EC 204 Micro II Note Week 4 L1 20/02/04

1. **Unreasonable**

When the only subgame is the game as a whole, pure strategy Nash equilibria are subgame perfect. However, sometimes PSNE does not make sense, like (Out, Fight).

2. Beliefs

- A system of beliefs

Definition 9.C.1: A *system of beliefs* μ in extensive form game Γ_E is a specification of a probability $\mu(x) \in [0, 1]$ for each decision node x in Γ_E such that

$$\textstyle\sum_{x \in H} \mu(x) = 1$$

for all information sets H.

  - Example:
- Figure 1. Insert from note (Ex1)
- Figure 2. Insert from note (EX2)

3. Sequentially rational at an information set given a system of beliefs

Definition 9.C.2: A strategy profile $\sigma = (\sigma_1, \ldots, \sigma_I)$ in extensive form game Γ_E is *sequentially rational at information set H given a system of beliefs* μ if, denoting by $\iota(H)$ the player who moves at information set H, we have

$$E[u_{\iota(H)} | H, \mu, \sigma_{\iota(H)}, \sigma_{-\iota(H)}] \geq E[u_{\iota(H)} | H, \mu, \tilde{\sigma}_{\iota(H)}, \sigma_{-\iota(H)}]$$

for all $\tilde{\sigma}_{\iota(H)} \in \Delta(S_{\iota(H)})$. If strategy profile σ satisfies this condition for *all* information sets H, then we say that σ is *sequentially rational given belief system* μ.

Thus, in Ex1 and Ex2, (L, m) and (Out, F) are not sequentially rational

4. Bayes' rule

   Prob(A|B) = Prob (A Intersect B) / Prob (B)

$$\text{Prob}\,(x \mid H, \sigma) = \frac{\text{Prob}\,(x \mid \sigma)}{\sum_{x' \in H} \text{Prob}\,(x' \mid \sigma)}.$$

5. Weak Perfect Bayesian Equilibrium in extensive game

Definition 9.C.3: A profile of strategies and system of beliefs (σ, μ) is a *weak perfect Bayesian equilibrium* (weak PBE) in extensive form game Γ_E if it has the following properties:

(i) The strategy profile σ is sequentially rational given belief system μ.

(ii) The system of beliefs μ is derived from strategy profile σ through Bayes' rule whenever possible. That is, for any information set H such that Prob $(H \mid \sigma) > 0$ (read as "the probability of reaching information set H is positive under strategies σ"), we must have

$$\mu(x) = \frac{\text{Prob}\,(x \mid \sigma)}{\text{Prob}\,(H \mid \sigma)} \quad \text{for all } x \in H.$$

- PBE is a strategy-belief pairs. Be explicit
- The relationship between the weak PBE and that of Nash equilibrium

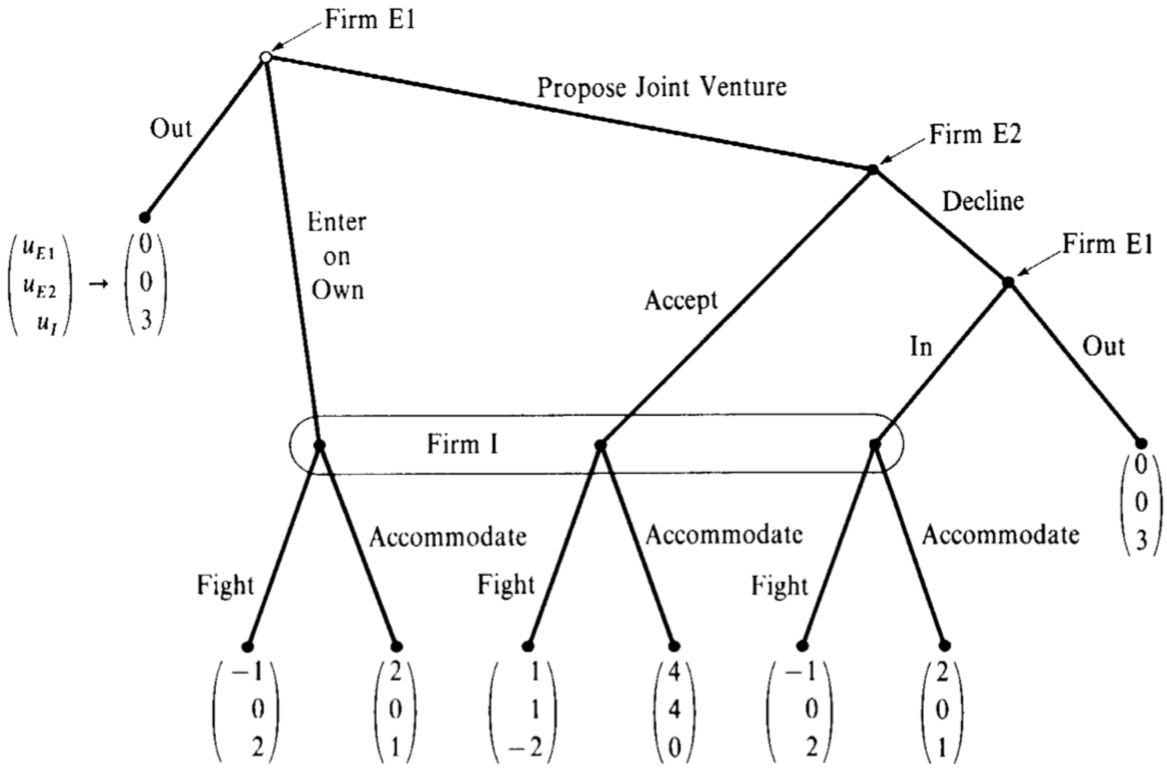
Proposition 9.C.1: A strategy profile σ is a Nash equilibrium of extensive form game Γ_E if and only if there exists a system of beliefs μ such that

(i) The strategy profile σ is sequentially rational given belief system μ *at all information sets H such that Prob* $(H \mid \sigma) > 0$.

(ii) The system of beliefs μ is derived from strategy profile σ through Bayes' rule whenever possible.

Insert proof here

- Example 9.C.2: Consider a joint venture entry game.



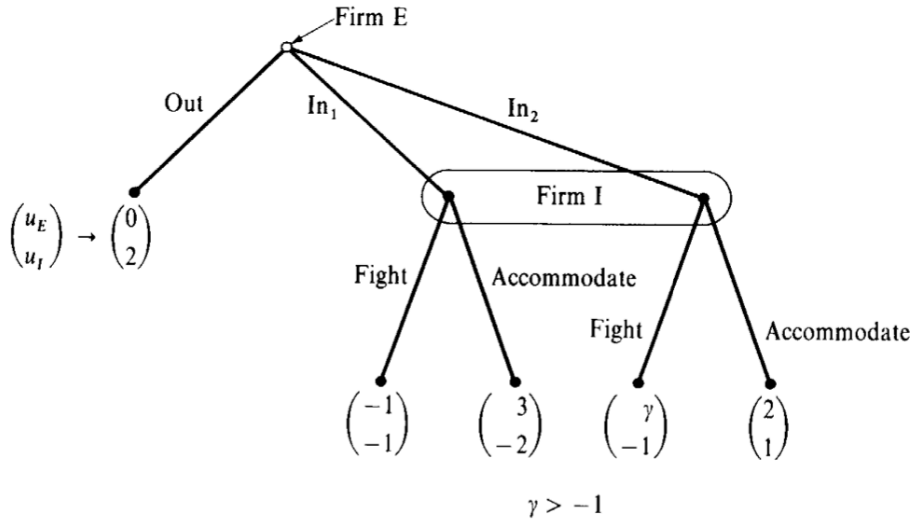
We conclude that Firm E2 must accept this venture.

System of beliefs: mu (that Firm I's information set is reached) = 1

Strategy ((purpose Joint venture, in if E2 declines), Accept, Accommodate))

Other SPNE: ((out, out if E2 declines), decline, fight if entry occurs)

- Example 9.C.3 the optimal strategy for player depends on another player's behavior.



- Restrict to the case where $\gamma > 0$

  Let μ_1 be Firm I's belief that Firm E choose In_1 if Entry has occurred

  Note that Firm I is willing to play "fight" iff

  $$-\mu_1 - (1 - \mu_1) > -2\mu_1 + (1 - \mu_1) \Rightarrow \mu_1 > 2/3$$

  If $\mu_1 > \frac{2}{3}$, then firm I must play fight. Then Firm E must be playing In_2 with probability 1. Then $\mu_1 = 0$, a contradiction

  If $\mu_1 < \frac{2}{3}$, then E play In_1 => $\mu_1 = 1$, a contradiction

  Hence, in any weak PBE of this game, we must have $\mu_1 = 2/3$

  And E should also be indifferent between In_2 and In_1

  => $\sigma_F = \frac{1}{\gamma+2}$

  This unique PBE (0, 2/3, 1/3), $(\frac{1}{\gamma+2}, 1\text{-}\frac{1}{\gamma+2})$, $\mu_1 = 2/3$