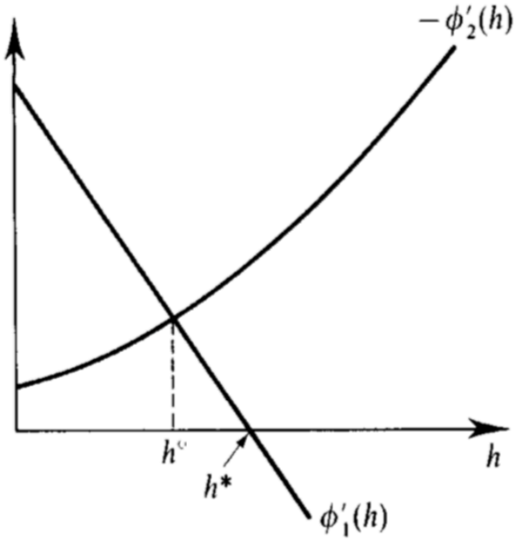
CH.11 Externality and Public Good

11.B A simple bilateral externality

1. Nonoptimality of the competitive outcome



2. Fostering Bargaining over Externalities: enforceable property rights

In this approach to the externality problem , we seek to insure that conditions are met for parties to themselves reach an optimal agreement on the level of externalities.

Setting:

- Two consumers, assume we assign the right to "externality-free" environment to consumer 2 (consumer 1 is unable to engage in the externality-producing activity without 2's permission)
- The bargaining takes a form in which c2 makes c1 a take-it-or-level-it offer, demanding a payment of *T* for externality *h*.
- C1 accepts iff

$$\phi_1(h) - T \geq \phi_1(0).$$

- C2's problem is to solve

$$\begin{aligned} \underset{h \geq 0,\, T}{\text{Max}} \quad & \phi_2(h) + T \\ \text{s.t. } & \phi_1(h) - T \geq \phi_1(0). \end{aligned}$$

The constraint is binding in any solution to this problem

$$T = \phi_1(h) - \phi_1(0).$$

Thus C2's problem becomes

$$\underset{h \geq 0}{\text{Max}} \quad \phi_2(h) + \phi_1(h) - \phi_1(0).$$

The solution is exactly the socially optimal solution.

- **Coase theorem [Coase (1960)]: If trade of the externality can occur, then bargaining will lead to an efficient outcome no matter how property rights are allocated.**
- Note that the existence of both well defined and enforceable property rights is essential for this type of bargaining to occur
- The consumer must know each others' preferences. In 11.E we will see that when agents are to some extent ignorant of each others' preferences, bargaining need not lead to an efficient outcome.

2. Externalities and Missing markets

Bargaining can generate an optimal outcome => a connection between externality and missing markets.

- Suppose the property rights are well defined and enforceable.
- A competitive market for the right to engage in the externality-generating activity exists.
- Assume C2 has the right to an externality-free environment.
- Let P_h denote the price of the right to engage in one unit of the activity.

Consumer 1's problem is

$$\underset{h_1 \geq 0}{\text{Max}}\ \phi_1(h_1) - p_h h_1,$$

which has the first-order condition

$$\phi_1'(h_1) \leq p_h, \quad \text{with equality if } h_1 > 0. \tag{11.B.6}$$

Consumer 2's problem is

$$\underset{h_2 > 0}{\text{Max}}\quad \phi_2(h_2) + p_h h_2,$$

which has the first-order condition

$$\phi_2'(h_2) \leq -p_h, \quad \text{with equality if } h_2 > 0. \tag{11.B.7}$$

In a competitive equilibrium, we must have h1 = h2

$$\phi_1'(h^{**}) \leq -\phi_2'(h^{**}), \quad \text{with equality if } h^{**} > 0.$$

Comparing this expression with (11.B.2), we see that h^{**} equals the optimal level h°. The equilibrium price of the externality is $p_h^* = \phi_1'(h^\circ) = -\phi_2'(h^\circ)$.

=> **If a competitive market exists for the externality, then optimality results.**
Thus we can see that externalities is inherently tied to the absence of certain competitive markets [Meade (1952) and Arrow (1969)]

- However this is unrealistic as prices are felt by many agents. Thus we hope in multilateral settings, price taking would be a more reasonable assumption and as a result a competitive market for the externality would lead to an efficient outcome (shown in 11.D)

11.C Public Goods

Definition: A *public good* is a commodity for which use of a unit of the good by one agent does not preclude its use by other agents

- Feature: nondepletable / excludable
- Ex. Knowledge / Patent System for excluding individuals from the use of knowledge
- A public good is not necessarily be desirable => public bads

1. **Conditions for Pareto Optimality**

Setting:

- I consumers and one public good (in additional to L traded goods of usual, private kind)
- Assume the quantity of the public good has no effect on the prices of L traded goods
- Each consumer's utility function is quasilinear with respect to the same numeraire, traded commodity.
- Let x be the quantity of the public good.
- $\phi_i(x)$, consumer i's utility from consuming the public good. Twice differentable, and $\phi_i''(x) < 0$ for all $x \geq 0$.
- Cost of supplying *q* units of the public good is *c(q)*. Assume twice differentiable and $c''(q) > 0$ for all $x \geq 0$
- To describe that a desirable public good whose production is costly.

$$\phi_i'(x) > 0 \text{ and } c'(x) > 0$$

The social planner's problem is to solve

$$\underset{q \geq 0}{\text{Max}} \sum_{i=1}^{I} \phi_i(q) - c(q).$$

The necessary and sufficient first-order condition for the optimal quantity q° is then

$$\sum_{i=1}^{I} \phi_i'(q^\circ) \leq c'(q^\circ), \quad \text{with equality if } q^\circ > 0. \tag{11.C.1}$$

=> **This is the classical optimality condition for a public good derived [Samuelson (1954; 1955)]**

$$\sum_i \boldsymbol{MB_i} = \boldsymbol{MC}$$

2. **Inefficiency of Private Provision of Public Good**

Consider a case in which the public good is provided by means of private purchases by consumers.

Setting:

- Each consumer i choose how much the public good to buy, at market price p.
- Treat supply side as consisting of a single profit-maximizing firm with cost function c(.) that chooses its production level taking the market price as given.

MWG Ch.11

At a competitive equilibrium we have x_i^* solves

$$\underset{x_i \geq 0}{\text{Max}} \quad \phi_i(x_i + \sum_{k \neq i} x_k^*) - p^* x_i.$$

F.O.C yields

$$\phi_i'(x_i^* + \sum_{k \neq i} x_k^*) \leq p^*, \quad \text{with equality if } x_i^* > 0.$$

Let x^* equal the sum of x_i^* we have

$$\phi_i'(x^*) \leq p^*, \quad \text{with equality if } x_i^* > 0. \qquad (11.C.3)$$

The firm's supply q^*, on the other hand, must solve $\text{Max}_{q \geq 0}\,(p^* q - c(q))$ and therefore must satisfy the standard necessary and sufficient first-order condition

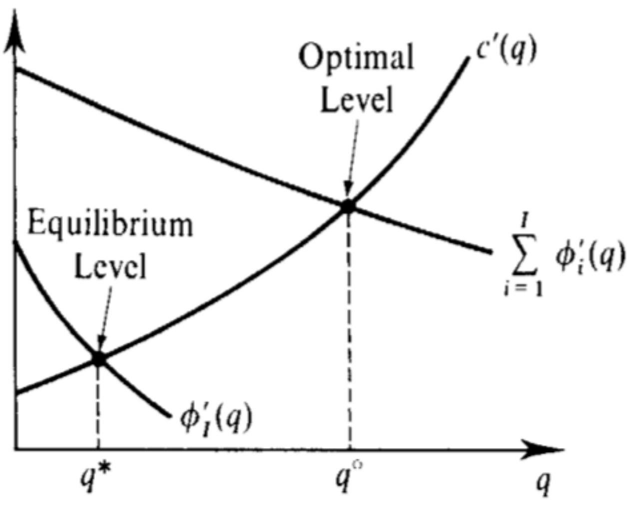
$$p^* \leq c'(q^*), \quad \text{with equality if } q^* > 0. \qquad (11.C.4)$$

At a competitive equilibrium, $q^* = x^*$. Thus, letting $\delta_i = 1$ if $x_i^* > 0$ and $\delta_i = 0$ if $x_i^* = 0$, (11.C.3) and (11.C.4) tell us that $\sum_i \delta_i[\phi_i'(q^*) - c'(q^*)] = 0$. Recalling that $\phi_i'(\cdot) > 0$ and $c'(\cdot) > 0$, this implies that whenever $I > 1$ and $q^* > 0$ (so that $\delta_i = 1$ for some i) we have

$$\sum_{i=1}^{I} \phi_i'(q^*) > c'(q^*). \qquad (11.C.5)$$

Compare this result with the optimal level of the public good provided

$$\sum_{i=1}^{I} \phi_i'(q^\circ) \leq c'(q^\circ), \quad \text{with equality if } q^\circ > 0. \qquad (11.C.1)$$



We see that the level of public good provided is too low, **$q^* < q^o$**

- **Each consumer's purchase of the public good provides a direct benefit not only to the consumer herself but also to every other consumer. However, she does not consider the benefits for others of her public good provision**
  => Free rider problem
  => Each consumer has an incentive to enjoy the benefits of the public good provided by others while providing it insufficiently herself.
  => Government intervention in the provision of public goods.

- **Optimal Subsidy/Tax:**
  example, suppose that there are two consumers with benefit functions $\phi_1(x_1 + x_2)$ and $\phi_2(x_1 + x_2)$, where x_i is the amount of the public good purchased by consumer i, and that $q^\circ > 0$. By analogy with the analysis in Section 11.B, a subsidy to each consumer i per unit purchased of $s_i = \phi'_{-i}(q^\circ)$ [or, equivalently, a tax of $-\phi'_{-i}(q^\circ)$
  If given two competitive eq lvels of the public good purchased by two consumers given subsidies, and p bar is the eq price, then consumer's purchases solve
  $$\text{Max}_{x_i \geq 0}\ \phi_i(x_i + \tilde{x}_j) + s_i x_i - \tilde{p} x_i,$$
  F.O.C yield
  $$\phi'_i(\tilde{x}_1 + \tilde{x}_2) + s_i \leq \tilde{p}, \text{ with equality of } \tilde{x}_i > 0.$$

Substituting for s_i, and using both condition (11.C.4) and the market-clearing condition that $\tilde{x}_1 + \tilde{x}_2 = \tilde{q}$, we conclude that $\tilde{q}$ is the total amount of the public good in the competitive equilibrium given these subsidies if and only if

$$\phi'_i(\tilde{q}) + \phi'_{-i}(q^\circ) \leq c'(\tilde{q}),$$

with equality for some i if $\tilde{q} > 0$. Recalling (11.C.1) we see that $\tilde{q} = q^\circ$. (Exercise 11.C.1 asks you to extend this argument to the case where $I > 2$; formally, we then have a multilateral externality of the sort studied in Section 11.D.)

- Note that this subsidy scheme require that **government know the benefits derived by consumers from public good.**

3. **Lindahl Equilibria *****

Private provision of public good => inefficiency
Market institution => optimality

Setting:
- For each consumer, we have a market for the public good "as experienced by the consumer". P_i is different across consumers.
- The firm produces a bundle of **I** goods with a fixed-proportions tech.

Given eq price P_i**, each consumer solves

$$\underset{x_i > 0}{\text{Max}} \quad \phi_i(x_i) - p_i^{**} x_i.$$

F.O.C yields the eq consumption level x_i s.t

$$\phi_i'(x_i^{**}) \le p_i^{**}, \quad \text{with equality if } x_i^{**} > 0.$$

The firm's problem is

$$\underset{q \ge 0}{\text{Max}} \quad \left(\sum_{i=1}^{I} p_i^{**} q \right) - c(q).$$

F.O.C yields output q** s.t

$$\sum_{i=1}^{I} p_i^{**} \le c'(q^{**}), \quad \text{with equality if } q^{**} > 0.$$

Two F.O.C results and the market-clearing condition $x_i = q^{**}$ imply that

$$\sum_{i=1}^{I} \phi_i'(q^{**}) \le c'(q^{**}), \quad \text{with equality if } q^{**} > 0.$$

Compare it with

$$\sum_{i=1}^{I} \phi_i'(q^{\circ}) \le c'(q^{\circ}), \quad \text{with equality if } q^{\circ} > 0. \qquad (11.C.1)$$

$$\Rightarrow \quad q^{**} = q^{o}$$

- This type of equilibrium in personalized markets for the public good is known as Lindahl eq. [Lindahl (1919), Milleron (1972)]
- Why we obtain efficiency? Because each consumer takes the price in her own personalized market as given, fully determines her own level of consumption of the public good => externalities are eliminated.

11.D Multilateral Externalities

The setting:

- We adopt a partial equilibrium approach and assume that agents take as given the price vector p of L traded goods.
- There are J firms that generate the externality in the process of production.
- Given price p, we can determine firm j's derived profit function ***Pi(hj)*** over the level of externality it generates **hj** >= 0.
- There are I consumers, quasilinear utility functions with respect a numeraire traded commodity

  Given price vector p, we denote by $\phi_i(\tilde{h}_i)$ consumer i's derived utility function over the amount of the externality $\tilde{h}_i$ she experiences. We assume that $\pi_j(\cdot)$ and $\phi_i(\cdot)$ are twice differentiable with $\pi_j''(\cdot) < 0$ and $\phi_i''(\cdot) < 0$. To fix ideas, we shall focus on the case where $\phi_i'(\cdot) < 0$ for all i, so that we are dealing with a negative externality.

1. **Depletable Externalities**

From 11.B, we have at any competitive eq,

$$\pi_j(h_j^*) \leq 0, \quad \text{with equality if } h_j^* > 0.^{13} \tag{11.D.1}$$

Any Pareto optimal allocation involves the level of externalities that solves

$$\begin{aligned} \underset{\substack{(h_1,\ldots,h_J)\geq 0 \\ (\tilde{h}_1,\ldots,\tilde{h}_I)\geq 0}}{\text{Max}} \quad & \sum_{i=1}^{I} \phi_i(\tilde{h}_i) + \sum_{j=1}^{J} \pi_j(h_j) \\ \text{s.t.} \quad & \sum_{j=1}^{J} h_j = \sum_{i=1}^{I} \tilde{h}_i. \end{aligned} \tag{11.D.2}$$

The constraint in (11.D.2) reflects the depletability of the externality: If $\tilde{h}_i$ is increased by one unit, there is one unit less of the externality that needs to be experienced by others. Letting μ be the multiplier on this constraint, the necessary and sufficient first-order conditions to problem (11.D.2) are

$$\phi_i'(\tilde{h}_i^\circ) \leq \mu, \quad \text{with equality if } \tilde{h}_i^\circ > 0,\ i = 1, \ldots, I, \tag{11.D.3}$$

and

$$\mu \leq -\pi_j'(h_j^\circ), \quad \text{with equality if } h_j^\circ > 0,\ j = 1, \ldots, J. \tag{11.D.4}$$

Conditions (11.D.3) and (11.D.4), along with the constraint in problem (11.D.2), characterize the optimal levels of externality generation and consumption. Note that they exactly parallel the efficiency conditions for a private good derived in Chapter 10, conditions (10.D.3) to (10.D.5), where we interpret $-\pi_j'(\cdot)$ as firm j's marginal cost of producing more of the externality. If well-defined and enforceable property rights can be specified over the externality, and if I and J are large numbers so that price taking is a reasonable hypothesis, then by analogy with the analysis of competitive markets for private goods in Chapter 10, a market for the externality can be expected to lead to the optimal levels of externality production and consumption in the depletable case.

2. Nondepletable Externalities

We now move to the case in which the externality is nondepletable. To be specific, assume that the level of the externality experienced by *each* consumer is $\sum_j h_j$, the total amount of the externality produced by the firms.

In an unfettered competitive equilibrium, each firm j's externality generation h_j^* again satisfies condition (11.D.1). In contrast, any Pareto optimal allocation involves externality generation levels $(h_1^\circ, \ldots, h_J^\circ)$ that solve

$$\underset{(h_1,\ldots,h_J)\geq 0}{\text{Max}} \quad \sum_{i=1}^{I} \phi_i(\textstyle\sum_j h_j) + \sum_{j=1}^{J} \pi_j(h_j). \tag{11.D.5}$$

This problem has necessary and sufficient first-order conditions for each firm j's optimal level of externality generation, h_j°, of

$$\sum_{i=1}^{I} \phi_i'(\textstyle\sum_j h_j^\circ) \leq -\pi_j'(h_j^\circ), \quad \text{with equality if } h_j^\circ > 0. \tag{11.D.6}$$

- Everyone facing the same amount of externalities

Condition 11.D.6 is exactly analogous to the optimality condition for a public good ($\sum_i \boldsymbol{MB_i} = \boldsymbol{MC}$)

- This is the same as what we discussed in private provision of public goods
- The free-rider problem reappears.
- The eq level of the negative externality will exceed its optimal level

- Pure market-based solutions, personalized or not are unlikely to work in the case of a **depletable** externalities. (due to the public nature of the externalities)

- Government using quotas/taxes
- With quotas, set an upper bound on each firm j's level of externality generation equal to its optimal level h^o
- Optimal-restoring tax

$$t_h = -\textstyle\sum_i \phi_i'(\sum_j h_j^\circ) \text{ per unit}$$

Given this tax, Firm j's problem is

$$\underset{h_j \geq 0}{\text{Max}} \quad \pi_j(h_j) - t_h h_j,$$

which has the necessary and sufficient first-order condition

$$\pi_j'(h_j) \leq t_h, \quad \text{with equality if } h_j > 0.$$

Given $t_h = -\sum_i \phi_i'(\sum_j h_j^\circ)$, firm j's optimal choice is $h_j = h_j^\circ$.

- We can use permits (specification of a quota on the total level of externalities and distribution of the number of tradeable externalities permits, each permit allows firm to generate one unit of externalities)

A partial market-based approach that can achieve optimality with a nondepletable multilateral externality involves specification of a quota on the *total* level of the externality and distribution of that number of *tradeable externality permits* (each permit grants a firm the right to generate one unit of the externality). Suppose that $h^\circ = \sum_j h_j^\circ$ permits are given to the firms, with firm j receiving $\bar{h}_j$ of them. Let p_h^* denote the equilibrium price of these permits. Then each firm j's demand for permits, h_j, solves $\text{Max}_{h_j \geq 0} (\pi_j(h_j) + p_h^*(\bar{h}_j - h_j))$ and so satisfies the necessary and sufficient first-order condition $\pi_j'(h_j) \leq p_h^*$, with equality if $h_j > 0$. In addition, market clearing in the permits market requires that $\sum_j h_j = h^\circ$. The competitive equilibrium in the market for permits then has price $p_h^* = -\sum_i \phi_i'(h^\circ)$ and each firm j using h_j° permits and so yields an optimal allocation. The advantage of this scheme relative to a strict quota method arises when the government has limited information about the $\pi_j(\cdot)$ functions and cannot tell which particular firms can efficiently bear the burden of externality reduction, although it has enough information, perhaps of a statistical sort, to allow the computation of the optimal aggregate level of the externality, h°.

---

h_j bar - hj => so one can buy or sell permits

11.E Private Information and Second Best Solutions

Private held or asymmetrically held information (the degree to which an agent is affected is only known to herself) can confound both centralized (quotas and taxes) and decentralized (bargaining) attempts to achieve optimality.

The Setting: firm type and consumer type

Suppose, then, that we can write the consumer's derived utility function from externality level h (see Section 11.B for more on this construction) as $\phi(h, \eta)$, where $\eta \in \mathbb{R}$ is a parameter, to be called the consumer's *type*, that affects the consumer's costs from the externality. Similarly, we let $\pi(h, \theta)$ denote the firm's derived profit given its type $\theta \in \mathbb{R}$. The actual values of θ and η are *privately observed*: Only the consumer knows her type η, and only the firm observes its type θ. The ex ante likelihoods (probability distributions) of various values of θ and η are, however, publicly known. For convenience, we assume that θ and η are independently distributed. As previously, we assume that $\pi(h, \theta)$ and $\phi(h, \eta)$ are strictly concave in h for any given values of θ and η.

1. **Decentralized Bargaining => nonoptimality ($c < b < T_c^*$)**

Consider the decentralized approach to the externality problem first. In general, bargaining in the presence of bilateral asymmetric information will *not* lead to an efficient level of the externality. To see this, consider again the case in which the consumer has the right to an externality-free environment, and the simple bargaining process in which the consumer makes a take-it-or-leave-it offer to the firm. For simplicity, we assume that there are only two possible levels of the externality, 0 and $\bar{h} > 0$, and we focus on the case of a negative externality in which externality level $\bar{h}$, relative to the level 0, is detrimental for the consumer and beneficial for the firm (the analysis is readily applied to the case of a positive externality).

It is convenient to define $b(\theta) = \pi(\bar{h}, \theta) - \pi(0, \theta) > 0$ as the measure of the firm's benefit from the externality-generating activity when its type is θ. Similarly, we let $c(\eta) = \phi(0, \eta) - \phi(\bar{h}, \eta) > 0$ give the consumer's cost from externality level $\bar{h}$. In this simplified setting, the only aspects of the consumer's and firm's types that matter are the values of b and c that these types generate. Hence, we can focus directly on the various possible values of b and c that the two agents might have. Denote by $G(b)$ and $F(c)$ the distribution functions of these two variables induced by the underlying probability distributions of θ and η (note that, given the independence of θ and η, b and c are independent). For simplicity, we assume that these distributions have associated density functions $g(b)$ and $f(c)$, with $g(b) > 0$ and $f(c) > 0$ for all $b > 0$ and $c > 0$.

Since the consumer has the right to an externality-free environment, in the absence of any agreement with the firm she will always insist that the firm set $h = 0$ (recall that $c > 0$). However, in any arrangement that guarantees Pareto optimal outcomes for all values of b and c, the firm should be allowed to set $h = \bar{h}$ whenever $b > c$.

Now consider the amount that the consumer will demand from the firm when her cost is c in exchange for permission to engage in the externality-generating activity. Since the firm knows that the consumer will insist on $h = 0$ if there is no agreement, the firm will agree to pay the amount T if and only if $b \geq T$. Hence, the consumer knows that if she demands a payment of T, the probability that the firm will accept her offer equals the probability that $b \geq T$; that is, it is equal to $1 - G(T)$. Given her cost $c > 0$ (and assuming risk neutrality), the consumer optimally chooses the value of T she demands to solve

$$\underset{T}{\text{Max}} \quad (1 - G(T))(T - c). \tag{11.E.1}$$

The objective function of problem (11.E.1) is the probability that the firm accepts the demand, multiplied by the net gain to the consumer when this happens $(T - c)$. Under our assumptions, the objective function in (11.E.1) is strictly positive for all $T > c$ and equal to zero when $T = c$. Therefore, the solution, say T_c^*, is such that $T_c^* > c$. But this implies that this bargaining process must result in a strictly positive probability of an inefficient outcome, since whenever the firm's benefit b satisfies $c < b < T_c^*$, the firm will reject the consumer's offer, resulting in an externality level of zero, even though optimality requires that $h = \bar{h}$.[17,18]

2. Quota and Tax

- When Marginal Damage of Externality (negative) is relatively horizontal (compared to marginal benefit), quota is better
- When Marginal Damage of Externality (negative) is relatively flat (compared to marginal benefit), tax is better

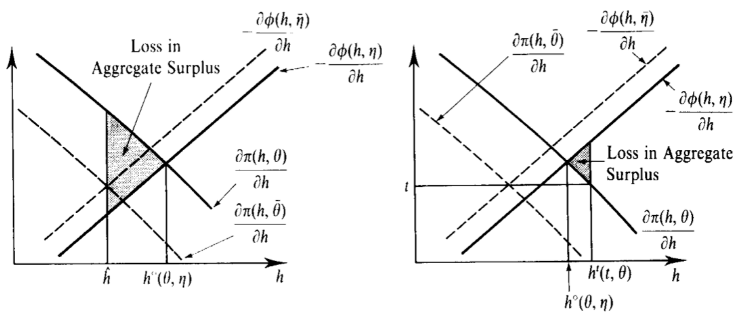


Figure 11.E.2 (left)
The loss in aggregate surplus under a quota for types (θ, η).

Figure 11.E.3 (right)
The loss in aggregate surplus under a tax for types (θ, η).

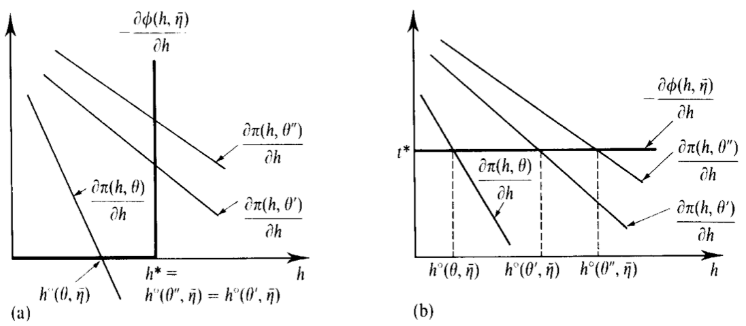


Figure 11.E.4
Two cases in which a quota or tax maximizes aggregate surplus for every realization of θ.
(a) Quota $\hat{h} = h^*$ maximizes aggregate surplus for all θ.
(b) Tax $t = t^*$ maximizes aggregate surplus for all θ.