RDocumentation

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stats (version 3.6.2)

Chisquare: The (non-central) Chi-Squared Distribution

Description

Density, distribution function, quantile function and random generation for the chi-squared (\(\chi^2\)) distribution with `df` degrees of freedom and optional non-centrality parameter `ncp`.

Usage

```
dchisq(x, df, ncp = 0, log = FALSE)
pchisq(q, df, ncp = 0, lower.tail = TRUE, log.p = FALSE)
qchisq(p, df, ncp = 0, lower.tail = TRUE, log.p = FALSE)
rchisq(n, df, ncp = 0)
```

Arguments

x, q	vector of quantiles.
p	vector of probabilities.
n	number of observations. If `length(n) > 1`, the length is taken to be the number required.
df	degrees of freedom (non-negative, but can be non-integer).

ncp	non-centrality parameter (non-negative).
log, log.p	logical; if TRUE, probabilities p are given as log(p).
lower.tail	logical; if TRUE (default), probabilities are \(P[X \le x]\), otherwise, \(P[X > x]\).

Value

`dchisq` gives the density, `pchisq` gives the distribution function, `qchisq` gives the quantile function, and `rchisq` generates random deviates.

Invalid arguments will result in return value `NaN`, with a warning.

The length of the result is determined by `n` for `rchisq`, and is the maximum of the lengths of the numerical arguments for the other functions.

The numerical arguments other than `n` are recycled to the length of the result. Only the first elements of the logical arguments are used.

Details

The chi-squared distribution with `df`\(= n \ge 0\) degrees of freedom has density $$f_n(x) = \frac{1}{{2}^{n/2} \Gamma (n/2)} {x}^{n/2-1} {e}^{-x/2}$$ for \(x > 0\). The mean and variance are \(n\) and \(2n\).

The non-central chi-squared distribution with `df`\(= n\) degrees of freedom and non-centrality parameter `ncp` \(= \lambda\) has density $$ f(x) = e^{-\lambda / 2} \sum_{r=0}^\infty \frac{(\lambda/2)^r}{r!}\, f_{n + 2r}(x)$$ for \(x \ge 0\). For integer \(n\), this is the distribution of the sum of squares of \(n\) normals each with variance one, \(\lambda\) being the sum of squares of the normal means; further,

\(E(X) = n + \lambda\), \(Var(X) = 2(n + 2*\lambda)\), and \(E((X - E(X))^3) = 8(n + 3*\lambda)\).

Note that the degrees of freedom `df`\(= n\), can be non-integer, and also \(n = 0\) which is relevant for non-centrality \(\lambda > 0\), see Johnson *et al* (1995, chapter 29). In that (noncentral, zero df) case, the distribution is a mixture of a point mass at \(x = 0\) (of size `pchisq(0, df=0, ncp=ncp)`) and a continuous part, and `dchisq()` is *not* a density with respect to that mixture measure but rather the limit of the density for \(df \to 0\).

Note that `ncp` values larger than about 1e5 may give inaccurate results with many warnings for `pchisq` and `qchisq`.

References

Becker, R. A., Chambers, J. M. and Wilks, A. R. (1988) *The New S Language*. Wadsworth & Brooks/Cole.

Johnson, N. L., Kotz, S. and Balakrishnan, N. (1995) *Continuous Univariate Distributions*, chapters 18 (volume 1) and 29 (volume 2). Wiley, New York.

See Also

Distributions for other standard distributions.

A central chi-squared distribution with \(n\) degrees of freedom is the same as a Gamma distribution with `shape` \(\alpha = n/2\) and `scale` \(\sigma = 2\). Hence, see `dgamma` for the Gamma distribution.

Examples

```
# NOT RUN {
require(graphics)

dchisq(1, df = 1:3)
pchisq(1, df =  3)
pchisq(1, df =  3, ncp = 0:4)  # includes the above

x <- 1:10
## Chi-squared(df = 2) is a special exponential distribution
all.equal(dchisq(x, df = 2), dexp(x, 1/2))
all.equal(pchisq(x, df = 2), pexp(x, 1/2))

## non-central RNG -- df = 0 with ncp > 0:  Z0 has point mass at 0!
```

```
Z0 <- rchisq(100, df = 0, ncp = 2.)
graphics::stem(Z0)

# }
# NOT RUN {
## visual testing
## do P-P plots for 1000 points at various degrees of freedom
L <- 1.2; n <- 1000; pp <- ppoints(n)
op <- par(mfrow = c(3,3), mar = c(3,3,1,1)+.1, mgp = c(1.5,.6,0),
          oma = c(0,0,3,0))
for(df in 2^(4*rnorm(9))) {
  plot(pp, sort(pchisq(rr <- rchisq(n, df = df, ncp = L), df = df, ncp = L)),
       ylab = "pchisq(rchisq(.),.)", pch = ".")
  mtext(paste("df = ", formatC(df, digits = 4)), line =  -2, adj = 0.05)
  abline(0, 1, col = 2)
}
mtext(expression("P-P plots : Noncentral  "*
                 chi^2 *"(n=1000, df=X, ncp= 1.2)"),
      cex = 1.5, font = 2, outer = TRUE)
par(op)
# }
# NOT RUN {
## "analytical" test
lam <- seq(0, 100, by = .25)
p00 <- pchisq(0,      df = 0, ncp = lam)
p.0 <- pchisq(1e-300, df = 0, ncp = lam)
stopifnot(all.equal(p00, exp(-lam/2)),
          all.equal(p.0, exp(-lam/2)))
# }
```