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1. (MWG Ch.12 B1) $\dfrac{p^m - c'(q^m)}{p^m}$ monopolist's price-cost margin

(a) Show the monopolist's price-cost margin is always equal to the inverse of price elasticity of demand at price p^m.

The monopolist's problem is

$$\max_{p} \; p \cdot x(p) - c(x(p))$$

F.O.C $\Rightarrow$ $x(p) + x'(p) \cdot p - c'(x(p)) \cdot x'(p) = 0$

we have a solution p^m satisfying the above equality

$$\Rightarrow x(p^m) + x'(p^m) \cdot p^m - c'(x(p^m)) \cdot x'(p^m) = 0$$

$x(p^m)$ is just q^m.

$$\Rightarrow q^m + q'^m \cdot p^m - c'(q^m) \cdot q'^m = 0$$

$$[p^m - c'(q^m)] \cdot q'^m = -q_m$$

$$\frac{p^m - c'(q^m)}{p_m} = \frac{-q_m}{q'^m} \cdot \frac{1}{p_m}$$

$$= \frac{1}{\dfrac{p^m \cdot q^{m\prime}}{-q_m}}$$

(By definition, $e\langle p \rangle = \dfrac{dQ/Q}{dP/P}$) $= \dfrac{1}{e\langle p_m \rangle}$ $\square$

(b) Argue that if $c'(x(P)) > 0$ at every $x(P)$, then $e_{<p_m>} > 1$ at the optimal p^m.

From (a), we know at p^m

$$\frac{p^m - c'(q^m)}{p^m} = \frac{1}{e_{<p_m>}}$$

From (a) we know that at p^m

$$\frac{p^m - c'(q^m)}{p^m} = \frac{1}{e_{<p_m>}} < \frac{p^m}{p^m} = 1$$

$$\Rightarrow e_{<p_m>} > 1$$

Thus $e_{<p_m>}$ always great than 1 $\square$

B2. A monopolist with $C(q) = cq$ $(c>0)$, facing demand $X(P) = \alpha P^{-\varepsilon}$, $\varepsilon > 0$

(a) show if $\varepsilon \leq 1$ then the optimal price is not well defined

$c'(q) = c$

$\Rightarrow$ The monopolist's problem is

$$\max_{P} \; P \cdot x(P) - c \cdot x(P)$$

$$\Rightarrow \max_{P} \; (P - c) \cdot \alpha P^{-\varepsilon}$$

$$\xRightarrow{\text{F.O.C}} \alpha P^{-\varepsilon} + (-\varepsilon) \cdot \alpha \cdot P^{-\varepsilon-1} \cdot P - c \cdot \alpha P^{-\varepsilon-1} \cdot (-\varepsilon) = 0$$

$$\alpha P^{-\varepsilon} - \varepsilon \cdot \alpha P^{-\varepsilon} + c \cdot \alpha P^{-\varepsilon-1} \cdot \varepsilon = 0$$

$$(1-\varepsilon) \alpha P^{-\varepsilon} + c \alpha P^{-\varepsilon-1} \cdot \varepsilon = 0$$

$$(1-\varepsilon) \alpha P^{-\varepsilon} = -c \alpha P^{-\varepsilon-1} \cdot \varepsilon$$

$$(1-\varepsilon) P = -c \varepsilon$$

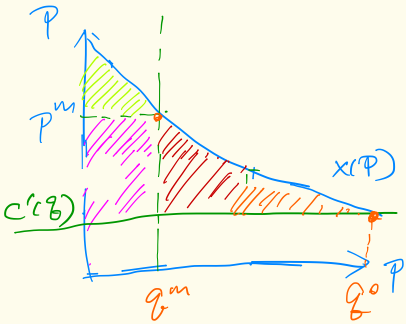
if $\varepsilon = 1 \Rightarrow 0 = c$ (positive) $\times$

$\varepsilon < 1 \Rightarrow P^m = \dfrac{c\varepsilon}{\varepsilon - 1} < 0$ $\times$

Thus if $\varepsilon \leq 1$, the optimal price is not well defined.

(b) Assume $\varepsilon > 1$. Derive p^m, q^m and $(p^m - c)/p^m$

Calculate DWL.

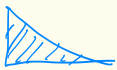


$$p^m = \frac{c\varepsilon}{\varepsilon - 1}$$

$$q^m = \alpha \cdot p^{m\,-\varepsilon} = \alpha \cdot \left(\frac{c\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon}$$

$$\frac{p^m - c}{p^m} = \frac{(\varepsilon - 1)c/\varepsilon - 1}{p^m} = \frac{c}{p^m}$$

DWL = 

=  −  − 

= consumer surplus at q^0 $(p = c)$

− Consumer plus at q^m − Producer surplus at q^m

$$= \int_c^\infty x(t)\,dt - \int_{p^m}^\infty x(t)\,dt - p^m \cdot q^m$$

$$= \int_c^\infty \alpha t^{-\varepsilon}\,dt - \int_{p^m}^\infty \alpha t^{-\varepsilon}\,dt - p^m \cdot q^m$$

$$= \frac{\alpha \cdot p^{-\varepsilon+1}}{-\varepsilon+1}\Bigg|_p^\infty - \int \frac{\alpha \cdot p^{-\varepsilon+1}}{-\varepsilon+1}\Bigg|_{p^m}^\infty - p^m \cdot q^m$$

$$= \frac{\alpha \cdot c^{-\varepsilon+1}}{\varepsilon - 1} - \frac{\alpha \cdot (p^m)^{-\varepsilon+1}}{\varepsilon - 1} - p^m \cdot q^m$$

$$= \frac{\alpha \cdot c^{-\varepsilon+1}}{\varepsilon - 1} - \frac{\alpha \cdot \left(\frac{c\varepsilon}{\varepsilon-1}\right)^{-\varepsilon+1}}{\varepsilon - 1} - \frac{c\varepsilon}{\varepsilon - 1} \cdot \alpha \cdot \left(\frac{c\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon}$$

$$= \frac{\alpha}{\varepsilon - 1}\left[c^{-\varepsilon+1} - \left(\frac{c\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon+1} - c\varepsilon \cdot \left(\frac{c\varepsilon}{\varepsilon - 1}\right)^{-\varepsilon}\right]$$

B6. Goverment tax/sub a monopolist who face inverse demand func $P(q)$ and has $c(q)$, both diff and $P(q)q - c(q)$ is concave in q.
What tax/sub per unit would led to efficiency?

The monopolist's problem is

$$\max_{q} \; P(q) - c(q) - Sq$$

$$\xRightarrow{\text{F.O.C}} \quad P'(q)\cdot q + P(q) - c'(q) - S = 0$$

Remember that in the competitive eq,

$$P = MC = c'(q)$$

$$\Longrightarrow \quad P'(q)\cdot q - S = 0$$

$$S = \underset{(<0)}{P'(q)}\cdot \underset{(>0)}{q} < 0.$$

Thus goverment should subsidize ~~this~~ monopolist $P'(q)\cdot q$ per unit.

B7. Demand for men $X_m(P) = a - \theta_m P$

Women $X_w(P) = a - \theta_w P$

$\theta_w < \theta_m$

Cost of production: C/unit.

(a) Suppose competitive market, find P & q at eq.

In competitive market eq,

$P^w = P^m = MC = c.$

$X_m(P) = a - \theta_m \cdot c \qquad X_w(P) = a - \theta_w \cdot c$

(b). Suppose firm A is a monopolist, if A can't discriminate w & m, find P^{mono}. On what condition $X_m, X_w > 0$

The monopolist's problem is:

$$\max_P \ (P-c)(2a - \theta_m P - \theta_w P)$$

$\xrightarrow{F.O.C}$ $2a - \theta_m P - \theta_w P + (P-c)(-\theta_m - \theta_w) = 0$

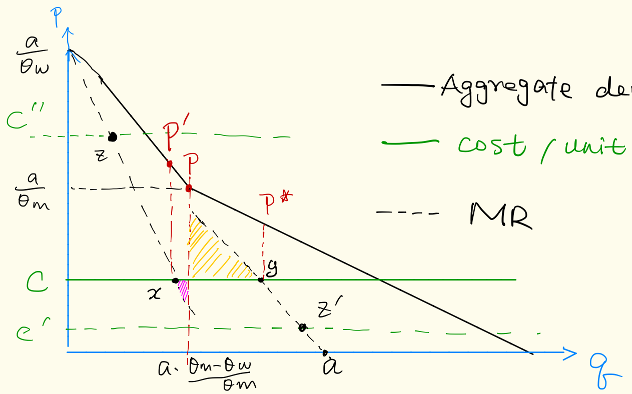
$$2a - \theta_m P - \theta_w P - \theta_m P - \theta_w P + c\cdot\theta_m + c\cdot\theta_w = 0$$

$$P^* = \frac{2a + c\cdot(\theta_m + \theta_w)}{2(\theta_m + \theta_w)} = \frac{a}{\theta_m + \theta_w} + \frac{c}{2}$$

However $\theta_m > \theta_w$,

we can raise the prive a bit above $\frac{a}{\theta_m}$ below $\frac{a}{\theta_w}$.

To see this, we compare P^* with $P' = \frac{a}{2\theta_w} + \frac{c}{2}$.



We see in the Graph, MR is discontinuous at point

$$q = a \cdot \frac{\theta_m - \theta_w}{\theta_m}.$$

if $MC = C$ intersects MR at one point

① left at z. Then $P > \frac{a}{\theta_m}$. So A only sell to women.

② right at z'. $P < \frac{a}{\theta_m}$. A sells to both men & women

if $MC = C$ intersects MR at two points (x, y)
A sells to both men & women.

From P' to P^*, [magenta hatched box] is lost and [yellow hatched box] is gained.

From P^* to P', we have a reverse effect on the welfare

Thus the profit-maximizing price is P^* or P' depending on the relative size of $\boxed{}$ and $\boxed{}$

If $\boxed{} > \boxed{}$, then $P' = \frac{a}{2\theta_w} + \frac{c}{2}$ is the price and both men & women consume a positive level of widgets.

If $\boxed{} < \boxed{}$, then $P^* = \frac{a}{\theta_w + \theta_m} + \frac{c}{2}$ is the price and both men & women consume a positive level of widgets.

(c) If A has produced X. What is the welfare maximizing way to distribute it?

The problem is:

$$\text{Max} \int_0^{q_m} P_m(x)\,dx + \int_0^{q_w} P_w(x)\,dx$$

subject to $q_m + q_w = X$

$P_m(x)$, $P_w(x)$ is the inverse demand func

we should have $P_m(q_m) = P_w(q_w)$. to maxmize the welfare

$$\Rightarrow \frac{a - q_m}{\theta_m} = \frac{a - q_w}{\theta_w}$$

$$q_m + q_w = X$$

$$\Rightarrow \frac{a - X + q_w}{\theta_m} = \frac{a - q_w}{\theta_w}.$$

$$a\theta_m - \theta_m \cdot q_w = a \cdot \theta_w - X \cdot \theta_w + q_w \cdot \theta_w.$$

$$q_w = \frac{(a - X)\theta_w - a \cdot \theta_m}{-(\theta_w + \theta_m)}$$

$$= \frac{(X - a)\theta_w + a \cdot \theta_m}{\theta_w + \theta_m}$$

$$q_m = X - q_w = \frac{a \cdot \theta_w + (X - a)\theta_m}{\theta_w + \theta_m}$$

(d) if A can discriminate, what price does it charge?

In the case of (b) [men & women positive level], does welfare ↑ or ↓ when discrimination ✓.

What about the case [one type served].

① If A can discriminate, the problem becomes.

$$\max_{P_m, P_w} (P_m - c)(a - \theta_m \cdot P_m) + (P_w - c)(a - \theta_w P_w).$$

$$\xRightarrow{F.O.C} \begin{cases} a - \theta_m \cdot P_m + (P_m - c) \cdot (-\theta_m) = 0 \\ a - \theta_w \cdot P_w + (P_w - c) \cdot (-\theta_w) = 0. \end{cases}$$

$$a + c\theta_m = P_m \cdot \theta_m + P_m \cdot \theta_m$$

$$\Rightarrow P_m = \frac{a + c \cdot \theta_m}{2\theta_m} = \frac{a}{2\theta_m} + \frac{c}{2}$$

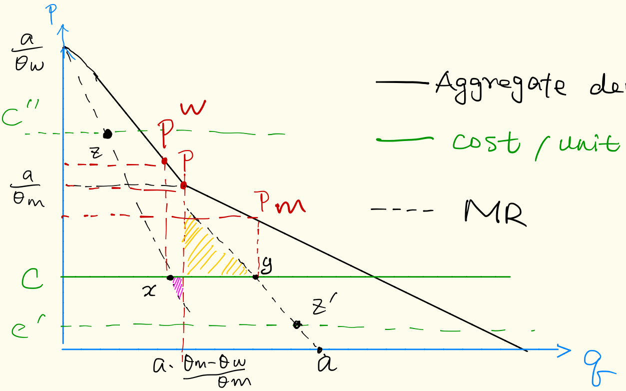
$$P_w = \frac{a}{2\theta_w} + \frac{c}{2}$$

Remember from (c) that $P_m(q^m) = P_w(q_w)$ (both parties are served)

However we cannot have $P_m(q^m) = P_w(q_w)$ here.

$\Rightarrow$ The Aggregate welfare measured by Marshallian Aggregate surplus falls when discrimination is allowed.

② In the other case in (b), A only sell to women



Now discrimination is allowed. there is surplus gained from selling to men (without affecting selling to women), marked by

Thus the welfare rises when discrimination is allowed

B8. Two Period Model.

A Firm is a monopolist. facing $P(q) = a - bq$ (each period)

cost: period 1 C_1/unit, period 2. $C_2 = C_1 - mq_1$/unit

Assume $a > c$, $b > m$, no discounting earnings.

(a) Find q_1^M, q_2^M.

The problem is.

$$\max_{q_1, q_2} \ (a - bq_1 - c)q_1 + (a - bq_2 - C_1 + mq_1)q_2$$

F.O.C $\Longrightarrow$

q_1: $a - 2bq_1 - C_1 + mq_2 = 0$

q_2: $a - 2bq_2 - C_1 = 0$

solving the system we have

$$q_2 = \frac{a - c_1}{2b}, \quad q_1 = \frac{a - C_1}{2b}\left(1 + \frac{m}{2b}\right)$$

(b) Social planner's problem. Any sense in which the planner's period 1 output is selecting so $P = MC$.

Clearly we should get q_2 at which the demand curve crosses the margin cost curve.

$$P(q_2) = a - bq_2 = C_1 - mq_1 \Rightarrow q_2 = \frac{a - C_1 + mq_1}{b}$$

Also $q_1 > \frac{a - c_1}{b}$ as the planner has an incentive to over produce in period

So No sense in selecting $P = MC$ in period 1.

(c) Given monopolist selecting P2 output, would the planner like the monopolist to increase q_1 above the value identified in (a)?

Period 2 surplus is

$$\frac{1}{2}(a-c_1)\cdot\Big(\underbrace{\frac{a-c_1+mq_1}{b}}_{\text{from (b)}}\Big)$$

Period 1 surplus is a constant minus DWL of overproduction

$$DWL = \frac{1}{2}\left[(a-bq_1)-c_1\right]\Big(\underbrace{\frac{a-c}{b}-q_1}_{\text{negative as } q_1 > \frac{a-c}{b}.}\Big)$$

So the social planner's problem is

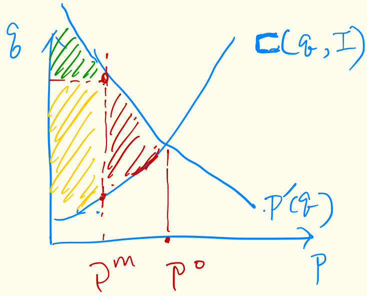
$$\max_{q_1} \frac{1}{2}(a-c_1)\cdot\left(\frac{a-c_1+mq_1}{b}\right)+\frac{1}{2}\left[(a-bq_1)-c_1\right]\cdot\left(\frac{a-c}{b}-q_1\right)$$

$$\xRightarrow{F.O.C} \quad q_1 = \frac{a-c_1}{b}\left(1+\frac{m}{2b}\right) > \underbrace{\frac{a-c_1}{2b}\left(1+\frac{m}{2b}\right)}_{q_1^M \text{ in (a).}}$$

Thus the answer is yes.

The intuition is that increasing q_1^M makes MB = MC in the first period, but MB will be higher in the next period $\Rightarrow$ welfare $\uparrow$

B9. Consider a monopolist in the market facing demand $P(q)$; he faces 2 decisions Investment I & quantity q. If I invested, cost is $c(I)$, $c'(I)<0$ & $c''(I)>0$. Assume the monopolist's utility func is concave in q & I.



(a) Derive F.O.C for choices.

The problem is

$$\max_{q,I} \left[P(q) - c(I)\right]\cdot q - I .$$

F.O.C $\Longrightarrow$

$$\left.\begin{aligned} q&: P'(q)\cdot q + P(q) - c(I) = 0 \\ I&: -c'(I)\cdot q - 1 = 0 \end{aligned}\right\} (*)$$

(b) Compare social planner's solution with solution in (a)

The problem now is:

$$\max_{q,I} \int_0^q P(x)dx - c(I)\cdot q - I .$$

F.O.C $\Longrightarrow$

$$\left.\begin{aligned} q&: P'(q) - c(I) = 0 \\ I&: -c'(I)\cdot q - 1 = 0 \end{aligned}\right\} (**)$$

Compare (**) with (*),

we get $q^o > q^m$

$I^o > I^m$ [since $1 = -c'(I)\,q$ and $-c'(I^o) < -c'(I^m)$]

(c) Compare q^M, I^M with q^*, I^* (social planner can only control I). (second best)

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The social planner's problem is

$$\max_{I} \int_0^q P(x)\,dx - C(I)q - I$$

And the monopolist's problem is

$$\max_{q} [P(q) - C(I)]q - I.$$

$$\xRightarrow{F.O.C} P'(q)\cdot q + P(q) - C(I) = 0.$$

Solving this system

$$L = \int_0^q P(x)\,dx - C(I)\cdot q - I - \lambda\cdot[P'(q)\cdot q + P(q) - C(I)] = 0.$$

$$\xRightarrow{F.O.C} q: P'(q) - C(I) - \lambda[P''(q)\cdot q + P'(q) + P'(q)] = 0 \quad (***)$$

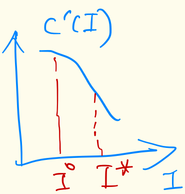
$$I: -C'(I)\cdot q + \lambda\cdot C'(I) - 1 = 0.$$

From (***) we see $P'(q^*) > P'(q^o) > P'(q^M)$

$$\Rightarrow q^* > q^o > q^M$$

and $-c'(I^*)\cdot q^* < -c'(I^o)\cdot q^*$

$$\Rightarrow I^* > I^o$$



Thus, In the second best, social planner choose an I^* greater ~~than~~ the optimal level I^o for the given output level to make monopolist produce more.