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Bayes Rule

Problem 1. Suppose you are a witness to a nighttime hit-and-run accident involving a taxi. All taxi cars are blue or green. You state that the taxi was blue. Extensive testing shows that, under dim lighting conditions, discrimination between blue and green is 60% reliable (i.e. $p(y = g \mid x = g) = p(y = b \mid x = b) = 1 - p(y = b \mid x = g) = 1 - p(y = g \mid x = b) = 0.6$).

(a) Given your statement as a witness and given that 8 out of 10 taxis are green, what is the probability of the taxi being blue?

(b) If 6 out of 10 taxis are green, what is the probability of the taxi being blue?

(c) Suppose now that there is a second witness who swears that the taxi is green. Unfortunately, he is color blind, so he has only a 50% chance of being right. How would this change the estimate from (b)?

(a) $P(x=g) = 8/10 \quad P(x=b) = 2/10$

$P(y=b) = P(y=b \mid x=b)\,P(x=b) + P(y=b \mid x=g)\,P(x=g)$

$= 0.6 \cdot 0.2 + (1-0.6)\cdot 0.8 = 0.44$

$$P(x=b \mid y=b) = \frac{P(y=b \mid x=b)\cdot P(x=b)}{P(y=b)}$$

$$= \frac{0.6 \cdot 0.2}{0.44}$$

$$\boxed{= 3/11 \text{ or } 0.272}$$

(b) $P(x=g) = 0.6 \qquad P(x=b) = 0.4$

$$P(y=b) = P(y=b|x=b)P(x=b) + P(y=b|x=g)\,P(x=g)$$
$$= 0.6 \cdot 0.4 + 0.4 \cdot 0.6$$
$$= \frac{12}{25} = 0.48$$

$$P(x=b|y=b) = \frac{P(y=b|x=b) \cdot P(x=b)}{P(y=b)}$$
$$= \frac{0.6 \cdot 0.4}{0.48}$$
$$\boxed{= 0.5}$$

(c) This would not change anything as the second witness does not provide any new useful information.

Bayes Filter

Problem 2. A vacuum cleaning robot is equipped with a cleaning unit to clean the floor. Furthermore, the robot has a sensor to detect whether the floor is clean or dirty. Neither the cleaning unit nor the sensor are perfect. From previous experience, you know that the robot succeeds in cleaning a dirty floor with a probability of

$$p(x_{t+1} = clean | x_t = dirty, u_{t+1} = vacuum - clean) = 0.8$$

where x_{t+1} is the state of the floor after having vacuum-cleaned, u_{t+1} is the control command, and x_t is the state of the floor before performing the action.

The probability that the sensor indicates that the floor is clean although it is dirty is given by

$$p(z = clean | x = dirty) = 0.2$$

and the probability that the sensor correctly detects a clean floor is given by

$$p(z = clean | x = clean) = 0.9$$

Unfortunately, you have no knowledge about the current state of the floor. However, after cleaning the floor the sensor of the robot indicates that the floor is clean.

(a) Compute the probability that the floor is still dirty after the robot has vacuum-cleaned it. (Hint: Assume that $p(x_0 = c) = 1 - p(x_0 = d) = q$).

(b) Which prior gives you a lower bound for that probability? (What is the corresponding q ?)

You can refer to the **Example.2.4.2** in **Probabilistic Robotics**

(a)

Given:

$$p(x_{t+1}=c \mid x_t=d, u_{t+1}=\text{vacuum-clean}) = 0.8$$

$$p(x_{t+1}=d \mid x_t=d, u_{t+1}=\text{vacuum-clean}) = 0.2$$

$$p(z=c \mid x_{t+1}=d) = 0.2 \qquad p(z=d \mid x_{t+1}=d) = 0.8$$

$$p(z=c \mid x_{t+1}=c) = 0.9 \qquad p(z=d \mid x_{t+1}=c) = 0.1$$

$$p(x_t=c) = a \qquad p(x_t=d) = 1-a$$

If floor initially clean, vacuuming should have no affect:

$$p(x_{t+1}=c \mid x_t=c, u_{t+1}=\text{vacuum-clean}) = 1$$

$$p(x_{t+1}=d \mid x_t=c, u_{t+1}=\text{vacuum-clean}) = 0$$

Using Bayes Filter to find prob of floor being dirty after vacuum-cleaning and detecting as clean:

$$p(x_{t+1}=d \mid z=c, u_{t+1}=\text{vacuum-clean})$$

$$= \eta\, p(z=c \mid x_{t+1}=d) \cdot \big(p(x_{t+1}=d \mid x_t=c, u=\text{vacuum-clean}) \cdot p(x_t=c) + p(x_{t+1}=d \mid x_t=d, u=\text{vacuum-clean}) \cdot p(x_t=d)\big)$$

$$= \eta \cdot 0.2 \cdot (0 \cdot a + 0.2 \cdot (1-a)) = (0.04 - 0.04a)\eta \quad (1)$$

Do same for floor being clean after vacuum and detected as clean:

$$p(x_{t+1}=c \mid z=c, u=\text{vacuum-clean})$$

$$= \eta p(z=c \mid x_{t+1}=c) \cdot \big(p(x_{t+1}=c \mid x_t=c, u=\text{vacuum-clean})\, p(x_t=c) + p(x_{t+1}=c \mid x_t=d, u=\text{vacuum-clean})\, p(x_t=d)\big)$$

$$= \eta \cdot 0.9 \cdot (1 \cdot q + 0.8 \cdot (1-q))$$

$$= \eta \cdot (0.18q + 0.72) \quad (2)$$

Since

$p(x_{t+1}=d \mid z=c, u=\text{vacuum-clean}) + p(x_{t+1}=c \mid z=c, u=\text{vacuum-clean}) = 1$,

use (1) and (2) to get η:

$$\eta(0.04-0.04q) + \eta \cdot (0.18q+0.72) = 1$$

$$\eta = \frac{1}{0.14q+0.76}$$

plug η into (1) to get answer:

$$p(x_{t+1}=d \mid z=c, u=\text{vacuum-clean}) = (0.04-0.04q) \cdot \frac{1}{0.14q+0.76}$$

$$\boxed{= \frac{0.04-0.04q}{0.76+0.14q}}$$

Since we have no knowledge of floor, assume $q = 0.5$

$$\boxed{= 0.024}$$

(b)

The prior that tells us the floor is clean gives us the lower bound. This is because if the floor is initially clean, then vacuum-cleaning will make no difference. The corresponding q is $q=1$, and the lower bound is $p(x_{t+1}=d \mid z=c, u=\text{vacuum-clean}) = 0$.