Math 136 **Real Analysis II**[1] **Spring 2020**
Prof. L. Tu

Textbooks: Patrick Fitzpatrick, *Advanced Calculus*, 2nd edition, American Mathematical Society, 2006. (ISBN-10: 0821847910) We will cover Chapters 4, 6, 13–19.

Jerrold E. Marsden and Michael J. Hoffman, *Elementary Classical Analysis*, 2nd edition, W. H. Freeman and Company, New York, 1993. (Lebesgue's Theorem and Fourier Analysis only).

Readings for January 15 and 22, 2020

§3.7 Quick review of definitions of limit of a function of one variable
§4.1 The algebra of derivatives for functions of one variable
§4.2 The chain rule in one variable (pp. 99–100 only)
§4.3 The mean value theorem and its geometric consequences

Readings for the weeks of January 27, 2020

§13.1 Limits of functions of several variables
§13.2 Partial derivatives
§13.3 The Mean Value Theorem in several variables

Problem Set 1

(100 points)

This homework is due on Wednesday, January 29 at the beginning of class. You may hand in the homework in up to 3:30 p.m., Friday, January 31, for a 10% penalty.

Part A. Please put the following problems (labeled Ax.) in your Apricot homework folder (marked with an A).

A1. (10 points) Let $A \subset \mathbb{R}^n$ and let $f : A \to \mathbb{R}^m$. Write $f = (f_1, f_2, f_3, \ldots, f_m)$ where $f_j = p_j \circ f$ is the j^{th} coordinate function of f (note that $p_j : \mathbb{R}^m \to \mathbb{R}$ is the j^{th} component function in $\mathbb{R}^m$ as defined on p. 279 of Fitzpatrick). Let $\mathbf{x}_*$ be a limit point of A and let $L = (\ell_1, \ell_2, \ldots, \ell_m) \in \mathbb{R}^m$. Prove that $\lim_{\mathbf{x}\to\mathbf{x}_*} f(\mathbf{x}) = L$ if and only if for each $j = 1, \ldots, m$, $\lim_{\mathbf{x}\to\mathbf{x}_*} f_j(\mathbf{x}) = \ell_j$.

A2. (8 points) Use the definition of derivative to compute the derivative of $f(x) = 1/(1+x^3)$ for $x \in \mathbb{R}$.

A3. (10 points) Let $f : [0,\infty) \to \mathbb{R}$ be defined by $f(x) = \sqrt{x}$ for $x \geq 0$. Find $f'(x)$ for $x > 0$ using the definition of derivative.

A4. (10 points) Let $f : [a,b] \to \mathbb{R}$ be continuous on $[a,b]$ and differentiable on (a,b). Assume f has k zeros (i.e., k points $x_1 < x_2 < \cdots < x_k$ in $[a,b]$ such that $f(x_j) = 0$ for $j = 1, 2, \ldots, k$).
Prove that f' has at least $k-1$ zeros in (a,b) (i.e., $k-1$ points $c_1 < c_2 < \cdots < c_{k-1}$ in (a,b) such that $f'(c_j) = 0$ for $j = 1, 2, \ldots, k-1$).

A5. (12 points) p. 108: # 1acd (provide either a mathematical justification or counterexample and explain why it is a counterexample)

(*Turn over for Part B.*)

Part B: Please put the following problems (labeled Bx.) in your Blue homework folder (marked with a B).

B1. (10 points) §4.1, p. 95: # 10.

B2. (10 points) §4.2, p. 101: # 5.

B3. (10 points) §4.2, p. 101: # 9.

B4. (10 points) §4.3, p. 108: # 6.

B5. (10 points) §4.3, p. 109: # 9.