THE HONG KONG UNIVERSITY OF SCIENCE & TECHNOLOGY

ELEC 3830 & BIEN 3310: Data Science in Neural Engineering

Homework 1

Due Date: 1pm, 28th Sept 2023

Your answers should be typed, not handwritten. You should submit a Word file or a pdf file (DO NOT UPLOAD .zip). Submissions are to be made via Canvas. ***Late submission is not allowed.***

Submitted File Name Format: name_ID_Homework1

Question 1 (15 points)

A random experiment has sample space $S = \{1,2,3,4\}$ with probabilities $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{4}, p_3 = \frac{1}{8}$, $p_4 = \frac{1}{8}$.

(a) Describe how this random experiment can be simulated using tosses of a fair coin.

(b) Describe how this random experiment can be simulated using an urn experiment.

(c) Describe how this experiment can be simulated using a deck of 52 distinct cards.

Question 2 (10 points)

Andy and Bob are playing a basketball shooting game. In each round they will each shoot for one ball. Anyone who makes the goal will get one point. After each round, if one of them has more points than the other, the game will end and the person with higher points wins. If they have the same points after one round, they will have a new round until one wins. Suppose the hit rate for Andy is p_1 and for Bob is p_2. What is the probability that Andy will win the game?

Question 3 (15 points)

A binary communication system transmits a signal X that is either a voltage signal +2 or a -2 voltage signal. A malicious channel reduces the magnitude of the received signal by the number of heads it counts in two tosses of a coin. Let Y be the resulting signal.

(a) Find the sample space.

(b) Find the set of outcomes corresponding to the event "transmitted signal was definitely +2"

(c) Describe in words the event corresponding to the outcome $Y = 0$.

Question 4 (10 points)

Let the events A and B have $P[A] = x, P[B] = y, and\ P[A \cup B] = z$. Use Venn diagrams to find $P[A \cap B]$, $P[A^c \cap B^c]$, $P[A^c \cup B^c]$, $P[A \cap B^c]$, $P[A^c \cap B]$

Question 5 (10 points)

Assume we have a random number generator that can generate a random number uniformly distributed in the range [0, 1]. If now we want to use this generator to generate a random number X that has a negative exponential distribution $f_X(x) = 3e^{-3x}(x \geq 0)$. Suppose the random number generator generates 0.75. What should be the value for X.

Question 6 (15 points)

Let X be a random variable with pmf $p_k = c/k^2$ for $k = 1,2, \ldots$

(a) Estimate the value of c numerically.
(b) Find $P[X > 4]$.
(c) Find $P[6 \leq X \leq 8]$.

Question 7 (15 points)

A radio transmitter sends a signal $s > 0$ to a receiver using three paths. The signals that arrive at the receiver along each path are:

$$X_1 = s + N_1, X_2 = s + N_2, X_3 = s + N_3$$

Where N_1, N_2 and N_3 are independent Gaussian random variables with zero mean and unit variance.

(a) Find the joint pdf of $X = (X_1, X_2, X_3)$. Are X_1, X_2 and X_3 independent random variables?
(b) Find the probability that the minimum of all three signals is positive.
(c) Find the probability that a majority of the signals are positive.

Question 8 (10 points)

Let U_1,U_2,U_3 be independent random variables with zero mean and variance 1. Find the linear MMSE estimator of S in terms of Z_1 and Z_2, and the corresponding MSE.

$$\begin{bmatrix} S \\ Z_1 \\ Z_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}$$

Homework1 notes

"Probability, Statistics, and Random Processes for Electrical Engineering"