Math 136 Real Analysis II Spring 2020
Prof. L. Tu

References: Jerrold E. Marsden and Michael J. Hoffman, *Elementary Classical Analysis*, 2nd edition, W. H. Freeman and Company, New York, 1993. Chapters 8 and 10 (Fourier Analysis) only.

Classes: Mon Wed 3:00–4:15 p.m. EDT

Next problem session: Tue, April 21, 6:30–7:30 p.m. EDT (Note the change of date and time because of Patriot's Day on Monday, April 20.)

Office hour: **Thu** 11–11:50 a.m. EDT

ARC Math 136 tutors office hours: Make appointments through Tutor Finder (go.tufts.edu/tutorfinder). Tutoring is now done over the internet.

Monday, April 20, 2020, Patriots' Day: No classes and no problem sessions

Readings for the week of April 13, 2020

Marsden and Hoffman

- §10.2 Orthogonal families
- §10.2 Bessel's inequality, Parseval's theorem
- §10.3 Mean completeness and pointwise convergence theorems (omit the proofs), computation of Fourier series (as done in class)
- §10.5 Omit this section from the syllabus.

Problem Set 10A (50 points)

(Due at 3:30 p.m., Thursday, April 23, 2020)

In these problems you may use the following integral formulas:

$$\int u\cos nu\,du = \frac{1}{n^2}\cos nu + \frac{u}{n}\sin nu + C,$$

$$\int u\sin nu\,du = \frac{1}{n^2}\sin u - \frac{u}{n}\cos nu + C,$$

$$\int u^2\sin nu\,du = \frac{2u}{n^2}\sin nu + \left(\frac{2}{n^3} - \frac{u^2}{n}\right)\cos nu + C,$$

$$\int u^2\cos nu\,du = \frac{2u}{n^2}\cos nu + \left(\frac{u^2}{n} - \frac{2}{n^3}\right)\sin nu + C.$$

A1. (10 points) (L^2 **space**) Let f be defined by the Fourier series

$$f(x) = \sum_{n=1}^{\infty}\frac{1}{n^2}\cos(nx).$$

(a) Prove that this series for f converges uniformly to f on $[0,2\pi]$. (*Hint*: Think about a test for the uniform convergence of a series you learned last term.)
(b) Show $f \in L^2([0,2\pi],\mathbb{C})$ and find an expression for $\|f\|_{L^2}$.

A2. (10 points) (**Gram-Schmidt Process**) Marsden §10.2, #2. Let $g_0, g_1, g_2, \ldots$ be linearly independent vectors in an inner product space. Inductively define

$$h_0 = g_0,\ \varphi_0 = \frac{h_0}{\|h_0\|}, \ldots,\ h_n = g_n - \sum_{k=0}^{n-1} \langle g_n, \varphi_n \rangle \varphi_k,\ \varphi_n = \frac{h_n}{\|h_n\|}, \ldots.$$

Show that $\varphi_0, \varphi_1, \varphi_2, \ldots$ are orthonormal. Why must we assume that the g's are linearly independent?

A3. (10 points) (**Orthonormal families on** $[0,\ell]$) Marsden §10.2, # 3ab.

A4. (10 points) (**Parseval's theorem**) Marsden §10.2, p. 560, # 4.

A5. (10 points) (**Trigonometric Fourier series**) Marsden §10.3, p. 569, # 1.

(End of Problem Set 10A)