

**NATIONAL UNIVERSITY OF SINGAPORE**  
**Department of Statistics and Applied Probability**

(2021/22) Semester 1

ST2334 Probability and Statistics

Tutorial 8

1. According to *Chemical Engineering Progress* (Nov, 1990), approximately 30% of all pipework failures in chemical plants are caused by operator error.
  - (a) What is the probability that out of the next 20 pipework failures at least 10 are due to operator error?
  - (b) What is the probability that no more than 4 out of 20 such failures are due to operator error?
  - (c) What is the probability that exactly 5 are operational errors out of 20 such failures?
  - (d) Repeat (b) if approximately 60% of all pipework failures are caused by operator error.
2. In testing a certain kind of truck tire over a rugged terrain, it is found that 25% of the trucks fail to complete the test without a blowout. Of the next 15 trucks tested, find
  - (a) The probability of zero blowouts.
  - (b) The probability of at least 8 blowouts.
  - (c) Expected number of blowouts.
  - (d) According to Chebyshev's theorem, what interval does the number of trucks having blowouts in the next 15 trucks fall with at least  $\frac{3}{4}$  chance?
3. Suppose that, on average, 1 person in 1000 makes a numerical error in preparing his or her income tax return. 10,000 forms are selected at random and examined.
  - (a) Find the probability that 6, 7, or 8 of the forms contain an error.
  - (b) Find the mean and variance of the number of persons among 10,000 who make an error in preparing their tax returns.
  - (c) According to Chebyshev's theorem, what interval does the number of persons who make errors in preparing their income tax returns among 10,000 persons fall with at least  $\frac{8}{9}$  chance?
4. The probability that a person, living in a certain city, owns a dog is estimated to be 0.3. Find the probability that the tenth person randomly interviewed in that city is the fifth one to own a dog.
5. A couple decides they will continue to have children until they have two males. Assuming that  $\Pr(\text{male}) = 0.5$ .
  - (a) What is the probability that their second male is their seventh child?
  - (b) What is the expected number of children for the couple?
6. Three people toss a fair coin and the odd man pays for coffee. If the coins all turn up the same, they are tossed again.
  - (a) Find the probability that fewer than 4 tosses are needed.
  - (b) Provide a general formula for the probability of at most  $x$  tosses are needed.
7. A secretary makes 2 errors per page, on average.
  - (a) Find the variance of the number of errors per page.
  - (b) What is the probability that on the next page he or she will make 4 or more errors? No errors?

8. Hospital administrators in large cities anguish about problems with traffic in emergency rooms in hospitals. For a particular hospital in a large city, the staff on hand cannot accommodate the patient traffic if there are more than 10 emergency cases in a given hour. It is assumed that patient arrival follows a Poisson process and historical data suggest that, on the average, 5 emergencies arrive per hour. Find the probability that
  - (a) In a given hour, there is no emergency.
  - (b) In a given hour the staff can no longer accommodate the traffic?
  - (c) More than 20 emergencies arrive during a 3-hour shift of personnel?
9. A notice is sent to all owners of a certain type of automobile, asking them to bring their cars to a dealer to check for the presence of a particular type of defect. Suppose that only 0.05% of the cars have the defect. Consider a random sample of 10000 cars.
  - (a) What are the expected value and standard deviation of the number of cars in the sample that have the defect?
  - (b) What is the (approximate) probability that at least 10 sampled cars have the defect?
  - (c) What is the (approximate) probability that no sampled cars have the defect?
10. Suppose that a large conference room for a certain company can be reserved for no more than 4 hours. The use of the conference room is such that both long and short conferences occur quite often. Assume that length  $X$  of a conference has a uniform distribution on the interval  $[0,4]$ .
  - (a) What is the probability density function of  $X$ ?
  - (b) What is the probability that any given conference lasts at least 3 hours?
  - (c) Find  $E(X)$  and  $V(X)$ .
11. The length of time for one individual to be served at a cafeteria is a random variable having an exponential distribution with a mean of 4 minutes. Find the probability that
  - (a) A person is served in more than 3 minutes.
  - (b) A person is served in less than 3 minutes.
  - (c) A person is served in less than 3 minutes on at least 4 of the next 6 days?

#### Answers to selected problems

- |     |   |                                      |                                 |                       |
|-----|---|--------------------------------------|---------------------------------|-----------------------|
| 1.  | (a) 0.0480  | (b) 0.2375                           | (c) 0.1789                      | (d) 0.000317          |
| 2.  | (a) 0.0134  | (b) 0.0173                           | (c) 3.75                        | (d) $1 \leq X \leq 7$ |
| 3.  | (a) 0.2657  | (b) 10; 9.99                         | (c) $1 \leq X \leq 19$          |                       |
| 4.  | 0.0515  |                                      |                                 |                       |
| 5.  | (a) 0.0469  | (b) 4                                |                                 |                       |
| 6.  | (a) 0.9844  | (b) $1 - \left(\frac{1}{4}\right)^x$ |                                 |                       |
| 7.  | (a) 2   | (b) 0.1429; 0.1353                   |                                 |                       |
| 8.  | (a) 0.0067  | (b) 0.0137                           | (c) 0.0830                      |                       |
| 9.  | (a) $\mu = 5, \sigma = 2.2355$  | (b) $\Pr(X \geq 10) \approx 0.0318$  | (c) $\Pr(X = 0) \approx 0.0067$ |                       |
| 10. | (a) $f_X(x) = \begin{cases} \frac{1}{4}, & 0 \leq x \leq 4, \\ 0, & \text{otherwise} \end{cases}$ | (b) 0.25                             | (c) 2; 1.3333                   |                       |
| 11. | (a) 0.4724  | (b) 0.5276                           | (c) 0.3968                      |                       |