EC204 Micro II HW 6 Liuyi YE

Ch12.

C4. Consider altering Bertrand duopoly model in which Case each firm's per unit cost is c_j. $c_1 < c_2$.

(a) PSNE

Obviously $P_2 \geq c_2 > c_1$, $P_1 \geq c_1$.

Thus Firm 1 can make a profit close to $(c_1 - c_2) q(c_2)$ by charging $c_2 + \varepsilon$.

So we charactrize this PSNE as.

$P_1 = c_2$, and $U_{Firm 1} = (c_2 - c_1) q(c_2)$.

C9. Consider 2-firm Cournot, constant r-to-scale c_j the per unit cost, $c_1 > c_2$.

$P(q) = a - bq$. $a > c_1$

(a) Derive NE. Under what condition there is only 1 firm producing? which one?

① The problem of firm J is.

$$\max_{q_j} \pi_j (q_j, q_{-j}) = [a - b(q_j + q_{-j}) - c_j] q_j$$

$\xrightarrow{F.O.C}$ $a - bq_j - 2bq_{-j} - c_j \leq 0$

$$\Rightarrow q_j^* = \frac{a + c_{-j} - 2c_j}{3b}.$$

Thus. we have the NE. (q_j^*, q_{-j}^*)

② only 1 firm produces.

Then the solution of the problem is $q_j = 0$.

iff $\left.\frac{\partial \pi(\cdot)}{\partial q_j}\right|_{q_j = 0} = a - bq_{-j} - c_j \leq 0$

$$q_{-j}^* = \frac{a - c_{-j}}{2b}$$

Thus the condition is $c_j \geq a - bq_{-j}^*$

$$= a - \frac{a - c_{-j}}{2}$$

$$= \frac{a + c_{-j}}{2}$$

③ The firm that produces 0 is firm 1.

Since when $c_j \geq \frac{a + c_{-j}}{2}$, $q_1^* = \frac{a + c_{-j} - 2c_j}{3b} \leq 0$.

which make sense since $c_1 > c_2$

(b) when 2 firm produce, how does eq output change when c1 changes

$$\pi_j^* = P(q_j^* + q_{-j}^*)q_j^* - c_j q_j^*$$

$$= \frac{a - 2c_j + c_{-j}}{9b}.$$

So if $c_1 \uparrow$, then

$q_1 \downarrow$ [from (a)], $q_2 \uparrow$.

$\pi_1 \downarrow$, $\pi_2 \uparrow$

(c) Now consider J firms.

show ratio of industry profits divided by Industry Revenue in NE is H/ε.

ε – The elasticity of the market demand curve at eq.

H – The Herfindahl index of concentration

$$= \Sigma_j \left(\frac{q_j^*}{Q^*}\right)^2$$

The problem is still the one in (a)., now

$$Q_{-j} = \sum_{k \neq j} q_k$$

$$\xrightarrow[\text{w.r.t } q_j]{\text{F.O.C}} \quad P - C_j = -P'(Q)\, q_j$$

$$= \frac{-P'(Q)\, Q}{P} \cdot P \cdot \frac{q_j}{Q}$$

$$= \frac{1}{\varepsilon} \cdot P \cdot \alpha_j$$

where. $Q = q_j + Q^{*}_{-j}$, $P = P(Q)$

and $\alpha_j = q_j / Q$ (the market share).

Thus The industry profit is.

$$\Pi = \sum_{j=1}^{J} \pi_j$$

$$= \sum (P - C_j)\, q_j = \sum P \cdot \frac{\alpha_j q_j}{\varepsilon}$$

$$= \sum P \frac{\alpha_j^2 Q}{\varepsilon}$$

$$= H \cdot \frac{PQ}{\varepsilon}$$

$$\Rightarrow \frac{\pi}{PQ} = \frac{H \cdot PQ / \varepsilon}{PQ} = H / \varepsilon$$

C13 Show when $v > c+3t$ in the Linear City Model, a Firm j's best response to P_{-j} results in all consumers purchasing from one of the two firms

The firm's problem is

$$\pi_j(P_j) = (P_j - c)\, x(P_j)$$

$$= (P_j - c) \cdot \frac{V - P_1}{t}$$

$$\xrightarrow{F.O.C} \pi_j'(P_j) = \frac{V + c - 2P_j}{t} \quad *$$

Suppose some consumers strictly prefer not to purchase.

So some consumers will not buy rather than buying from Firm j

$\Rightarrow P_j > V - t$ (**) (for consumers at $z=1$)

$$\xrightarrow{*\,\&\,**} \pi_j'(P_j) < \frac{c+2t-V}{t} < 0$$

Thus This is not the best response $\square$

D2

Stationary SPNE outcome. iff $\delta \geq \frac{J-1}{J}$.

Π^* is the total profit at eq.

A Firm can slightly lower the price by ε, and can steal demand and obtain profit close to Π^*.

In later periods, This firm gets 0.

If not, gets $\sum_{t=0}^{\infty} \delta^t \Pi^* / J$

The deviation happens iff.

$$\sum \delta^t \pi^* / J \geq \pi^*$$

$$\Rightarrow \frac{1}{1-\delta} \pi \geq \pi \cdot J.$$

$$\frac{1}{J} \geq 1 - \delta.$$

$$\delta \geq \frac{J-1}{J}$$

as $J \uparrow$, $\delta \uparrow$.

So with more Firms in the market, the difficulty of sustaining collusion $\uparrow$

E2 prove that π_J is ↓ in J under Assumption A1 – A3 of Prop. 12.E.1

$$\pi_j = P(Jq_j)\,q_j - c(q_j)$$

$$\frac{d\pi_j}{dJ} = \left[P(Jq_j) - c'(q_j)\right]\frac{dq_j}{dJ} + P'(Jq_j)\cdot q_j\,\frac{dJq_j}{dJ}$$

By A3. $P(Jq_j) - c'(q_j) > 0.$

A2. $\frac{dq_j}{dJ} < 0.$

A1. $\frac{dJq_j}{dJ} > 0.$

$$\Rightarrow \frac{d\pi_j}{dJ} < 0.$$ Thus π_j ↓ in J