Math 136 Real Analysis II Spring 2020
Prof. L. Tu

Textbook: Patrick Fitzpatrick, *Advanced Calculus*, 2nd edition, American Mathematical Society, 2006. (ISBN-10: 0821847910) We will cover Chapters 4, 6, 13–19.

Problem Set 7B (50 points)

(Due at 3 p.m., Wednesday, March 25, 2020)

Please submit the solutions as a single pdf file through Canvas.

B1. (10 points) (**The integrals of two functions that are equal except on a set of Jordan content 0**)
§18.3, p. 497: # 9

B2. (10 points) (**Integral on a closed rectangle versus on its interior**)
§18.3, p. 497: # 11. Let I be a generalized rectangle in $\mathbb{R}^n$ and let the function $f\colon I \to \mathbb{R}$ be integrable. Denote the interior of I by D. Show that the restriction $f\colon D \to \mathbb{R}$ is integrable and that

$$\int_I f = \int_D f.$$

B3. (10 points) (**A function that is constant on a subset**)
Let $g : \mathbb{R}^n \to \mathbb{R}$.

(a) Assume g is constant on an open set $\mathscr{O} \subset \mathbb{R}^n$. Prove that $g : \mathbb{R}^n \to \mathbb{R}$ is continuous at all $\mathbf{x} \in \mathscr{O}$.

(b) Now, assume $g : \mathbb{R}^n \to \mathbb{R}$ is constant on an *arbitrary set O in* $\mathbb{R}^n$. Is g necessarily continuous for all $\mathbf{x} \in O$? Either prove this or provide a counterexample.

B4. (10 points) (**Closure and interior of a set of Jordan content 0**)
Let $S \subset \mathbb{R}^n$ have Jordan content zero.

(a) Prove that the closure of S, $\mathrm{cl}(S)$, has Jordan content zero.
(b) Prove that $\mathrm{int}(S) = \emptyset$.

B5. (10 points) (**Jordan content 0 versus volume 0**)
Let A be a bounded subset of $\mathbb{R}^n$. Prove that A has Jordan content zero if and only if A has volume and $\mathrm{vol}(A) = 0$. (*Hint:* Lemma 18.29 makes one direction very simple.)

(End of Problem Set)