NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2021/22) Semester 1 ST2334 Probability and Statistics Tutorial 4

1. From a box containing 4 one-cent coins and 2 five-cent coins, 3 coins are selected at random *without replacement*. Let X denote the total amount of the selected coins.
   (a) What are the possible values for X?
   (b) Find the probability function of X.

2. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin.
   (a) What are the possible values for W?
   (b) Find the probability function of W assuming the coin is fair.
   (c) Find the probability function of W assuming the coin is biased so that a head is twice as likely to occur as a tail.

3. Determine the value c so that the following function can serve as a probability function of a discrete random variable X.

$$f_X(x) = \begin{cases} c(x^2+4), & x = 0, 1, 2, 3; \\ 0, & \text{otherwise.} \end{cases}$$

4. A contractor is required by the city planning department to submit 1, 2, 3, 4, or 5 forms (depending on the nature of the project) in applying for a building permit. Let $Y =$ the number of forms required of the next application. The probability that y forms are required is known to be proportional to y — that is, $f_Y(y) = ky$ for $y = 1, 2, \cdots, 5$.

   (a) What is the value of k?
   (b) What is the probability that at most three forms are required?
   (c) What is the probability that between two and four forms (inclusive) are required?
   (d) Find the cumulative distribution function (c.d.f.) of Y.

5. An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let $X =$ the number of months between successive payments. The c.d.f. of X is as follow.

$$F_X(x) = \begin{cases} 0, & x < 1, \\ 0.30, & 1 \le x < 3, \\ 0.40, & 3 \le x < 4, \\ 0.45, & 4 \le x < 6, \\ 0.60, & 6 \le x < 12, \\ 1, & 12 \le x. \end{cases}$$

   (a) What is the probability function of X?
   (b) Using the c.d.f., compute $\Pr(3 \le X \le 6)$ and $\Pr(4 \le X)$.

6. Consider the probability density function

$$f_X(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Evaluate the constant k.

(b) Find the cumulative distribution function $F_X(x)$ and use it to evaluate $\Pr(0.3 < X < 0.6)$.

7. Suppose the distance X between a point target and a shot aimed at the point in a coin-operated target game is a continuous random variable with p.d.f.

$$f_X(x) = \begin{cases} \frac{3}{4}(1 - x^2), & -1 \le x \le 1, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute $\Pr\left(-\frac{1}{2} < X < \frac{1}{2}\right)$.

(b) Compute $\Pr\left(X < -\frac{1}{4} \text{ or } X > \frac{1}{4}\right)$.

(c) Find the c.d.f. of X.

8. The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution

$$F_X(x) = \begin{cases} 0, & x \le 0, \\ 1 - e^{-8x}, & x > 0. \end{cases}$$

(a) Find the probability of waiting less than 12 minutes between successive speeders.

(b) Find the probability density function of X.

Answers to selected problems

1. (a) 3, 7 and 11.
   (b)

x	3	7	11
$f_X(x)$	1/5	3/5	1/5

2. (a) 3, −1, 1, 3.
   (b)

w	3	1	–1	–3
$f_W(w)$	1/8	3/8	3/8	1/8

   (c)

w	3	1	–1	–3
$f_W(w)$	8/27	12/27	6/27	1/27

3. $c = 1/30$.
4. (a) $k = 1/15$.
   (b) 0.4.
   (c) 0.6.
   (d)

$$F_Y(y) = \begin{cases} 0, & y < 1; \\ 1/15\,, & 1 \le y < 2; \\ 3/15, & 2 \le y < 3; \\ 6/15, & 3 \le y < 4; \\ 10/15, & 4 \le y < 5; \\ 1, & 5 \le y. \end{cases}$$

5. (a)

x	1	3	4	6	12
$f_X(x)$	0.3	0.1	0.05	0.15	0.4

   (b) 0.3, 0.6.
6. (a) $3/2$.
   (b)

$$F_X(x) = \begin{cases} 0, & x < 0; \\ x^{3/2}, & 0 \le x \le 1; \\ 1, & x \ge 1. \end{cases}$$

   $F_X(x) = 0$ for $x < 0$; $F_X(x) = x^{3/2}$ for $0 \le x < 1$; $F_X(x) = 1$ for $x \ge 1$.
   (c) 0.3004.
7. (a) 0.6875.
   (b) 0.6328.
   (c)

$$F_X(x) = \begin{cases} 0, & x < -1; \\ (2 + 3x - x^3)/4, & -1 \le x \le 1; \\ 1, & x > 1. \end{cases}$$

8. (a) 0.7981.
   (b)

$$f_X(x) = \begin{cases} 8e^{-8x}, & x \ge 0; \\ 0, & x < 0. \end{cases}$$