Ch 8. Simultaneous Move Games

Dominated & Dominant Strategies

Goal: Determine how a game is likely to be played?

We only discuss ps not ms for now.

PD

		2: DC	2: C
1	~~DC~~	~~-2, 2~~	~~-10, -1~~
	C	-1, -10	-5, -5

For P1, C is dominant strategy

Strictly Dominant strategy: bs regardless of what other players do.

Defn $s_i \in S_i$ is strictly dominant if $\forall s_i' \neq s_i$, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$.
$\forall s_{-i} \in S_{-i}$.

Discuss PD. not best outcome for both.

Externality: Going from ~~C→DC~~ DC → C, gain 1 but impose 8 on other player.

		2	
		L	R
1	U	1,−1	−1,1
	M	−1,1	1,−1
	D	−2,5	−3,2

D is strictly dominated. by M and U.

<u>Strictly Dominated</u> strategy for i is

if $\exists\ s_i' \in S_i$ s.t. $\forall\ s_{-i} \in S_{-i}$,

$$u_i(s_i', s_{-i}) > u_i(s_i, s_{-i}).$$

ie s_i' strictly dominates s_i.

Strictly dominant is same as strictly dominating every other strategy.

<u>Weakly Dominated</u> s_i weakly dominated if

$\exists\ s_i'$ s.t. $\forall\ s_{-i} \in S_{-i}$,

$$u_i(s_i', s_{-i}) \geq u_i(s_i, s_{-i})$$

with strict inequality for some s_{-i}.

<u>Weakly dominant</u> $\Rightarrow$ if it weakly dominates every other strategy in S_i.

1 \ 2	L	R
U	5,1	4,0
M	6,0	3,1
D	6,4	4,4

U weakly dominated by D.
M " " " D

D weakly dominant.

Cant eliminate weakly dominated strategies
eg 1 may play M if he is sure 2 plays L.

1 is DA's brother

<u>Iterated Deletion</u>

DA's brother

1 \ 2	DC	C
DC	0, -2	-10, -1
C	-1, -10	-5, -5

If 2 plays C, 1 plays C eq^m (C, C)
Now 1 does not have dominant strategy.

<u>Common knowledge</u>.

If players rational, they must not play strictly dominated strategies. eg in <u>PD</u>.
In PD DA brother, need more assumptions:
P2 rational; P1 knows P2 rational.

When we delete strictly dominated strategies, order of deletion doesn't matter.
May matter for weakly dominated strategies.

MS.

$\sigma_i \in \Delta(S_i)$ strictly dominated if

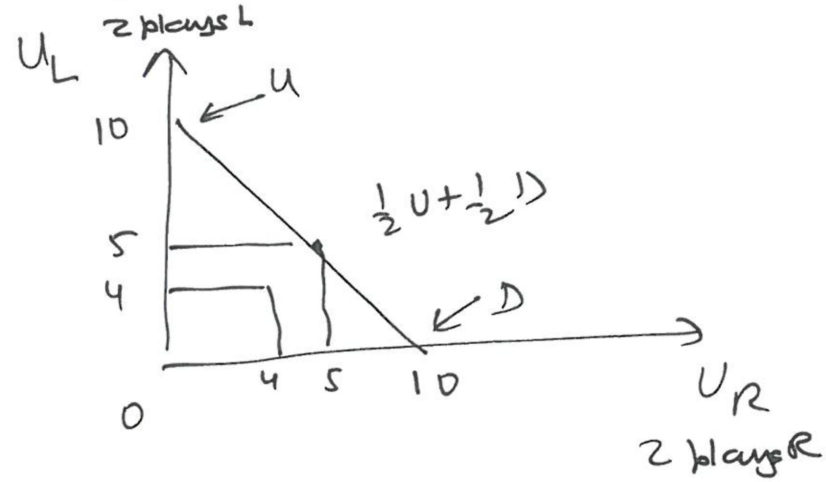
$\exists\, \sigma_i' \in \Delta(S_i)$ s.t. $\forall\, \sigma_{-i} \in \prod_{j \neq i} \Delta(S_j)$,

$$u_i(\sigma_i', \sigma_{-i}) > u_i(\sigma_i, \sigma_{-i}).$$

σ_i' strictly dominates σ_i.

σ_i strictly dominant if it dominates every other strategy.

1 \ 2	L	R
U	10, 1	0, 4
M	4, 2	4, 3
D	0, 5	10, 2



$\frac{1}{2}U + \frac{1}{2}D$ strictly dominates ~~U &~~ M

PA will never play M in any pure/mixed strategy.

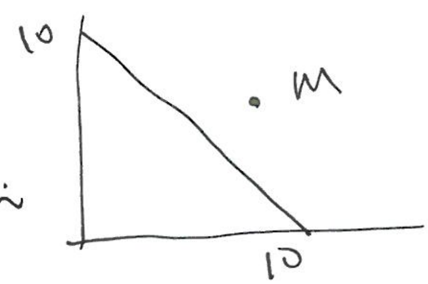
Read Prop 8B1.

If ps s_i is strictly dominated, then so is every ms that assigns it +ve probability

Exercise 8B6.

1 \ 2	L	R
U	10, 1	0, 4
M	6, 2	6, 3
D	0, 5	10, 2

ps not dominated by ms playin U, D always dom by M.



M dominates $\frac{1}{2}U+\frac{1}{2}D$.

even tho U, D undominated ps (by any ps)

Steps :

1. Eliminate all strictly dominated ps.
   (dom by ps & ms).

   Get $S_i^U \subset S_i$.

2. Eliminate any ms in $\Delta(S_i^U)$ that are dominated.

leave set of undominated ps & ms in $\Delta(S_i^U)$.

Do 8B1
8B3

Rationalizable strategies

Use common knowledge to delete more strategies (than strictly dominated strategies).

Rationalizable strategy: those strategies blessed when structure of game & CK are used.

σ_i is br for player i for strategies σ_{-i} if

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i}) \quad \forall \sigma_i' \in \Delta(S_i)$$

A strictly dominated strategy is not a br.

but one not strictly dominated also may not be br.

Every strategy that remains must be br to <u>something</u> rival plays.

What is set of rationalizable strategies?

1 \ 2	b_1	b_2	b_3	b_4
a_1	0,7	2,5	7,0	0,1
a_2	5,2	3,3	5,2	0,1
a_3	7,0	2,5	0,7	0,1
a_4	0,0	0,−2	0,0	10,−1

2: b_4 strictly dominated by $0.5 b_2 + 0.5 b_3$.

If b_4 eliminated, a_4 is strictly dom by a_2.

Set of rationalizable strategies:

P1: $\{a_1, a_2, a_3\}$

P2: $\{b_1, b_2, b_3\}$

Chain of justification: a_1 br to b_3 br to a_3 br to a_1.

b_2 br to a_2. a_2 br to b_2.

<u>Nash Eq^m</u>

P1 \ P2	l	m	r
U	5,3	0,4	3,5
M	4,0	5,5	4,0
D	3,5	0,4	5,3

(M, m) is NE. no profitable deviation.

Set of rationalizable strategies?

Defn of NE: $s = (s_1, \ldots, s_I)$ is NE if

$\forall i, \quad u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \;\forall\, s_i' \in S_i.$

NE is br to strategies actually played.

Set of NE $\subset$ set of rationalizable strategies.

Multiple NE: (Tom Schelling)

		P1 ES	P1 GC
P2	ES	100, 100	0, 0
P2	GC	0, 0	100, 100

NE based on mutually correct expectations.

Why NE reasonable?

1. Based on rational inference (but that is rationalizability).
2. If players think there is unique outcome, it must be NE.
3. Focal point of game. (Meeting in NYC).
4. Self-enforcing agreement (pre-play comm).
5. Social convention in repeated games. (walking on left)

MSNE : $\sigma = (\sigma_1, \ldots, \sigma_I)$ is NE if

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i}) \quad \forall \sigma_i' \in \Delta(S_i).$$

eg.

		2: H	2: T
1	H	$-1, 1$	$1, -1$
	T	$1, -1$	$-1, 1$

H&T with $p = 0.5$.

MSNE in NY game

		Schelling: ES	Schelling: GC	
Thomas	ES	100, 100	0, 0	$10/11$
	GC	0, 0	1000, 1000	$1/11$
		$1-\sigma_S$	σ_S	

Thomas

$$100(1-\sigma_S) + 0\,\sigma_S = 0(1-\sigma_S) + 1000\,\sigma_S$$

$$EU_{ES} = EU_{GC}$$

$$100 - 100\sigma_S = 1000\sigma_S \Rightarrow \sigma_S = 1/11.$$

$$1-\sigma_S = 10/11.$$

S plays ES with $10/11$ & GC with $1/11$.

same with T.

T's expected payoff $= 100\left(\frac{10}{11}\right)\left(\frac{10}{11}\right) + 1000\left(\frac{1}{11}\right)\left(\frac{1}{11}\right)$

$$= \frac{10000}{121} + \frac{1000}{121} = \frac{11000}{121} = \frac{1000}{11}$$

T's EU of playing ES with prob 1

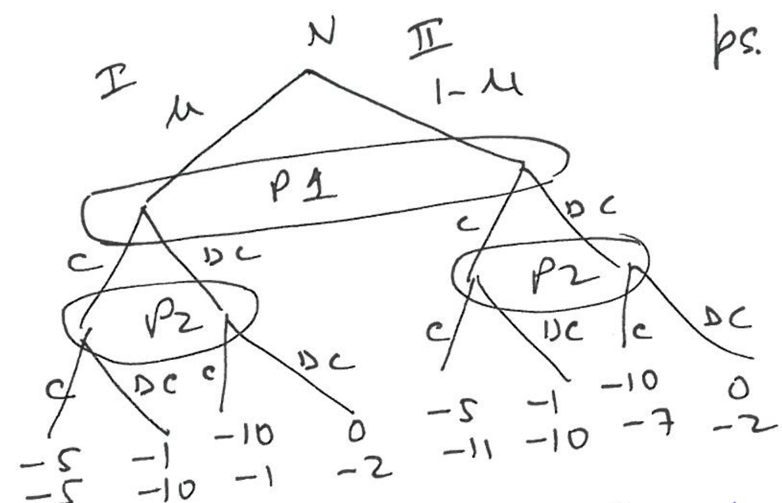
$$= 100\left(\frac{10}{11}\right) = \frac{1000}{11}$$

PS NE are degenerate MSNE.

Read Prop 8D1. Every ps in a ms profile gives same payoff.

Games of Incomplete Information

Until now, we looked only at games of complete information. Now we study incomplete information. Nature moves first. Players only observe their own type. However all players have prob dist. about rv, which is common knowledge. 2 types of P2.



ps. P1 strategies: {C, DC}

P2 " :

{C/I, C/II}
C/I, DC/II
DC/I, C/II
DC/I, DC/II

P2 (type I, μ):

P1 \ P2	DC	C
DC	0, -2	-10, -1
C	-1, -10	-5, -5

μ

P2 (type II, $1-\mu$):

P1 \ P2	DC	C
DC	0, -2	-10, -7
C	-1, -10	-5, -11

$1-\mu$ 1 2

P2 plays C
P1 plays C.
(C, C)

(DC, DC)

payoff fns: $u_i(s_i, s_{-i}, \theta_i)$, $\theta_i \in \Theta_i$

observed by player i.

jt pdf $F(\theta_1, -, \theta_I)$ is CK.

$\Theta = \Theta_1 \times \Theta_2 \times - - \times \Theta_I$.

Bayesian game $[I, \{S_i\}, \Theta, F(\cdot)]$.

ps NE: $\forall i = 1, -, I$,

$$\tilde{u}_i(s_i(\cdot), s_{-i}(\cdot)) \geq \tilde{u}_i(s_i'(\cdot), s_{-i}(\cdot))$$

$$\forall s_i'(\cdot) \in S_i .$$

P1 play DC if $E\pi(DC) \geq E\pi(C)$

$$(1-u)0 - 10(u) \geq -5(u) - 1(1-u)$$

$$-10u \geq -5u - 1 + u$$

$$1 \geq 6u \Rightarrow u \leq 1/6.$$

1 Play DC if $u < 1/6$

C if $u > 1/6$..

Alphabeta Consortium

Cost of new invention $c \in (0,1)$

Firm of i type $\theta_i \in [0,1]$.

Benefit of invention to i: θ_i^2.

		2: 1	2: 0
1	1	$\theta_1^2-c, \theta_2^2-c$	θ_1^2-c, θ_2^2
1	0	θ_1^2, θ_2^2-c	$0, 0$

$s_i(\theta_i)=1$ develop
$\quad\;\; =0$ not develop

i develops "zigger" iff

$$\theta_i^2 - c \geq \theta_i^2 \cdot \text{Prob}(\theta_j(s_j)=1) + 0 \cdot \text{Prob}(\theta_j(s_j)=0).$$

$$\theta_i^2\left[1-\text{Prob}(\theta_j(s_j)=1)\right] \geq c$$

$$\theta_i \geq \left[\frac{c}{1-\text{Prob}(\theta_j(s_j)=1)}\right]^{1/2}$$

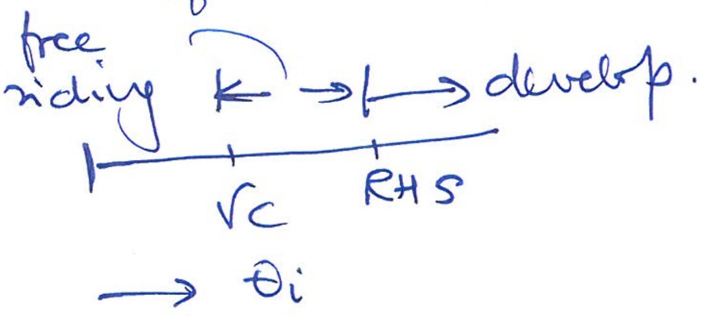
So firm i develops if $\theta_i \geq$ RHS
not if $\theta_i <$ RHS

Let $\hat{\theta}_1, \hat{\theta}_2$ be these cutoff values of θ_i.

$\therefore$ invest if $\theta_1 \geq \hat{\theta}_1$
$\theta_2 \geq \hat{\theta}_2$.

If only one firm in industry,
develop iff $\theta_i^2 - c > 0 \Rightarrow \theta_i = \sqrt{c}$

If $\text{Prob}(s_j(\theta_j) = 1) \geq 0$, then RHS $\geq \sqrt{c}$.



If $\sqrt{c} \leq \theta_i \leq$ RHS
no zigger is developed.

$\hat{\theta}_1, \hat{\theta}_2 \in (0,1)$ cutoffs

$\text{Prob}(s_j(\theta_j) = 1) = 1 - \hat{\theta}_j$

$$\therefore \quad \hat{\theta}_1 = \left(\frac{c}{\hat{\theta}_2}\right)^{1/2}$$

$$\Rightarrow \quad \hat{\theta}_1^2 \hat{\theta}_2 = c$$

Symmetry, $\hat{\theta}_1 \hat{\theta}_2^2 = c$

$$\Rightarrow \quad \hat{\theta}_1 = \hat{\theta}_2 = c^{1/3} = \theta^*$$ identical cutoff values

Prob both firms develop $(1-\theta^*)^2$

no " " θ^{*2}

only one "

= i develops + j develops

$\theta^*(1-\theta^*) + \theta^*(1-\theta^*)$

$= 2\theta^*(1-\theta^*)$