

NATIONAL UNIVERSITY OF SINGAPORE
DEPARTMENT OF STATISTICS & APPLIED PROBABILITY
ST2334 PROBABILITY AND STATISTICS
SEMESTER N, AY 20YY/20YY

Mid-Semester Test (Sample B)

Name _____

Matric No. _____

Tutorial Group _____ Seat No. _____

Score:

Instructions

- This paper consists of 10 multiple-choice questions (that are worth 1 mark each) and 3 structured questions.
- Please write your answers for Questions 1 to 10 in the table below:

Question	1	2	3	4	5	6	7	8	9	10
Answer										

Multiple Choice Questions

- Placebos are a tool for
 - sampling
 - blinding
 - control
 - randomization
- Find the sample standard deviation of the numbers 4, 1, 8, 14, 7, 12, 16, 17, 10.
 - 5.1
 - 5.4
 - 26.1
 - 29.4
- Given that events A and B are mutually exclusive and $P(A) = 0.5$ and $P(B) = 0.5$, are A and B independent?
 - no
 - yes
 - cannot be determined
- Which of the following is not true of statistics?
 - Statistics is used to answer questions with 100% certainty.
 - Statistics involves collecting and summarizing data.
 - Statistics can be used to organize and analyze information.
 - Statistics is used to draw conclusions using data.

5. From a deck of 52 cards, we draw cards at random, one at a time, without replacement until only cards of one suit are left. Find the probability that the cards left are all spades. (Pick the option closest to the answer.)
 - A. 0.2
 - B. 0.4
 - C. 0.6
 - D. 0.8
6. A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?
 - A. 199,584
 - B. 23,950,080
 - C. 2,874,009,600
 - D. 344,881,152,000
7. A market researcher randomly selects 100 homeowners under 60 years of age and 200 homeowners over 60 years of age. What sampling technique was used?
 - A. simple random
 - B. cluster
 - C. convenience
 - D. stratified
8. A medical journal published the results of an experiment on insomnia. The experiment investigated the effects of a controversial new therapy for insomnia. Researchers measured the insomnia levels of 92 adult women who suffer moderate conditions of the disorder. After the therapy, the researchers again measured the women's insomnia levels. The differences between the pre- and post-therapy insomnia levels were reported. What is the treatment in this experiment?
 - A. the 92 adult women who suffer from insomnia
 - B. the disorder (insomnia or no insomnia)
 - C. the therapy
 - D. the differences between the pre- and post-therapy insomnia levels
9. In distributions that are skewed to the right, what is the relationship of the mean, median, and mode?
 - A. mean > median > mode
 - B. median > mean > mode
 - C. mode > median > mean
 - D. mode > mean > median
10. Your teacher tells you your test score was the 3rd quartile for the class. Which is true?
 - I. You got 75% on the test.
 - II. You can't really tell what this means without knowing the standard deviation.
 - III. You can't really tell what this means unless the class distribution is nearly Normal.
 - A. none of these
 - B. I only
 - C. II only
 - D. III only
 - E. II and III

Structured Questions

11. Let A and B be two events. If $P(A) = P(B) = 1$, what is the value of $P(AB)$?

[5 marks]

12. Abang and Adik go on a long and potentially boring road trip. To while away time, Abang suggests a bet to Adik. They will focus on the last two *digits* on the car number plates of all the cars they see travelling in the opposite direction. For example, both SCF6045P and GU2745X will yield “45” while SX18Z will give “18”, and so on. Abang bets that at least two of the next ten cars would yield the same outcome. (If there is only one digit on the number plate, for example, SGP6R, we will interpret it as “06”, while if there are no digits on the number plate, for example, SIRGGP, we will interpret it as “00”.)

Should Adik take up the bet? Explain. Also state any assumptions you are making.

[5 marks]

13. Four fair dice are rolled. Suppose the numbers that appeared on the dice are A , B , C , and D respectively.

(i) What is the probability that the numbers are different?

[3 marks]

(ii) If no two dice land on the same number, what is the probability that $A < B < C < D$?

[3 marks]

(iii) What is $P(A < B < C < D)$?

[4 marks]

[END OF PAPER]

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- Find the sample standard deviation of the numbers 4, 1, 8, 14, 7, 12, 16, 17, 10.
 - 5.1
 - 5.4 ✓
 - 26.1
 - 29.4
- Given that events A and B are mutually exclusive and $P(A) = 0.5$ and $P(B) = 0.5$, are A and B independent?
 - no ✓ $P(A)P(B) = 0.25 \neq 0 = P(AB)$
 - yes
 - cannot be determined
- Which of the following is not true of statistics?
 - Statistics is used to answer questions with 100% certainty. ✓
 - Statistics involves collecting and summarizing data.
 - Statistics can be used to organize and analyze information.
 - Statistics is used to draw conclusions using data.

5. From a deck of 52 cards, we draw cards at random, one at a time, without replacement until only cards of one suit are left. Find the probability that the cards left are all spades. (Pick the option closest to the answer.)
- A. 0.2 ✓ by symmetry, the answer is 1/4.
 - B. 0.4
 - C. 0.6
 - D. 0.8
6. A dance class consists of 22 students, of which 10 are women and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?
- A. 199,584
 - B. 23,950,080 ✓ pick 5 men, 5 women then pair them off $\Rightarrow C_5^{10} \times C_5^{12} \times 5!$
 - C. 2,874,009,600
 - D. 344,881,152,000
7. A market researcher randomly selects 100 homeowners under 60 years of age and 200 homeowners over 60 years of age. What sampling technique was used?
- A. simple random
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Structured Questions

11. Let A and B be two events. If $P(A) = P(B) = 1$, what is the value of $P(AB)$?

[5 marks]

Solution:

We know that $P(A \cup B) \leq 1$.

It is also clear that

$$P(A \cup B) \geq P(A) = 1,$$

So $P(A \cup B) \geq 1$ as well, which gives $P(A \cup B) = 1$.

The Inclusion-Exclusion Principle gives

$$P(A \cup B) = P(A) + P(B) - P(AB) \iff 1 = 1 + 1 - P(AB).$$

Thus $P(AB) = 1$.



Common Errors



- (i) Some might write that $P(AB) = P(A)P(B)$, which is not necessarily true, unless we are told that A and B are independent.
- (ii) Some of you might wrongly claim or assume that if $P(A) = 1$, then $A = S$, the sample space.

This is not true. Consider $X \sim U(0, 1)$ and the event

$$A = \{0 < X < 1/2\} \cup \{1/2 < X < 1\}.$$

Then $P(A) = 1$ but $A \neq S = (0, 1)$.

We learnt from lectures that if S is the sample space, then $P(S) = 1$. We did not say that if $P(A) = 1$, then $A = S$.

12. Abang and Adik go on a long and potentially boring road trip. To while away time, Abang suggests a bet to Adik. They will focus on the last two *digits* on the car number plates of all the cars they see travelling in the opposite direction. For example, both SCF6045P and GU2745X will yield “45” while SX18Z will give “18”, and so on. Abang bets that at least two of the next ten cars would yield the same outcome. (If there is only one digit on the number plate, for example, SGP6R, we will interpret it as “06”, while if there are no digits on the number plate, for example, SIRGGP, we will interpret it as “00”.)

Should Adik take up the bet? Explain. Also state any assumptions you are making.

[5 marks]

Solution:

This is the birthday problem with 100 days and $n = 10$.

The probability that we have at least two of the next twenty cars yielding the same outcome is given as

$$1 - \frac{100 \times 99 \times \dots \times 91}{100^{10}} \approx 1 - 0.628 = 0.372.$$

Adik should take up the bet since Abang's probability of winning is less than 0.5.

We make the assumption that each of the 100 numbers 00 to 99 are equally likely to appear as the last two digits of a car number plate.

13. Four fair dice are rolled. Suppose the numbers that appeared on the dice are A , B , C , and D respectively.

(i) What is the probability that the numbers are different?

[3 marks]

Solution:

Let E denote the event that the numbers are different.

The required probability is given as

$$P(E) = \frac{6 \times 5 \times 4 \times 3}{6^4} = \frac{5}{18} \approx 0.278.$$

(ii) If no two dice land on the same number, what is the probability that $A < B < C < D$?

[3 marks]

Solution:

If no two dice land on the same number, there are $4! = 24$ ways the numbers could have appeared. Only one, however, would result in the order $A < B < C < D$. Thus the required probability is

$$P(A < B < C < D | E) = 1/24 \approx 0.0417.$$



A Technical Note



Note that there is no need to consider what the four numbers that appeared are. Even if the question is changed to “Four fair dice, each with 24 faces (numbered from 1 to 24), are rolled.”, the same argument will work for this part.

(iii) What is $P(A < B < C < D)$?

[4 marks]

Solution:

We condition on whether the numbers obtained are different or not.

We know that $P(E) = 5/18$ from part (i). It is also clear that

$$P(A < B < C < D | E^c) = 0,$$

while from part (ii) we know that

$$P(A < B < C < D | E) = 1/24.$$

Thus the required probability is

$$\begin{aligned} P(A < B < C < D) &= P(A < B < C < D | E)P(E) + P(A < B < C < D | E^c)P(E^c) \\ &= 1/24 \times 5/18 + 0 \\ &= 5/432 \\ &\approx 0.0116. \end{aligned}$$

[END OF PAPER]