ELEC 3210
Introduction to Mobile Robotics
Lecture 13

(Machine Learning and Infomation Processing for Robotics)

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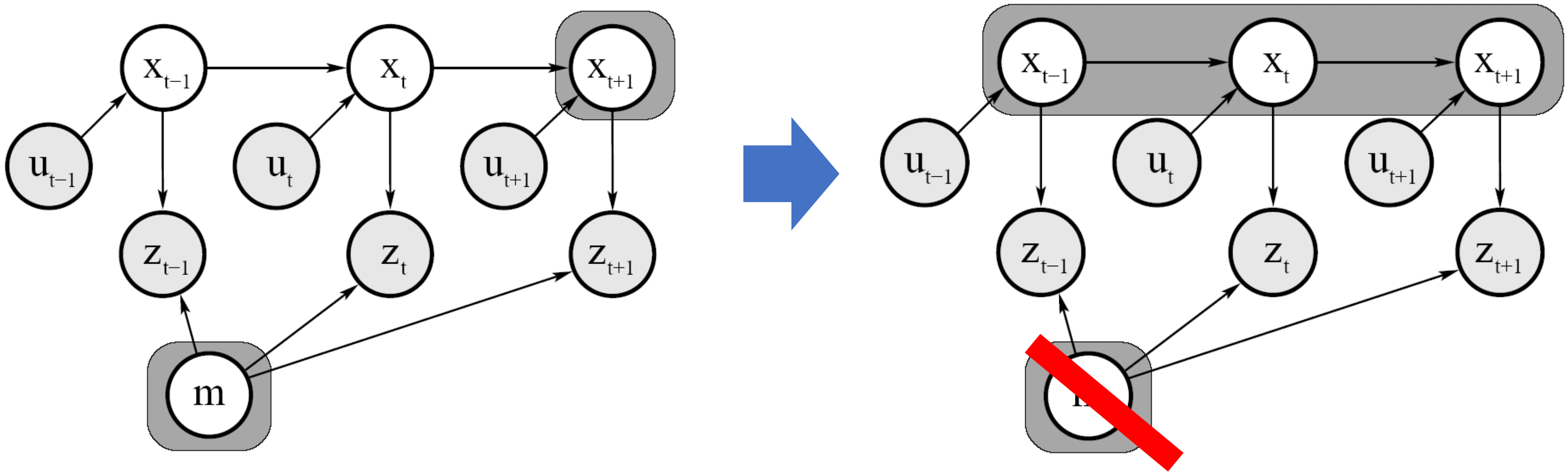
Recap 12 - Pose Graph SLAM

- Achieve global consistent mapping with loops
- From recursive filter to batch processing

$$p(x_t, m \mid z_{1:t}, u_{1:t})$$

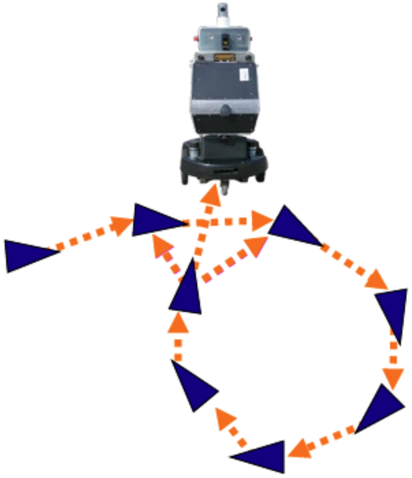
$$p(x_{1:t}, \cancel{m} \mid z_{1:t}, u_{1:t})$$



Courtesy: Thrun, Burgard, and Fox

Recap L12 - Pose Graph SLAM

- Every **node** in the graph corresponds to a pose of the robot during mapping
- Every **edge** between two nodes corresponds to a spatial constraint between them
- Build the graph and find a node configuration that **minimize** the error introduced by the constraints

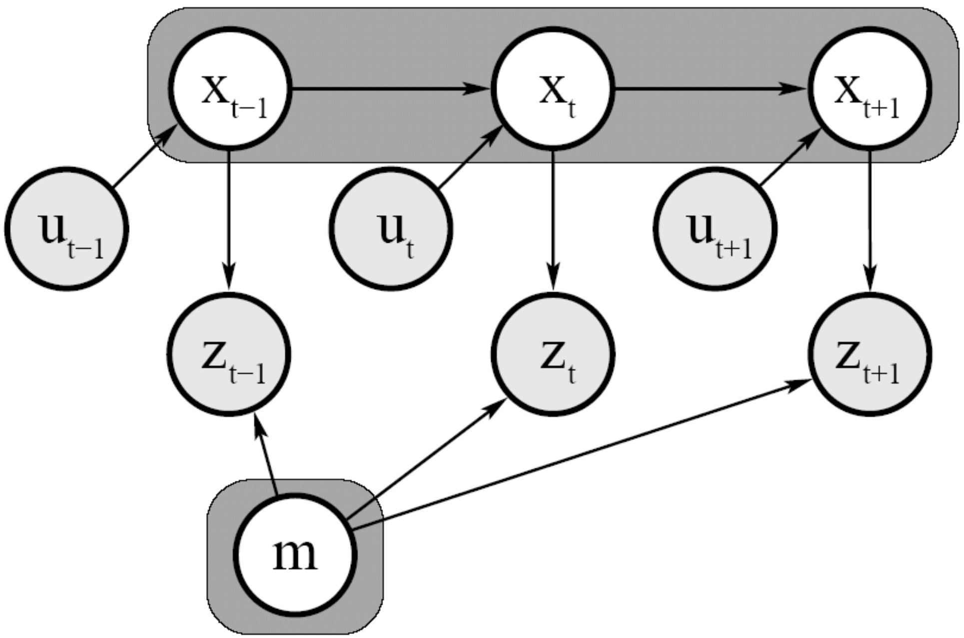


Courtesy: Cyrill Stachniss

Topic Today

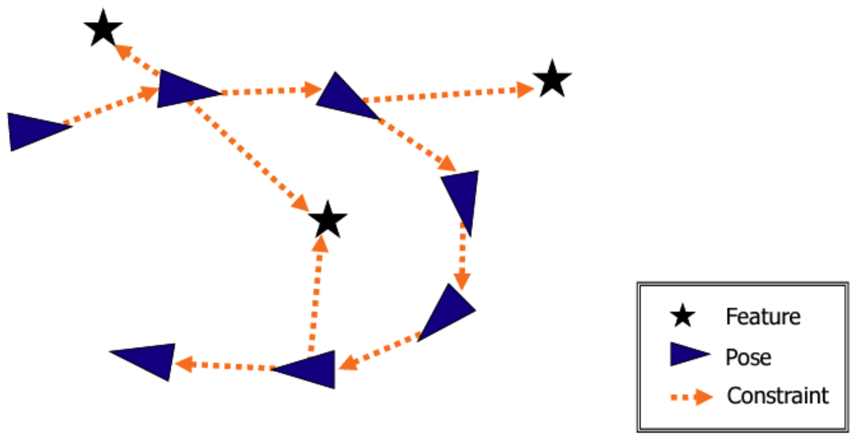
- Graph SLAM with Landmarks
- Estimate both landmark map and robot poses

$$p(x_{1:t}, m \mid z_{1:t}, u_{1:t})$$



Graph with Landmarks

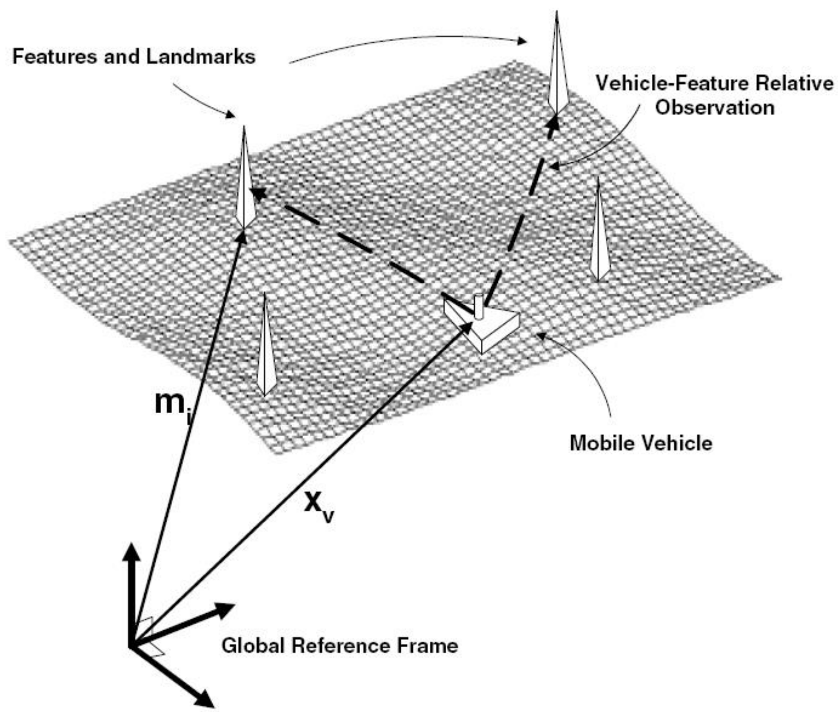
- Nodes can represent:
  - Robot poses
  - Landmark locations
- Edges can represent:
  - Landmark observations
  - Odometry measurements
- Today we mainly focus on error function design for the graph



Courtesy: Cyrill Stachniss

Deal with 2D Landmarks

- Landmark is a (x,y) - point in the world
- Relative observation in (x,y)



Courtesy: Cyrill Stachniss

Landmarks Observation

- Expected observation (x-y sensor)

$$\widehat{\mathbf{z}}_{ij}\left(\mathbf{x}_i, \mathbf{x}_j\right) = \mathbf{R}_i^T\left(\mathbf{x}_j - \mathbf{t}_i\right)$$

robot

landmark

robot translation

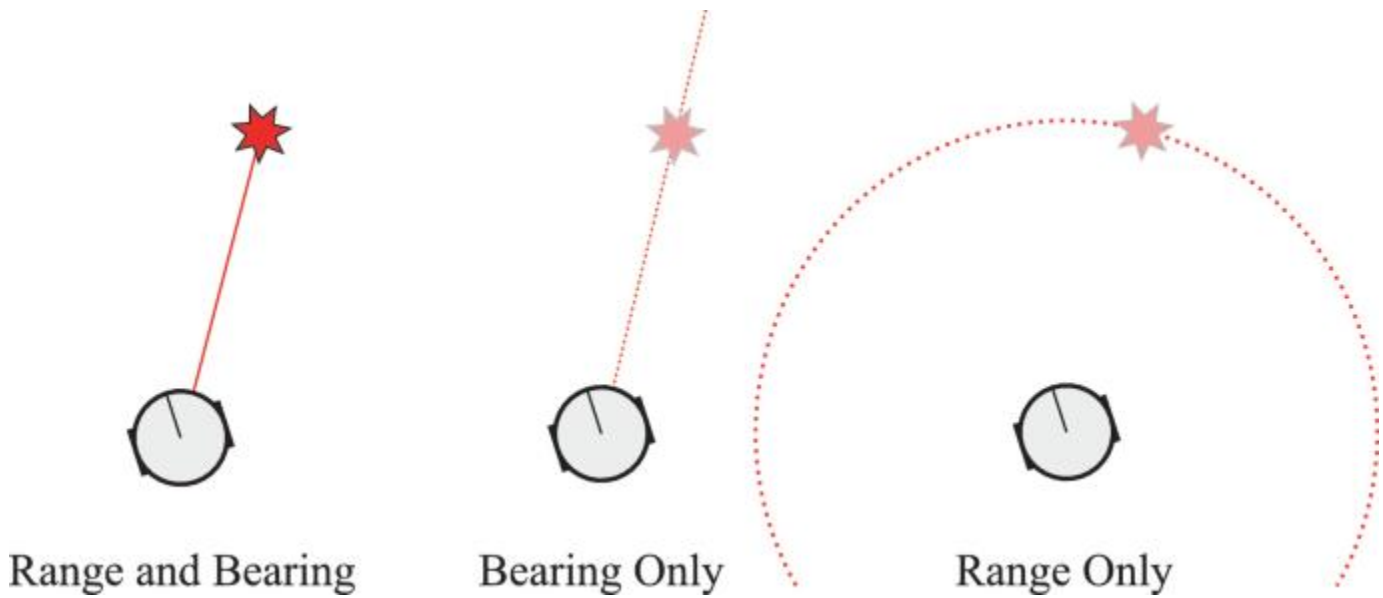
- Error function

$$\begin{aligned}\mathbf{e}_{ij}\left(\mathbf{x}_i, \mathbf{x}_j\right) &= \widehat{\mathbf{z}}_{ij} - \mathbf{z}_{ij} \\ &= \mathbf{R}_i^T\left(\mathbf{x}_j - \mathbf{t}_i\right) - \mathbf{z}_{ij}\end{aligned}$$

Courtesy: Cyrill Stachniss

Recap L10 - EKF SLAM

- Range-Bearing Observation
- Detect a landmark with the laser scan
  - like clustering the laser points and detect with a classifier
- Range (meter) and orientation (degree/rad) respected to the robot
- What if we have bearing-only observation?



Bearing-Only Observations

- Bearing Only Observations
- The robot observe only the bearing (orientation towards the landmark)
- Observation function

$$\hat{\mathbf{z}}_{ij}\left(\mathbf{x}_i, \mathbf{x}_j\right) = \operatorname{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i)_{.y}}{(\mathbf{x}_j - \mathbf{t}_i)_{.x}} - \theta_i$$

robot

landmark

robot-landmark angle

robot orientation

- Error function

$$\mathbf{e}_{ij}\left(\mathbf{x}_i, \mathbf{x}_j\right) = \operatorname{atan} \frac{(\mathbf{x}_j - \mathbf{t}_i)_{.y}}{(\mathbf{x}_j - \mathbf{t}_i)_{.x}} - \theta_i - \mathbf{z}_j$$

Courtesy: Cyrill Stachniss

The Rank of the Matrix H

- What is the rank of $\mathbf{H}_{ij}$ for a 2D landmark-pose constraint?
  - The blocks of $\mathbf{J}_{ij}$ are a 2x3 matrices
  - $\mathbf{H}_{ij}$ cannot have more than rank 2

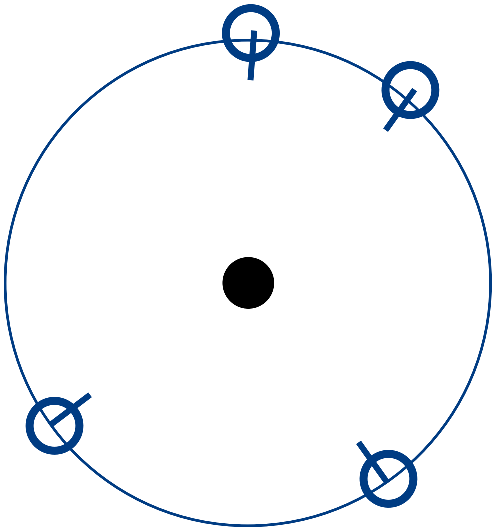
    $$\operatorname{rank}\left(J^T J\right) = \operatorname{rank}\left(J^T\right) = \operatorname{rank}(J)$$

- What is the rank of $\mathbf{H}_{ij}$ for a bearing-only constraint?
  - The blocks of $\mathbf{J}_{ij}$ are a 1x3 matrices
  - $\mathbf{H}_{ij}$ has rank 1

Courtesy: Cyrill Stachniss

Where is the Robot?

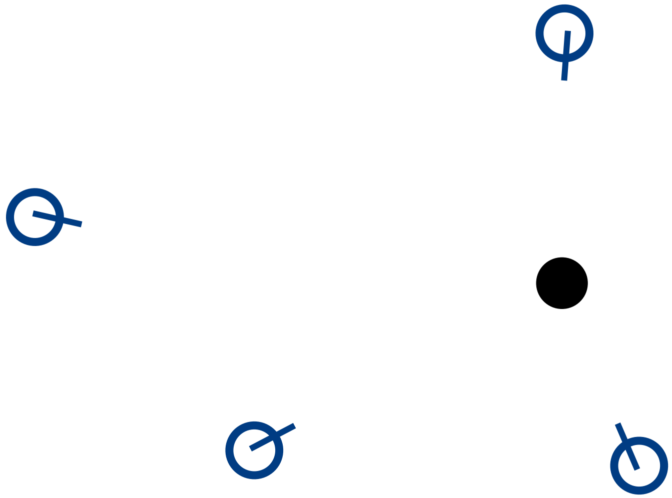
- Robot observes one landmark (x,y)
- Where can the robot be?
  - The robot can be somewhere on a circle around the landmark
  - It is a 1D solution space (constrained by the distance and the robot’s orientation)



Courtesy: Cyrill Stachniss

Where is the Robot?

- Robot observes one landmark with bearing-only
- Where can the robot be?
  - The robot can be anywhere in the x-y plane
  - It is a 2D solution space (constrained by the robot's orientation)



Courtesy: Cyrill Stachniss

Rank

- In landmark-based SLAM, the system can be under-determined
- The rank of $\mathbf{H}$ is less or equal to the sum of the ranks of the constraints
- To determine a unique solution, the system must have full rank

- **Question**
- How many 2D landmark observations are needed to resolve for a robot pose?
- How many bearing-only observations are needed to resolve for a robot pose?

Courtesy: Cyrill Stachniss

Under-Determined Systems

- No guarantee for a full rank system
  - Landmarks may be observed only once
  - Robot might have no odometry
- We can still deal with these situations by adding a “damping” factor to $\mathbf{H}$
- Instead of solving $\mathbf{H}\Delta\mathbf{x} = -\mathbf{b}$

  we solve

$$(\mathbf{H} + \lambda\mathbf{I})\Delta\mathbf{x} = -\mathbf{b}$$

- The damping factor $\lambda\mathbf{I}$ makes the system positive definite
- It adds an additional constraints that “drag” the increments towards 0
- Wighted sum of Gauss Newton and Steepest Descent

Courtesy: Cyrill Stachniss

Simplifed Levenberg Marquardt

- Damping to regulate the convergence using backup/restore actions

```
x: the initial guess
while (! converged)
  λ = λ_init
  <H,b> = buildLinearSystem(x);
  E = error(x)
  x_old = x;
  Δx  = solveSparse( (H + λ I) Δx = -b);
  x += Δx;
  If (E < error(x)){
    x  = x_old;
    λ *= 2;
} else { λ /= 2; }
```

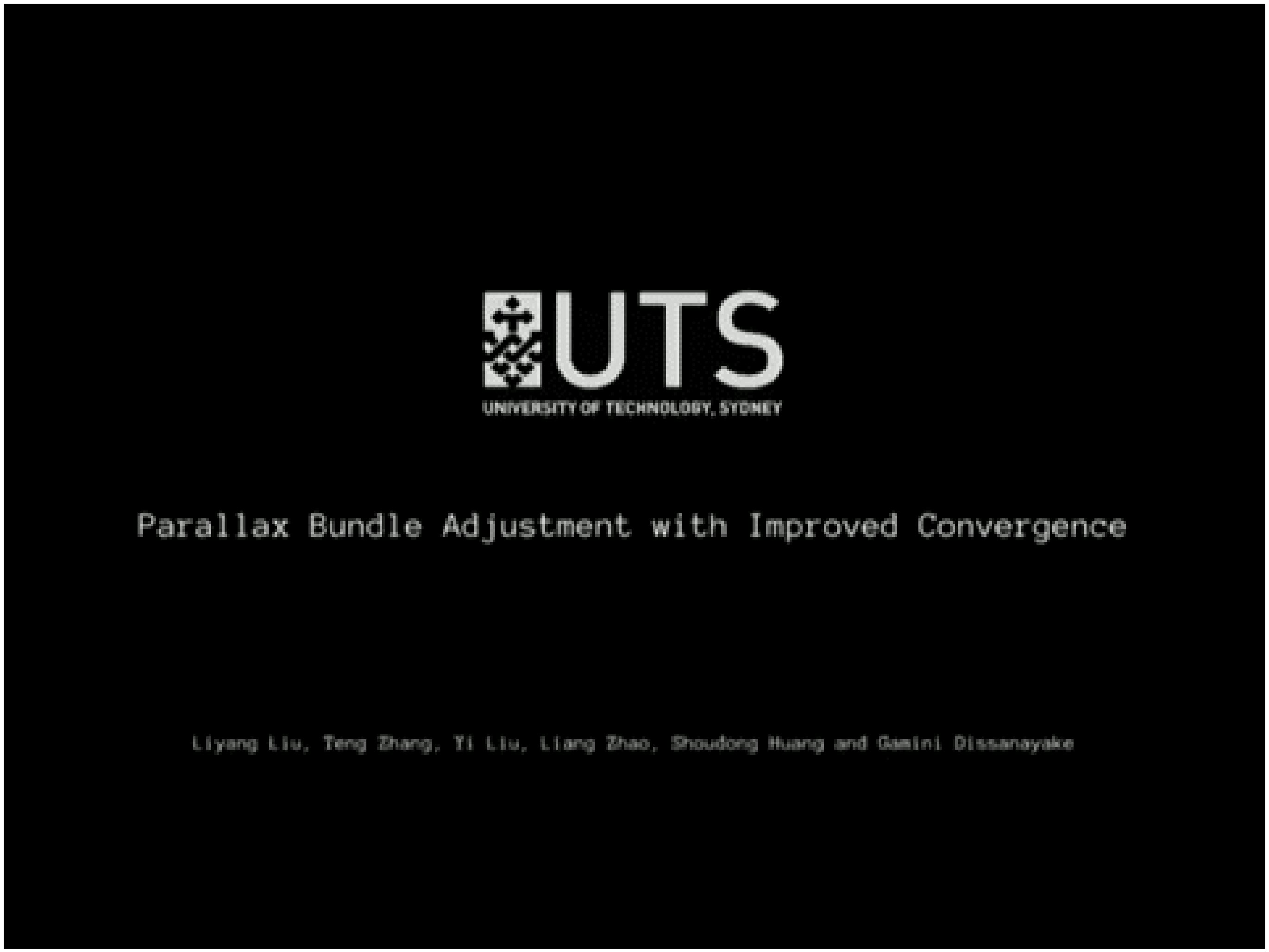
Courtesy: Cyrill Stachniss

Bundle Adjustment

- Often Levenberg Marquardt
- No notation of odometry (pose-pose)
- An offline pose graph with landmarks problem
  - SLAM is an online problem
- 3D reconstruction based on images taken at different views
  - LiDAR Bundle Adjustment is also popular in recent years
- Minimizes the projection error in the 2D image plane
- Developed in photogrammetry during 1950s

Courtesy: Cyrill Stachniss

Visual Bundle Adjustment



Courtesy: UTS RI

LiDAR Bundle Adjustment

Courtesy: HKU Mars Lab

Next Lecture

- Visual Perception 1
  - Feature Detection and Matching