EC204 Micro II Spring 2020

HW3
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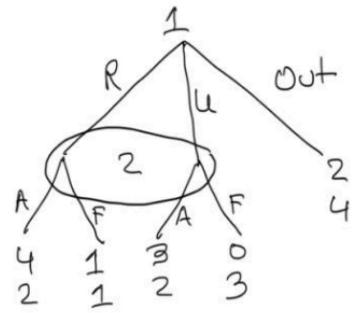
1. Find all perfect Bayesian equilibria of the following game:



Solution:
Let *m* and *1-m* be Player 2's beliefs that Player 1 has chosen R and U respectively
Player 2 will choose A over F when

$$2m + (1 - m) \geq m + 3(1 - m)$$

$$=> m \geq \frac{1}{2}$$

If $m > \frac{1}{2}$, Player 2 plays *A*. 1 must play *R*. The beliefs are consistent.

If $m = \frac{1}{2}$, Player 2 is indifferent between *A* and *F*. Player 1 must play *R*, then m = 1. A contradiction
If $m < \frac{1}{2}$, Player 2 plays *F*. Player 1 must play *Out*. This may be a weakly PBE but never a PBE.
Thus we have this PBE: $(m \geq \frac{1}{2}$, (R, A))

2. Tesla wants to buy GM. However GM's true value is unknown to Tesla and is supposed to be $x, where Tesla thinks x is distributed uniformly between 0 and 100 (in million dollars). GM knows its value. Tesla makes an offer to pay GM $y to merge. It can choose any value for y. If GM accepts, the firms merge and the value of the new combined firm which is owned by Tesla is 1.5x. That is, if GM accepts, Tesla's payoff is 1.5x-y and GM's payoff is y. If GM rejects the offer, Tesla gets $0 and GM has a value of $x. Find a NE of this game.

Solution:
Consider the case when GM accepts the offer (if y > x) . So, if GM's market value is below y, then GM will accept it.
Given x is distributed uniformly between 0 and 100, the expected value of GM in this case is

$$\mathrm{E(x)} = (\mathrm{x} + 0)/2 = \mathrm{x}/2$$

For Tesla, the expected payoff is

$$1.5E(x) - y > 0 \Rightarrow \frac{1.5x}{2} - y > 0$$

This is obviously impossible since y > x

Thus, there is a NE (Tesla makes an offer, GM declines)

3. Consider the case below where row player country 1 decides whether to keep its nukes or destroy them and country 2 has to decide whether to spy on country 1 or not. Country 1 below may be aggressive with probability ß and non-aggressive with probability 1- ß. The aggressive type likes its nukes and if there is spying, it leads to war which country 1 wins and is costly to country 2. When country 1 is non-aggressive, in this case there is no war, but only a scandal. The payoffs are given below. What is the Bayesian Nash Equilibrium? Assume ß<0.2.

Country 1 is aggressive (ß)

P2

P1	Spy	No Spy
Keep	10,-9	5,-1
Destroy	0,2	0,2

Country 1 is non aggressive (1- ß)

P2

P1	Spy	No Spy
Keep	-1,1	1,-1
Destroy	0,2	0,2

Solution:

Observe that *(Keep, No Spy)* and *(Destroy, Spy)* are NEs in two games.

If Country 1 is aggressive, then it will stick to *Keep* because *Destroy* is strictly dominated by Keep.

If Country 1 is non aggressive, it will play *Destroy*. Since it assumes Country 2 rationally behaves and play *Spy* because *No Spy* is weakly dominated by *Spy,* then Country 1 plays *Destroy* accordingly.

Thus Country 2's best response given Country 1's strategy is to

Play *Spy* if $-9\beta + 2(1-\beta) > -\beta + 2(1-\beta)$

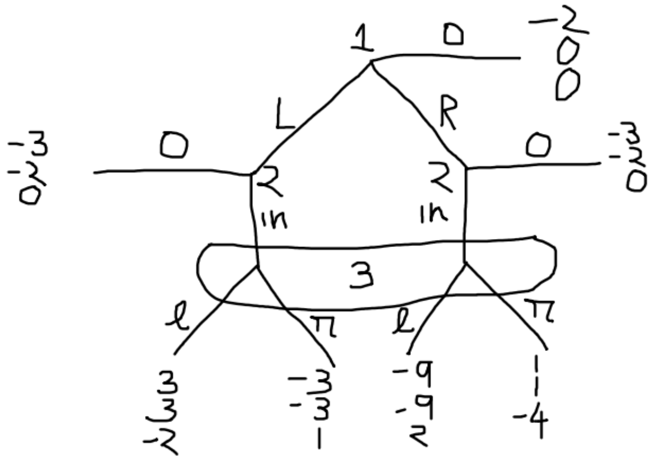
However, this can't be true for $\beta \epsilon [0, 0.2)$

So, Country 2 must play No Spy

There is a BNE

($\beta \epsilon [0, 0.2)$, (Country 1 plays *Keep* if it is aggressive type, plays *Destroy* if it is non-aggressive type), Country 2 plays *No Spy*)

4. Consider the game below. Sorry my apple pen was not available so I had to draw it crudely with my fingers! Player 1 can play Out or L or R. Player 2 can play In or Out. Player 3 plays l or r. Find a perfect Bayesian equilibrium that is consistent with a system of beliefs of each player.



Solution:

First assume Player 3's beliefs about Player 1 has chosen *L* and Player 2 has chosen *In* is μ, and Player 1 has chosen *R* and Player 2 has chosen *In* is $1-\mu$.
Player 3 plays *l* when

$$-2\mu + 2(1-\mu) > \mu - 4(1-\mu) => \ \mu < 2/3$$

If $\mu < 2/3$, Player 3 plays *l,* Player 2 must play *In*, Player 1 must play *L* => $\mu = 1$, a contradiction

If $\mu > 2/3$, Player 3 plays *r,* Player 2 must play *In*, Player 1 must play *R* => $\mu = 0$, a contradiction.

If $\mu = 2/3$, Player 3 is indifferent between *l* and *r*. Player 2 plays *In*.
Assume Player 3 plays *l* and *r* with σ_l and $1 - \ \sigma_l$ respectively.
Player 1 should be indifferent between *L* and *R* if

$$3\sigma_l + (1-\sigma_l) * (-3) = 9\sigma_l + (1-\sigma_l) * 1 => \ \sigma_l = 1/4$$

So, we have this PBE: $\mu = 2/3$, ((2/3, 1/3, 0), In, (1/4, 3/4)).

(We simply cannot check the cases in which Player 1 plays *Out* or Player 2 plays *Out* so in these cases there can't be any PBE)