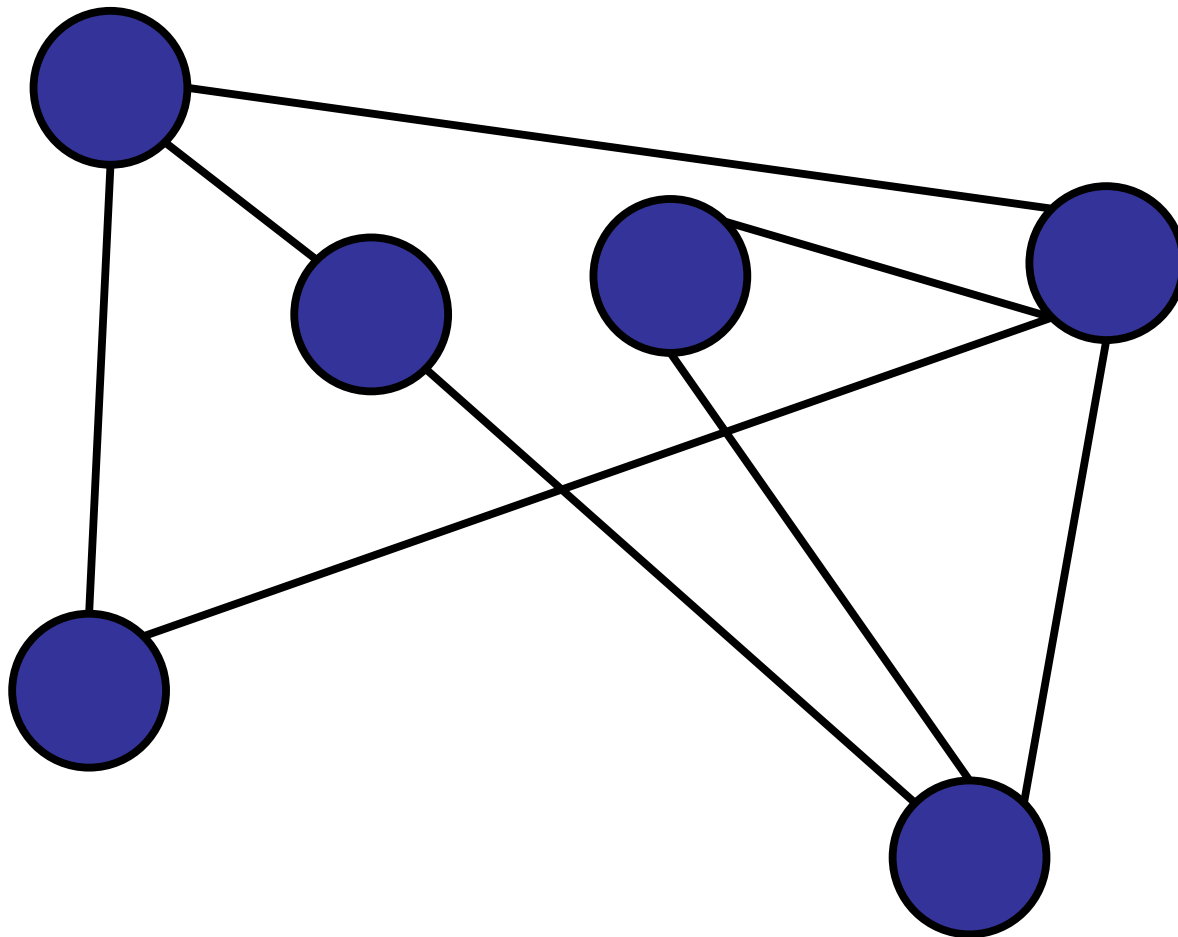
Is it a graph?

- ✓ 1. Yes
- 2. No.



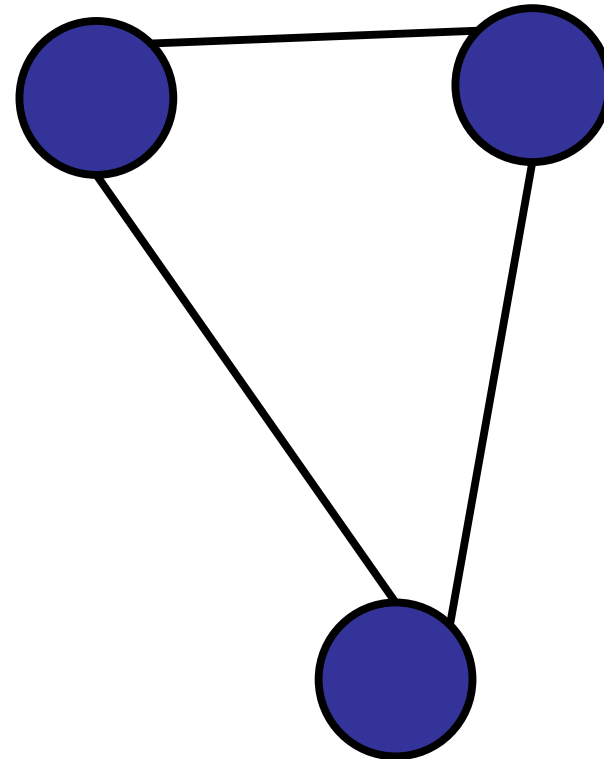
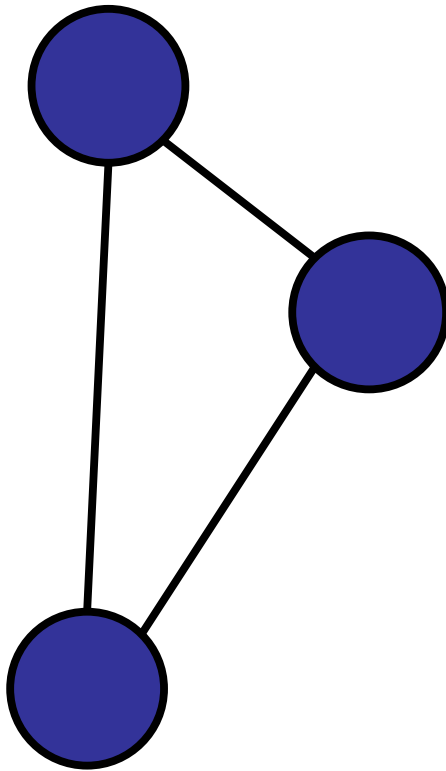
Is it a graph?

- ✓ 1. Yes
- 2. No.



Is it a graph?

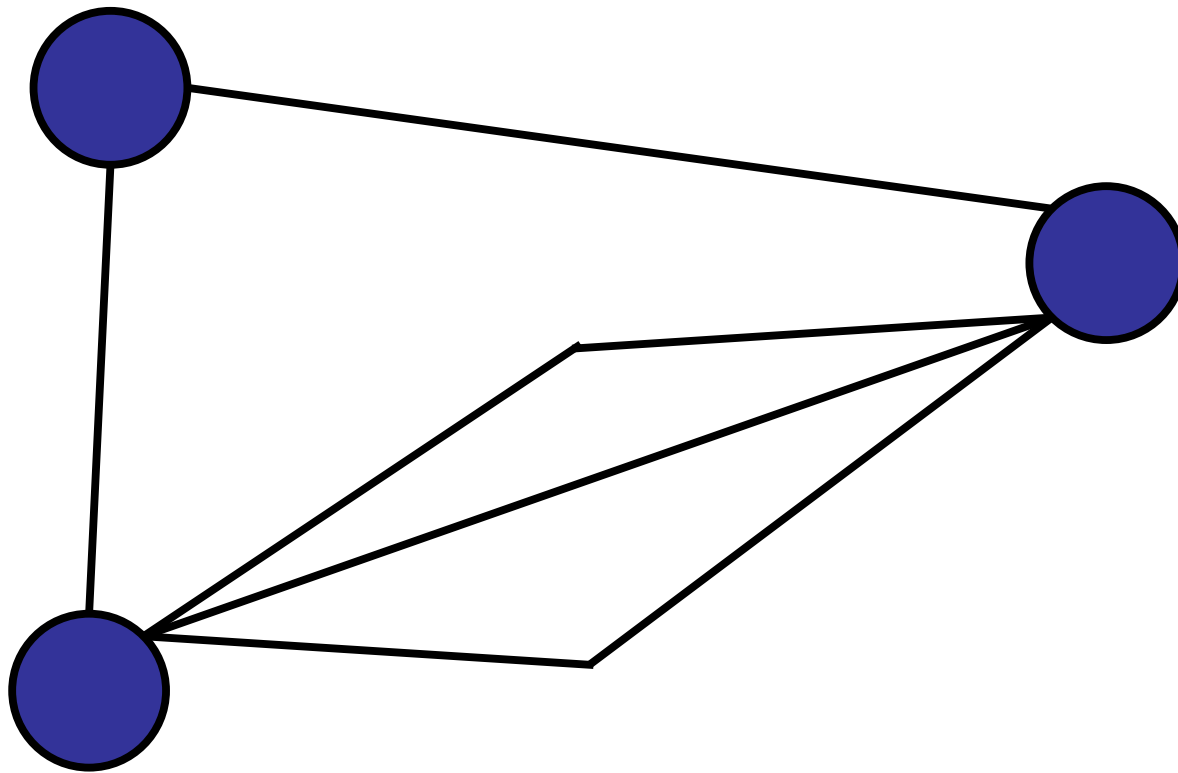
- ✓ 1. Yes
- 2. No.



Is it a graph?

1. Yes

✓ 2. No.



Is it a graph?

- ✓ 1. Yes
- 2. No.

What is a graph?

Graph consists of two types of elements:

- Nodes (or vertices)
 - At least one.
- Edges (or arcs)
 - Each edge connects two nodes in the graph
 - Each edge is unique.

What is a **hypergraph**?

Graph consists of two types of elements:

- Nodes (or vertices)
 - At least one.
- Edges (or arcs)
 - Each edge connects ≥ 2 nodes in the graph
 - Each edge is unique.

(Not common in CS2040S)

What is a multigraph?

Graph consists of two types of elements:

- Nodes (or vertices)
 - At least one.
- Edges (or arcs)
 - Each edge connects two nodes in the graph
 - Two nodes may be connected by more than one edge.

(Rare in CS2040S.)

What is a graph?

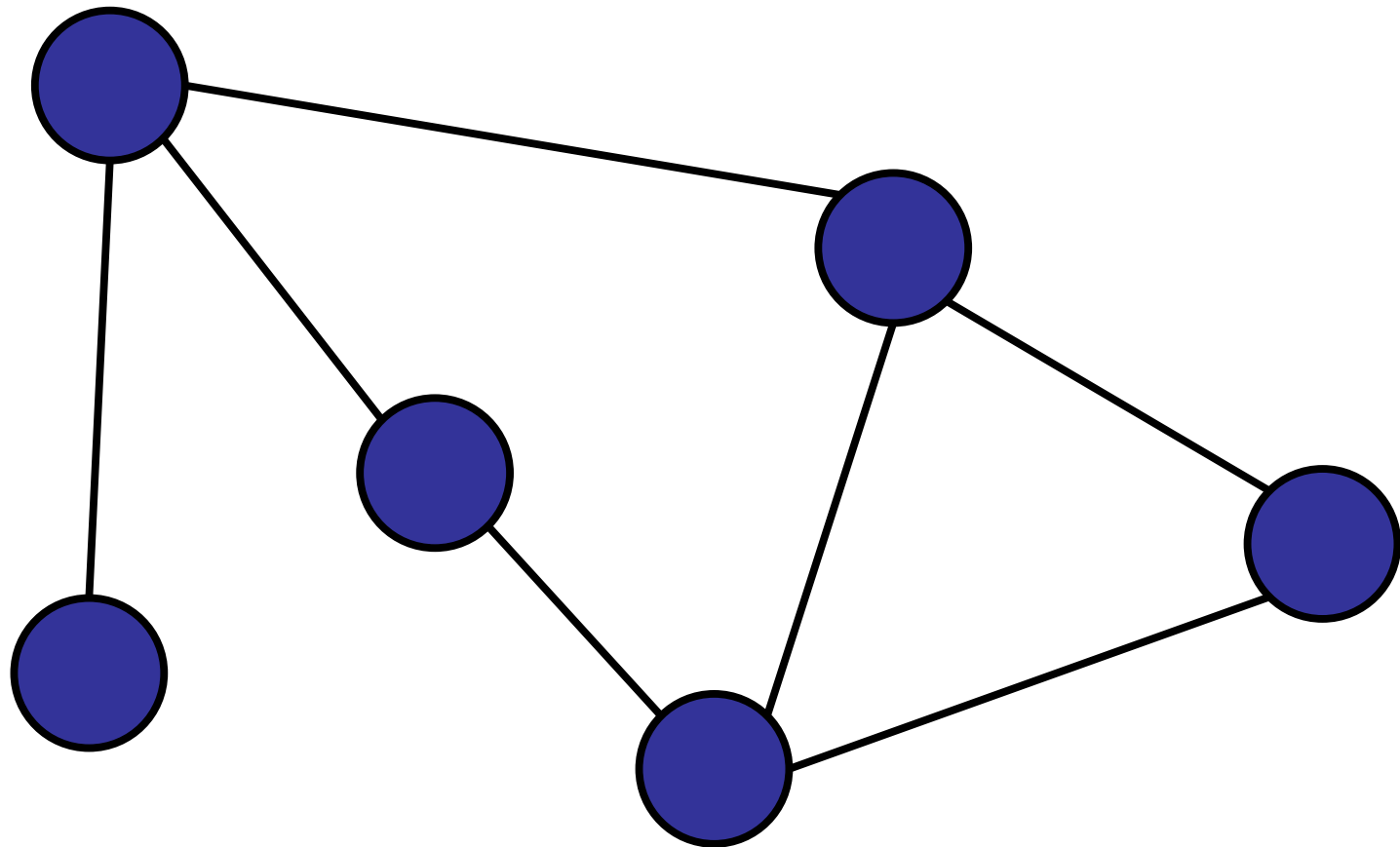
Graph $G = \langle V, E \rangle$

- V is a set of nodes
 - At least one: $|V| > 0$.
- E is a set of edges:
 - $E \subseteq \{ (v,w) : (v \in V), (w \in V) \}$
 - $e = (v,w)$
 - For all $e_1, e_2 \in E : e_1 \neq e_2$

Graph Terminology

Connected:

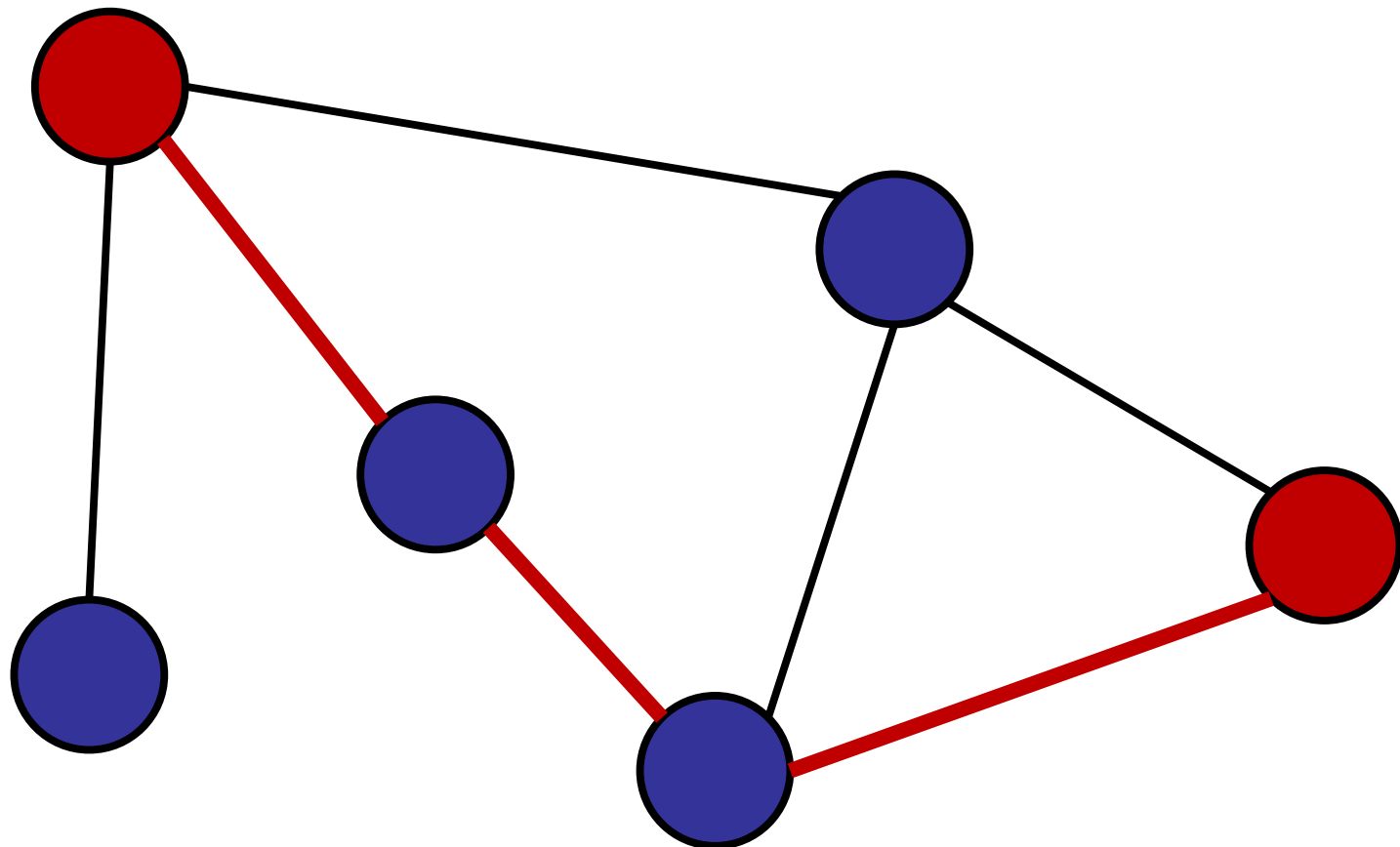
- Every pair of nodes is connected by a path.



Graph Terminology

Connected:

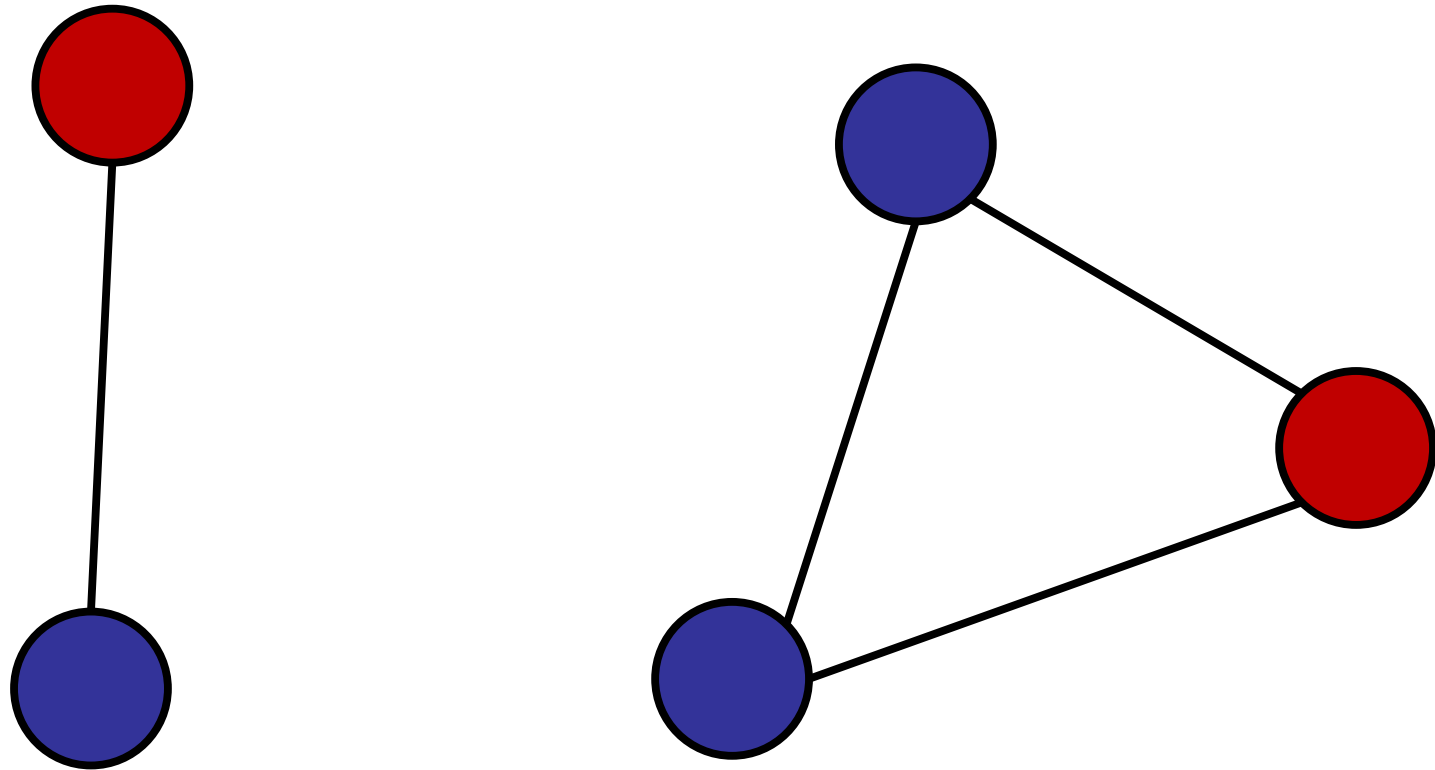
- Every pair of nodes is connected by a path.



Graph Terminology

Disconnected:

- Some pair of nodes is not connected by a path.

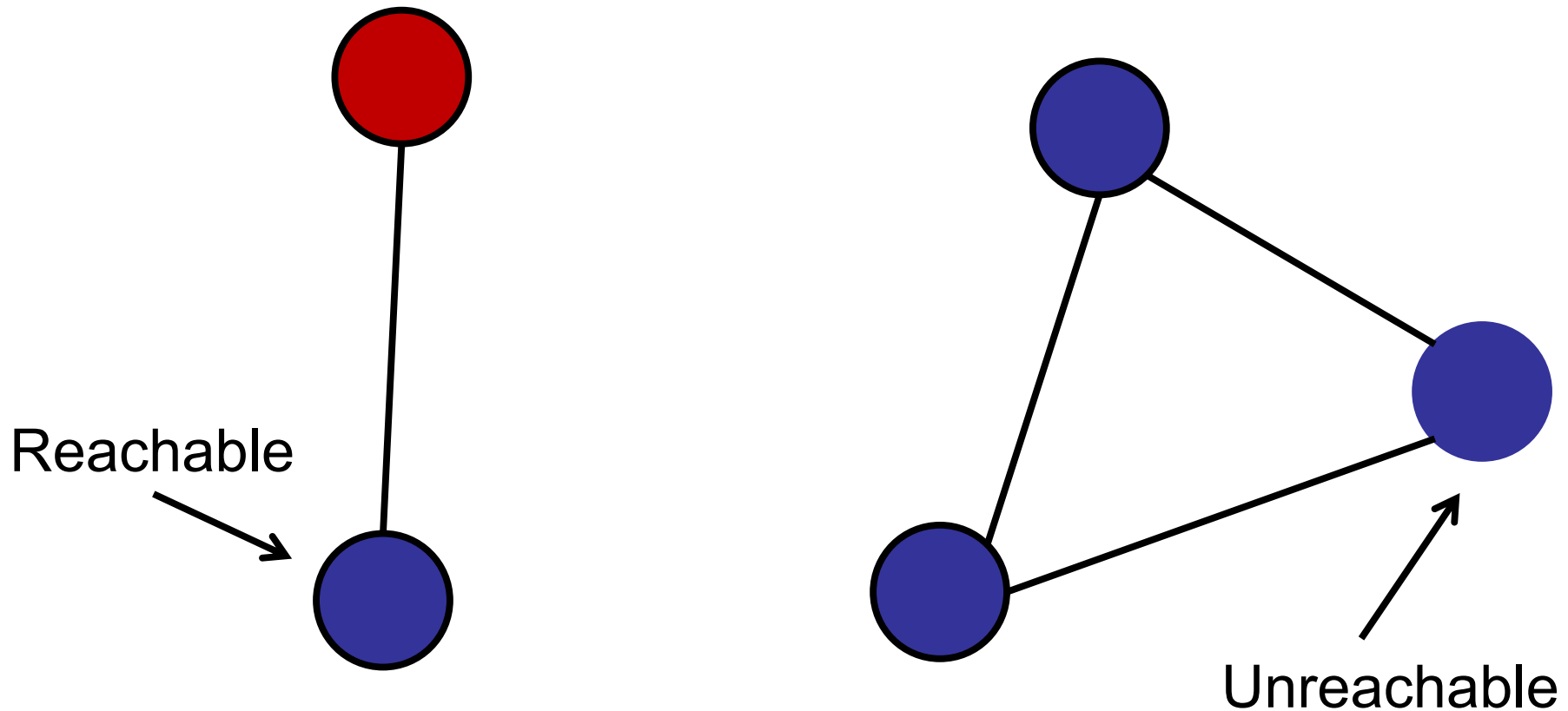


- Two **connected components**.

Graph Terminology

Disconnected:

- Some pair of nodes is not connected by a path.

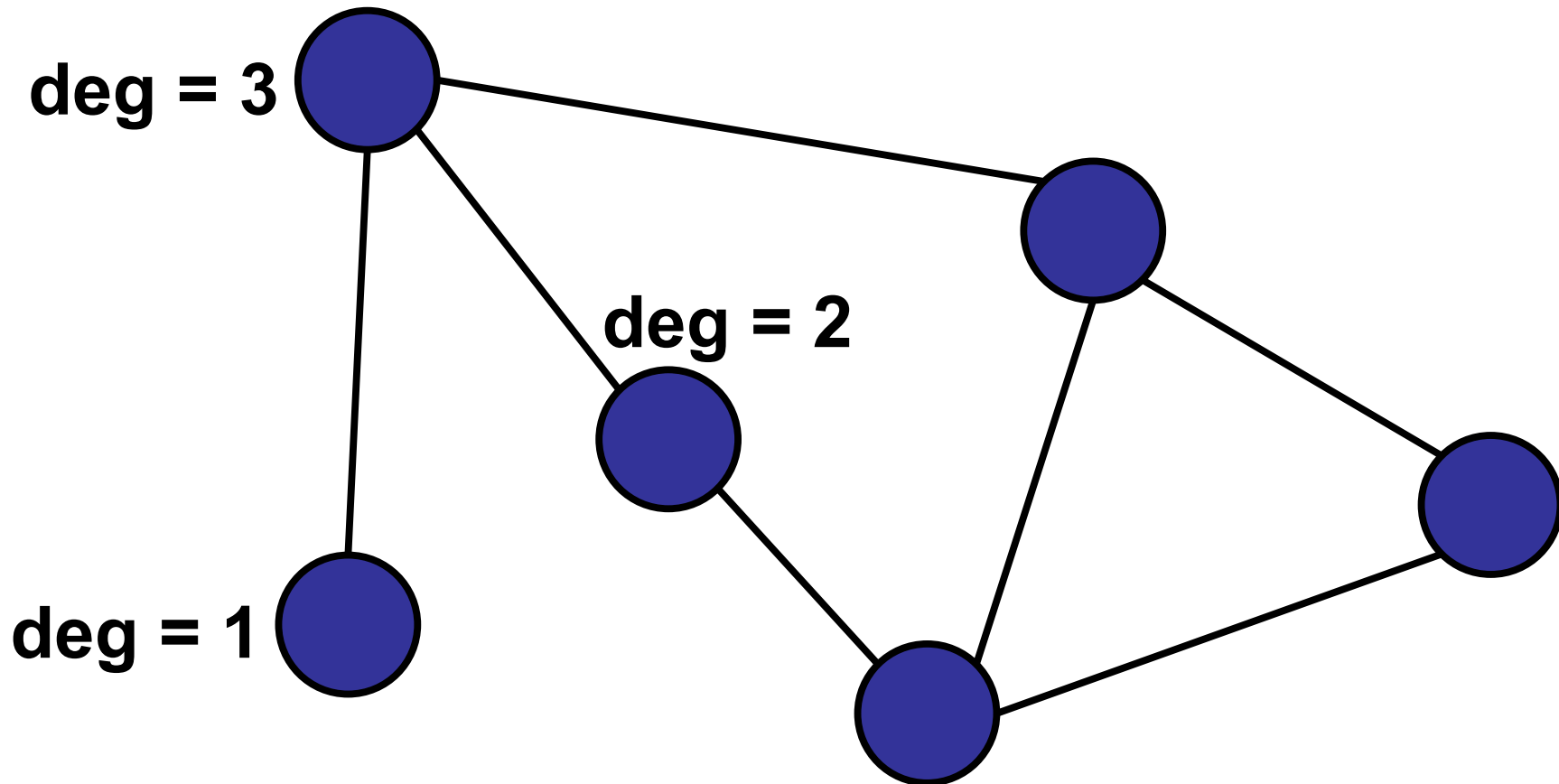


- Two **connected components**.

Graph Terminology

Degree of a node:

- Number of **adjacent** edges.

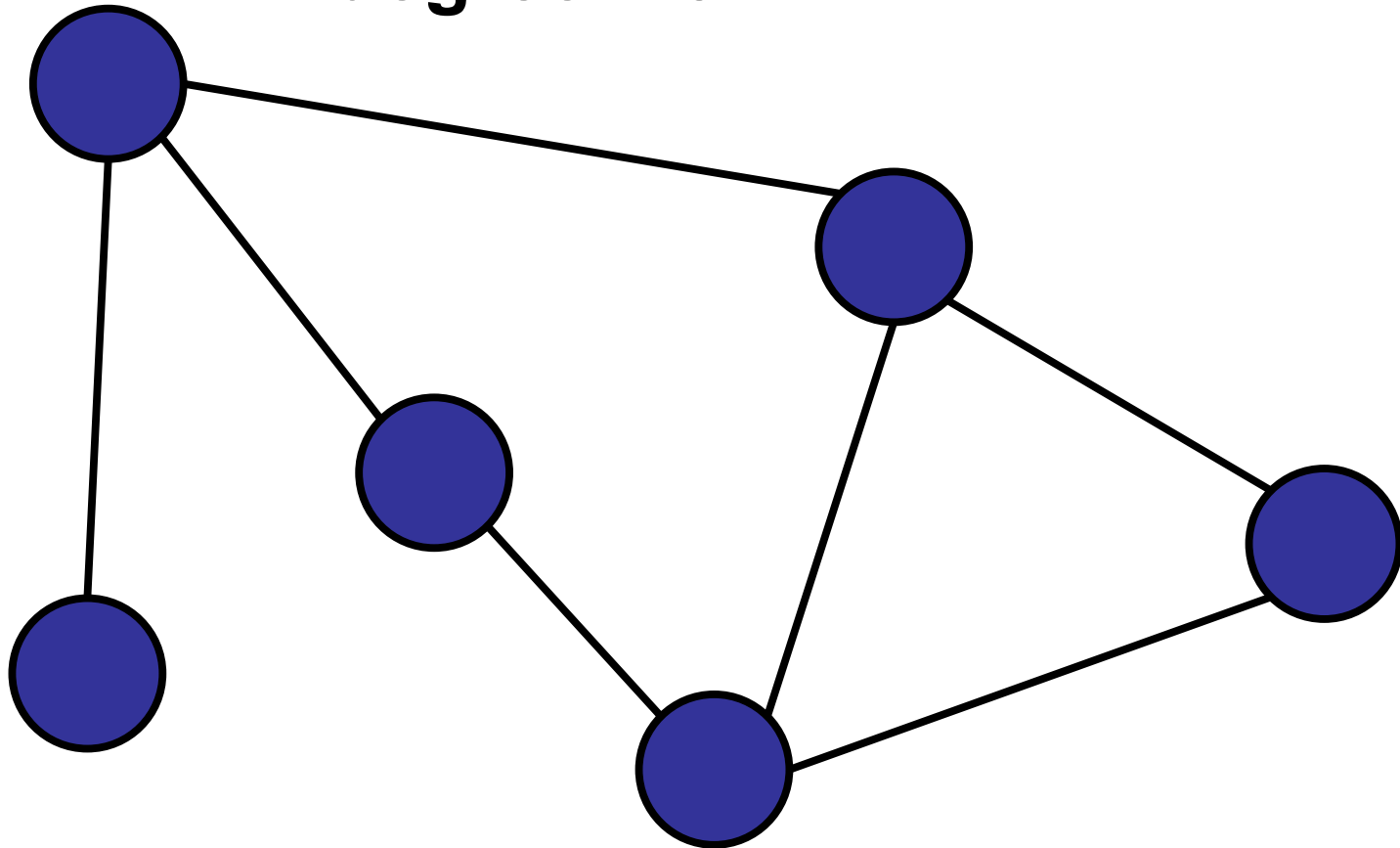


Graph Terminology

Degree of a graph:

- Maximum number of **adjacent** edges.

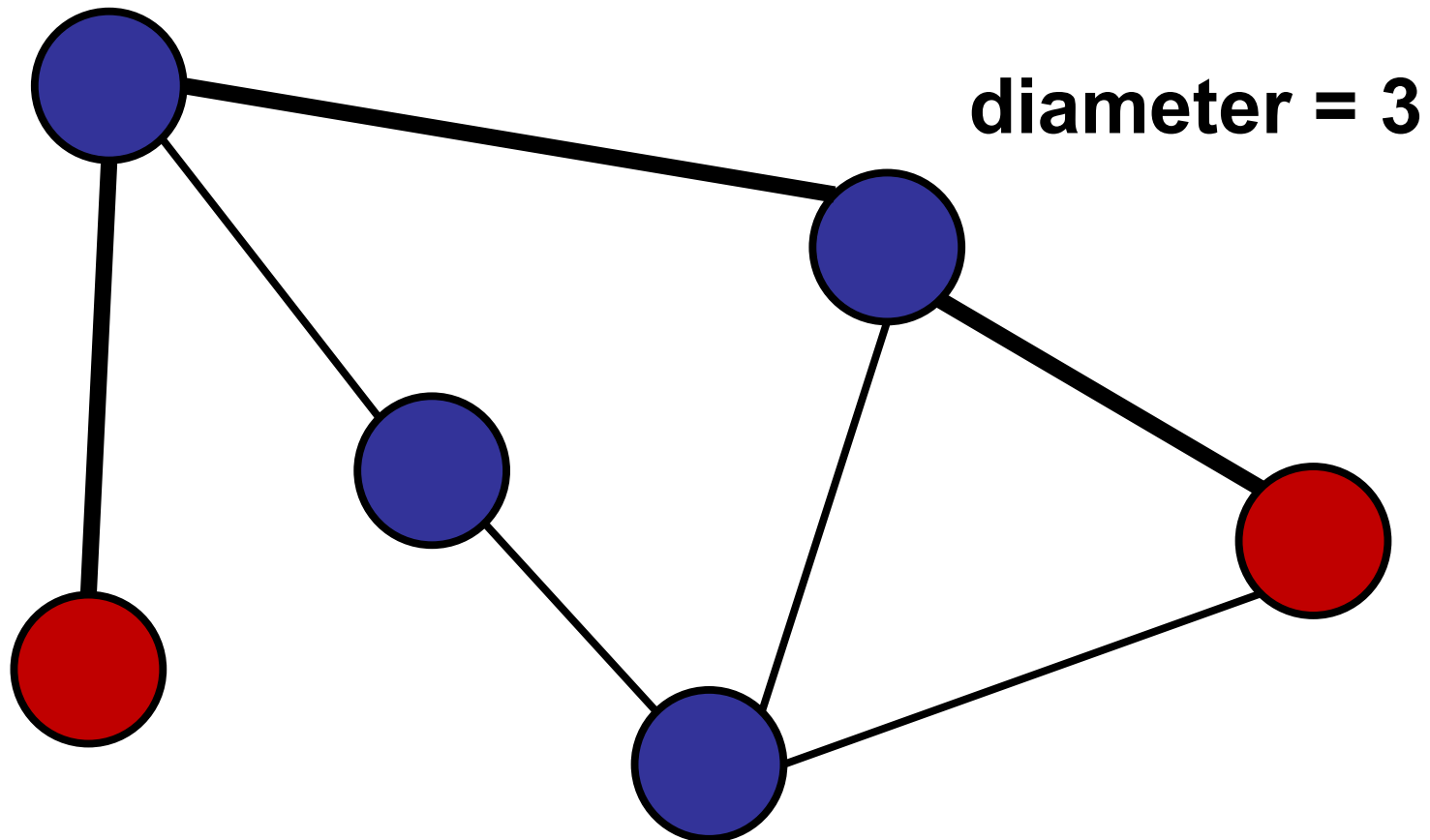
degree = 3



Graph Terminology

Diameter:

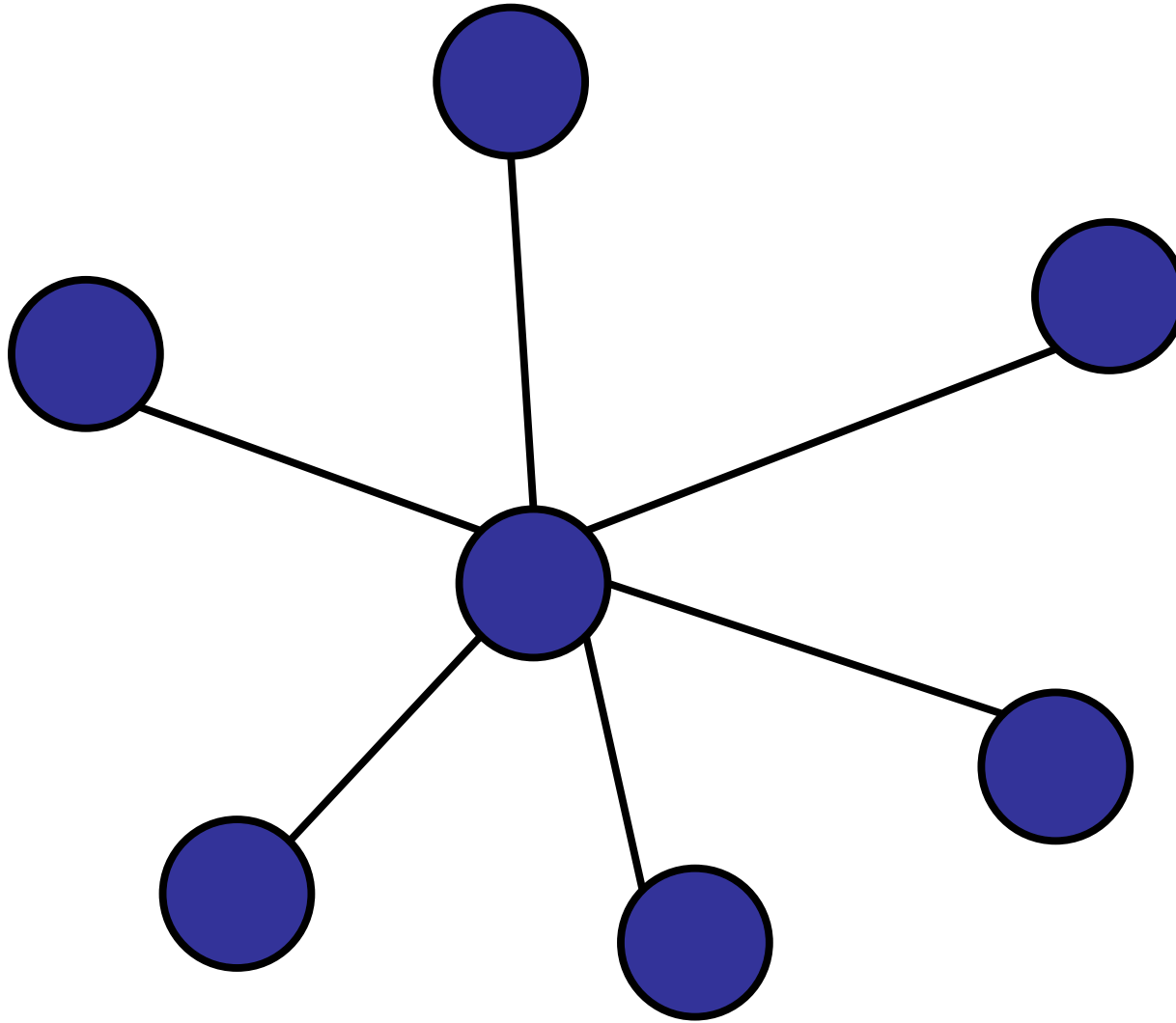
- Maximum distance between two nodes, following the shortest path.



Special Graphs

Special Graphs

Star



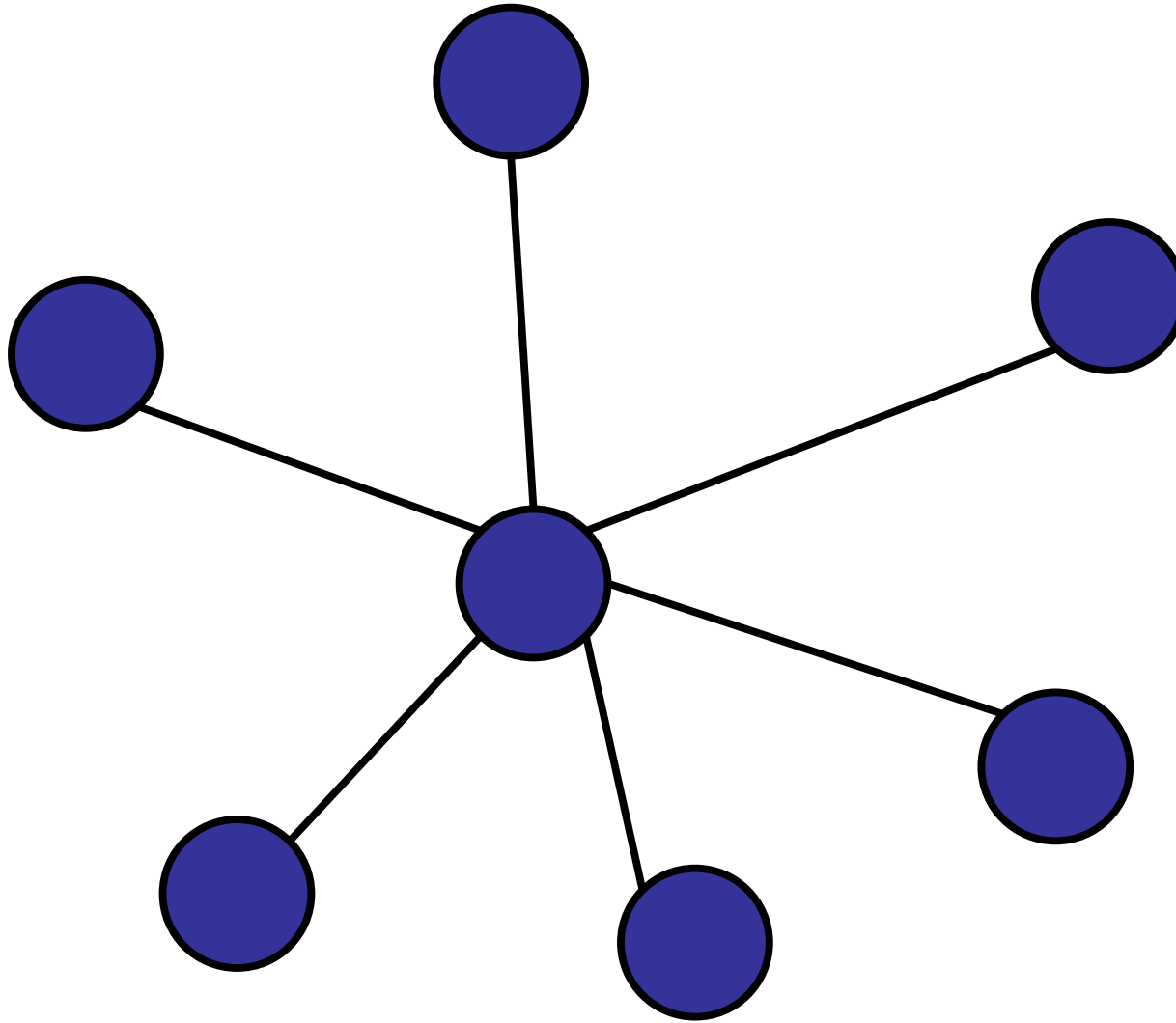
One central node, all edges connect center to edges.

Degree of n-node star is:

- 1. 1
- 2. 2
- 3. $n/2$
- 4. $n-2$
- ✓ 5. $n-1$
- 6. n

Special Graphs

Star



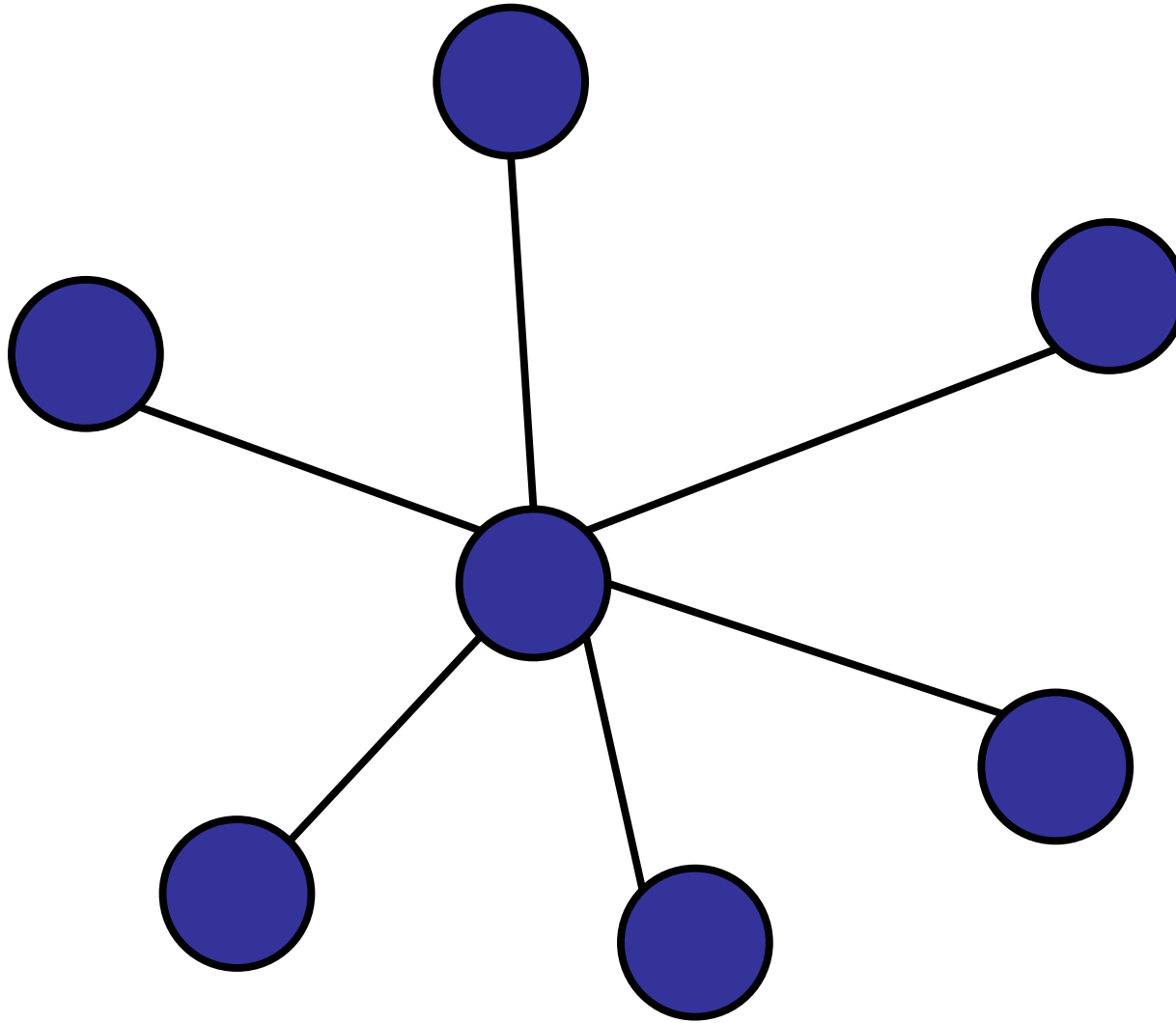
One central node, all edges connect center to edges.

Diameter of n-node star:

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Special Graphs

Star

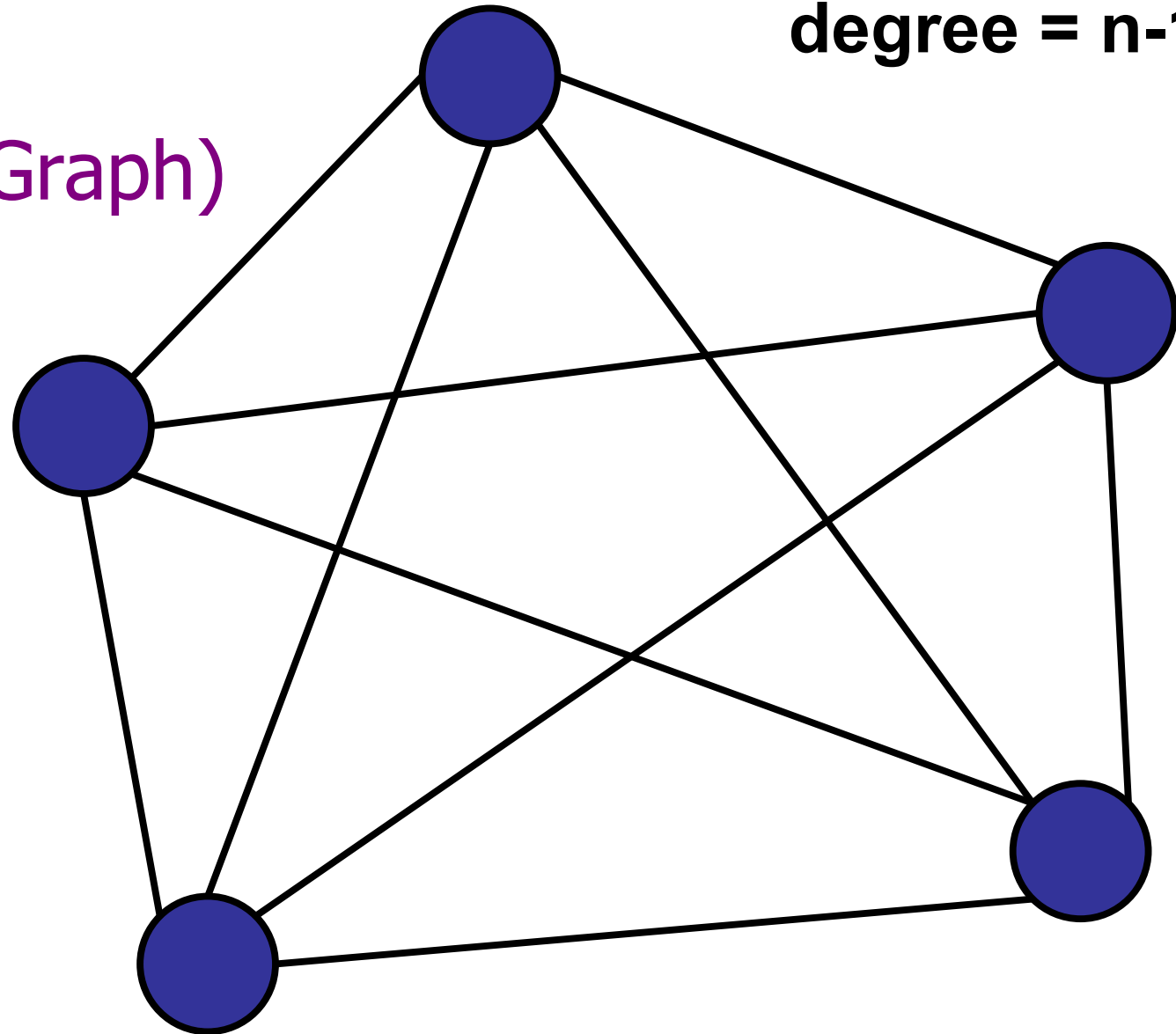


One central node, all edges connect center to edges.

Special Graphs

Clique
(Complete Graph)

diameter = 1
degree = $n-1$



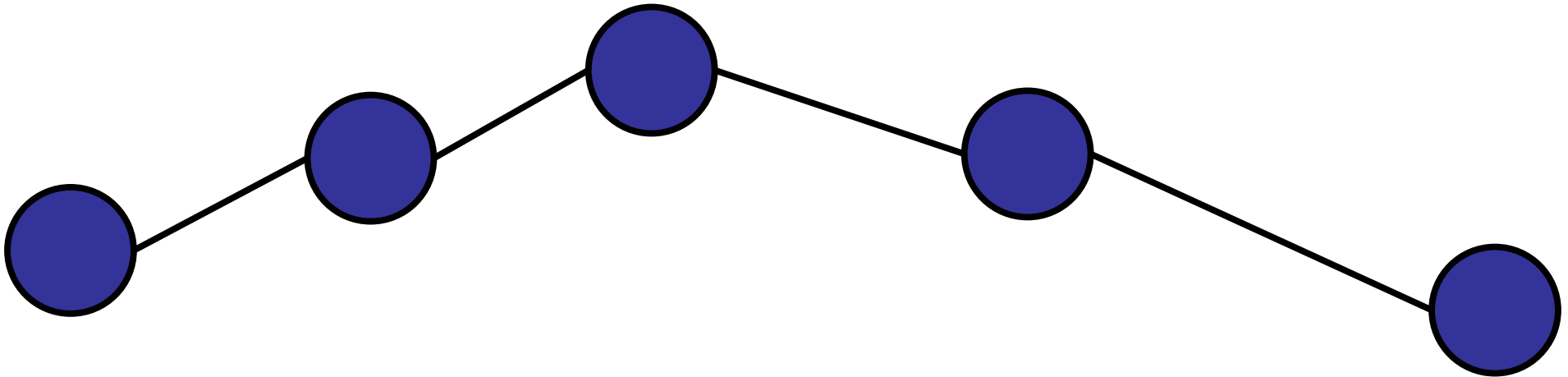
All pairs connected by edges.

Special Graphs

Line (or path)

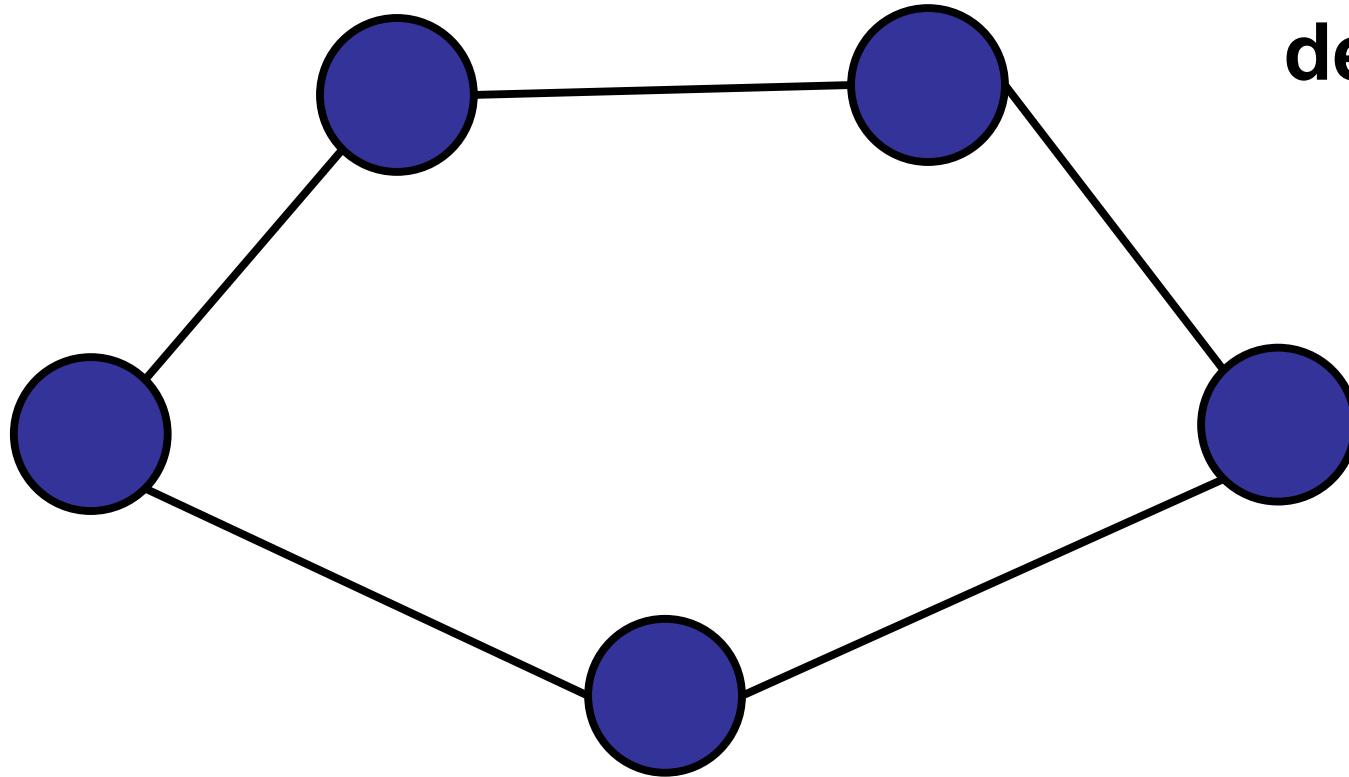
diameter = $n-1$

degree = 2



Special Graphs

Cycle



diameter = $n/2$

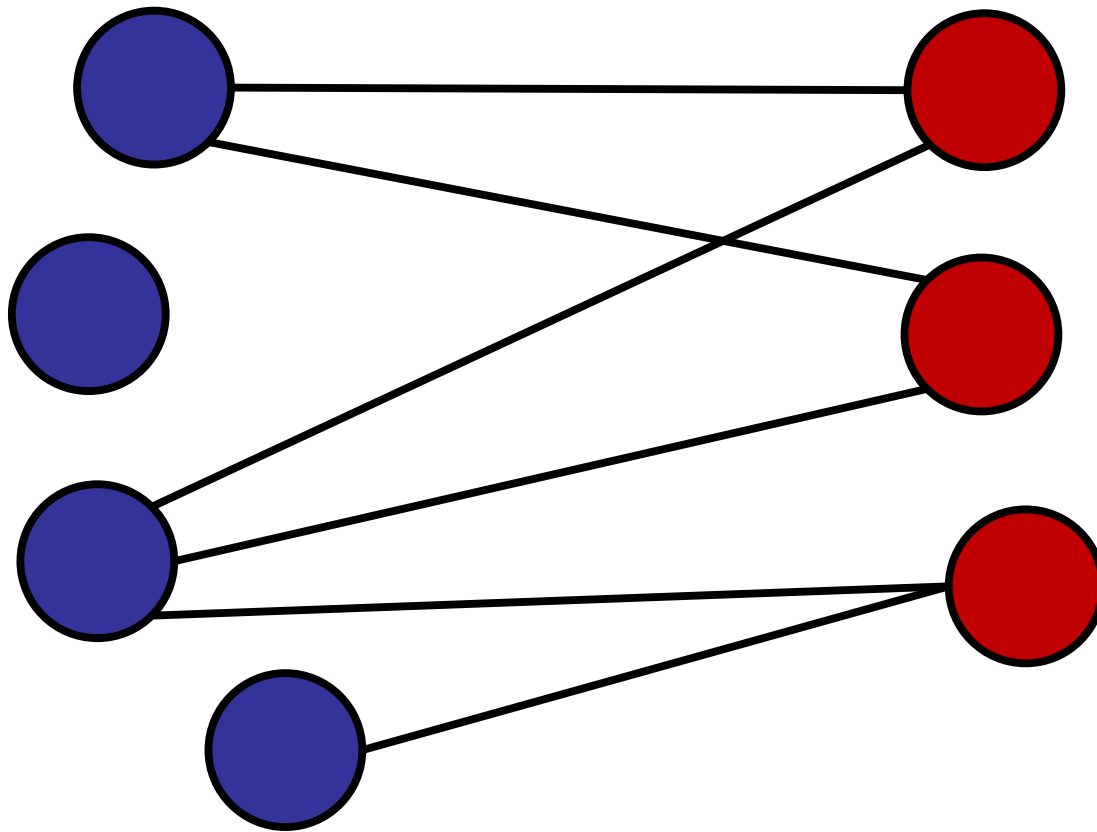
or

diameter = $n/2 - 1$

degree = 2

Special Graphs

Bipartite Graph



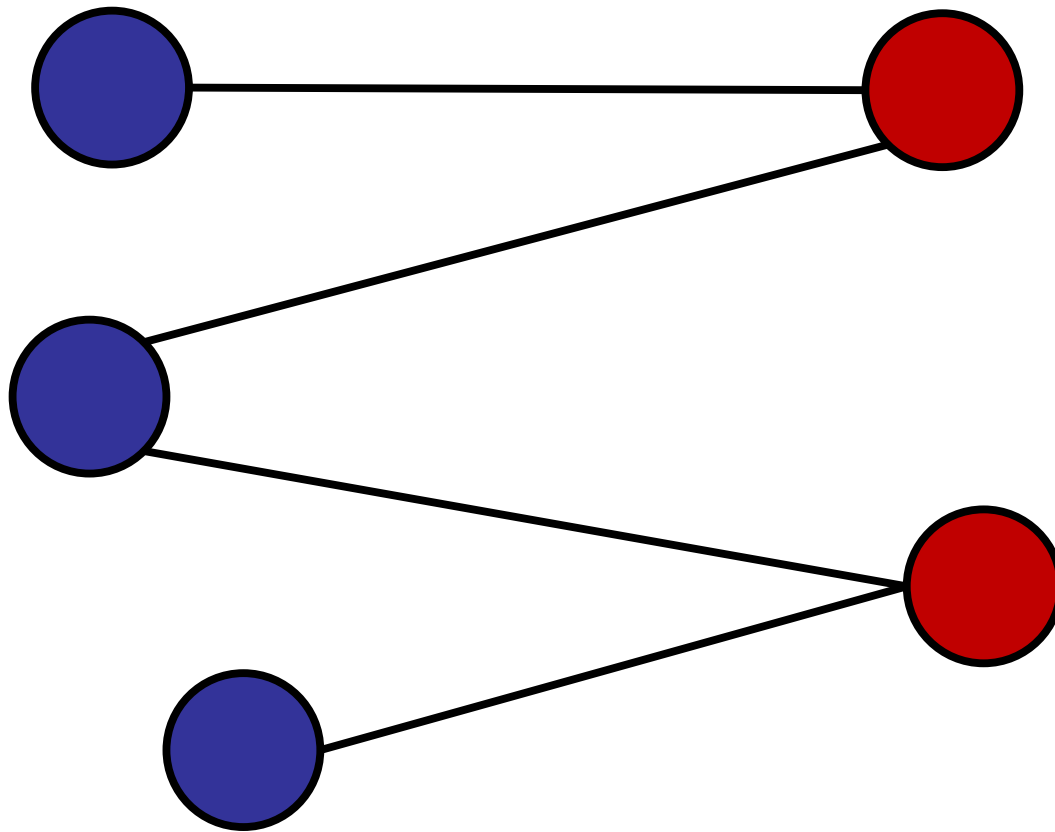
Nodes divided into two sets with no edges between nodes in the same set.

Max. diameter of n -node bipartite graph is:

1. 1
2. 2
3. $n/2$
4. $n-2$
- ✓ 5. $n-1$
6. n

Special Graphs

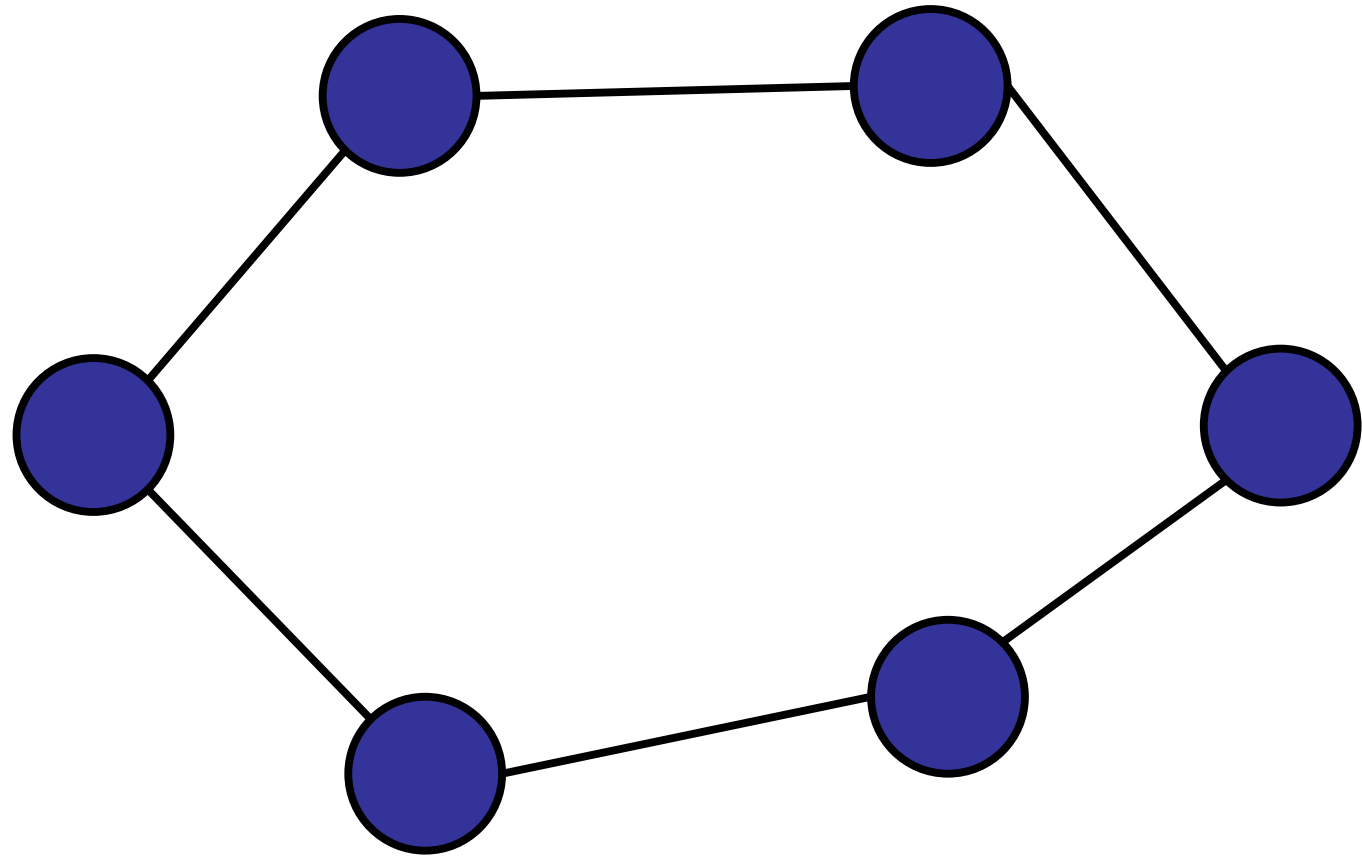
Bipartite Graph



Nodes divided into two sets with no edges between nodes in the same set.

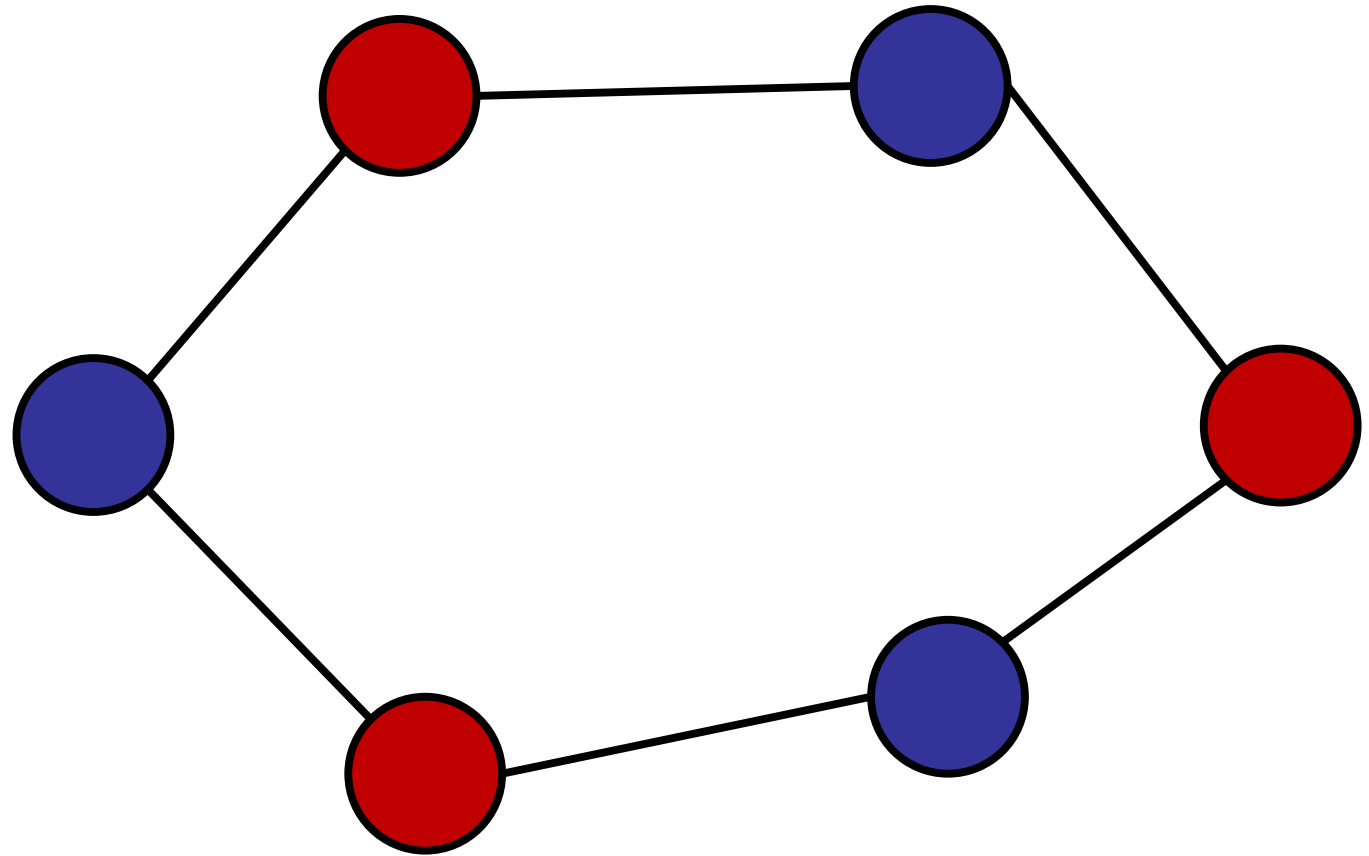
Is it bipartite?

- ✓ 1. Yes
- 2. No



Is it bipartite?

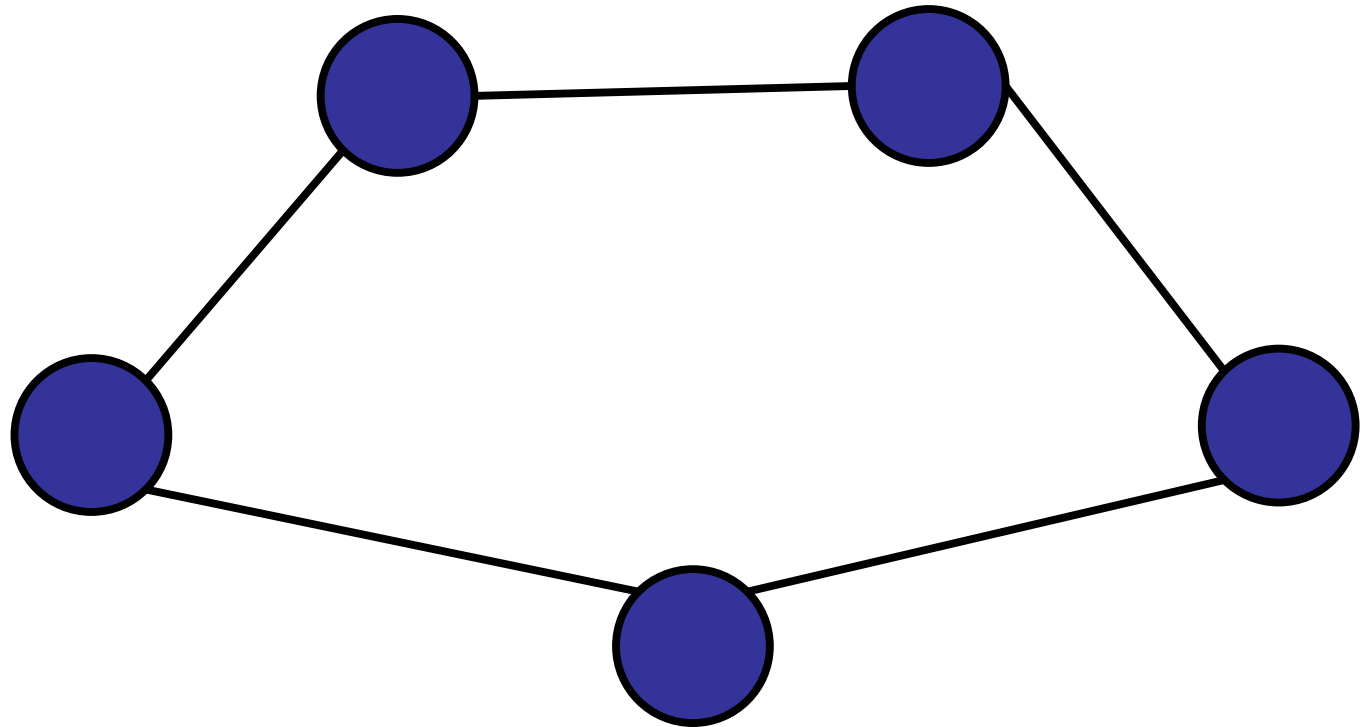
- ✓ 1. Yes
- 2. No



Is it bipartite?

1. Yes

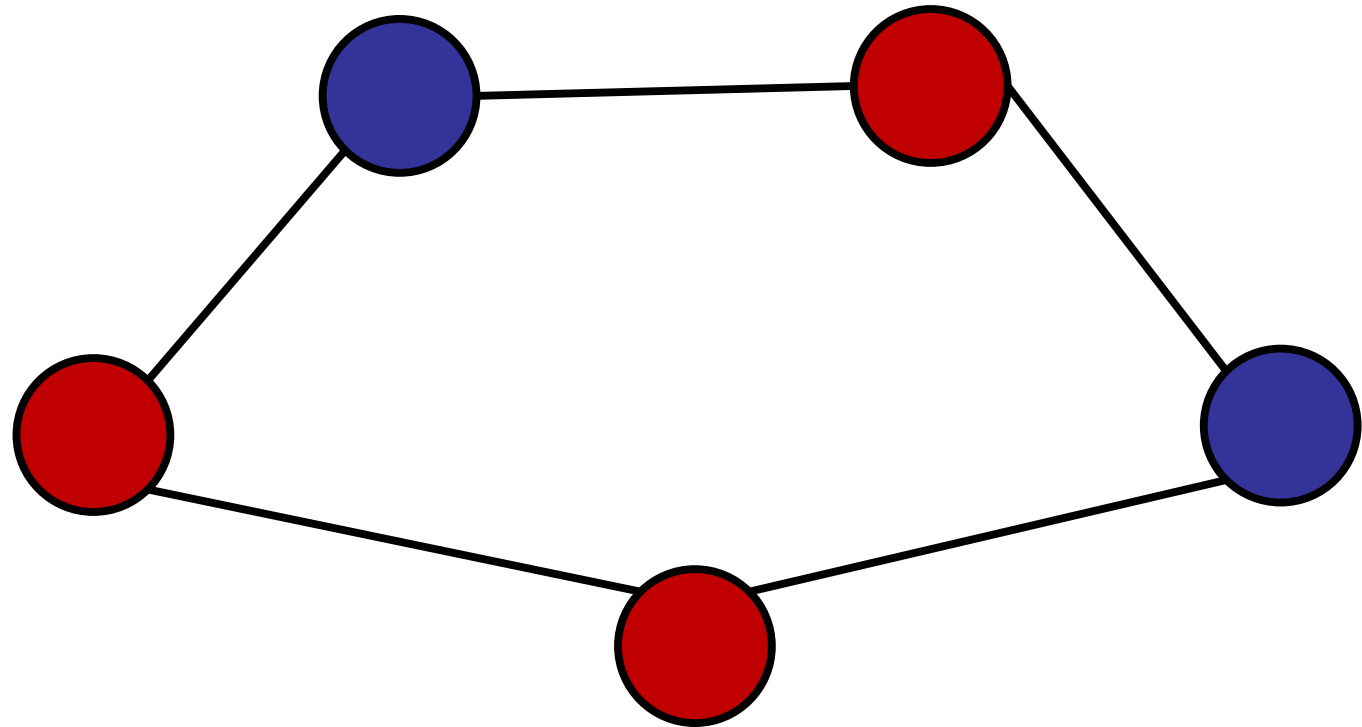
✓ 2. No



Is it bipartite?

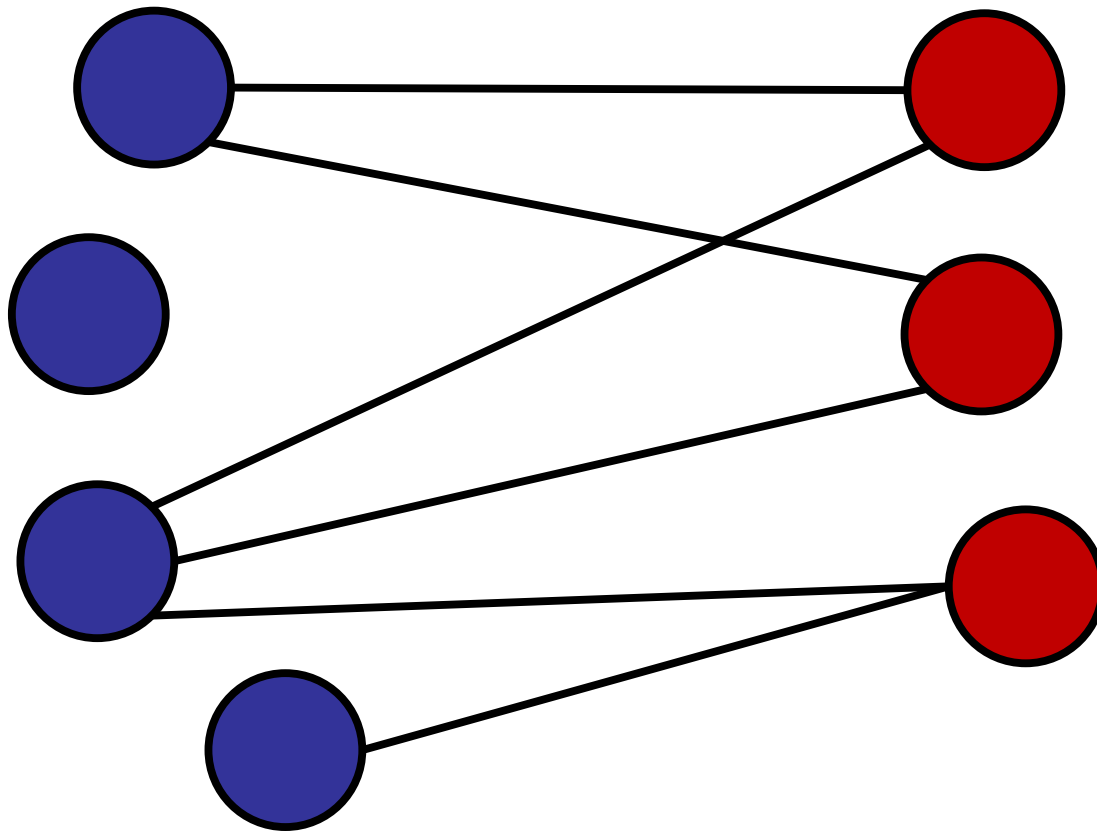
1. Yes

✓ 2. No



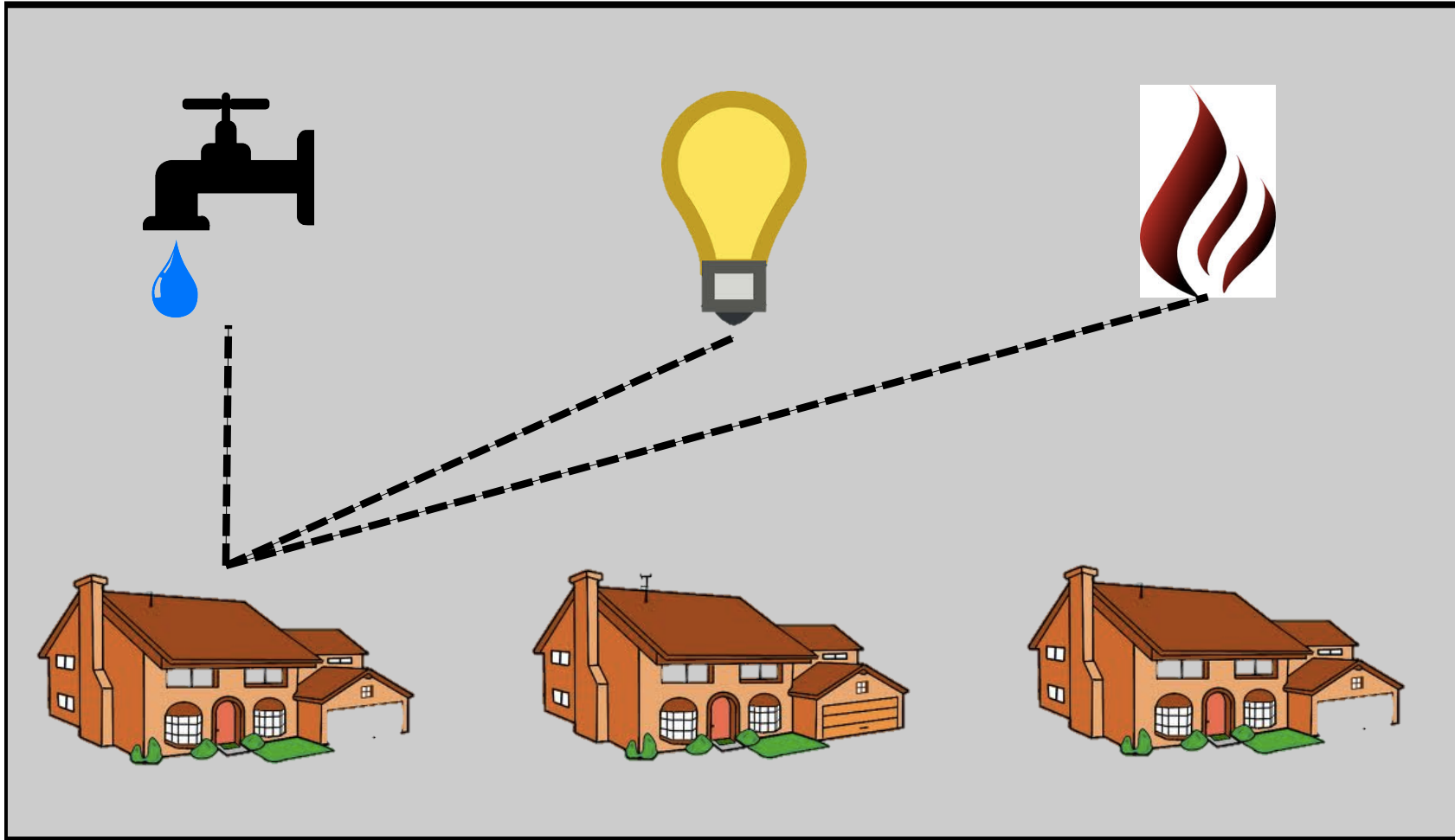
Special Graphs

Bipartite Graph



Nodes divided into two sets with no edges between nodes in the same set.

Puzzle



Connect each house to all three utilities (water, electricity, gas).
Do not let any of the cables or pipes cross.
(Or show that it is impossible.)