Math 136 Real Analysis II Spring 2020
Prof. L. Tu

Textbook: Patrick Fitzpatrick, *Advanced Calculus*, 2nd edition, American Mathematical Society, 2006. (ISBN-10: 0821847910) We will cover Chapters 4, 6, 13–19.

Prof. Tu's office hours: Wed 11–11:50 a.m. in BP-206

Prof. Tu's problem session: Mon 7:30–8:30 p.m.

ARC Math 136 tutors office hours: Make appointments through Tutor Finder (go.tufts.edu/tutorfinder).

Readings for the weeks of March 9, 2020

§18.3 Jordan domains

§18.3 Volume

March 14–22, 2020: Spring Break!

Readings for the weeks of March 23, 2020

§19.1 Fubini's theorem

§19.2 Change of variables formula (without proof)

§9.4 Uniformly convergent sequences of integrable functions and of differentiable functions (pp. 250–252)

Problem Set 7A (50 points)

(Due at 3 p.m., Wednesday, March 25, 2020)

Please submit the solutions as a single pdf file through Canvas.

A1. (10 points) (**Monotonicity for upper integrals and for lower integrals**)
Prove that if $f : A \to \mathbb{R}$ and $g : A \to \mathbb{R}$ are bounded and $f \leq g$, then $\overline{\int}_A f \leq \overline{\int}_A g$. (NOTE: a similar proof shows $\underline{\int}_A f \leq \underline{\int}_A g$.)

A2. (10 points) (**Jordan content 0**)
§18.2, p. 488: # 2. Show that the set of real numbers $\{1/n \mid n \in \mathbb{N}\}$ has Jordan content 0.

A3. (10 points) (**The boundary of a generalized rectangle**)
§18.2, p. 489: # 9. Show that the boundary of a generalized rectangle has Jordan content 0.

(*Turn over for more problems.*)

A4. (10 points) (**An ellipsoid**)

For positive numbers a, b, and c, show that the ellipsoid

$$\{(x,y,z) \in \mathbb{R}^3 \mid \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1\}$$

has Jordan content 0.

A5. (10 points)

(a) (**The boundary of a union**)

§18.3, p. 496: # 3. For two subsets D_1 and D_2 of $\mathbb{R}^n$, show that

$$\partial(D_1 \cup D_2) \subseteq \partial D_1 \cup \partial D_2.$$

(b) (**Union of two Jordan domains**)

S 18.3, p. 496: # 4. Prove that the union of two Jordan domains is a Jordan domain.