

NATIONAL UNIVERSITY OF SINGAPORE
Department of Statistics and Applied Probability

(2021/2022) Semester 1

ST2334 Probability and Statistics

Tutorial 2

1. The probability that a Singapore company will set up a factory in City A is 0.7. The probability that it will set up a factory in City B is 0.4 and the probability that it will set up in either City A or City B or both is 0.8. What is the probability that the company will set up a factory
 - (a) in both cities?
 - (b) in neither city?
2. Suppose there are 500 applicants for five equivalent positions at a factory and the company is able to narrow the field to 25 equally qualified applicants. Six of the finalists are minority candidates. Assume that the five who are chosen are selected at random from this final group of twenty-five.
 - (a) In how many ways can the selection be made?
 - (b) What is the probability that none of the minority candidates is hired?
 - (c) What is the probability that no more than one minority candidates are hired?
3. Consider 5-card poker hands dealt from a standard 52 card deck. Two important events are
$$A = \{\text{You draw a flush (5 cards from the same suit)}\}$$
$$B = \{\text{You draw a straight (values of the 5 cards are in sequence, e.g., 9 of diamonds, 10 of hearts, jack of hearts, queen of spades and king of spades), assuming that aces can be high or low}\}$$
[Note: We consider a straight flush (5 consecutive cards of the same suit) is not a straight]
If you are dealt a 5-card hand, find the following probability
 - (a) $\Pr(A)$
 - (b) $\Pr(B)$
4. There are two intersections with traffic lights along the route taken a motorist in driving to work. The probability that he must stop at the first light is 0.4, the probability that he stops at the second light is 0.5, and the probability that he must stop at least one of the two lights is 0.6. What is the probability that he must stop
 - (a) at both lights?
 - (b) at exactly one light?
 - (c) at neither light?
 - (d) at the second light given that he has stopped at the first light?Is the event stopping at first traffic light independent of the event stopping at the second traffic light? Justify your answer.
5. Consider 9-digit numbers where each digit is one of the 10 integers $0, 1, \dots, 9$.
 - (a) What is the probability that no two consecutive digits are the same in a randomly selected 9-digit number?
 - (b) What is the probability that 0 appears as a digit for a total of 3 times in a randomly selected 9-digit number?

6. Player A has entered a golf tournament but it is not certain whether B will enter. Player A has probability $1/6$ of winning the tournament if player B enters, and probability $3/4$ of winning if player B does not enter the tournament. If the probability that player B enters is $1/3$, find the probability that player A wins the tournament.
7. A soft-drink bottling company maintains records concerning the number of unacceptable bottles of soft drink obtained from the filling and capping machines. Based on the past data, the probability that a bottle came from machine I and was nonconforming is 0.01, and the probability that a bottle came from machine II and was nonconforming is 0.025. Half the bottles are filled on machine I and the other half are filled on machine II. If a filled bottle of soft drink is selected at random, what is the probability that
 - (a) it is a nonconforming bottle?
 - (b) it was filled on machine II?
 - (c) it was filled on machine II and is a conforming bottle?
 - (d) It was filled on machine I or is a conforming bottle?
 - (e) Suppose you know that the bottle was produced on machine I. What is the probability that it is nonconforming?
 - (f) Suppose you know that the bottle is nonconforming. What is the probability that it was produced on machine I?
 - (g) Explain the difference in the answers to (e) and (f).
8. 75% of all women who submit to pregnancy tests are really pregnant. A certain pregnancy test gives a false positive result with probability 0.02 and a valid positive result with probability 0.99. [Note: A false positive is where the pregnancy test is positive, when in fact the woman is not pregnant.]
 - (a) If a particular woman's test is positive, what is the probability that she is really pregnant?
 - (b) What is the probability that she is not pregnant given that the test is negative?

Answers to selected problems:

1. (a) $\Pr(A \cap B) = 0.3$. (b) $\Pr(A' \cap B') = 0.2$.
2. (a) ${}_{25}C_5 = 53130$. (b) ${}_6C_0 \times {}_{19}C_5 / {}_{25}C_5 = 0.2189$. (c) $({}_6C_1 \times {}_{19}C_4) / {}_{25}C_5 + ({}_6C_0 \times {}_{19}C_5) / {}_{25}C_5 = 0.6566$.
3. (a) ${}_4C_1 \times ({}_{13}C_5 \times ({}_{13}C_0)^3) / {}_{52}C_5 = 0.00198$. (b) $({}_{10}C_1) \times [({}_4C_1)^5 \times ({}_4C_0)^8 - 4] / ({}_{52}C_5) = 0.003925$.
4. $A_i = \{\text{Stop at Light } i\}$. (a) $\Pr(A_1 \cap A_2) = 0.3$. (b) $\Pr((A_1 \cap A_2') \cup (A_1' \cap A_2)) = 0.3$. (c) $\Pr(A_1' \cap A_2') = 0.4$. (d) $\Pr(A_2 | A_1) = 0.75$.
5. (a) $({}_9C_1)^9 / ({}_9C_1 \times ({}_{10}C_1)^8) = 0.4305$. (b) ${}_8C_3 \times ({}_9C_1)^6 / ({}_9C_1 \times ({}_{10}C_1)^8) = 0.0331$.
6. $\Pr(A) = \Pr(A|B) \Pr(B) + \Pr(A|B') \Pr(B') = 5/9$.
7. $M_i = \{\text{Filled on Machine } i\}$, $N = \{\text{Non-conforming}\}$. (a) $\Pr(N) = 0.035$. (b) $\Pr(M_2) = 0.5$. (c) $\Pr(M_2 \cap N') = 0.475$. (d) $\Pr(M_1 \cup N') = 0.975$. (e) $\Pr(N|M_1) = 0.02$. (f) $\Pr(M_1|N) = 0.2857$.
8. $P = \{\text{Pregnant}\}$, $T = \{\text{Test } +\}$. $\Pr(P|T) = 0.9933$. (b) $\Pr(P'|T') = 0.9703$.