HW4 Externalités

11.B.1 Positive Externalité

11.B.1[B] (M. Weitzman) On Farmer Jones' farm, only honey is produced. There are two ways to make honey: with and without bees. A bucket full of artificial honey, absolutely indistinguishable from the real thing, is made out of 1 gallon of maple syrup with one unit of labor. If the same honey is made the old-fashioned way (with bees), k total units of labor are required (including bee-keeping) and b bees are required per bucket. Either way, Farmer Jones has the capacity to produce up to H buckets of honey on his farm.

The neighboring farm, belonging to Smith, produces apples. If bees are present, less labor is needed because bees pollinate the blossoms instead of workers doing it. For this reason, c bees replace one worker in the task of pollinating. Up to A bushels of apples can be grown on Smith's farm.

Suppose that the market wage rate is w, bees cost p_b per bee, and maple syrup costs p_m per gallon. If each farmer produces her maximal output at the cheapest cost to her (assume the output prices they face make maximal production efficient), is the resulting outcome efficient? How does the answer depend on k, b, c, w, p_b, and p_m? Give an intuitive explanation of your result. Up to how much would Smith be willing to bribe Jones to produce honey with bees? What would happen to efficiency if both farms belonged to the same owner? How could the government achieve efficient production through taxes?

a. Socially optimal outcome

Jones will choose to produce honey with bees if and only if the cost of using bees is lower than that of using maple syrup, i.e.

$$H(\,bp_b + kw) < H(p_m + w)$$
$$=> p_m > bp_b + kw - w \;\; (1)$$

The social planner will choose bee production if and only if

$$H(\,bp_b + kw) + Aw - \frac{b}{c}w * H < H(p_m + w) + Aw$$
$$=> p_m > bp_b + kw - \frac{b}{c}w \;(2)$$

By simple observation, whatever values k, p_b and w take, the inequality (2) is still valid when (1) is satisfied.

Thus we know that if ratio $\frac{b}{c} \leq 1$, then the Jones' choice of using bee production if socially optimal. If $\frac{b}{c} > 1$, then Jones' choice of using maple syrup is not socially optimal. Intuitively, this means if the efficiency of using bee is higher compared to that of using worker is higher, then the socially optimal outcome is achieved using bee production.

The values of p_m and p_b will affect whether Jones' choice is socially optimal as illustrated in (1) and (2)

b. Compensation for positive externality

Smith is willing to compensate Jones for using bees production with a payment of $H\frac{b}{c}w$ (the difference between Smith's cost under social optimal condition and Smith's cost under Jones' cost minimum condition)

c. Merging

If two farms belonged to the same owner, then the cost can be internalized, thus the social optimal outcome is achieved.

d. Tax/Subsidy for optimal outcome

Finally, government (social planner) can achieve optimal outcome by taxing Smith a lump sum payment and subsidize Jones' bee production, and the amount equals $H\frac{b}{c}w$.

11.B.4 Offset Negative Externality

11.B.4[B] Consider again the two-consumer externality problem studied in Section 11.B. Suppose that consumer 2 can take some action, say $e \in \mathbb{R}$, that affects the degree to which she is affected by the externality, so that we now write her derived utility function as $\phi_2(h, e) + w_2$. To fix ideas, let h be a negative externality, and suppose that $\partial^2\phi_2(h, e)/\partial h \partial e > 0$, so that increases in e reduce the negative effect of the externality on the margin. Suppose that both h and e can in principle be taxed or subsidized. Should e be taxed or subsidized in the optimal tax scheme? Why or why not?

Since the setting is that increases in e reduce the negative effect of the negative externality on the margin, for a given level of h, **we would foresee that consumer 2 will choose an optimal e^o that maximizes her utility function, and the social optimal outcome is also achieved at this time.**

In this case, there is no need to subsidize e since it will not affect the social optimal outcome (consumer 1's utility function is not a function of e, thus increase in e does not have spillover effect)

11.C.1

11.C.1[A] Consider the model discussed in Section 11.C, in which I consumers privately purchase a public good. Identify per-unit subsidies $s_1, \ldots, s_I$, such that when each consumer i faces subsidy rate s_i, the total level of the public good provided is optimal.

From 11.C we know that the necessary and sufficient first-order condition for the optimal quantity q^o is then

$$\sum_{i=1}^{I} \phi_i'(q^\circ) \le c'(q^\circ), \quad \text{with equality if } q^\circ > 0.$$

At an interior solution, we have $\sum_i \phi_i'(q^\circ) = c'(q^\circ)$, which means the sum of consumers' marginal benefits from the public good is set equal to its marginal cost.

Since in this setting there is only one public good, **then the per unit subsidy s_i facing consumer *i* should be set to the sum of consumers' marginal benefits excluding *i*'s marginal benefit to have the optimal quantity q^o.**

Thus we have

$$s_i = \sum_{j \neq i} \phi_i'(q^o)$$

11.D.3

11.D.3[B] Consider an industry composed of $J > 1$ identical firms that act as price takers. The price of their output is p, and the prices of their inputs are unaffected by their actions. Suppose that partial equilibrium analysis is valid and that the aggregate demand for their product is given by the function $x(p)$. The industry is characterized by "learning by doing," in that each firm's total cost of producing a given level of output is declining in the level of total industry output; that is, each firm j has a twice-differentiable cost function of the form $c(q_j, Q)$ for $Q = \sum_j q_j$, where $c(\cdot)$ is strictly increasing in its first argument and strictly decreasing in its second. Letting subscripts denote partial derivatives, assume that $c_q + Jc_Q > 0$ and $(1/n)c_{qq} + 2c_{qQ} + nc_{QQ} > 0$ for $n = 1$ and J. Compare the equilibrium and optimal industry output levels. Interpret. What tax or subsidy restores efficiency?

11.D.3 The Problem of firm J is to solve.

$$\max_{q_j} \pi_j(q_j, q_{-j}) = P \cdot q_j - C(q_j, Q).$$

$$Q = q_j + \sum_{k \neq j} q_k.$$

F.O.C $\Longrightarrow$ $P - C_q(q_j, Q) - C_Q(q_j, Q) = 0.$

S.O.C $\Longrightarrow$ $-C_{qq}(q_j, Q) - 2C_{Qq}(q_j, Q) - C_{QQ}(q_j, Q)$

and this term strictly smaller than 0 ~~by~~

assumption $(\frac{1}{n}) \cdot C_{qq} + 2C_{qQ} + nC_{QQ} > 0$

for $n = 1$ & J.

Thus we have a unique solution $Q^* = J \cdot q^*$

F.O.C $\Rightarrow P = C_q(q_j^*, Q^*) + C_Q(q_j^*, Q^*) \geq 0$

(by assumption)

A social planner's problem is.

$$\max_{q_1 \cdots q_J} \int_0^Q P(x)dx - \sum_{k=1}^{J} C(q_k, Q).$$

F.O.C $\Longrightarrow$ $P(Q) - C_q(\cdot) - C_Q(\cdot)$

$$- \sum_{k \neq J} C(\cdot) = 0.$$

Let q° denote the social optimal solution

$$P(Q^\circ) = C_q(\frac{Q^\circ}{J}, Q^\circ) + C_Q(\cdot) + \sum_{k=1}^{J-1} C(\cdot)$$

Then we know $P(Q^o) < C_q(*) + C_Q(*)$
Since we already know that $P'^{(Q)} < 0$,

$$\frac{d}{dQ}\left(C_q(q_j,Q) + C_Q(q_j,Q)\right) = \frac{1}{J}(C_{qq}(.) + (J+1)C_{qQ}(.) + JC_{QQ}(.) > 0$$

We claim that **the optimal total industry output $\mathbf{Q^o > Q^*}$**

The interpretation: since this industry is characterized as "learning by doing", each individual firm does not see the positive externality created and thus the total industry output will be less than the optimal output.

To **restore efficiency**, s = - $\sum_{k=1}^{j-1} C_Q(Q^o/J, Q^o) = -(J-1)\, C_Q(Q^o/J, Q^o)$, then the solution of Firm j is $q^* = \frac{Q^o}{J}$, which makes Q^* equal the socially optimal output Q^o.

11.D.4

11.D.4[B] Reconsider the nondepletable externality example discussed in Section 11.D, but now assume that the externalities produced by the J firms are not homogeneous. In particular, suppose that if $h_1, \ldots, h_J$ are the firms' externality levels, then consumer i's derived utility is given by $\phi_i(h_1, \ldots, h_J) + w_i$ for each $i = 1, \ldots, I$. Compare the equilibrium and efficient levels of $h_1, \ldots, h_J$. What tax/subsidy scheme can restore efficiency? Under what condition should each firm face the same tax/subsidy rate?

a. Restore efficiency

The social planner's problem is to solve

$$\underset{h_1, h_2 \ldots h_j}{Max} \sum_{i}^{I} \phi_i(h_1 \ldots h_j) + \sum_{j=1}^{J} \pi_i(h_j)$$

$$\xrightarrow{F.O.C} \sum_i^I \phi_i'(.) + \pi_i'(h_j) \leq 0 \text{ with equality if } h_j > 0$$

we have h_j^o satisfies the F.O.C

At any competitive equilibrium, each firm j will wish to set the externality-generating activity at the level h_j^* such that

$$\pi_j(h_j^*) \leq 0, \quad \text{with equality if } h_j^* > 0.^{13}$$

Then we must set tax rate facing firm j equals $-\sum_i^I \phi_i'(h_1^o, h_2^o \ldots h_j^o)$, which is the sum of the marginal benefit of producing more externalities facing firm j. Then the F.O.C of the objective function is

$$\xrightarrow{F.O.C} \sum_i^I \phi_i'(.) + \pi_i'(h_j) - \sum_i^I \phi_i'(h_1^o, h_2^o \ldots h_j^o) \leq 0 \text{ with equality if } h_j > 0$$

$$=> \pi_i'(h_j) \leq 0 \text{ with equality if } h_j > 0$$

Thus, efficiency is restored.

b. Firm facing the same tax rate

It is easy to see that each firm will face the same tax rate if and only if $-\sum_i^I \phi_i'(h_1^o, h_2^o \ldots h_j^o)$ are same for each firm.

11.E.3 and 11.E.4

11.E.3[B] Show that truth-telling is the consumer's only weakly dominant strategy in the (Groves–Clarke) revelation mechanism studied in Section 11.E.

11.E.4[A] In text.

Exercise 11.E.4: Show that in the tax-subsidy part of the mechanism above we could add, without affecting the mechanism's truth-telling or optimality properties, an additional payment to each agent that depends in an arbitrary way on the other agent's announcement.

11.E.3 Solution

We already know that under Groves-Clarke revelation mechanism truth telling is guaranteed.
Suppose there exists another weakly dominant strategy that is not truth telling.
However this cannot be true simply because if there is a pair of (b, c') that does not reflect the true values, then c is not the best response.
To see this, if not truth telling, a consumer may announce c',
If c'> b > c, the consumer should announce c' instead of c.
If c' < b < c, the consumer should choose c' instead of c
We find a contradiction to the best response. Thus, truth telling is the only weakly dominant strategy in revelation mechanism.

11.E.4 Solution

If we add an additional payment to each agent that depends in an arbitrary way on the other agent's announcement, it will not change the mechanism's truth-telling or optimality properties.
To see this, for consumer, we add up a term in the consumer's utility function (a term equals an additional payment of the firm), the analysis in the book still follows. The solution of consumer's problem is still the same since in the F.O.C the derivative of this term is 0.