12B. Monopoly Pricing

1. Basic Model

We study the pricing behavior of a profit-maximizing monopolist, a firm that is the only producer of a good.

- The demand for this good at price p is given by the function *x(p), x(p)* is continuous and strictly decreasing at all p s.t x(p) > 0
- For convenience, assume there exists a price (ensures an optimal solution to the monopolist's problem exists)
  $\bar{p} < \infty$ such that $x(p) = 0$ for all $p \geq \bar{p}$.
- Suppose the monopolist knows the demand function for its product and can produce output level *q* at cost of *c(q)*

The monopolist's problem is:

$$\underset{p}{\text{Max}} \quad px(p) - c(x(p)).$$

An equivalent problem is:

$$\underset{q \geq 0}{\text{Max}} \quad p(q)q - c(q).$$

Assume p(.) and c(.) are continuous and twice differentiable at all q >= 0,

We focus on the quantity formulation for the monopolist's problem

$$p'(q^m)\, q^m + p(q^m) \leq c'(q^m), \quad \text{with equality if } q^m > 0. \tag{12.B.3}$$

The left-hand side of (12.B.3) is the *marginal revenue* from a differential increase in q at q^m, which is equal to the derivative of revenue $d[p(q)q]/dq$, while the right-hand side is the corresponding marginal cost at q^m. Since $p(0) > c'(0)$, condition (12.B.3) can be satisfied only at $q^m > 0$. Hence, under our assumptions, *marginal revenue must equal marginal cost* at the monopolist's optimal output level:

$$p'(q^m)q^m + p(q^m) = c'(q^m). \tag{12.B.4}$$

Thus **MR = MC** and **P > MC**

DWL Loss can be calculated by

$$\int_{q^m}^{q^*} [p(s) - c'(s)]\, ds > 0,$$

2. **Monopoly Pricing with Linear Inverse Demand Function and Constant Returns to Scale**

Example 12.B.1: *Monopoly Pricing with a Linear Inverse Demand Function and Constant Returns to Scale.* Suppose that the inverse demand function in a monopolized market is $p(q) = a - bq$ and that the monopolist's cost function is $c(q) = cq$, where $a > c \geq 0$ [so that $p(0) > c'(0)$] and $b > 0$. In this case, the objective function of the monopolist's problem (12.B.2) is concave, and so condition (12.B.4) is both necessary and sufficient for a solution to the monopolist's problem. From condition (12.B.4), we can calculate the monopolist's optimal quantity and price to be $q^m = (a - c)/2b$ and $p^m = (a + c)/2$. In contrast, the socially optimal (competitive) output level and price are $q^\circ = (a - c)/b$ and $p^\circ = p(q^\circ) = c$. ■

Notice that the monopolist's quantity < socially optimal quantity, and monopolist's price > socially optimal price

- If the monopolist were to able perfectly discriminate among its customers, then the monopoly quantity distortion disappear.

12.C Static Model of Oligopoly

1. **Bertrand Model of Price Competition:**

Proposition 12.C.1: There is a unique Nash equilibrium (p_1^*, p_2^*) in the Bertrand duopoly model. In this equilibrium, both firms set their prices equal to cost: $p_1^* = p_2^* = c$.

Exercise 12.C.1: Show that in any Nash equilibrium of the Bertrand model with $J > 2$ firms, all sales take place at a price equal to cost.

2. **Cournot Model of Quantity Competition**

Proposition 12.C.2: In any Nash equilibrium of the Cournot duopoly model with cost $c > 0$ per unit for the two firms and an inverse demand function $p(\cdot)$ satisfying $p'(q) < 0$ for all $q \geq 0$ and $p(0) > c$, the market price is greater than c (the competitive price) and smaller than the monopoly price.

- Since the Bertrand model assumes that firms compete on price and not output quantity, it predicts that a duopoly is enough to push prices down to marginal cost level, meaning that a duopoly will result in perfect competition.

- Neither model is necessarily "better." The accuracy of the predictions of each model will vary from industry to industry, depending on the closeness of each model to the industry situation.

- Cournot Duoplicy with a Linear Inverse Demand Function and Constant Returns to Scale

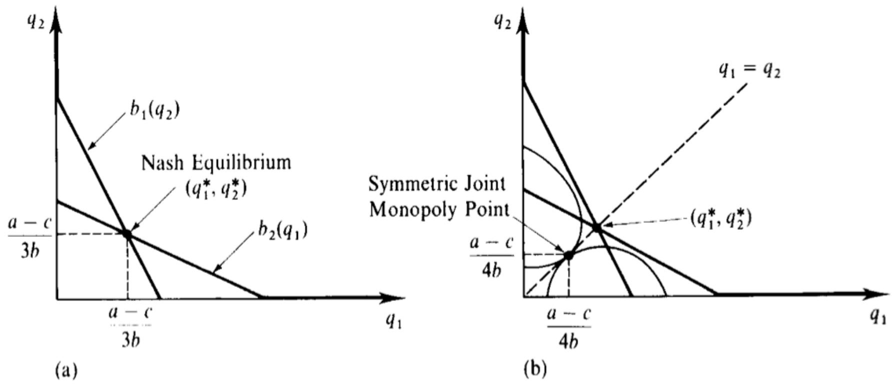


Figure 12.C.2
Nash equilibrium in the Cournot duopoly model of Example 12.C.1.

3. J > 2 Identical Firms

Suppose now that we have $J > 2$ identical firms facing the same cost and demand functions as above. Letting Q_J^* be aggregate output at equilibrium, an argument parallel to that above leads to the following generalization of condition (12.C.5):

$$p'(Q_J^*)\frac{Q_J^*}{J} + p(Q_J^*) = c. \tag{12.C.6}$$

Price -> MC as n -> infinity

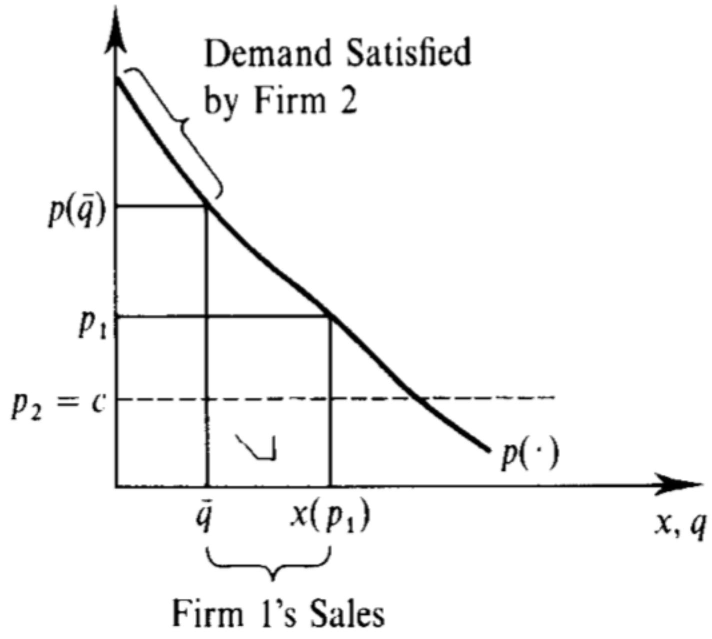
- J = 1, MR = C. / J = infinity, P = C

- The Cournot model displays **a gradual reduction in market power** as the number of firms increases.

4. Capacity Constraints and Decreasing Returns to Scale

- A modification to Bertrand model.

To see how capacity constraints can affect the outcome of the duopoly pricing game, suppose that each of the two firms has a constant marginal cost of $c > 0$ and a capacity constraint of $\bar{q} = \frac{3}{4}x(c)$. As before, the market demand function $x(\cdot)$ is continuous, is strictly decreasing at all p such that $x(p) > 0$, and has $x(c) > 0$.



- With capacity constraint, **competition will not generally drive price down to cost**, shown in Edgeworth (1897)

5. **Product Differentiation**

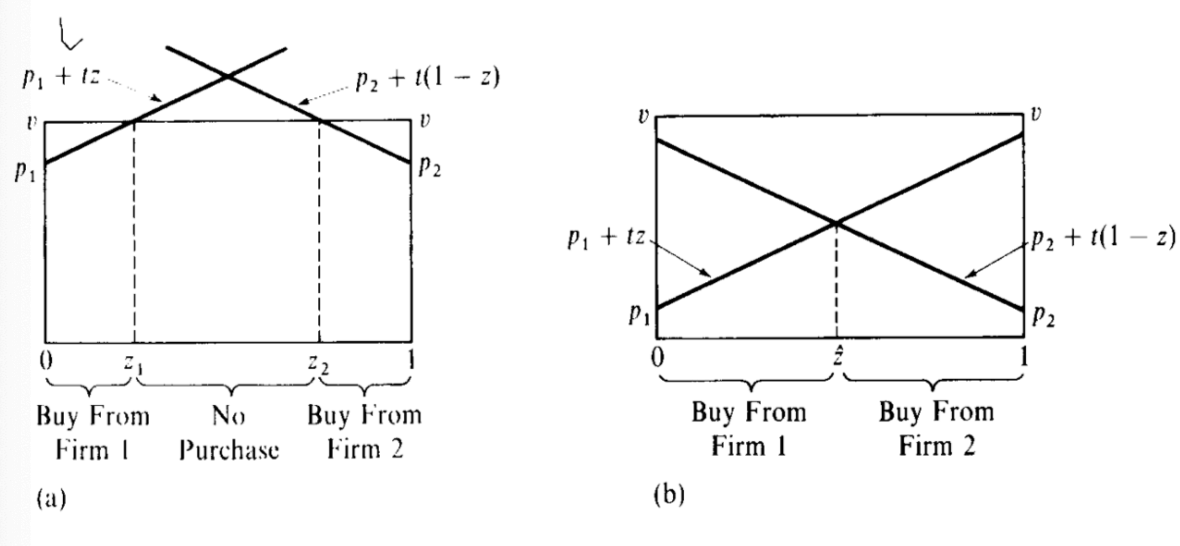
- Uniqueness of its product

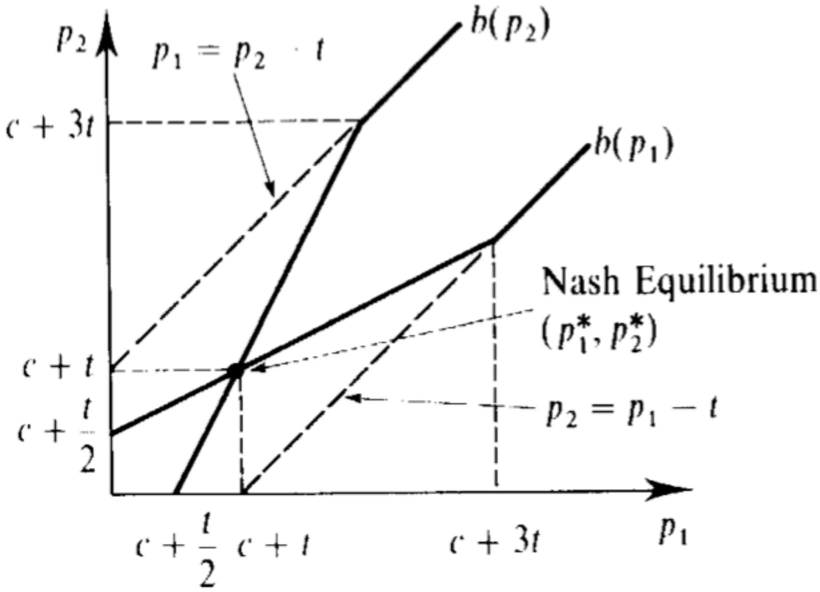
  each firm j takes its rivals' price choices $\bar{p}_{-j}$ as given and chooses p_j to solve

$$\underset{p_j}{\text{Max}} \quad (p_j - c)x_j(p_j, \bar{p}_{-j}).$$

Note that as long as $x_j(c, \bar{p}_{-j}) > 0$, firm j's best response necessarily involves a price in excess of its costs ($p_j > c$) because it can assure itself of strictly positive profits by setting its price slightly above c. Thus, in the presence of product differentiation, equilibrium prices will be above the competitive level. As with quantity competition and capacity constraints, the presence of product differentiation softens the strongly competitive result of the Bertrand model.

- One example: The Linear City Model





In this Nash equilibrium, each firm has sales of $M/2$ and a profit of $tM/2$. Note that as t approaches zero, the firms' products become completely undifferentiated and the equilibrium prices approach c, as in the Bertrand model. In the other direction, as the travel cost t becomes greater, thereby increasing the differentiation between the firms' products, equilibrium prices and profits increase.

12.D Repeated Interaction

The previous models are static models, and the 'static' nature is not realistic.

In this section, we consider the simplest type of dynamic model in which these concerns arise. Two identical firms complete for sales repeatedly, with competition in each period t described by the Bertrand model. When they do so, the two firms know all the prices that have been chosen (by *both* firms) previously. There is a discount factor $\delta < 1$, and each firm j attempts to maximize the discounted value of profits, $\sum_{t=1}^{\infty} \delta^{t-1} \pi_{jt}$, where π_{jt} is firm j's profit in period t. The game that this situation gives rise to is a dynamic game (see Chapter 9) of a special kind: it is obtained by repeated play of the same static simultaneous-move game and is known as a *repeated game*.

- Finitely repeated game
  (c, c) is a the unique SPNE by backward induction

- Infinitely repeated game

Things can change dramatically, however, when the horizon is extended to an infinite number of periods (this is known as an *infinitely repeated game*). To see this, consider the following strategies for firms $j = 1, 2$:

$$p_{jt}(H_{t-1}) = \begin{cases} p^m & \text{if all elements of } H_{t-1} \text{ equal } (p^m, p^m) \text{ or } t = 1 \\ c & \text{otherwise.} \end{cases} \tag{12.D.1}$$

In words, firm j's strategy calls for it to initially play the monopoly price p^m in period 1. Then, in each period $t > 1$, firm j plays p^m if in every previous period both firms have charged price p^m and otherwise charges a price equal to cost. This type of strategy is called a *Nash reversion strategy*: Firms cooperate until someone deviates, and any deviation triggers a permanent retaliation in which both firms thereafter set their prices equal to cost, the one-period Nash strategy. Note that if both firms follow the strategies in (12.D.1), then both firms will end up charging the monopoly price in every period. They start by charging p^m, and therefore no deviation from p^m will ever be triggered.

Proposition 12.D.1: The strategies described in (12.D.1) constitute a subgame perfect Nash equilibrium (SPNE) of the infinitely repeated Bertrand duopoly game if and only if $\delta \geq \frac{1}{2}$.

Proposition 12.D.2: In the infinitely repeated Bertand duopoly game, when $\delta \geq \frac{1}{2}$ repeated choice of any price $p \in [c, p^m]$ can be supported as a subgame perfect Nash equilibrium outcome path using Nash reversion strategies. By contrast, when $\delta < \frac{1}{2}$, any subgame perfect Nash equilibrium outcome path must have all sales occurring at a price equal to c in every period.

- Proposition 12.D.2 tells us the set of SPNE of the repeated Bertrand game grows as the discounting factor delta grows large.

- Folk Theorem:

> In fact, a general result in the theory of repeated games, known as the *folk theorem*, tells us the following: In an infinitely repeated game, *any feasible discounted payoffs that give each player, on a per-period basis, more than the lowest payoff that he could guarantee himself in a single play of the simultaneous-move component game can be sustained as the payoffs of an SPNE if players discount the future to a sufficiently small degree*. In Appendix A, we provide a more precise statement and extended discussion of the folk theorem for general repeated games. Its message is clear: Although infinitely repeated games allow for cooperative behavior, they also allow for an *extremely wide range* of possible behavior.

12.E Entry

- 12.D review repeated interactions

1. Eq Entry with Cournot competition

$\cancel{p^m}$ $p \in [c, p^m]$ can be an SPNE when $\delta \geq 1/2$

$\frac{p^m - c}{2}$

J firms $\Rightarrow$ $p \in [c, p^m]$ is SPNE iff $\delta \geq \frac{J-1}{J}$

$J = 2 \Rightarrow \delta \geq 1/2$.

Entry   Make # firms endogenous

2 stage game

1: Firms entry or not ?
if in, set up cost $= K$.

2. Firms play game

Stage 2: π_J = profit of a firm given J firms

eq^m of $J^* \Rightarrow$

$\pi_{J^*} \geq K$; $\pi_{J^*+1} < K$

u want to switch to a microphone or speaker

Typically, we expect that π_J is decreasing in J and that $\pi_J \to 0$ and $J \to \infty$. In this case, there is a unique integer $\hat{J}$ such that $\pi_J \geq K$ for all $J \leq \hat{J}$ and $\pi_J < K$ for all $J > \hat{J}$, and so $J^* = \hat{J}$ is the unique equilibrium number of firms.[21,22]

Cournot Model

$c(q) = cq$, $p(q) = a - bq$ $a > c \geq 0$, $b > 0$

$q_J = \frac{a-c}{b(J+1)}$ (?) $q_j + \cancel{(J-1)}\, q_k$

$J = 2 \Rightarrow q_2 = \frac{a-c}{3b}$ ✓

$\pi_J = [a - \underbrace{b(Jq_J)}_{b} - c]\, q_J$

$= \left[(a-c) - \frac{bJ(a-c)}{b(J+1)}\right] q_J$

$\left[\frac{b(a-c)(J+1) - bJ(a-c)}{b(J+1)}\right] q_J$

ant to switch to a

$$\pi_J = \left(\frac{a-c}{J+1}\right)^2 \left(\frac{1}{b}\right).$$

$\pi_J \to 0$ as $J \to \infty$

$$Jq_J = \frac{(a-c)J}{b(J+1)} = \frac{a-c}{b} \cdot \frac{J}{J+1}$$

$$\to \frac{a-c}{b} \text{ as } J \to \infty.$$

Comp Qty

$$\pi_{\tilde{J}} = K$$

$$K = \frac{(a-c)^2}{(\tilde{J}+1)^2} \frac{1}{b} \Rightarrow (\tilde{J}+1)^2 = \frac{(a-c)^2}{bK}$$

$$\tilde{J} = \frac{a-c}{\sqrt{bK}} - 1$$

t to switch to a
phone or speaker

Thus the eq level of the # of the firms = largest integer that is less than or equal to J titled

If K = 0, J titled goes to infinity

2. Entry with Bertrand Competition

Eq^m Entry w/ Bertrand?

$c(q) = cq$, $p(q) = a - bq$

$\pi_1 = \pi^m \quad (J=1)$

$\pi_J = 0 \quad \forall J \geq 2.$

If $\pi^m > K$, SPNE $\Rightarrow J^* = 1.$

Example 12.E.2: *Equilibrium Entry with Bertrand Competition.* Suppose now that competition in stage 2 of the two-stage entry game takes the form of the Bertrand model studied in Section 10.C. Once again, $c(q) = cq$, $p(q) = a - bq$, $a > c \geq 0$, and $b > 0$. Now $\pi_1 = \pi^m$, the monopoly profit level, and $\pi_J = 0$ for all $J \geq 2$. Thus, assuming that $\pi^m > K$, the SPNE must have $J^* = 1$ and result in the monopoly price and quantity levels. Comparing this result with the result in Example 12.E.1 for the Cournot model, we see that the presence of more intense stage 2 competition here actually *lowers* the ultimate level of competition in the market! ■

- **Eq Entry with Cournot Competition / Bartrand Competition**
- **J* = infinity / J* = 1**

3. Entry and Welfare

Let q_J be the symmetric equilibrium output per firm when there are J firms in the market. As usual, the inverse demand function is denoted by $p(\cdot)$. Thus, $p(Jq_J)$ is the price when there are J active firms; and so $\pi_J = p(Jq_J)q_J - c(q_J)$, where $c(\cdot)$ is the cost function of a firm after entry. We assume that $c(0) = 0$.

We measure welfare here by means of Marshallian aggregate surplus (see Section 10.E). In this case, social welfare when there are J active firms is given by

$$W(J) = \int_0^{Jq_J} p(s)\,ds - Jc(q_J) - JK. \qquad (12.E.5)$$

Entry & Welfare:

q_J = eq^m output

$p(Jq_J)$, J = # of firms in industry

$\pi_J = p(Jq_J)\,q_J - c(q_J)$ $\quad cq_J$

Surplus. $W(J) = \int_0^{Jq_J} p(s)ds$

$W(J) = aJq_J - \frac{b}{2}(Jq_J)^2 - Jcq_J - JK$

f/g

$\frac{fg' - gf'}{g^2}$

$a\boxed{\frac{(a-c)}{b}\frac{J}{J+1}} - \frac{b}{2b^2}(a-c)^2\left(\frac{J}{J+1}\right)^2 - JK$

$- Jc\frac{(a-c)}{b(J+1)}$

$\frac{(a-c)^2}{b}$

Exam.

From George to Everyone:
yes. Ch 11 & Ch 12 AB

- Second stage playing Bertrand => J* = 1

4. **Compute the number of firms at W'(J bar) = 0**

Example 12.E.3: Consider the Cournot model of Example 12.E.1. For the moment, ignore the requirement that the number of firms is an integer, and solve for the number of firms $\bar{J}$ at which $W'(\bar{J}) = 0$. This gives

$$(\bar{J} + 1)^3 = \frac{(a - c)^2}{bK}. \tag{12.E.6}$$

If $\bar{J}$ turns out to be an integer, then the socially optimal number of firms is $J^o = \bar{J}$. Otherwise, J^o is one of the two integers on either side of $\bar{J}$ [recall that $W(\cdot)$ is concave]. Now, recall from (12.E.4) that $\pi_J = (1/b)[(a - c)/(J + 1)]^2$. As noted in Example 12.E.1, if we let $\tilde{J}$ be the real number such that

$$(\tilde{J} + 1)^2 = \frac{(a - c)^2}{bK}, \tag{12.E.7}$$

the equilibrium number of firms is the largest integer less than or equal to $\tilde{J}$. From (12.E.6) and (12.E.7), we see that

$$(\tilde{J} + 1) = (\bar{J} + 1)^{3/2}.$$

if $J^o = \bar{J} = 2$, $\Rightarrow \tilde{J} \cong 4.2 \Rightarrow J^* = 4$.

- **The # of firms enter is higher than the socially optimal #**

5. The nature of entry bias

- The nature of entry bias [Mankiw and Whinston (1986)]

Mankiw/Whinston model.

A1 $\quad J q_J \geq J' q_{J'}$, if $J > J'$

A2 $\quad q_J \leq q_{J'}$, if $J > J'$ (business stealing)

A3 $\quad p(J q_J) - c'(q_J) \geq 0 \quad \forall J$

- The sec and condition : **business stealing**

Conditions (A1) and (A3) are straightforward: (A1) requires that aggregate output increases (price falls) when more firms enter the industry, and (A3) says that price is not below marginal cost regardless of the number of firms entering the industry. Condition (A2) is more interesting. It is the assumption of *business stealing*. It says that when an additional firm enters the market, the sales of existing firms fall (weakly). Hence, part of the new firm's sales come at the expense of existing firms. These conditions are satisfied by most, although not all, oligopoly models. [In the Bertrand model, for example, condition (A3) does not hold.]

Then we have this proposition

Proposition 12.E.1: Suppose that conditions (A1) to (A3) are satisfied by the post-entry oligopoly game, that $p'(\cdot) < 0$, and that $c''(\cdot) \geq 0$. Then the equilibrium number of entrants, J^*, is at least $J^o - 1$, where J^o is the socially optimal number of entrants.[23]

Proof: The result is trivial for $J^\circ = 1$, so suppose that $J^\circ > 1$. Under the assumptions of the proposition, π_J is decreasing in J (Exercise 12.E.2 asks you to show this). To establish the result, we therefore need only show that $\pi_{J^\circ - 1} \geq K$.

To prove this, note first that by the definition of J° we must have $W(J^\circ) - W(J^\circ - 1) \geq 0$, or

$$\int_{Q_{J^\circ - 1}}^{Q_{J^\circ}} p(s)\,ds - J^\circ c(q_{J^\circ}) + (J^\circ - 1)c(q_{J^\circ - 1}) \geq K,$$

where we let $Q_J = Jq_J$. We can rearrange this expression to yield

$$\pi_{J^\circ - 1} - K \geq p(Q_{J^\circ - 1})q_{J^\circ - 1} - \int_{Q_{J^\circ - 1}}^{Q_{J^\circ}} p(s)\,ds + J^\circ[c(q_{J^\circ}) - c(q_{J^\circ - 1})].$$

Given $p'(\cdot) < 0$ and condition (A1), this implies that

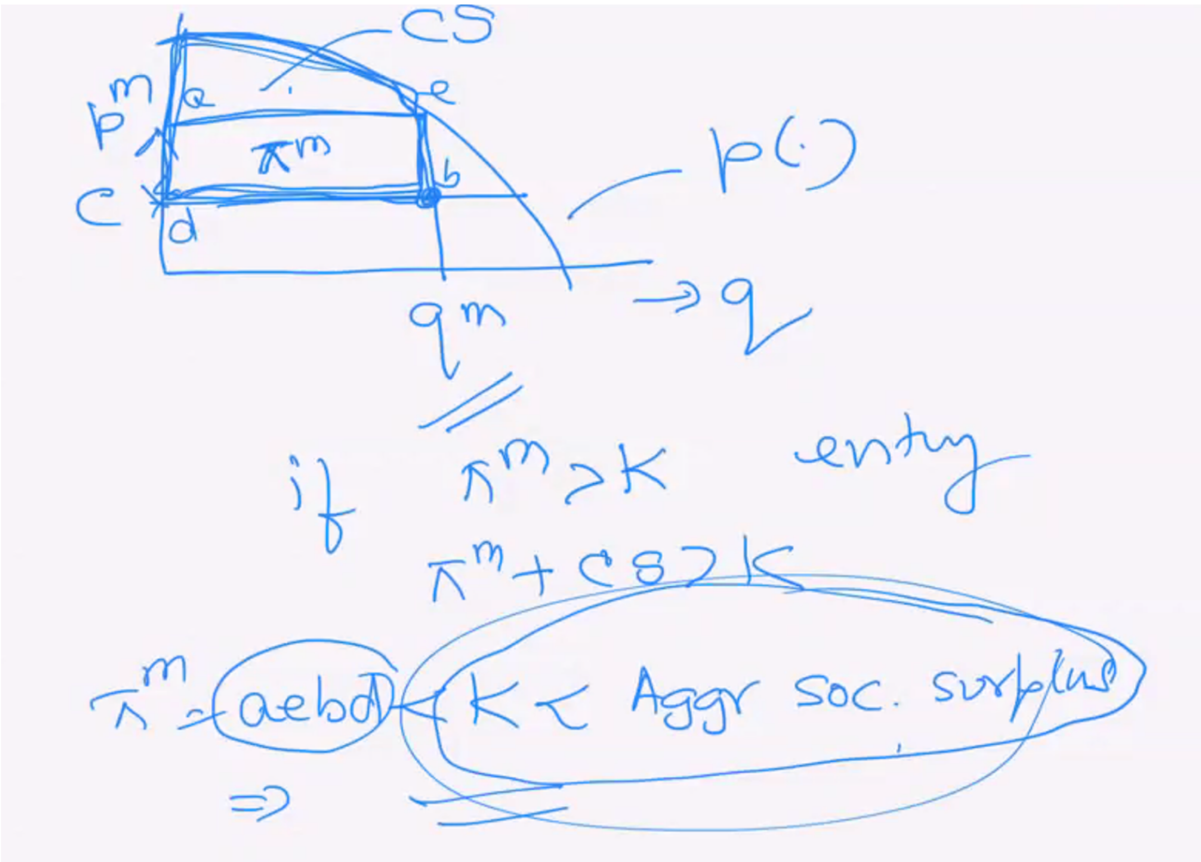
$$\pi_{J^\circ - 1} - K \geq p(Q_{J^\circ - 1})[q_{J^\circ - 1} + Q_{J^\circ - 1} - Q_{J^\circ}] + J^\circ[c(q_{J^\circ}) - c(q_{J^\circ - 1})]. \quad (12.E.8)$$

But since $c''(\cdot) \geq 0$, we know that $c'(q_{J^\circ - 1})[q_{J^\circ} - q_{J^\circ - 1}] \leq c(q_{J^\circ}) - c(q_{J^\circ - 1})$. Using this inequality with (12.E.8) and the fact that $q_{J^\circ - 1} + Q_{J^\circ - 1} - Q_{J^\circ} = J^\circ(q_{J^\circ - 1} - q_{J^\circ})$ yields

$$\pi_{J^\circ - 1} - K \geq [p(Q_{J^\circ - 1}) - c'(q_{J^\circ - 1})]J^\circ(q_{J^\circ - 1} - q_{J^\circ}).$$

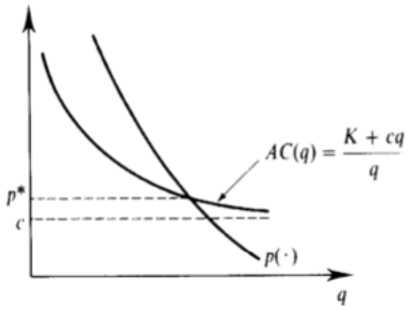
Conditions (A2) and (A3) then imply that $\pi_{J^\circ - 1} \geq K$.[24] ■

- Additional Entry steal others' business.



- Under entry here

6. **Simultaneous Game** (One stage Entry Model with Bertrand Competition)



- Firm can rent capital, thus fixed cost become variable cost. (no profitable)

  => Entry more agreesively and a lower equilibrium price (Contestable market)

7. Entry Deterance (appendix B MWG P423)

Entry Deterrence

$x(p)$

I. I chooses capacity K_I
unit cost $\$r$

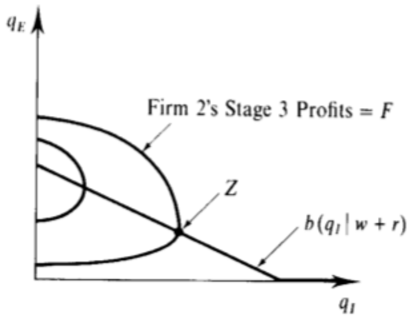
II. E enter or stay away
entry cost F

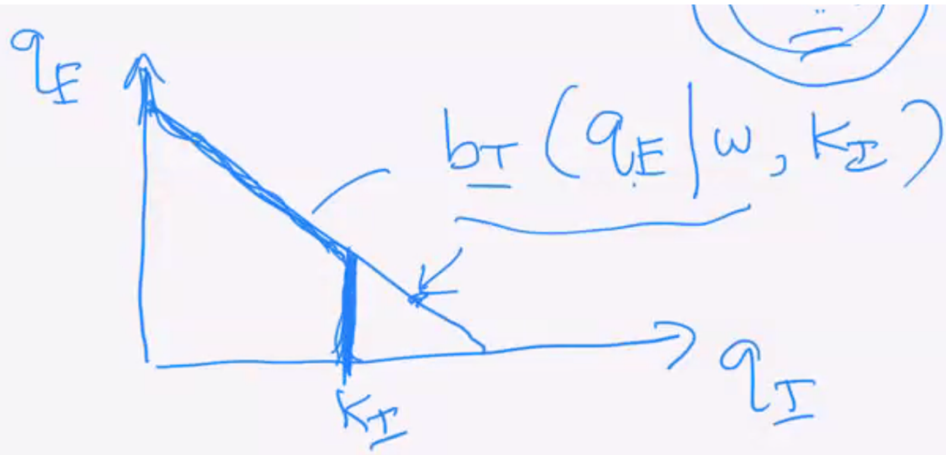
III If entry occurs, choose q_I, q_E

E's cost $(w + r)$

labor

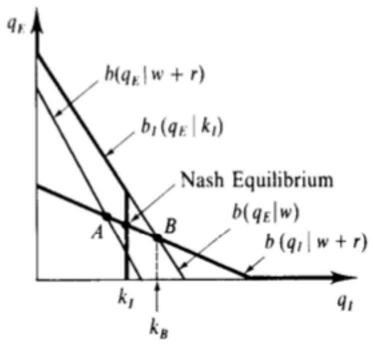
capacity

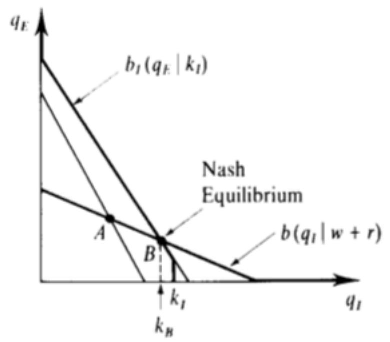




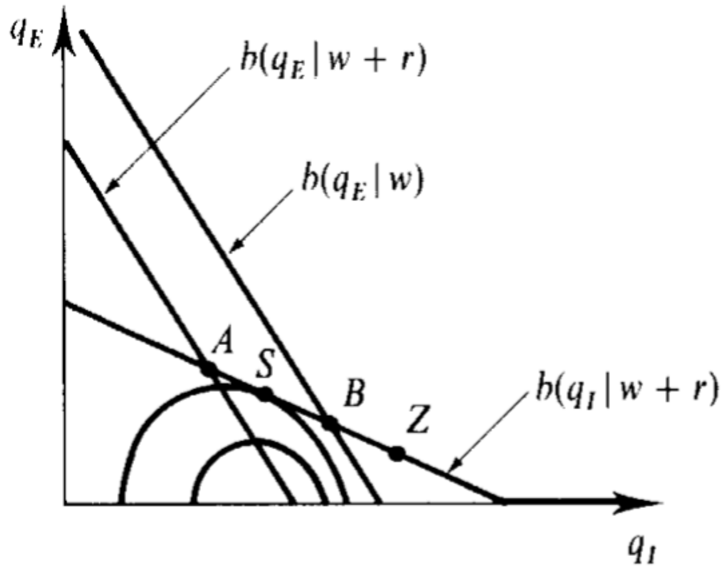
I's br: $b_I(q_E \mid k_I)$

$$= \min\{ b(q_E \mid w), k_I \}$$



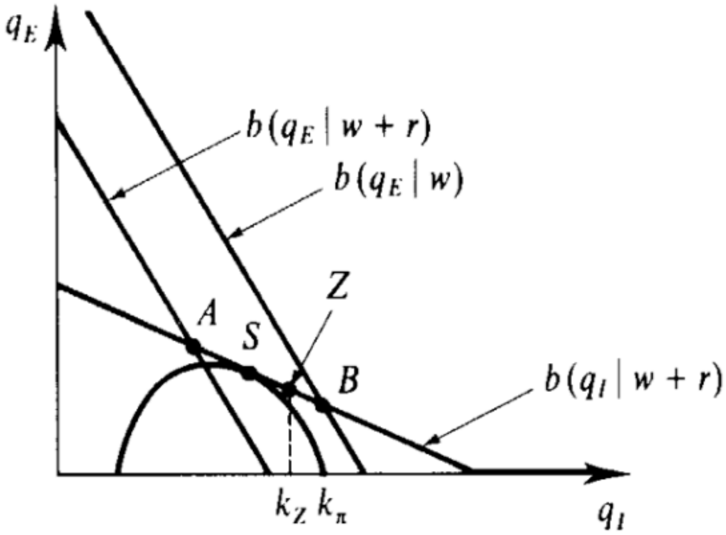


- **When Entry is blockaded**. Firm I achieves its best possible outcome
- **Entry deterrence is impossible**: strategic entry accommodation.
- Firm I's first mover advantage allows it to earn a higher profits than the otherwise identical firm E.



- If S point lies to the right of point B. then Firm I is unable to credibly commit to produce the output associate with point S. The optimal capacity choice is K_I = K_B

- **When deterrence is possible but not inevitable**



Firm I compare the profit at (K_Z, 0) and (K_pi, 0), then make choices accordingly.

- Note that if deterrence is optimal, then **even though entry does not occur its threat nevertheless has an effect on the market outcome**, raising the level of output and welfare relative to a situation in which no entry is possible