Math 136 | Real Analysis II | Spring 2020
Prof. L. Tu

Textbook: Patrick Fitzpatrick, *Advanced Calculus*, 2nd edition, American Mathematical Society, 2006. (ISBN-10: 0821847910) We will cover Chapters 4, 6, 13–19.

Readings for the weeks of January 27, 2020

§13.1 Limits of functions of several variables

§13.2 Partial derivatives

§13.3 The mean value theorem in several variables and directional derivatives

Readings for the weeks of February 3, 2020

§14.1 First-order approximation, tangent planes, and affine functions

§15.2 The derivative matrix and the differential

§15.3 The chain rule

§15.1 is a review of linear algebra. We will assume you know the material in this section and not discuss it. If you are rusty in linear algebra, you should read through it.

Problem Set 2 (100 points)

(Due Wednesday, February 5, 2020)

Part A: Please put the following problems in Homework Folder A.

A1. (10 points) Let $f : \mathbb{R}^2 to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}.$
Either find $\lim_{(x,y)\to 0} f(x,y)$ or prove the limit does not exist.

A2. (10 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{x^2y^2}{x^4+y^4} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}.$
Either find $\lim_{(x,y)\to 0} f(x,y)$ or prove the limit does not exist.

A3. (10 points) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{xy^3}{x^2+y^6} & (x,y) \neq 0 \\ 0 & (x,y) = 0 \end{cases}.$
Either find $\lim_{(x,y)\to 0} f(x,y)$ or prove the limit does not exist.

A4. (10 points) §13.2, p. 361: # 3 (f_x, f_y exist but are not continuous)

A5. (10 points) §14.1, p. 379: # 14 (a limit)

(*Turn over for Part B.*)

Part B. Please put the following problems in your Homework Folder B.

B1. (10 points) §13.2, p. 362: # 5 (f_x, f_y identically zero on $\mathbb{R}^2 \Rightarrow f$ constant; note that f_x, f_y are not assumed continuous.)

B2. (10 points) §13.2, p. 362: # 13 (f_{xy}, f_{yx} exist but are not equal)

B3. (10 points) §13.3, p. 370: # 6 (mean-value theorem in several variables)

B4. (10 points) §13.3, p. 371: # 7 (*Hint*: Use §13.2, # 5.)

B5. (10 points) Let D be an open subset of $\mathbb{R}^n$ and a a point in D. Suppose $g\colon D \to \mathbb{R}$ is a real-valued function on D. Use the ε-δ criterion for limits to prove that

$$\lim_{x\to a} g(x) = 0 \quad \Leftrightarrow \quad \lim_{x\to a} |g(x)| = 0.$$