EC204 Micro II HW7. GE & Labor Market Model

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1. Two agents. with utility

$$V_1(P_1, P_2, y) = \ln y - a\ln P_1 - (1-a)\ln P_2$$

$$V_2(P_1, P_2, y) = \ln y - b\ln P_1 - (1-b)\ln P_2$$

with initial endowment for 1 & 2 is

$$\omega_1 = (x=1, y=1), \quad \omega_2 = (x=1, y=1)$$

Calculate market clearing prices

Answer: The demand for the good 1 is (from Micro I)

For
$$\begin{cases} \text{Agent 1:} & x^1(P_1, P_2, y) = \dfrac{ay}{P_1} \\ \text{Agent 2:} & x^2(P_1, P_2, y) = \dfrac{by}{P_1} \end{cases}$$

The wealth of Agent 1 & 2:

$$y = P_1 + P_2 \quad \& \quad y = P_1 + P_2.$$

The aggregated demand is

$$x^1 + x^2 = \frac{ay + by}{P_1} = \frac{(a+b)(P_1+P_2)}{P_1}$$

$$= a+b + \frac{(a+b)P_2}{P_1}$$

since the aggregated supply is 2

$$\Rightarrow a+b+(a+b)\frac{P_2}{P_1} = 2$$

$$\frac{P_2}{P_1} = \frac{2}{a+b} - 1.$$

2. Consider an economy with 15 consumers and 2 goods. c3 has an utility func.

$$U_3(X_3^1, X_3^2) = \ln X_3^1 + \ln X_3^2$$

At a certain Pareto-efficient allocation X^* c^3 holds $(10, 5)$. What are the competitive prices that support x^*?

Answer: We know in equilibrium

$$MRS_3^{xy} = -\frac{MU_x}{MU_y} = -\frac{\frac{1}{X_3^1}}{\frac{1}{X_3^2}} = -\frac{X_3^2}{X_3^1}$$

$$= -\frac{P_1}{P_2}$$

Since we also know $x_3^1 = 10$, $x_3^2 = 5$.

$$\Rightarrow \frac{-5}{10} = -\frac{P_1}{P_2}$$

$$\Rightarrow \frac{P_1}{P_2} = \frac{1}{2}$$

3. $U_A = x_1 + x_2$

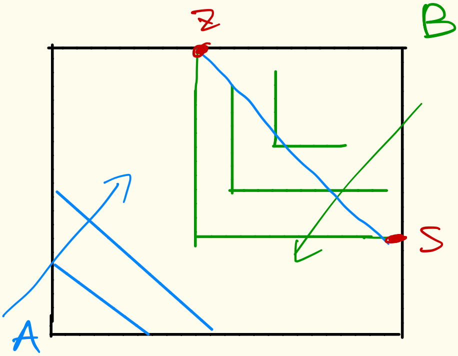
$U_B = \max\{x_1, x_2\}$

A & B have identical endowments $(\frac{1}{2}, \frac{1}{2})$

- Find the eq relationship between P_1 & P_2
- what is the eq allocation

Answer:

①



We see that there will be corner solution

Also, both A & B weigh good 1 & 2 equally

$\Rightarrow P_1 = P_2$.

② Since we have corner sol.

we are at either Z or S.

$\Rightarrow$ One person holds $(0, 1)$

and the other holds $(1, 0)$

4. An economy. 2 firms 2 consumers

C1 owns F1. It produces guns $g = 2x$.

C2 owns F2. It ——— butter $b = 3x$.

C1 & C2 each own 10 barrels of oil.

$U_{c_1}(g,b) = g^{0.4} b^{0.6}$. $U_{c_2}(g,b) = 10 + 0.5\ln g + 0.5\ln b$.

- Find market clearing prices
- How many guns / butter each consumes.
- How much oil each use?

Answer: ① F1 maximizes

$$\underbrace{P_g}_{\text{Price of } g} \cdot \underbrace{2y_{o_1}}_{\text{Produced quantity}} - P_o \cdot y_{o_1}$$

Price of oil

oil that F1 owns.

$$= (2P_g - P_o) \cdot y_{o_1}$$

F2 maximizes

$$(3P_b - P_o) \cdot y_{o_2}.$$

Since $U_{c_1}(g,b) = g^{0.4} b^{0.6}$

By Lagrange

$$\Longrightarrow L(g_1, b_1, \lambda) = \alpha \ln g^{0.4} + (1-\alpha)\ln b^{0.6} - \lambda \cdot (P_g \cdot g + P_b \cdot b - w_1)$$

$$\xRightarrow{F.O.C} P_g \cdot g = \frac{\alpha}{1-\alpha} P_b \cdot b$$

$\Rightarrow$ C^1's demand is $\left(0.4 \cdot \frac{W_1}{P_g}, 0.6 \frac{W_1}{P_b}\right)$

C^2's demand is $\left(0.5 \frac{W_2}{P_g}, 0.5 \frac{W_2}{P_g}\right)$

Also, $w_1 = w_2 = 10 P_o$

$$\Rightarrow \left(0.4 \frac{w_1}{P_g} + 0.5 \frac{w_2}{P_g},\ 0.6 \frac{w_1}{P_b} + 0.5 \frac{w_2}{P_b} \right)$$

$$= (\underbrace{y_{g_1}}_{\text{quantity produced}}, \overset{=0}{y_{b_1}}) + (\overset{=0}{y_{g_2}}, y_{b_2})$$

Since $y_{b_1} = y_{g_2} = 0$

$$\Rightarrow y_{g_1} = 0.4 \times \frac{10 P_o}{P_g} + 0.5 \times \frac{10 P_o}{P_g} = \frac{9 P_o}{P_g}$$

$$y_{b_2} = \frac{11 P_o}{P_b}$$

Also, Firm maximizes profit $\Rightarrow P_o = 2 P_g$, $P_o = 3 P_b$.

(2) C1 consumes $\left(\frac{2 \times 10 P_o}{5 \cdot \frac{1}{2} P_o}, \frac{3 \times 10 P_o}{5 \cdot \frac{1}{3} P_o} \right) = (8, 18)$

C2 consumes $\left(\frac{10 P_o}{2 \cdot \frac{1}{2} P_o}, \frac{10 P_o}{2 \cdot \frac{1}{3} P_o} \right) = (10, 15)$

(3) $y_{g_1} = \frac{9 P_o}{P_g} = \frac{9 P_o}{\frac{1}{2} P_o} = 18.$

$y_{b_2} = \frac{11 P_o}{P_b} = \frac{11 P_o}{\frac{1}{3} P_o} = 33$

$y^1_{oil} = 18/2 = 9$, $\quad y^2_{oil} = 33/3 = 11$

5. MWG 13.B3

Consider a positive selection version of 13.B in which $r(\cdot)$ is a cont, strictly ↓ func of θ.
Let density of worker of type θ be $f(\theta)$ with $f(\theta) > 0$ for θ in $[\underline{\theta}, \bar{\theta}]$.

(a) show the more capable workers are the ones choosing to work at any given wage.

(b) show if $r(\theta) > \theta$ for all θ, the resulting competitive eq is pareto efficient.

(c) sps $\exists$ a $\hat{\theta}$ s.t $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$.
Show any competitive eq with strictly positive employment <u>necessarily</u> involves too much employment relative to the pareto allocation of workers.

Answer

(a). Sps the wage is w.
Thus individuals with $r(\theta) \le w$ will work
sps $\exists$ θ^* s.t $r(\theta) = w$.
more capable or high type
Thus people whose type higher than θ^* will work simply because $r(\theta) < r(\theta^*)$ $[r(\cdot) \downarrow \text{in } \theta]$

(b). if $r(\theta) > \theta$ for all θ.

Firm offers $\bar{\theta}$. but $r(\bar{\theta}) > \bar{\theta}$.

$\Rightarrow$ No one works, regardless of type

$\Rightarrow$ it is Pareto-efficient

(c) In the Pareto-efficient outcome, individuals with $\hat{\theta} \leq \theta \leq \bar{\theta}$ work.

The wage is $w = \hat{\theta} = r(\hat{\theta})$

If Firm offers $w = r(\hat{\theta})$.

$E[\theta \mid r(\theta) < w] > w$ since $r(\cdot) \downarrow$ in θ.

Thus, demand $>$ supply.

The firm wants to hire more workers.