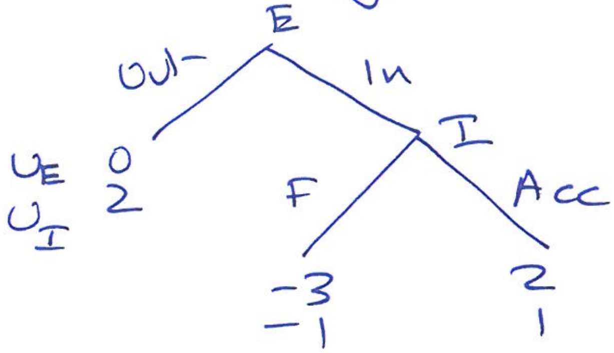
Dynamic Games Ch 9

Sequentail Rationality:

Predation Game:



		I	
		F/In	Acc/In
E	Out	0, 2	0, 2
	In	-3, -1	2, 1

2 psNE

{Out, F/In} ← Is it reasonable

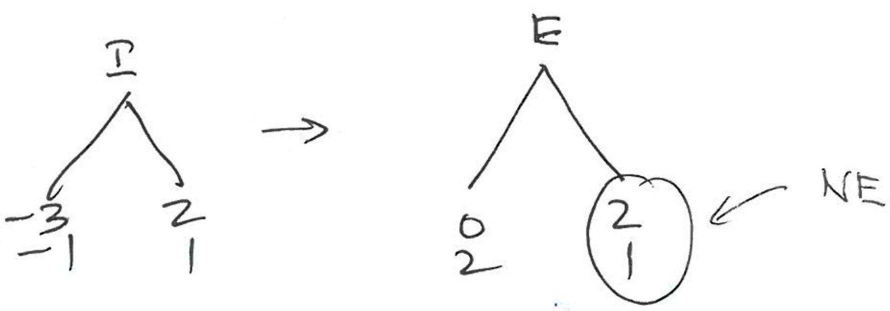
{In, Acc/In}.

IF called upon, I will play Acc.

∴ I's play not sequentially rational.

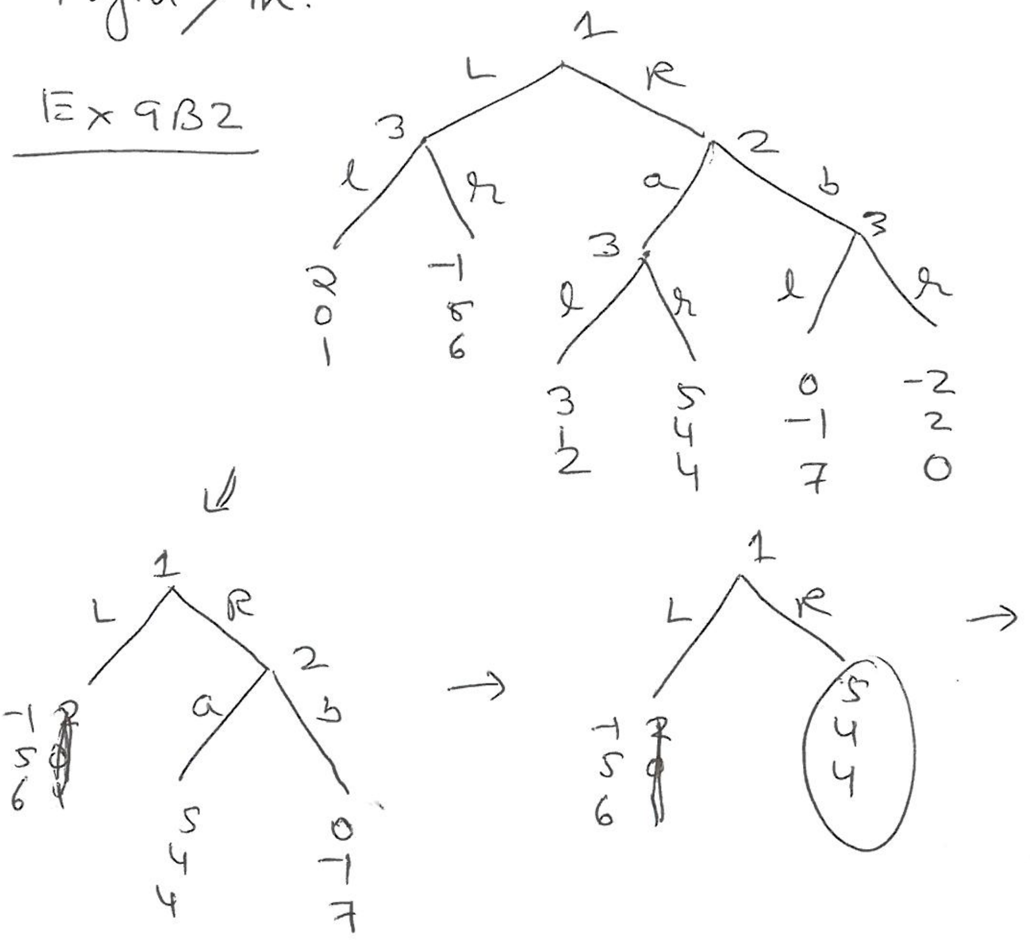
Stategy should be optimal at every point in game tree.

Use principle of "backward induction":



Players must satisfy principle of "sequential rationality.". must play optimally at every node. Rules out Fight/In.

Ex 9B2

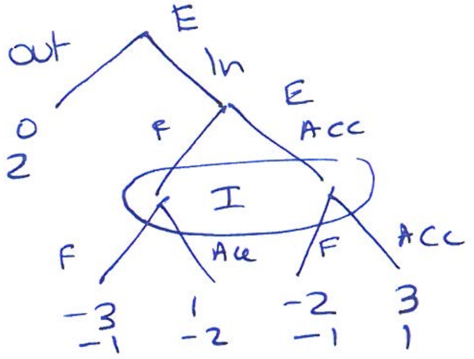


Qs. Are There other NE?

Every finite game of perfect info has a pSNE that can be derived thru backward induction.

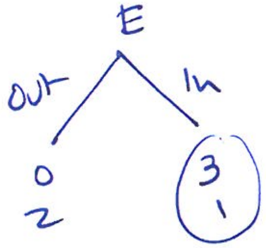
If no player has same payoff at terminal nodes, Unique NE.

Ex 9B3



NE in subgame

E \ I	Acc	F
Acc	3, 1	-2, -1
F	1, -2	-3, -1



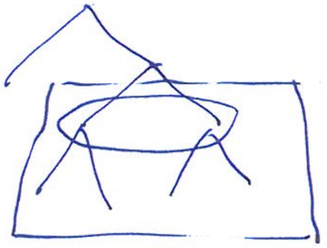
But check normal form of whole game:

E \ I	F	Acc
Out/F	0, 2	0, 2
Out/Acc	0, 2	0, 2
In/F	-3, -1	1, -2
In/Acc	-2, -1	3, 1

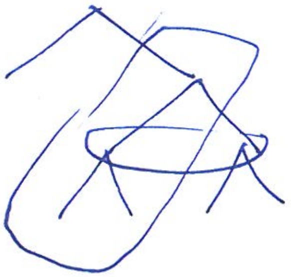
3 psNE

Subgame 1. Game begins w/ decision node, contains all successor nodes, and none other.

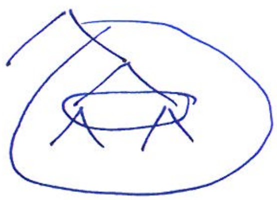
2. If decision node is in subgame, all inf sets containing that subgame are.



not s/game



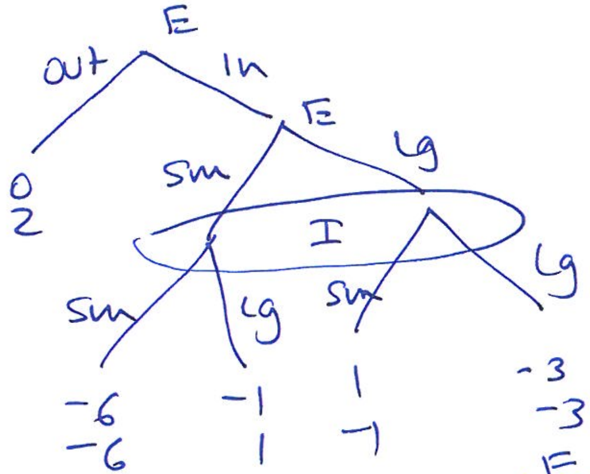
not s/game



sub game. Whole game is s/game.

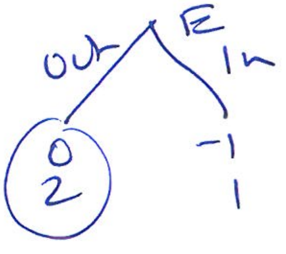
Prop. Every finite game of perfect information ~~has~~ has a ~~psNE~~. ps SPNE.
Unique if no player has same payoffs at any two terminal nodes.

Niche Choice Game



		I	
		Sm	Lg
E	Sm	-6,-6	-1, 1
	Lg	1,-1	-3,-3

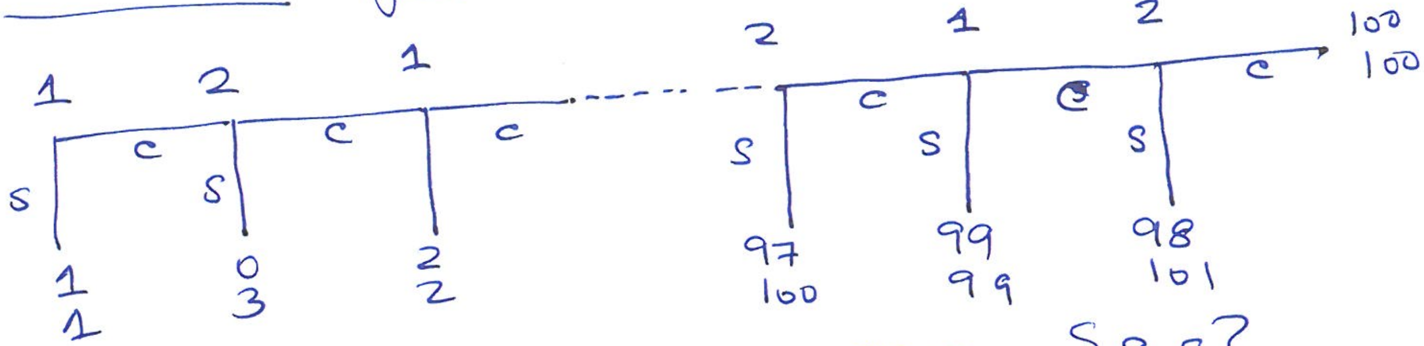
NE



SPNE :

{out, small, large
In, large, small.}

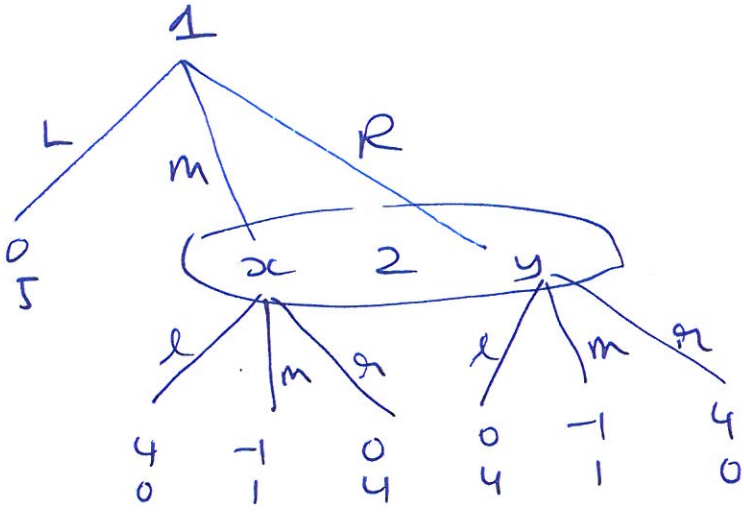
Centipede game   Is SPNE reasonable?

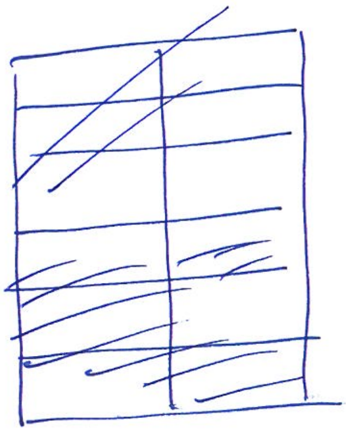


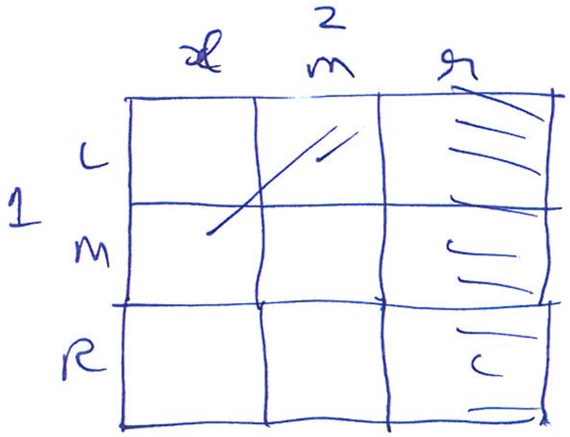
Backward induction: SPNE {S,S}

Beliefs and sequential Rationality

No strict subgame.
many NE







2 NE

		l	m	r
1	L	0,5	0,5	0,5
	M	4,0	-1,1	0,4
	R	0,4	-1,1	4,0

PSNE = ?. {M, m}

it is NE & SPNE since no strict subgame.

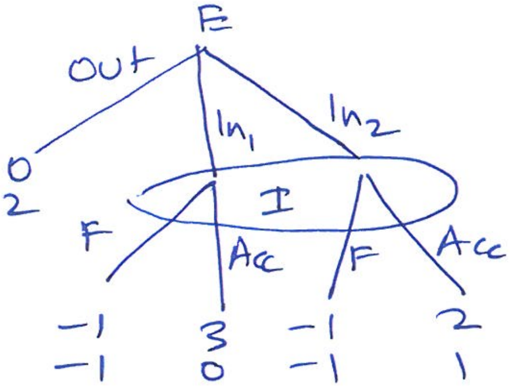
shd 2 play m?

let 2 assume p1 chose M with prob $p(x)$, R with $p(y)$. $p(x)+p(y)=1$.

Eπ to 2 are	$4p(y)$	1	$4p(x)$
	of choosing l	choose m	choose r.

=> 2 never chooses m bc m is dominated whatever value of $p(x), p(y)$.

Beliefs & Seq Rationality



I

		F	Acc
E	out	0,2	0,2
	In_1	-1,-1	3,0
	In_2	-1,-1	2,1

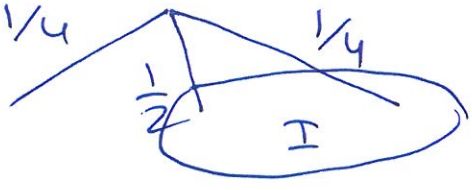
2 psNE

No proper subgame

technically both psNE are SPNE!

System of beliefs by each player

What does I think E will do?

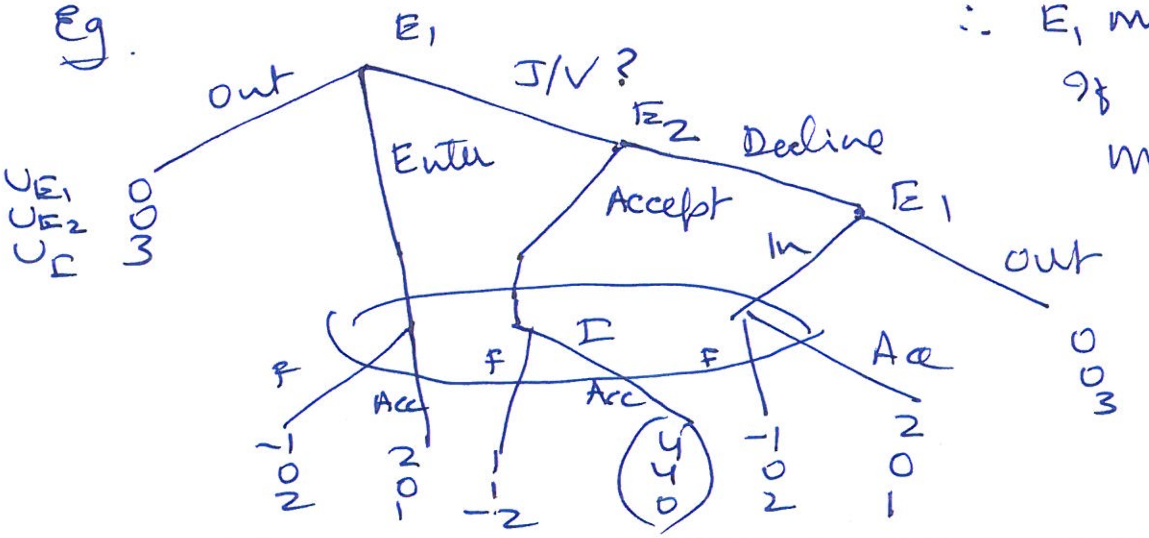


I should play Acc if Entry occurs.

{out, F} should not be a PBE.

If Entry occurs, Acc is dominant strategy for I.

Eg.



E_2 must accept.

∴ E_1 must offer J/V

If I plays, I must be at middle node with prob 1.

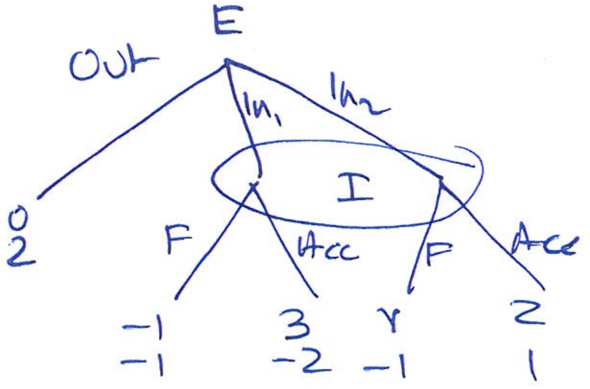
So must acc.

PBE: { ~~out,~~ I/N, In, Accept, Acc }

& system of beliefs μ { middle node of I's inf set with prob 1 }.

Other SPNE: { out, out, Decline, fight }

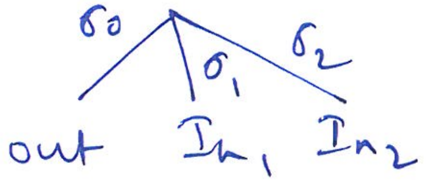
Above, beliefs were easy to identify.



Assume $r > 0$.

σ_F = prob I fights.

I believes In_1 occurred conditional on entry with prob $\underline{\underline{\mu_1}}$.



prob with which E chooses his actions.

$r > -1$. So In_1 does not dominate In_2

I fights iff $-1\mu_1 + (-1)(1-\mu_1) > -2(\mu_1) + 1(1-\mu_1)$

$$-\mu_1 - 1 + \mu_1 > \cancel{-2}\,2\mu_1 + 1 - \mu_1$$

$$\cancel{-1 > -2 + 1 + \mu_1} \qquad -1 > -3\mu_1 + 1$$

$$\cancel{> \mu_1} \qquad \mu_1 > \frac{2}{3}.$$

But if $\mu_1 > \frac{2}{3}$, I plays F

∴ E plays In_2 with prob 1. $(r > 0)$

∴ $\mu_1 = 0$. ✗.

Sps $\mu_1 < 2/3$. I plays Acc.

E plays In_1 with prob 1. ∴ $\mu_1 = 1$ X.

∴ PBE. $\mu_1 = 2/3$.

That is $\sigma_1 = 2\sigma_2$.

E is indiff between In_1 & In_2.

$$\therefore \quad -1\sigma_F + 3(1-\sigma_F) = v\sigma_F + 2(1-\sigma_F)$$

$$-\sigma_F + 3 - 3\sigma_F = v\sigma_F + 2 - 2\sigma_F$$

$$-4\sigma_F + 3 = \sigma_F(v-2) + 2$$

$$1 = 4\sigma_F + \sigma_F(v-2)$$

$$1 = \sigma_F(v+2)$$

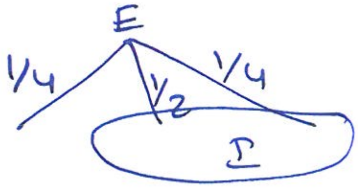
$$\sigma_F = 1/v+2.$$

Unique BE: $(\sigma_0, \sigma_1, \sigma_2) = (0, 2/3, 1/3)$

$\sigma_F = 1/v+2$

$\mu_1 = 2/3$.

9-10



Prob of being on left for I

$$= \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{4}} = \frac{2}{3}.$$

on right = 1/3.