Ch 11. Externalities

2 agent partial eq^m model:

p = price of good, $p \in \mathbb{R}^L$, L goods.

consumer i's wealth is w_i.

utility fn of i is $u_i(x_{1i}, x_{2i}, \ldots, x_{Li}, h)$

$x_1, \ldots, x_L$ goods, h externality, $h \in \mathbb{R}^+$.

h action taken by agent 1.

But $\dfrac{\partial u_2}{\partial h} \neq 0$. (+ve or neg ext.).

Indirect utility fn $v_i(p, w_i, h) = \max\limits_{x_i \geq 0} u_i(x_i, h)$

s/t $p \cdot x_i \leq w_i$.

Assume quasi-linear utility,

$v_i(p, w_i, h) = \phi_i(p, h) + w_i$

Since p is unaffected by h, we can write

$\phi_i(h)$, with $\phi_i''(h) < 0$.

If consumers were firms, $\phi_i(h)$ will be replaced by $\pi_i(h)$.

Comp Eq^m w/ Externality:

Each agent max utility subject to p & w_i.

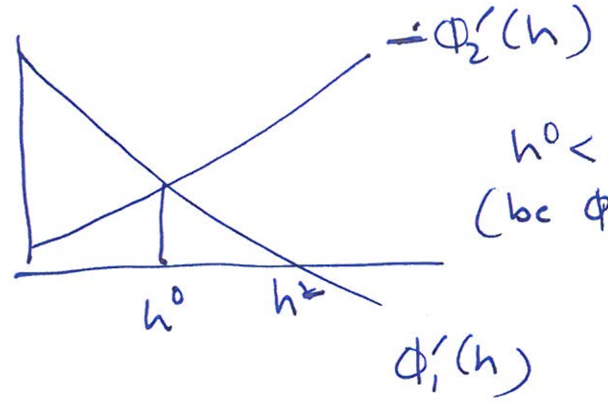
$\Rightarrow$ $\phi_1'(h^*) \leq 0$ (= if $h^* > 0$).

Optimal level of h:

$$\max_{h\geq 0} \quad \phi_1(h)+\phi_2(h)$$

$\Rightarrow \quad \phi_1'(h^0) \leq -\phi_2'(h^0) \quad (= \text{ if } h^0>0).$

If $\phi_2'(h^0)<0$, Then $\phi_1'(h^0) = -\phi_2'(h^0)>0$.



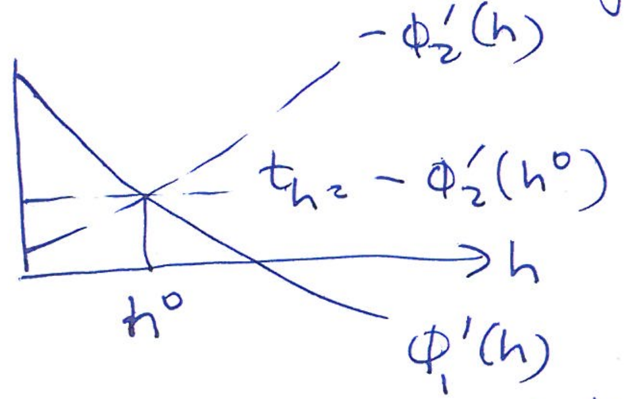
$h^0<h^*$.
(bc $\phi_1'(h^*)=0$)

If $\phi_2'(h)>0$ (+ve ext)
$\phi_1'(h^0) = -\phi_2'(h^0)<0$

<u>Traditional</u> <u>Solutions</u>

<u>Quotas</u> & <u>Taxes</u>:

1. Mandate $h\leq h^0 \Rightarrow h=h^*$.
2. Tax the externality (Pigouvian taxes)



Then 1's problem is

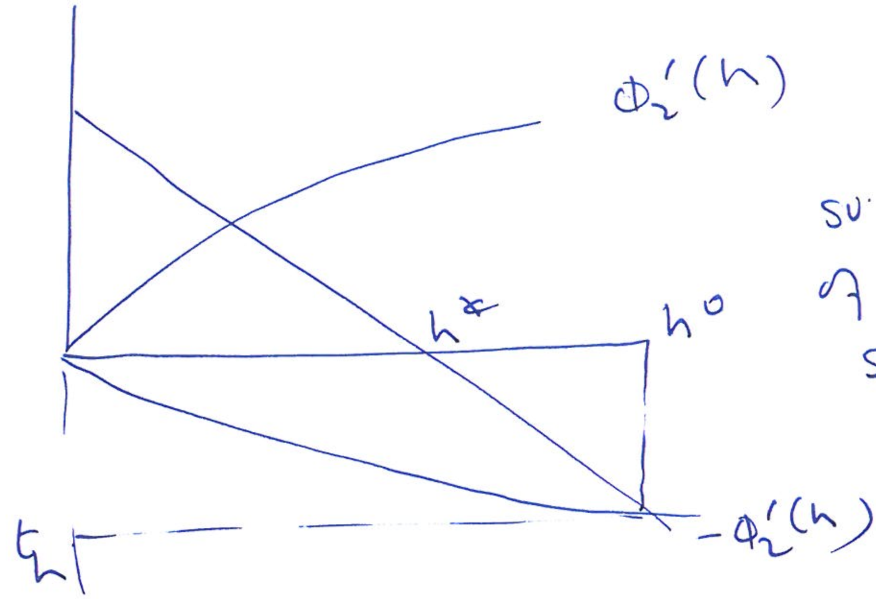
$$\max_{h\geq 0} \phi_1(h) - t_h \cdot h$$

$\Rightarrow \phi_1'(h) \leq t_h \; (= \text{ if } h>0)$

To implement h^0, set $t_h = -\phi_2'(h^0)>0$.

tax = MEC at h^0.

For +ve externality, $t_h = -\phi_2'(h^0)<0$ is a subsidy.



For neg ext, govt can pay subsidy for every unit of h below h^*, $s_h = -\Phi_2'(h^0) > 0$.

1 $\max_h \quad \Phi_1(h) + s_h(h^* - h)$

$\Rightarrow \quad \Phi_1(h) - s_h h + \underbrace{s_h h^*}_{\text{payment}}$

$\Rightarrow \quad \Phi_1'(h) = s_h$
$\qquad = -\Phi_2'(h^0)$

Subsidy + lumpsum transfer = tax on h^0.
on $h^* - h$

Tax, subsidy, quota all equivalent.
taxing output does not work. eg. fisheries.

Taxes, quotas yield same outcome, but information requirements are high.

Bargaining: 2 has rights to "externality-free" environment. 2 makes 1 a take-it-or-leave-it offer. 2 demands T to generate h.
1 accepts iff $\Phi_1(h) - T \geq \Phi_1(0)$

2 chooses (h, T) to solve

$$\max_{h \geq 0, T} \Phi_2(h) + T \quad \text{s/t} \quad \Phi_1(h) - T \geq \Phi_1(0)$$

So given 1 $\Phi_1(0) \Rightarrow \Phi_1(h) - T = \Phi_1(0)$

2's optimal offer solves

$$\max_{h \geq 0} \Phi_2(h) + \Phi_1(h) - \Phi_1(0)$$

$$\Rightarrow \quad \Phi_2'(h) + \Phi_1'(h) \leq 0$$ same as h^0.

1 has rights to pollute:

1 produces at h^*.

2 pays $T \lessgtr 0$ for $h < h^*$.

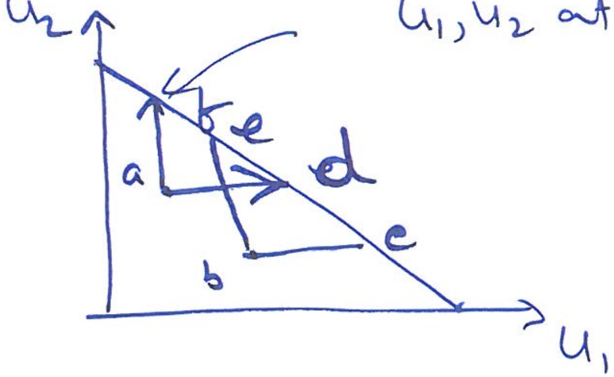
1 agrees iff $\Phi_1(h) - T \geq \Phi_1(h^*)$ $(T < 0)$

2 solves $\max_{h, T} \Phi_2(h) + T$

$$\Rightarrow \quad \max_{h \geq 0} \Phi_2(h) + \Phi_1(h) - \Phi_1(h^*) \quad \Rightarrow h^0.$$

Property rights affect final wealth but not allocation (Coase Theorem)

In Case 1.



a = ext level $h = 0$

b = " " $h = h^c$

2 has barg power [f, e]

1 has power [d, c]

Other bargaining procedures may yield solutions in set [f, d] & [e, c].

In both cases we examined, 2 had power and put 1 to reservation utility.

Imp Property rights must be well-defined.

Consumers need to know each other's preferences but govt need not.

- mergers of firms if both agents are firms internalizes the externality.
- Externality needs to be measured ⇒ include in costs.

Markets for pollution

Markets to pollute: let p_h be price of a permit to pollute; 2 has right to clean env^t. Then 1 solves

$$\max_{h_1 \geq 0} \phi_1(h_1) - p_h \cdot h_1$$

$$\Rightarrow \quad \phi_1'(h_1) \leq p_h \quad (= \text{ if } h_1 > 0).$$

2 solves: how many to sell:

$$\max_{h_2 \geq 0} \phi_2(h_2) + p_h h_2$$

$$\Rightarrow \quad \phi_2'(h_2) \leq -p_h \quad (= \text{ if } h_2 > 0)$$

Comp Eq^m: mkt must clear

$$\Rightarrow \quad h_1 = h_2.$$

$$\Rightarrow \quad \phi_1'(h^{**}) \leq p_h \leq -\phi_2'(h_2^{**})$$
$$(= \text{ if } h_2^{**} > 0),$$

$$\Rightarrow \quad h^{**} = h^0$$
$$p_h^* = \phi_1'(h^0) = -\phi_2'(h^0).$$

Their utilities: $\phi_1(h^0) - p_h^* h^0$

$\phi_2(h^0) + p_h^* h^0.$

Mkt solves the ext. problem.

We need <u>many</u> agents for CE to work.

Public Goods

They are non depletable.

Exclusion not possible or costly.

Many agents consume one unit.

L goods, 1 public good

quasi-linear utility

$\phi_i(x)$, $\phi_i''(x) < 0$.

$c(q)$ cost of supplying q units.

Desirable public good (public bad)

$\phi_i'(x) > 0$, $c'(\cdot) > 0$.

(Public bad: $\phi_i'(\cdot) < 0$, $c_i'(\cdot) < 0$)

abatement costly

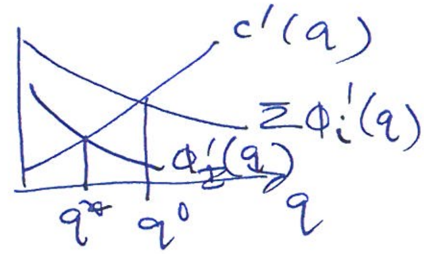
P-O max Aggr surplus:

$$\max_{q \geq 0} \sum_i \phi_i(q) - c(q)$$

$$\Rightarrow \sum_{i=1}^{I} \phi_i'(q^0) \leq c'(q^0). \quad (= \text{ if } q^0 > 0)$$

$$\sum_i MB_i = MC$$

(Samuelson condition)



Inefficiency:

Each firm maximizes

$$\max_{x_i} \ \phi_i(x_i + \sum_{k \neq i} x_k^*) - p^* \cdot x_i$$

$$\Rightarrow \quad \phi_i'(x_i^* + \sum_{k \neq i} x_k^*) \leq p^* \quad (= \text{ if } x_i^* > 0)$$

Let $x^* = \sum_i x_i^*$, $\therefore \quad \phi_i'(x^*) \leq p^* \quad (= \text{ if } x_i^* > 0)$.

Firm supply: $\max_q \ p^* q - c(q)$

$$\Rightarrow \quad p^* \leq c'(q^*). \quad (= \text{ if } q^* > 0)$$

Inefficiency $\Rightarrow$ $q^* < q^0$. (See graph)

Free rider problem.

Taxes / Subsidies — what is optimal subsidy?

$$\max_{x_1} \ \phi_1(x_1 + \tilde{x}_2) + s_2 x_1 - \tilde{p} x_1$$

$$\Rightarrow \quad \phi_1'(\tilde{x}_1 + \tilde{x}_2) + s_1 \leq \tilde{p}.$$

$$\Rightarrow \quad \phi_1'(\tilde{x}) + s_1 \leq \tilde{p}.$$

If $s_2 = \phi_2'(q^0)$ Then

$$\phi_1'(\tilde{x}_1) + \phi_2'(q_0) \leq \tilde{p}$$

$\Rightarrow$ $\tilde{x}_1$ same condition as

$$\sum \phi_i'(q^0) \leq \tilde{p}.$$

Lindahl Equilibria

"Personalized" prices of public goods can work in a market.

Each consumer's consumption is a distinct good:

$$\max_{x_i} \; \phi_i(x_i) - p_i^{**} x_i$$

Each person is charged (different) price p_i^{**}

$$\Rightarrow \quad \phi_i'(x_i^{**}) \le p_i^{**}$$

Firm produces good q to solve

$$\max_{q \geq 0} \; \left(\sum_{i=1}^{I} p_i^{**} q\right) - c(q)$$

$$\Rightarrow \quad \sum_{i=1}^{I} p_i^{**} \le c'(q^{**})$$

$\therefore$ Mkt clearing: $\sum \phi_i'(q^{**}) \le c'(q^{**})$

$\Rightarrow$ Each person consumes efficient or q° amount.

Difficult to implement: must be able to exclude.

Multilateral Externalities:

Many agents. (generate or feel)

depletable or non-depletable

(garbage) (air pollution climate change)

↓ ↓

Mkts work Mkts dont work

Assume: firms pollute & consumers feel.

homogeneous externalities: indiff to source of origin.

Depletable

J firms generate externality

$h_j \geq 0$, Profits $\pi_j(h_j)$, $\pi_j''(h_j) < 0$

I consumers with quasi-linear utility fns

$\phi_i(\tilde{h}_i)$, $\phi_i'' < 0$. $\phi_i'(\cdot) < 0$, neg externality

P-O allocation: $(\tilde{h}_1^o, \ldots, \tilde{h}_I^o; h_1^o, \ldots h_J^o)$ solves

$$\max_{\substack{(\tilde{h}_1, \ldots, \tilde{h}_I) \\ (h_1, \ldots, h_J)}} \sum_I \phi_i(\tilde{h}_i) + \sum_J \pi_j(h_j)$$

$$\text{s.t. } \sum_J h_j = \sum_I \tilde{h}_i \qquad (1)$$

$$\Rightarrow \sum_I \phi_i(\tilde{h}_i) + \sum_J \pi_j(h_j) + \lambda\left[\sum_J h_j - \sum_I \tilde{h}_i\right] \qquad (2)$$

$$\Rightarrow \quad \phi_i'(\tilde{h}_i^o) \leq \lambda \qquad (3)$$

$$\lambda \leq -\pi_j'(h_j^o)$$

①, ②, ③ determine allocation.

Same as pvt good: $MB = MC$.

At CE, each firm chooses h_j s.t.

$$\pi_j'(h_j^*) \le 0 \quad (= \text{ if } h_j^* > 0).$$

Non depletable Externalities:

Each agent feels $\sum_j h_j$ (total pollution)

CE (same): $\pi_j'(h_j^*) \le 0$.

P-O allocation: $(h_1^0, \ldots, h_J^0)$ solves

$$\max_{h_1, \ldots, h_J} \sum_{i=1}^{I} \phi_i\Big(\sum_j h_j\Big) + \sum_{j=1}^{J} \pi_j(h_j)$$

$$\Rightarrow \sum_{i=1}^{I} \phi_i'\Big(\sum_j h_j^0\Big) \le -\pi_j'(h_j^0) \quad (= \text{ if } h_j^0 > 0).$$

Same as public good $\sum MB \le MC$.

Not bilateral externality, so public good problem appears. There will be free-riding & too much pollution.

Given perfect information, quotas and taxes can be used.

Quota: Set $h_j \le h_j^o \ \forall j \in \{1, \dots, J\}$

Tax: Set $t_h = -\sum \phi_i'(\sum_j h_j^o)$ per unit externality. Firm j solves

$$\max_{h_j} \ \hat{\pi}_j(h_j) - t_h \cdot h_j$$

$$\Rightarrow \quad \hat{\pi}_j'(h_j) \le t_h = \sum_i \phi_i'(\sum_j h_j^o) \quad \text{ensures}$$

$$h_j = h_j^o \ \forall j.$$

Tradable Permits: Specify a quota & distribute as tradable permits.

Each permit = one unit of electricity

Let $h^o = \sum_j h_j^o$ permits distributed.

Firm j receives $\overline{h_j}$ permits.

Eqm price of permits p_h^*. Firm buys h_j.

Solves $\max_{h_j} \ \hat{\pi}_j(h_j) + p_h^*(\overline{h_j} - h_j)$

$$\Rightarrow \quad \hat{\pi}_j'(h_j) \le p_h^* \quad (= \text{if } h_j > 0).$$

Mkt clearing $\sum_j h_j = h^o$.

Recall $-\hat{\pi}_j'(h_j^o) = \sum_i \phi_i'(\sum_j h_j^o)$ so

$$p_h^* = -\sum_i \phi_i'(h^o).$$

Each firm uses h_j^o permits, sells $(\overline{h_j} - h_j^o)$

Adv : Firm has aggr info on h^0 but not on firm j.

Private Information

Consumer's utility fn is $\phi(h, \eta)$, $\eta \in \mathbb{R}$ a parameter, consumer type

$\pi(h, \theta)$, $\theta \in \mathbb{R}$ firm type

θ, η privately observed.

prob dist of θ, η publicly known.

$\pi(h, \theta)$ and $\phi(h, \eta)$ concave in ~~θ, η~~ h, fixed θ, η.

Decentralized Bargaining:

Only 2 levels of h: 0 & $\bar{h} > 0$.

Neg externality.

Let $b(\theta) = \pi(\bar{h}, \theta) - \pi(0, \theta) > 0$. Firm benefit

$c(\eta) = \phi(0, \eta) - \phi(\bar{h}, \eta) > 0$ consumer's cost

Let $G(b)$ & $F(c)$ be prob dist of b & c, generated by θ & η; b & c are indep.

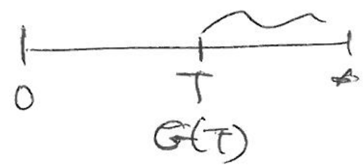
Let the density fns be $g(b)$ & $f(c)$ with $g(b) > 0$ and $f(c) > 0$ $\forall\ b > 0, c > 0$.

Let consumer has right to clean env't.
She likes $h=0$. But if
$b>c$, P-O $\Rightarrow$ $h=\bar{h}$.

Bargaining solution:

Firm agrees to ban $T>0$ if $b \geq T$.
Consumer sets T s.t. ~~Prob~~ firm accepts, ie
Prob $(b \geq T)$. Prob firm accepts if $1-G(T)$

Consumer solves

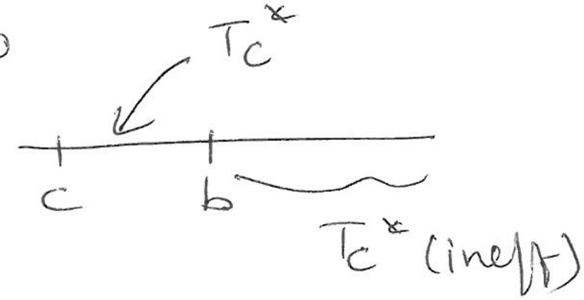


$$\max_T \; (1-G(T))(T-c)$$

$$\Rightarrow \quad 1-G(T)+(T-c)(-G'(T))=0$$
$$[1-G(T)]=(T-c)\,g(T)$$

Soln: If $1-G(T) > 0$
So if $T>c$, obj fn >0
$T=c$, $=0$

$\therefore$ Soln $T_c^* > c$.



But if b s.t. $c<b<T_c^*$,
firm will reject offer so $h=0$.
Even though P-O $\Rightarrow$ $h=h^*$.

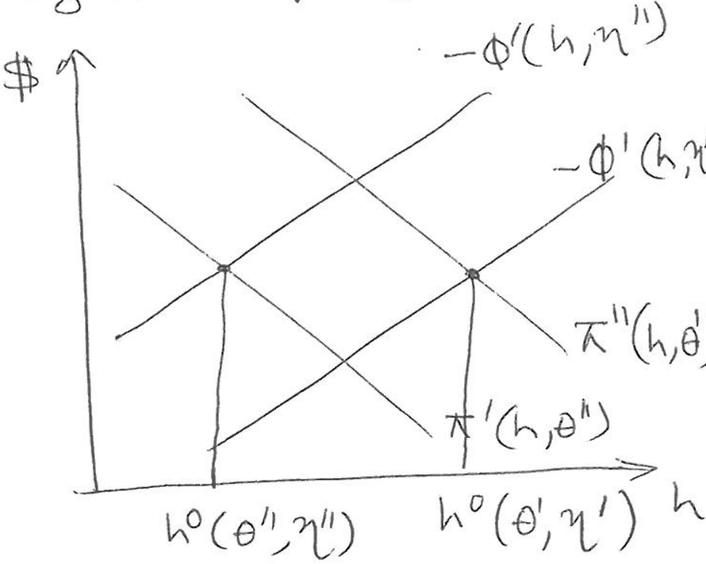
Asymmetric info may not lead to P-O outcome.

Quotas & Taxes

Aggr Surplus = $\phi(h,\eta) + \pi(h,\theta)$

Optimal h depends on realization of η & θ.

let $h^\circ(\theta,\eta)$ See fig.



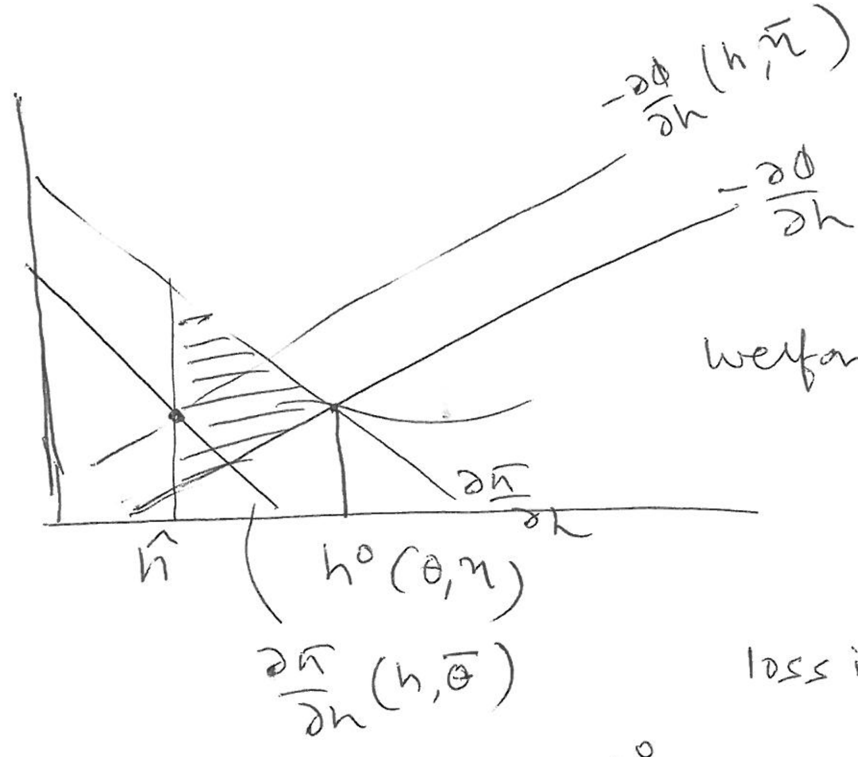
Quota fix $h = \hat{h}$.

Firm chooses

$$\max_h \pi(h,\theta)$$

s/t $h \le \hat{h}$.

$\Rightarrow$ soln: $h^q(\hat{h},\theta)$.



Quota set with mean values $\bar{\theta}, \bar{\eta}$

Welfare loss

Quota $\hat{h}$

P-O h^o

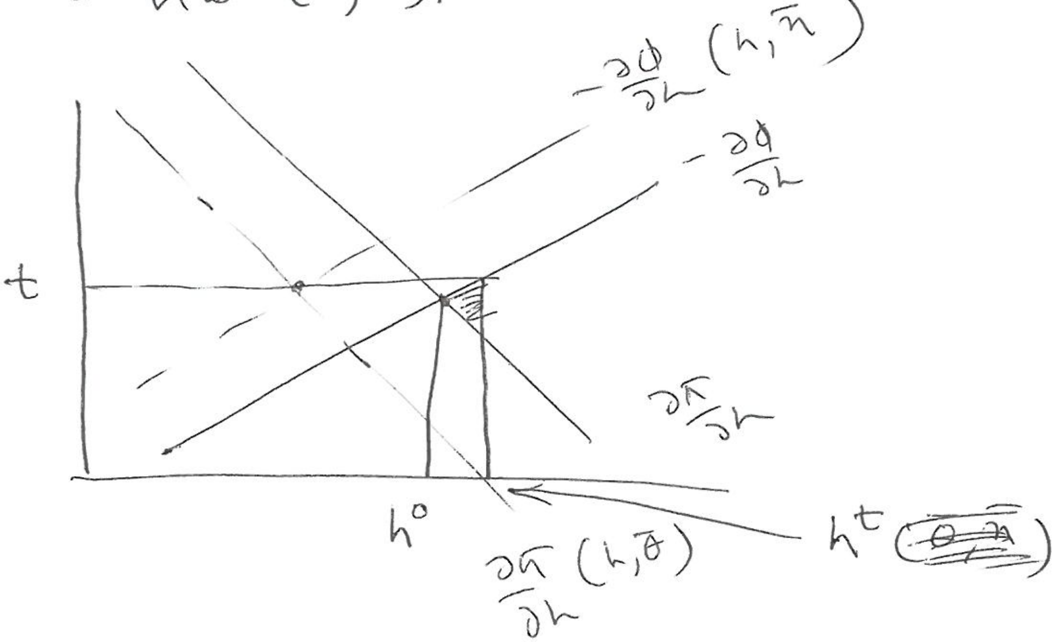
loss in aggr surplus

$$= \int_{\hat{h}}^{h^o} \left(\frac{\partial\pi}{\partial h} + \frac{\partial\phi}{\partial h}\right) dh.$$

Tax : \$t / unit of h:

Firm chooses $\max_h \bar{\pi}(h,\theta) - th$

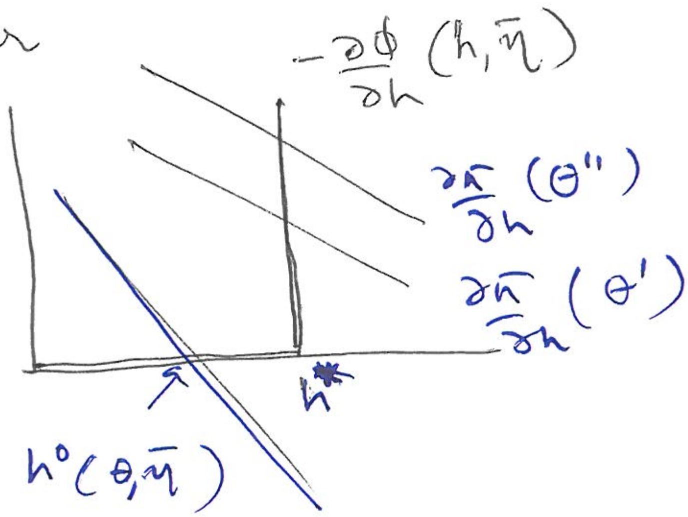
$\Rightarrow h^t(t,\theta)$.



Both tax and quota respond to firm MB, not consumer MD.

Which is better, tax or quota?

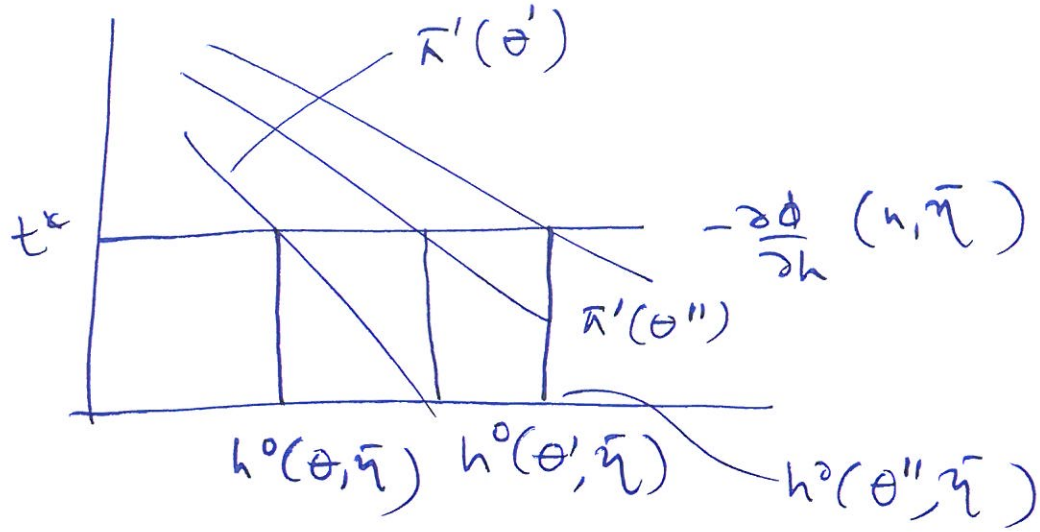
If consumer MD is insensitive, then quota better



Set quota $\hat{h} = h^*$.

tax

MD indep^t of h:



tax at t^* better than quota.

Its relative:

c sensitive
F not
} Quota

c not
F sensitive
} tax.

General Policy Mechanisms

Revelation Mechanism: 2 levels $0, \bar{h}$. $h = \{0, \bar{h}\}$.

Firm announces $\hat{b}$.

Consumer " $\hat{c}$.

Govt announces: $h = \hat{h}$ iff $\hat{b} > \hat{c}$.

tax firm $\hat{c}$, subsidize consumer $\hat{b}$.

This is a truth-telling mechanism.

Sps consumer's true type is c.

If $\hat{b} > c$, consumer wants $\underline{h = \hat{h}}$ since

$$\text{Utility} = \begin{cases} h = 0 & 0 \\ h = \hat{h} & \hat{b} - c > 0. \end{cases}$$

$\therefore$ optimal announcement $\hat{c} < \hat{b}$.

$\therefore$ $\hat{c} = c < \hat{b}$.

If $\hat{b} \le c$, consumer wants $h = 0$ since

$$\text{Utility} = \begin{cases} h = 0 & 0 \\ h = \bar{h} & \hat{b} - c \le 0. \end{cases}$$

$\therefore$ consumer wants $h = 0$. $\therefore$

$\hat{c} = c \geq \hat{b} \Rightarrow$ ✓

$\therefore \forall \hat{b}$, TT optimal for consumer.

Do it for firm. (Groves-Clark mechanism).

Problem: Budget may not balance

if $b > c$. deficit $= b - c > 0$.