NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2021/22) Semester 1 ST2334 Probability and Statistics Tutorial 3

1. For customers purchasing a full set of tires at a particular tire store, consider the events
   A = {tires purchased were made in the United States},
   B = {purchaser has tires balanced immediately},
   C = {purchaser requests front-end alignment}.
   Denote the compliments of A, B, and C by A', B', and C' respectively. Assume the following unconditional and conditional probabilities:
   $\Pr(A) = 0.75$, $\Pr(B|A) = 0.9$, $\Pr(B|A') = 0.8$, $\Pr(C|A \cap B) = 0.8$ and $\Pr(C|A' \cap B) = 0.7$.
   (a) Compute $\Pr(A \cap B \cap C)$.
   (b) Compute $\Pr(B)$.
   (c) Compute $\Pr(A|B)$, the probability of a purchase of U.S. tires given that balancing was requested.
   (d) Compute $\Pr(B \cap C)$.
   (e) Compute $\Pr(A|B \cap C)$, the probability of a purchase of U.S. tires given that both balancing and an alignment were requested.

2. Even with strong advertising programs, new products are often unsuccessful. A company that produces a variety of household items found that only 18% of the new products it introduced over the last 10 years have become profitable. When two new products were introduced during the same year, only 5% of the time did both products become profitable. Suppose the company plans to introduce two new products, A and B, next year. Assume that the percentages just cited define the probability of success.
   (a) If product B is profitable, what is the probability that product A becomes profitable?
   (b) If at least one of the products will be profitable, what is the probability that the profitable product is A?

3. Total quality management (TQM) is a management philosophy and system of management techniques to improve product and service quality and worker productivity. TQM involves such techniques as teamwork, empowerment of workers, improved communication with customers, evaluation of work processes, and statistical analysis of processes and their output. One hundred local companies were surveyed and it was found that 30 had implemented TQM. Among the 100 companies surveyed, 60 reported an increase in sales last year. Of those 60, 20 had implemented TQM. Suppose one of the 100 surveyed companies is to be selected for additional analysis.
   (a) What is the probability that a firm that implemented TQM is selected? That a firm whose sales increased is selected?
   (b) Are the two events {TQM implemented} and {Sales increased} independent or dependent? Explain.
   (c) Suppose that instead of 20 TQM-implementers among the 60 firms reporting sales increases, there were 18. Now are the events {TQM implemented} and {Sales increased} independent or dependent? Explain.

4. A company uses three different assembly lines — A_1, A_2, and A_3 — to manufacture a particular component. Of those manufactured by line A_1, 5% need rework to remedy a

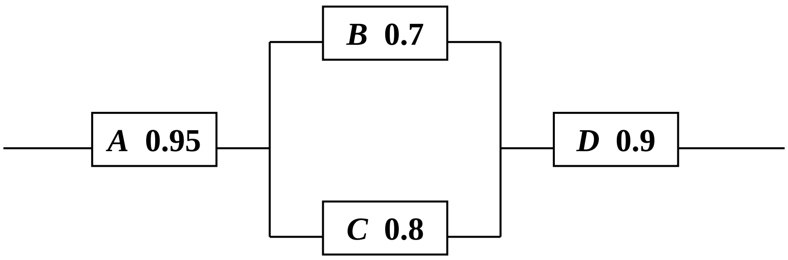
defect, whereas 8% of A_2's components need rework, and 10% of A_3's components need rework. Suppose that 50% of all components are produced by line A_1 while 30% are produced by line A_2, and 20% come from line A_3. If a randomly selected component needs rework, what is the probability that it came

(a) from line A_1?
(b) from line A_2?
(c) from line A_3?

5. A box contains the following 4 slips of paper, each having exactly the same dimensions: (1) win prize 1; (2) win prize 2; (3) win prize 3; (4) win prizes 1, 2, and 3. One slip of paper will be randomly selected. Let A_1 = {win prize 1}, A_2 = {win prize 2}, and A_3 = {win prize 3}.

(a) Check that A_1 and A_2 are independent, that A_1 and A_3 are independent, and that A_2 and A_3 are also independent (that is *pairwise* independence).
(b) "Verify" that $\Pr(A_1 \cap A_2 \cap A_3) \neq \Pr(A_1)\Pr(A_2)\Pr(A_3)$, so the three events are *not* mutually independent.

6. Consider the system of components connected as in the following picture. The system works if components *A* and *D* work and either components *B* or *C* work. The components work independently of one another and the probabilities that *A*, *B*, *C*, and *D* work are 0.95, 0.7, 0.8 and 0.9 respectively.



(a) Find the probability that the entire system works, and
(b) Find the probability that the component *C* does not work, given that the entire system works.

7. Sixty percent of all vehicles examined at a certain emissions inspection station pass the inspection. Assume that successive vehicles pass or fail independently of one another.

(a) Find the probability that all of the next three vehicles inspected pass.
(b) Find the probability that at least one of the next three vehicles inspected fails.
(c) Find the probability exactly one of the next three vehicles inspected passes.
(d) Given that at least one of the next three vehicles passes inspection, what is the probability that all three pass?

8. A real estate agent has 8 keys to open several new homes. For any given house, there is only 1 key which will open the house. If 30% of these homes are usually left unlocked, what is the probability that the real estate agent can get into a specific home if the agent selects 3 keys at random before leaving the office?

<u>Answers to selected problems</u>

1. (a) $\Pr(A)\Pr(B|A)\Pr(C|A\cap B) = 0.54.$
   (b) $\Pr(A)\Pr(B|A) + \Pr(A')\Pr(B|A') = 0.875.$
   (c) $\Pr(A\cap B)\,/\Pr(B) = 0.7714.$
   (d) $\Pr(B\cap C) = 0.68.$
   (e) $\Pr(A|B\cap C) = 0.7941.$
2. (a) $\Pr(A|B) = 0.2777.$
   (b) $\Pr(A|C) = 0.5806.$
3. A = {TQM implemented}, B = {sales increased}
   (a) $\Pr(A) = 0.3.$ $\Pr(B) = 0.6.$
   (b) $\Pr(A|B)\Pr(B) = 0.2.$
   (c) $\Pr(A|B)\Pr(B) = 0.18.$
4. B = {Need rework}
   (a) $\Pr(A_1|B) = 0.3623.$
   (b) $\Pr(A_2|B) = 0.3478.$
   (c) $\Pr(A_3|B) = 0.2898.$
5. (a) $\Pr(A_1\cap A_2) = 1/4.$
   (b) $\Pr(A_1\cap A_2\cap A_3) = 1/4.$
6. (a) $\Pr(A\cap(B\cup C)\cap D) = 0.8037.$
   (b) 0.1489.
7. A_i = {i-th vehicle passes the inspection}.
   (a) $\Pr(A_1\cap A_2\cap A_3) = 0.216.$
   (b) $\Pr(A_1\cup A_2\cup A_3) = 0.784.$
   (c) $\Pr(A_1\cap A_2'\cap A_3') + \Pr(A_1'\cap A_2\cap A_3') + \Pr(A_1'\cap A_2'\cap A_3) = 0.288.$
   (d) 0.2308.
8. 0.5625.