EC204 Micro II 2020 Spring

HW2 2020/02/05
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1. Find all pure and mixed strategy Nash Equilibria for the following game:

		P2	
P1		L	R
	U	2,1	0,0
	D	1,0	0,2

Solution:

1) PSNE: There are two PSNE (U, L) and (D, R) because there is no profitable deviation for both players in either situation.
2) MSNE: (0, 2/3)
   Suppose P1 play U and D with probability *q* and *(1-q)* respectively, P2 play L and R with probability *p* and *(1-p)* respectively.
   When P1 is indifferent between U and D, we have
   $$u_1(U,(p,1-p)) = u_1(D,(p,1-p))$$
   $$\Rightarrow 2p = p \quad \Rightarrow p = 0$$
   When P2 is indifferent between L and R, we have
   $$u_2(L,(q,1-q)) = u_2(D,(q,1-q))$$
   $$\Rightarrow q + 0 * (1-q) = 0 + 2(1-q) \quad \Rightarrow q = 2/3$$
   Thus, we have a MSNE (0, 1, 2/3, 1/3), in short we write (0, 2/3)

2. Consider the following game with two players. There is a dollar on the table. If both players try to grab it, both get fined one dollar each. If only one person grabs it, s(he) gets the dollar. If no one grabs it, each gets nothing.
(a) Construct the normal form and find NE in pure and mixed strategies.

			p	1-p
			player 2	
			G	NG
q		G	-1, -1	1, 0
1-q	player 1	NG	0, 1	0, 0

Solution:

PSNE: (NG, NG), since there is no profitable deviation for both players.
MSNE: (1/2, 1/2)

For player 1,
$$u_1(G,(p,1-p)) = u_1(NG,(p,1-p))$$
$$\Rightarrow -p + 1 - p = 0 \quad \Rightarrow p = 1/2$$
For player 2, we have
$$u_2(G,(q,1-q)) = u_2(NG,(q,1-q))$$
$$\Rightarrow -q + 1 - q = 0 \quad \Rightarrow q = 1/2$$
Thus we have a MSNE (1/2, 1/2, 1/2, 1/2), in short we write (1/2, 1/2)

3. Consider the following 2 player game.

		P2	
		a	b
P1	c	12,2	3,9
	d	5,8	4,2

a. Are there any weakly or strictly dominated strategies?
b. Are there any pure strategy NE?
c. Are there any mixed strategy NE?
d. Now suppose we change the payoff of player 2 when she plays a and player 1 plays d, from 8 to 6.
e. Re-compute all the pure and mixed strategy NE. Compare the answers from the previous version of the game and comment.

Solution:

a. <u>There is no weakly or strictly dominated strategies.</u>
Since the utility gained from player 1's strategy *c* when player 2 plays a is 12, which is greater than the utility gained from player 1's strategy *d (5).* However, the utility gained from player 1's strategy *c* when player 2 plays *b* is 3, which is less than the utility gained from player 1's strategy *d (4).* Thus, there is no weakly dominated strategies or strictly dominated strategies for player 1.
Apply the same logic we come to the conclusion that there is no weakly dominated strategies or strictly dominated strategies for Player 2.

b. <u>There is no pure strategy NE.</u>
To see this, if P1 knows P2 plays a, P1 will play c. If P2 knows that P1 knows that P2 plays a, then P2 will play b. Apply this logic, when we are at (c, b), P1 will want to play d, then we move to (d, b). When we are at (d, b), P2 will want to play a then we move to (d, a). when we are at (d, a), P1 will have an incentive to play c so we are again at (c, a). Thus, there is no PSNE.

c. Yes<u>. (1/8, 6/13)</u>
Suppose P1 play *c* and *d* with probability *q* and *(1-q)* respectively, P2 play *a* and *b* with probability *p* and *(1-p)* respectively

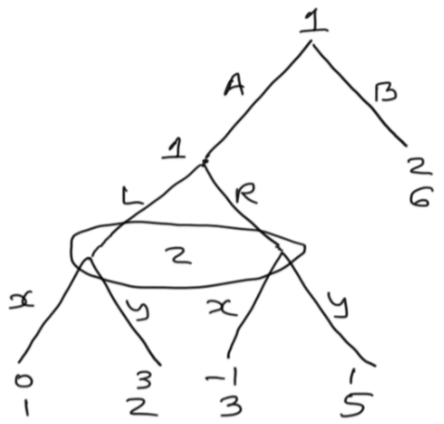
$$u_1(c,(p,1-p)) = u_1(d,(p,1-p))$$
$$\Rightarrow 12p + 3 - 3p = 5p + 4 - 4p \Rightarrow p = 1/8$$
$$u_2(a,(p,1-p)) = u_2(b,(p,1-p))$$
$$\Rightarrow 2q + 8 - 8q = 9q + 2 - 2q \Rightarrow q = 6/13$$

Thus, we have a MSNE (1/8, 7/8, 6/13, 7/13). In short (1/8, 6/13)

d, e. <u>There is still no PSNE.</u> The same logic from (b) applies here.
Use the same method from (c) we compute a MSNE (1/8, 4/11)

When the payoff of player 2 when she plays a and player d decreases, she will assign a lower probability to strategy a (from 2/5 to 4/11) and assign a higher probability to strategy b so she can be indifferent from P2's strategies a and b.

4. Consider the following game of imperfect information:



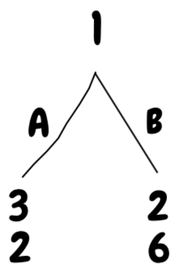
a. Find the subgame perfect Nash Equilibrium. Are there other NE that are not subgame perfect?

Solution:

a. <u>SPSE: (A, L, y)</u>
We use backward induction, first we solve the subgame. The normal form is the following.

			p	1-p
			player 2	
			x	y
q		L	0, 1	3, 2
1-q	**player 1**	R	-1, 3	1, 5

We see that (L, y) is a PSNE because there is no profitable deviation for both players.

Now we look at this 

, player 1 will choose strategy A because the payoff she gets from A is 3, which is greater than 2, the payoff she gets from playing B.

b. Other NE (not sub perfect): (1/3, 0)

The normal form of the game is the following

			p	1-p
			player 2	
			x	y
q1		AL	0, 1	3, 2
	player 1	AR	-1, 3	1, 5
q2		BL	2, 6	2, 6
1-q1-q2		BR	2, 6	2, 6

First, we observe that AR will never be played since it is strictly dominated by BL and BR.

Suppose P1 play *AL, BL and BR* with probability *q1*, q2 and 1-q1-q2 respectively, P2 play *x* and *y* with probability *p* and *(1-p)* respectively

$$u_1(x,(p,1-p)) = u_1(y,(p,1-p))$$
=> $0+3(1-p) = 2P+2-2p = 2P+2-2p$ => p = 1/3

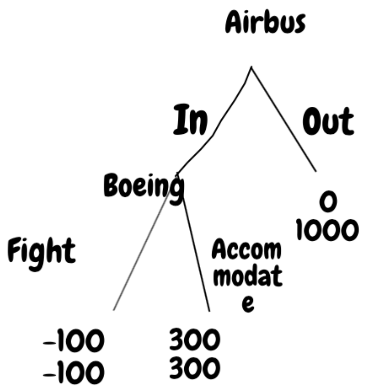
$$u_2(x,(q1,q2,1-q1-q2)) = u_1(y,(q1,q2,1-q1-q2))$$
=> $q1+6q2+6(1-q1-q2) = 2q1+6q2+6(1-q1-q2)$ => q1 = 0

Thus, we have a MSNE (1/3, 2/3, 0, q*, 1-q*) where q* is in [0,1]
In short, we have a MSNE (1/3, 0)

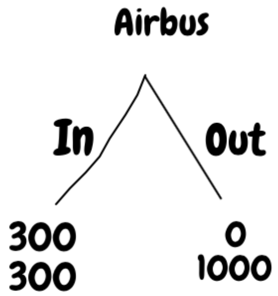
5. Airbus wants to enter the market for Dreamliner aircraft now only made by Boeing. If Airbus stays out, it earns zero profit and Boeing makes $1 billion. If Airbus enters the market, Boeing can Fight it by waging a price war or Accommodate and do nothing. Fighting will generate losses for each company worth 100 million. Accommodating will yield profits of 300 million each. This is a game of perfect information.

(a) Draw the extensive form game and solve for the subgame perfect NE.
(b) Are there other NE in pure and mixed strategies?

(a) The extensive form is like the following
SPNE: (In, Accommodate)



We use backward induction. First, we solve the subgame, obviously in the subgame, Boeing will choose to play *Accommodate* because 300 > -100.
Then Airbus will play *In* since 300 > 0.



(b) First, we show the normal form of this game

		Boeing	
		Fight	Accommodate
	In	-100, -100	300, 300
Airbus	Out	0, 1000	0, 1000

PSNE (not SP): (Out, Fight). It is easy to see that when we are at (Out, Fight), there is no profitable deviation for both players. (Although this does not make sense)

MSNE: (3/4, 0)

Suppose Airbus play *In and Out* with probability *q and 1- q* respectively, Boeing play *Fight* and *Accommodate* with probability *p* and *(1-p)* respectively

$$u_A\big(In, (p, 1-p)\big) = u_A(Out, (p, 1-p))$$
$$\Rightarrow -100p + 300 - 300p = 0 \Rightarrow p = 3/4$$

$$u_B\big(Fight, (q, 1-q)\big) = u_B(Accommodate, (q, 1-q))$$
$$\Rightarrow -100q + 1000 - 1000q = 300q + 1000 + 1000q \Rightarrow q = 0$$

Thus, we have a MSNE (3/4, 1/4, 0, 1)

In short, we write it as (3/4, 0)