ELEC 3210
Introduction to Mobile Robotics
Lecture 2

(Machine Learning and Infomation Processing for Robotics)

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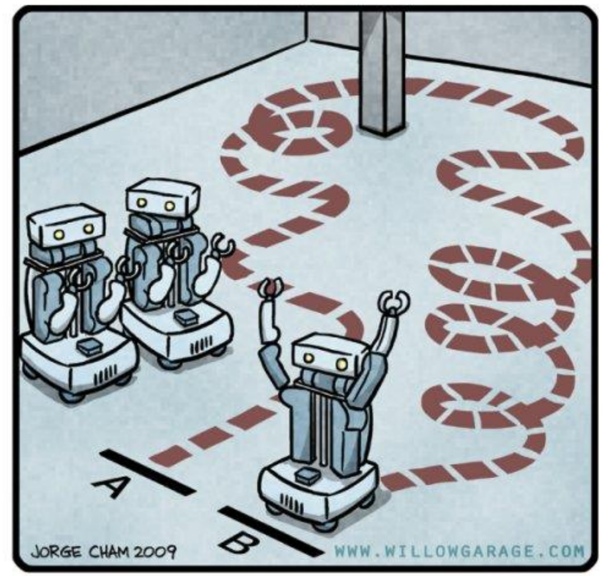
About

- Autonomous Navigation
- Wheeled Mobile Robot
- Sening and Estimation
  - Kalman Filter
  - Particle Filter
  - Graph Optimization
  - **Place Recognition**
- Motion Planning
  - Path Planning
  - Trajectory Planning
- **Visual Perception**
- Frontiers of Mobile Robotics
- 2D Laser-based ROS Projects

R.O.B.O.T. Comics



"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

Not About

- Soft Robotics
- Mechanics
- Machine Learning
- Swarm
- Vehicular Communication
- Robot Motion and Control (ELEC 4220 Prof Fumin Zhang)
- Robotic Manipulation (ELEC 4220 Prof Fumin Zhang)
- Drones (ELEC 5660 Prof Shaojie Shen)
- Visual-Inertial SLAM (ELEC 5660 Prof Shaojie Shen)
- etc

Grade (Tentative)

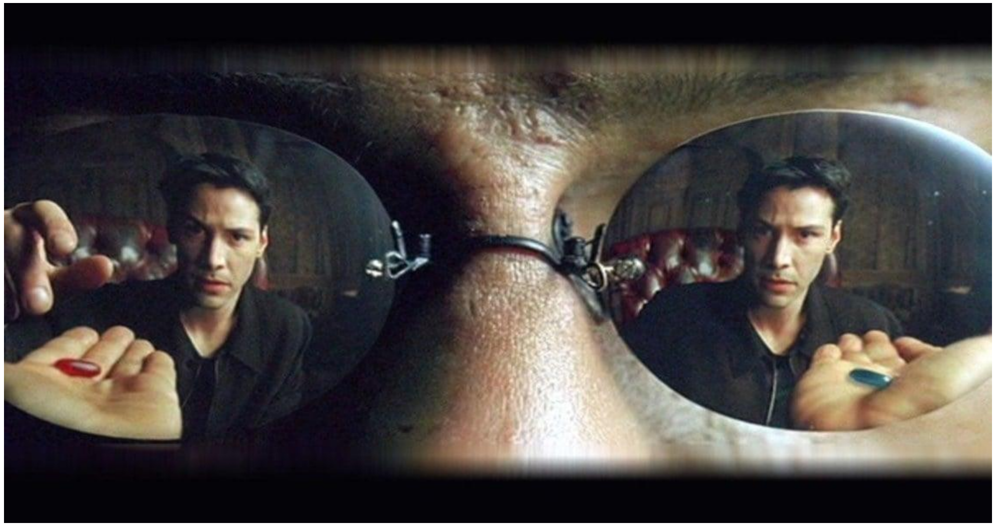
- Quiz 20%
  - Randomly in lectures
  - 1 page A4 paper
  - Maybe 4~6 times

- Homework 30%
  - Submit after letures in due time
  - Maybe 3 times

- Group Projects
  - Proj 1 10%
  - Proj 2 20%
  - Proj 3 20%

Requirements

- Love Robots ☺
- Basic Math
  - Linear Algebra
  - Probability
- Programming (Important!)
  - C++
  - Linux + ROS (**R**obot **O**perating **S**ystem)

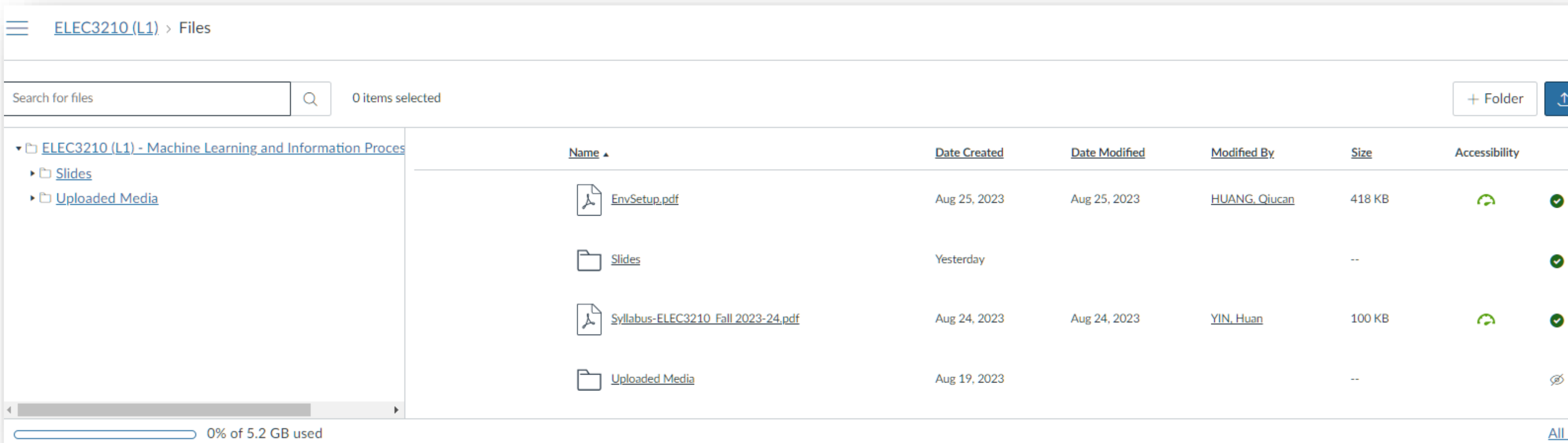


C++ Online Resources

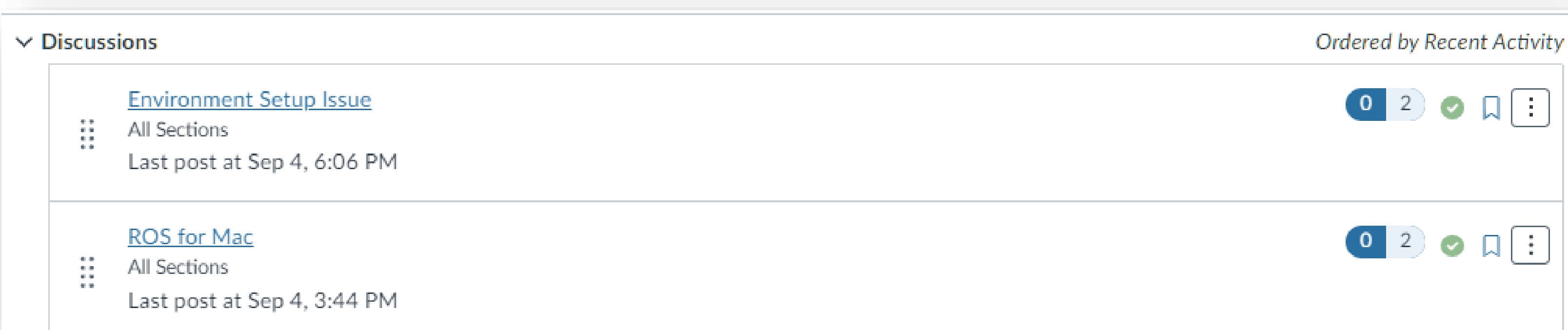
- Thanks to Prof. Cyrill Stachniss and his team

Canvas

- Files

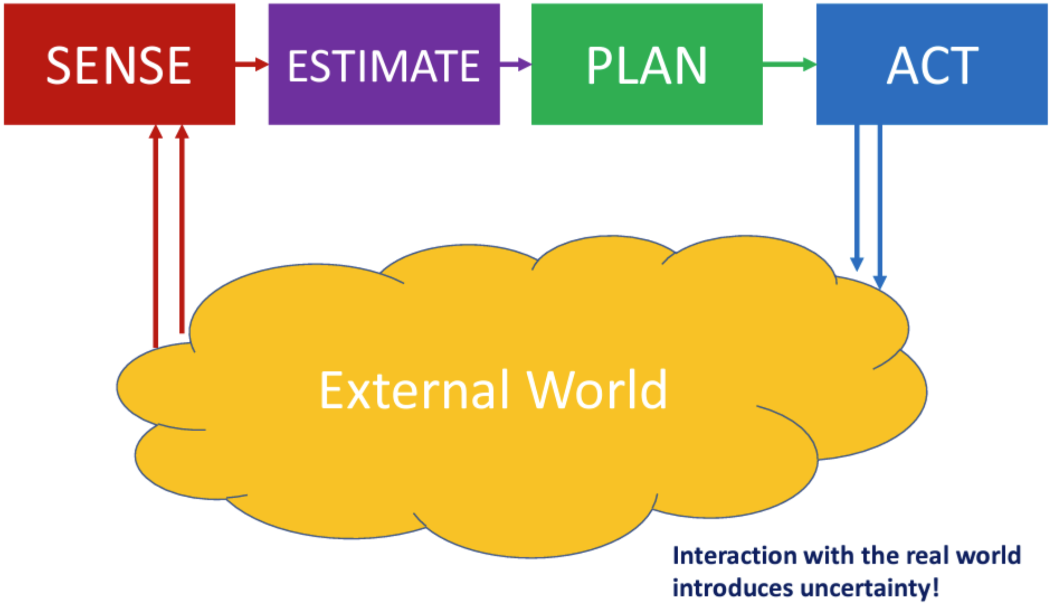


- Discussions



Robot Navigation Paradigm

- Sensing&Estimation - Estimate current and past robot pose
- Planning - Generate future robot pose
- Control - Stabilize robot pose



Courtesy: Nikolay Atanasov, PyPose Video

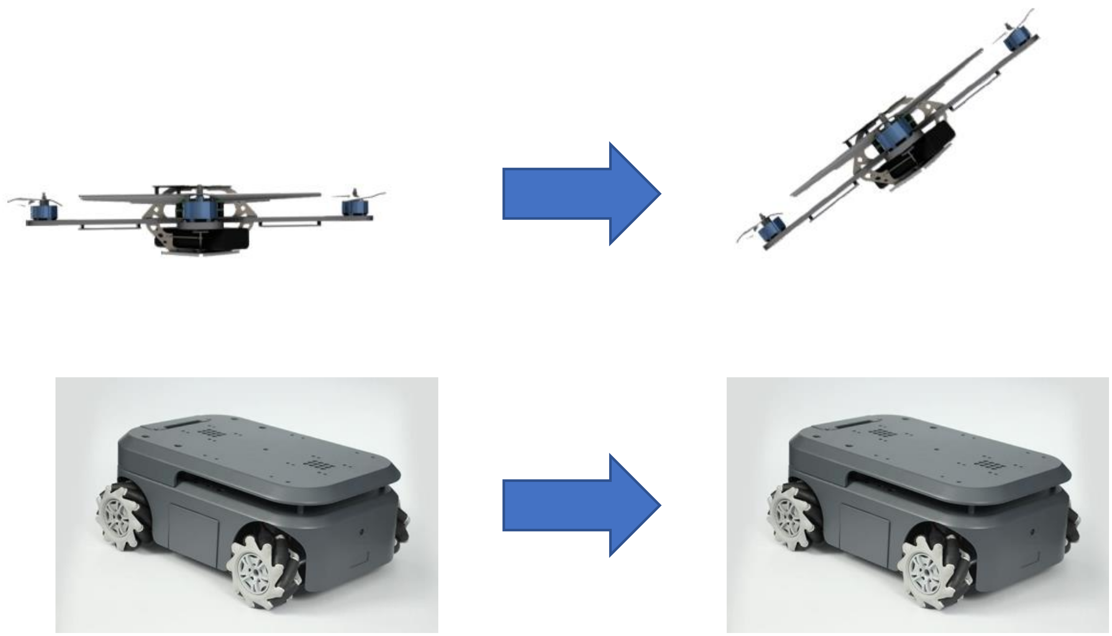
Outline

- Pose
  - Math
  - Concepts

Rigid Body Displacement

Rigid Body

- Two distinct positions and orientations of the same rigid body
  - Let $\mathbf{p}$ and $\mathbf{q}$ be two points on a rigid body
  - $\|\boldsymbol{p}(t) - \boldsymbol{q}(t)\| = \|\boldsymbol{p}(0) - \boldsymbol{q}(0)\| = \text{constant}$



Courtesy: Shaojie Shen

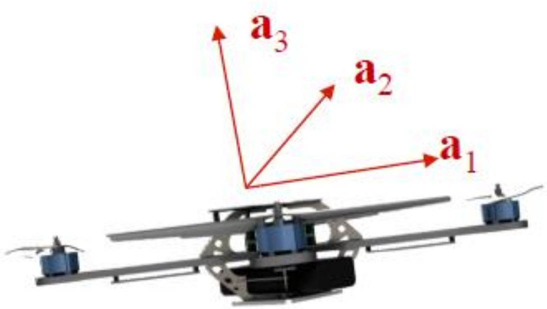
Soft Robotics

- Soft Robotics

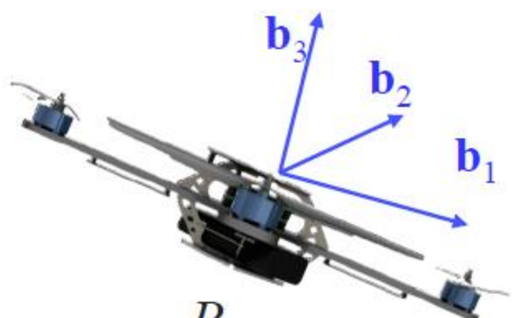
Courtesy: Nature

Reference Frame

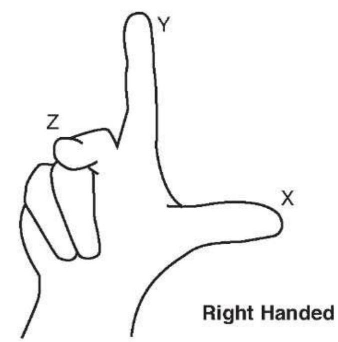
- Pose = position (translation) + orientation (rotation)
- We associate any position and orientation with a reference frame
  - We use **right-handed** coordinate frames
  - We can find three linearly independent vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ that are basis vectors for reference frame A
  - We can write any vector as a linear combination of basis vectors in either frame $\boldsymbol{v} = \boldsymbol{v}_\mathbf{1}\mathbf{a}_\mathbf{1} + v_2\mathbf{a}_2 + v_3\mathbf{a}_3$



A



B



Courtesy: Shaojie Shen

Notation

- **Be Aware of Potential Confusion!!!**

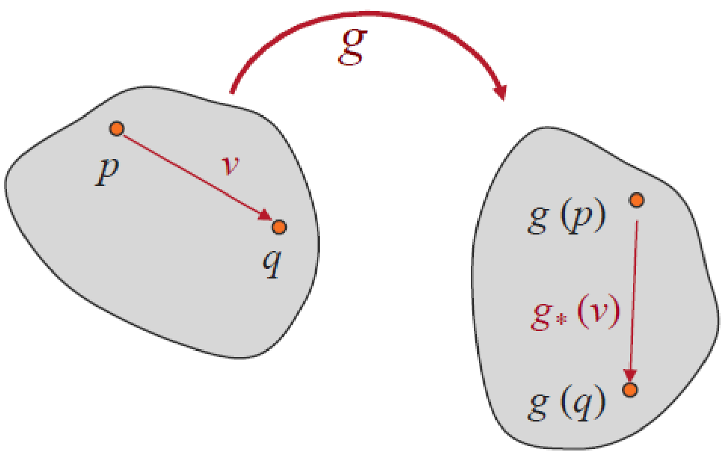
- Vectors
  - $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{a}$, …
- Matrices
  - $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, …
- Reference frames
  - A , B , C , …
  - a , b , c , …
- Transformations
  - ${}^{A}\mathbf{A}_{B}$, ${}^{A}\mathbf{R}_{B}$ $\cdots$
  - $\mathbf{A}_{ab}$, $\mathbf{R}_{ab}$ $\cdots$
  - $g_{ab}(.)$, $h_{ab}(.)$ $\cdots$

Courtesy: Shaojie Shen

Rigid Body Displacement

- A displacement of a transformation of points
  - Transformation (g) of points induces an action (g*) on vectors

$$g_*(\mathbf{v}) = g(\mathbf{q}) - g(\mathbf{p})$$



Courtesy: Shaojie Shen

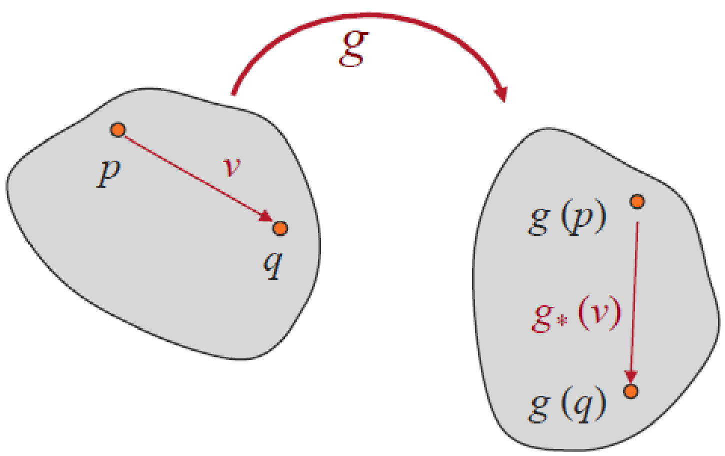
Rigid Body Displacement

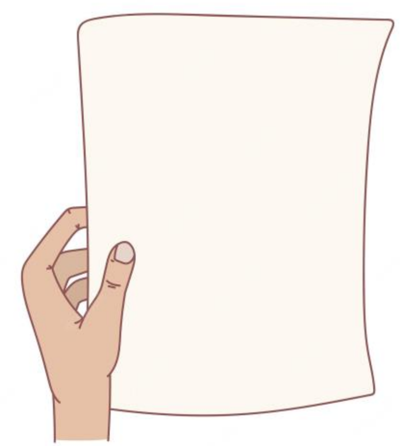
- Length are preserved

$$\|g(\mathbf{q})-g(\mathbf{p})\|=\|\mathbf{q}-\mathbf{p}\|$$

- Cross products are preserved

$$g_*(\mathbf{v})\times g_*(\mathbf{w})=g_*(\mathbf{v}\times\mathbf{w})$$

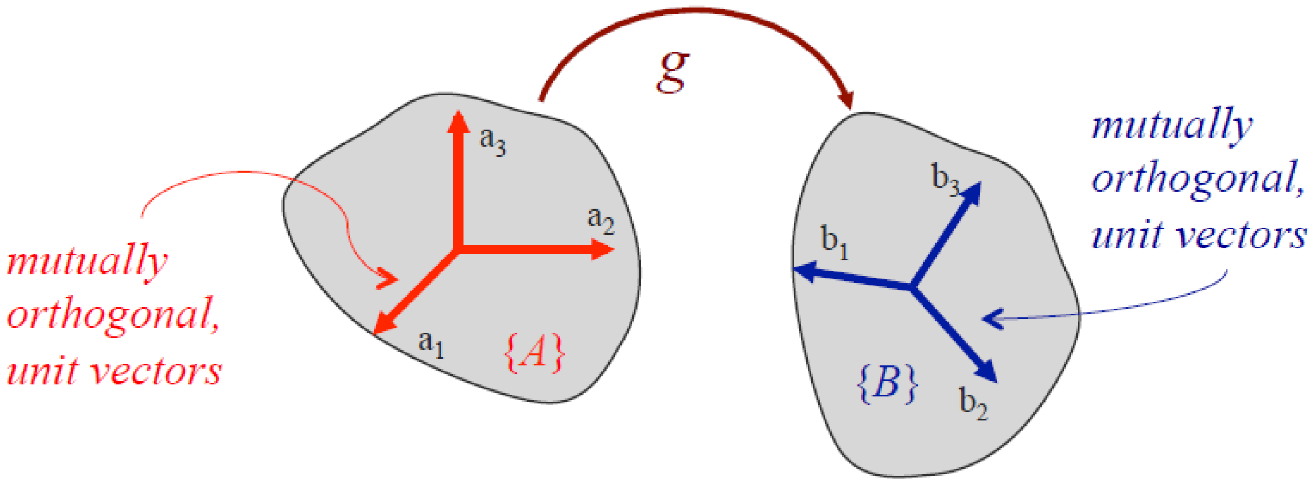




Courtesy: Shaojie Shen

Rigid Body Displacement

- Orthogonal vectors are mapped to orthogonal vectors



Courtesy: Shaojie Shen

Rigid Body Displacement

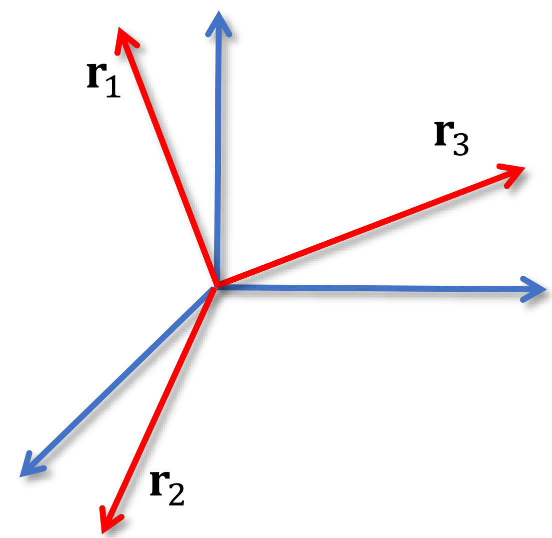
- Rigid body displacements are transformations that satisfy two important properties:
  - Lengths are preserved
  - Cross products are preserved

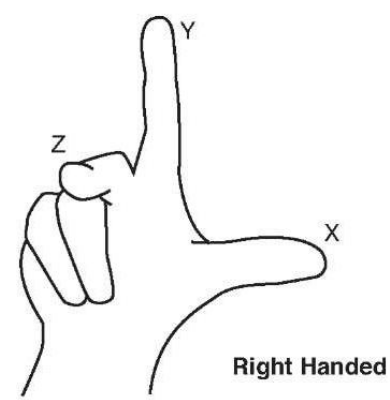
- Rigid body transformations and rigid body displacements are often used interchangeably
  - Transformations generally used to describe relationship between reference frames attached to different rigid bodies.
  - Displacements describe relationships between two positions and orientation of a frame attached to a displaced rigid body

Rotation

Rotations

- Coordinate frames are right-handed
- Principle axes of frame *A*:
  - $\mathbf{x} = [1\ 0\ 0]^T$
  - $\mathbf{y} = [0\ 1\ 0]^T$
  - $\mathbf{z} = [0\ 0\ 1]^T$
- Principle axes of frame *B*:
  - $\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab} \subset \mathbb{R}^3$
- The Rotation Matrix:
  - $\mathbf{R}_{ab} = [\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}]$
  - Coordinates of principle axes of *B* related to *A*

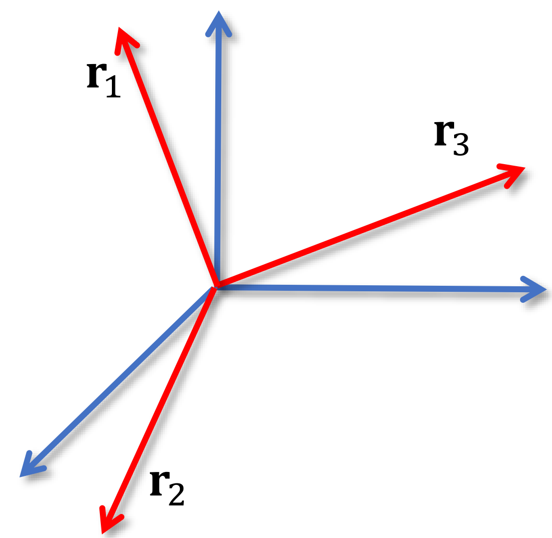




Courtesy: Shaojie Shen

Properties of a Rotation Matrix

- Let $\mathbf{R} = [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]$ be a rotation matrix
- Orthogonal:
  - ${\mathbf{r}_i}^T \cdot \mathbf{r}_j = \begin{cases} 0 \text{ if } i \neq j \\ 1 \text{ if } i = j \end{cases}$
  - $\mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}$
- Special orthogonal:
  - $\det\mathbf{R} = {\mathbf{r}_1}^T \cdot (\mathbf{r}_2 \times \mathbf{r}_3) = {\mathbf{r}_1}^T \cdot \mathbf{r}_1 = 1$



- The set of all rotations forms the Special Orthogonal Group
  - Special orthogonal group
  - 3D rotations: $SO(3)$
  - 2D rotations: $SO(2)$
  - $SO(n) = \{\mathbf{R} \in \mathbb{R}^{n\times n} | \mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}, \det\mathbf{R} = 1\}$

Courtesy: Shaojie Shen

Properties of a Rotation Matrix

$(G,\cdot)$ is a group if:

1) $g_1, g_2 \in G \Rightarrow g_1 \cdot g_2 \in G$
2) $\exists!\, e \in G,\ s.t.\ g \cdot e = e \cdot g = g,\ \forall g \in G$
3) $\forall g \in G,\ \exists!\ g^{-1} \in G,\ s.t.\ g \cdot g^{-1} = g^{-1} \cdot g = e$
4) $g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3$

Group examples:

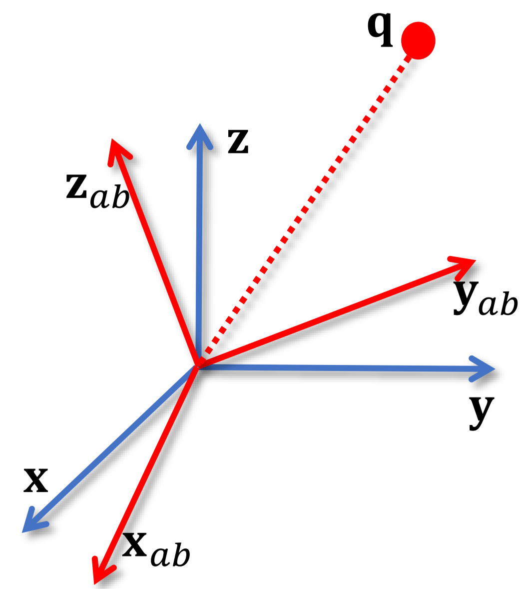
1. The set of all integers with addition operation
2. The set of all real numbers with arithmetic operations

- $SO(3) = \{\mathbf{R} \in \mathbb{R}^{3\times 3} | \mathbf{R} \cdot \mathbf{R}^T = \mathbf{I}, \det\mathbf{R} = 1\}$
- $SO(3)$ is a group under the operation of matrix multiplication
  1. Closure: If $\mathbf{R}_1, \mathbf{R}_2 \in SO(3)$, then $\mathbf{R}_1 \cdot \mathbf{R}_2 \in SO(3)$
  2. Identity: The identity matrix is the identity element
  3. Inverse: If $\mathbf{R} \in SO(3)$, then $\mathbf{R}^{-1} \in SO(3)$
  4. Associativity: $\mathbf{R}_1 \cdot (\mathbf{R}_2 \cdot \mathbf{R}_3) = (\mathbf{R}_1 \cdot \mathbf{R}_2) \cdot \mathbf{R}_3$

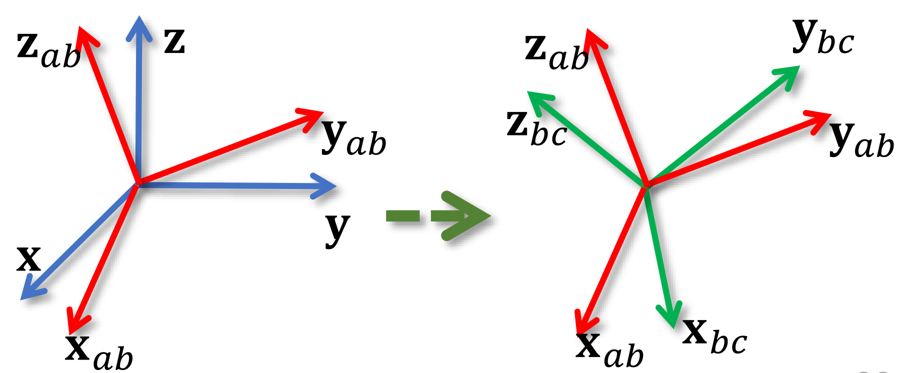
Courtesy: Shaojie Shen

Properties of a Rotation Matrix

- A transformation that rotates the coordinates of a point from frame *B* to frame *A*
  - Let $\mathbf{q}_b = [x_b, y_b, z_b]^T \in \mathbb{R}^3$ be coordinate of point $\mathbf{q}$ in frame *B*
  - Let $\mathbf{q}_a$ be coordinate of point $\mathbf{q}$ in frame *A*
  - $\mathbf{q}_a = x_b \cdot \mathbf{x}_{ab} + y_b \cdot \mathbf{y}_{ab} + z_b \cdot \mathbf{z}_{ab} = [\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}] \begin{bmatrix} x_b \\ y_b \\ z_b \end{bmatrix} = \mathbf{R}_{ab} \cdot \mathbf{q}_b$



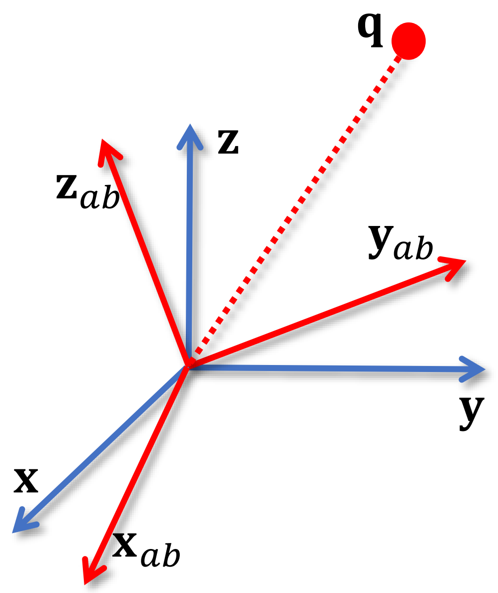
- Composition Rule
  - $\mathbf{R}_{ac} = \mathbf{R}_{ab} \cdot \mathbf{R}_{bc}$



Courtesy: Shaojie Shen

Rotation is a Rigid Body Transformation

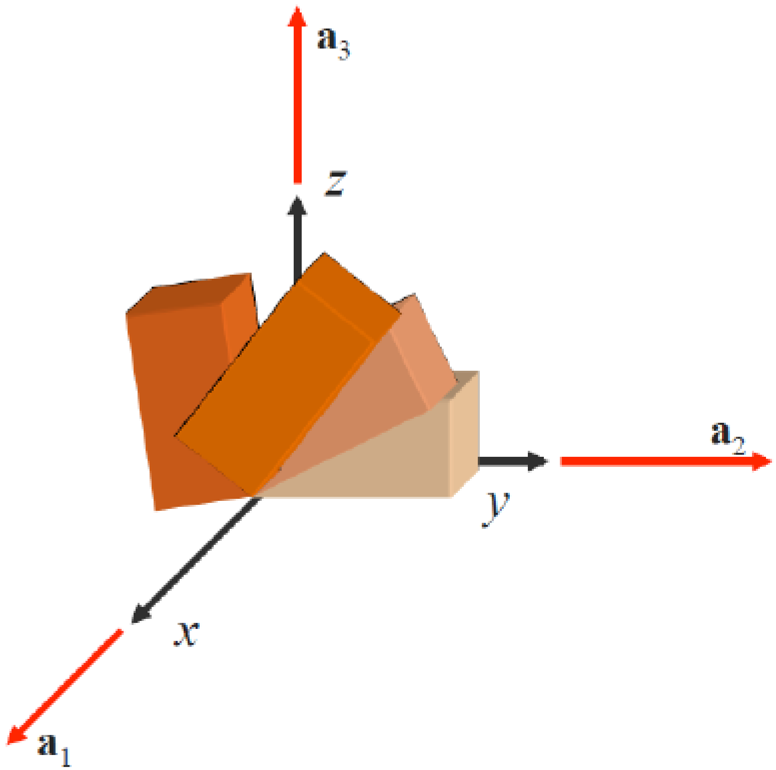
- $\mathbf{R}_{ab} = [\mathbf{x}_{ab}, \mathbf{y}_{ab}, \mathbf{z}_{ab}]$ preserves:
  - Length: $\|\mathbf{R}_{ab}(\mathbf{p}_b - \mathbf{q}_b)\| = \|\mathbf{p}_b - \mathbf{q}_b\|$
  - Cross product: $\mathbf{R}_{ab}(\mathbf{v} \times \boldsymbol{w}) = (\mathbf{R}_{ab}\mathbf{v}) \times (\mathbf{R}_{ab}\mathbf{w})$

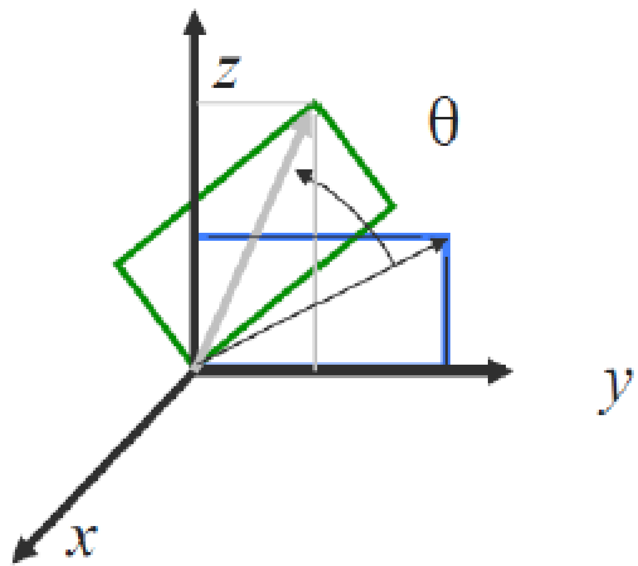


Courtesy: Shaojie Shen

Example - Rotation

$$\mathbf{R}_{\mathrm{x}}(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & cos\theta & -sin\theta\\ 0 & sin\theta & cos\theta\end{bmatrix}$$



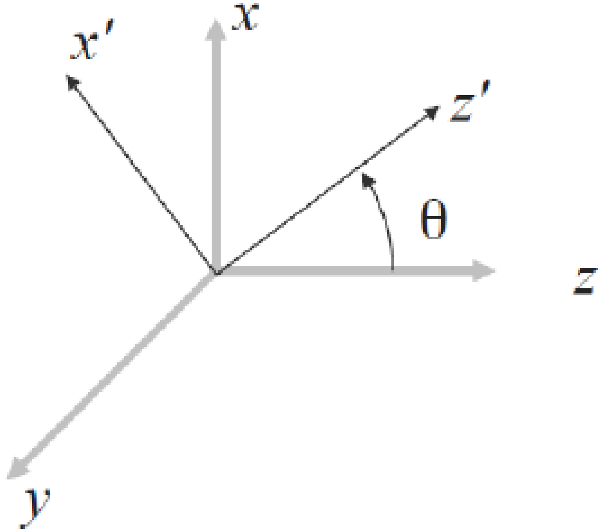


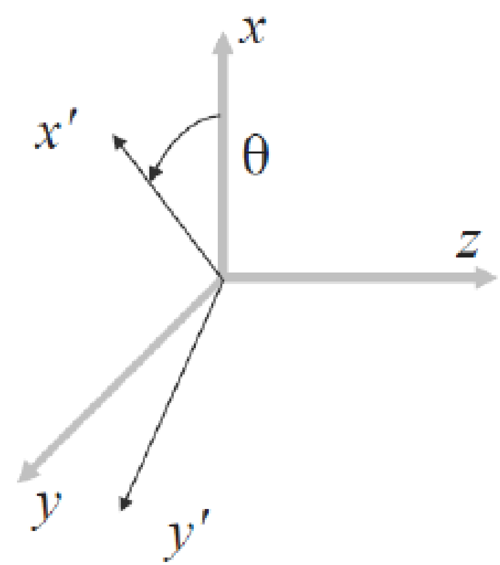
Courtesy: Shaojie Shen

Example - Rotation

$$\mathbf{R}_y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

$$\mathbf{R}_z(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

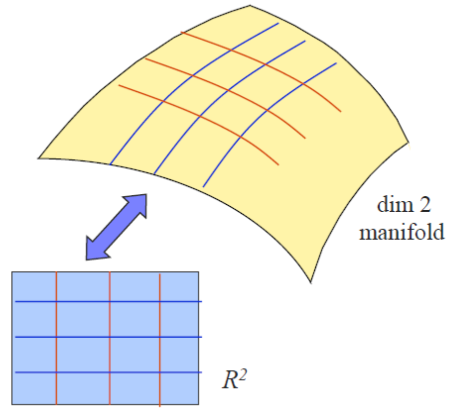


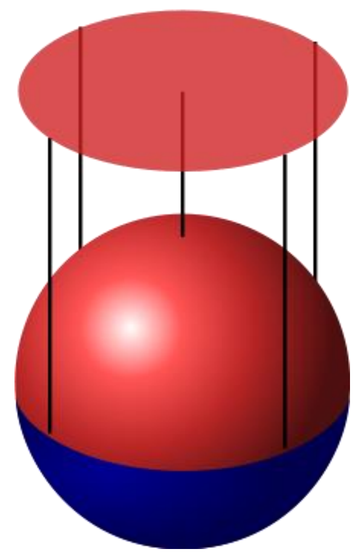


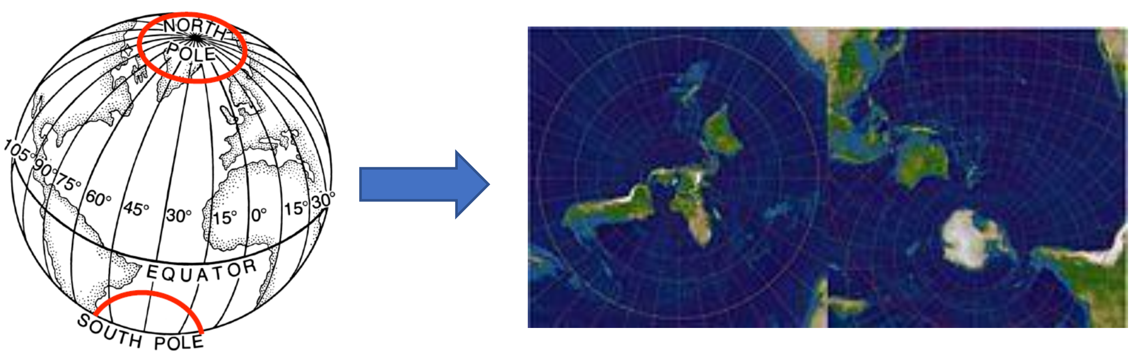
Courtesy: Shaojie Shen

Properties of Rotation

- $SO(3)$ is a continuous group
  - The multiplication operation is a continuous operation
  - The inverse is a continuous function
- $SO(3)$ is a smooth manifold
  - A manifold of dimension n is a set M which is locally resembled to Euclidean space $\mathbb{R}^n$ near each point
  - Example: sphere is a differentiable manifold that is locally resembled to $\mathbb{R}^2$





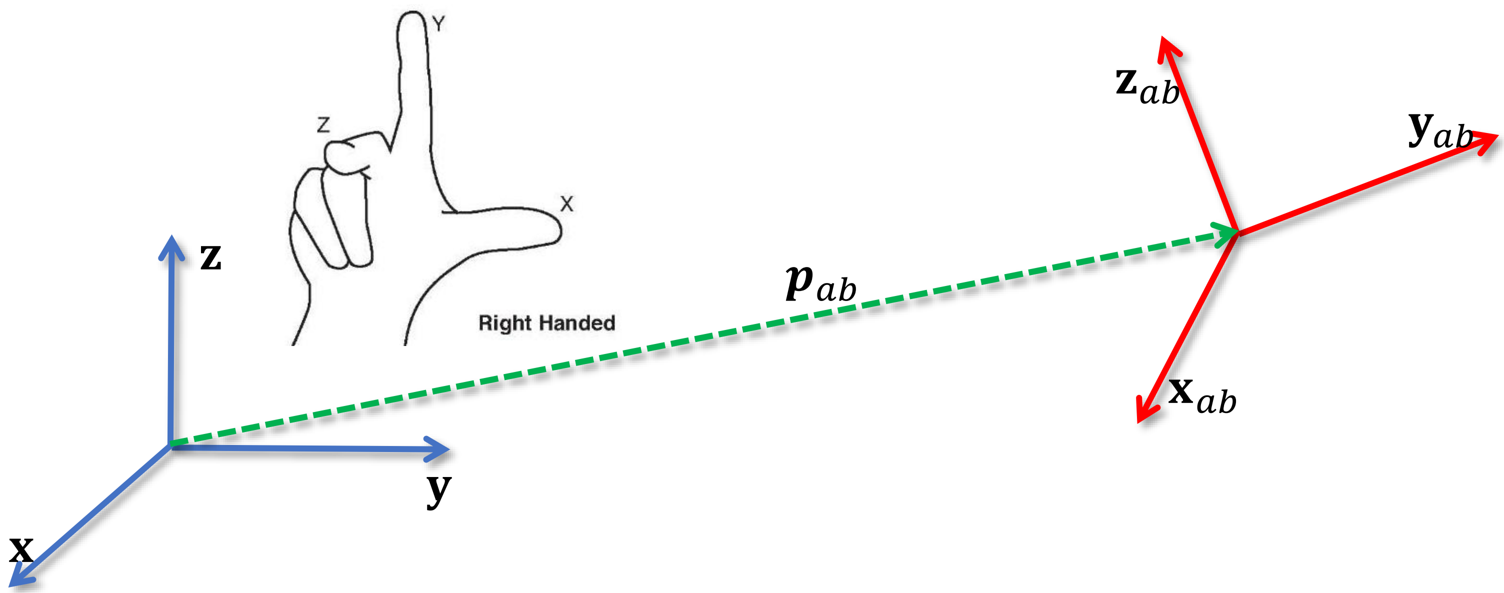


Courtesy: Shaojie Shen

Rigid Body Motion

Rigid Body Motion

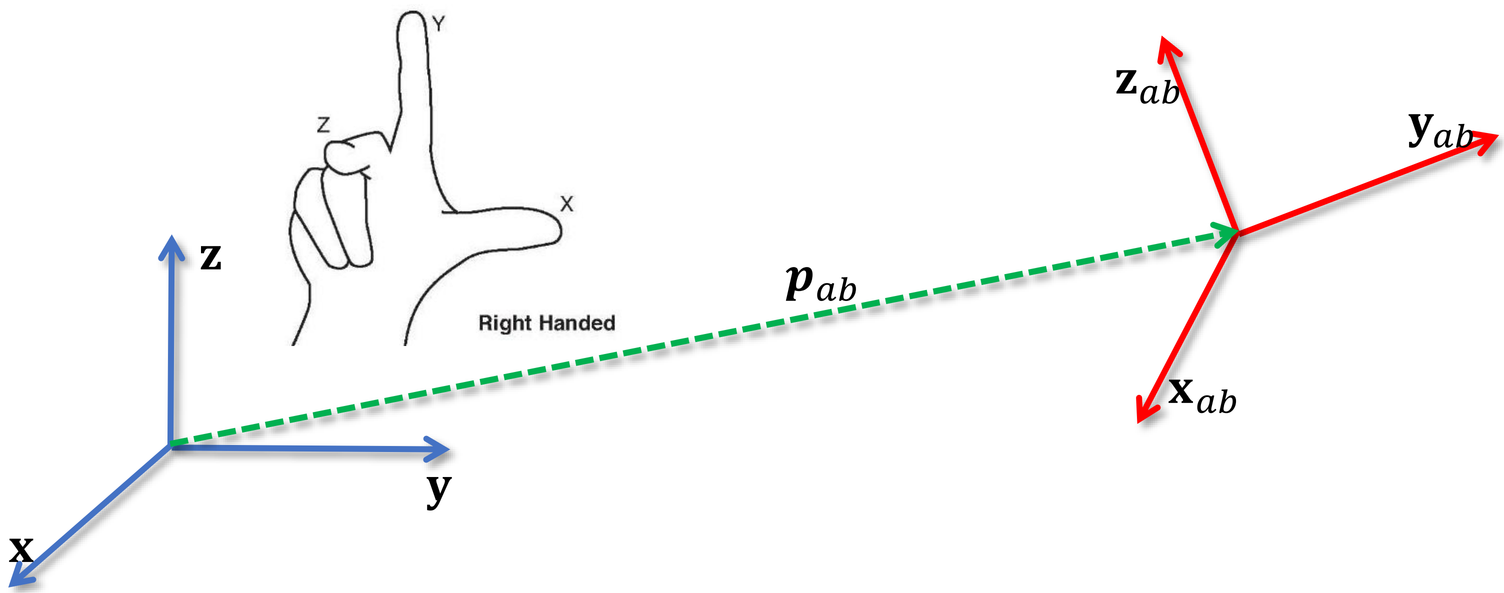
- General rigid body motions that includes both translation and rotation forms the product space of $\mathbb{R}^3$ and $SO(3)$. Denoted as $SE(3)$ – **Special Euclidean group**.
  - $SE(3) = \{(\boldsymbol{p}, \boldsymbol{R}) : \boldsymbol{p} \in \mathbb{R}^3, \boldsymbol{R} \in SO(3)\} = \mathbb{R}^3 \times SO(3)$



Courtesy: Shaojie Shen

Rigid Body Motion

- Special Euclidean group:
  - $SE(3) = \{(\boldsymbol{p}, \boldsymbol{R}) : \boldsymbol{p} \in \mathbb{R}^3, \boldsymbol{R} \in SO(3)\} = \mathbb{R}^3 \times SO(3)$
- Transformation of a point between different coordinate frames:
  - $\boldsymbol{p}^a = \boldsymbol{R}_{ab}\boldsymbol{p}^b + \boldsymbol{p}_{ab} = g_{ab}(\boldsymbol{p}^b)$



Courtesy: Shaojie Shen

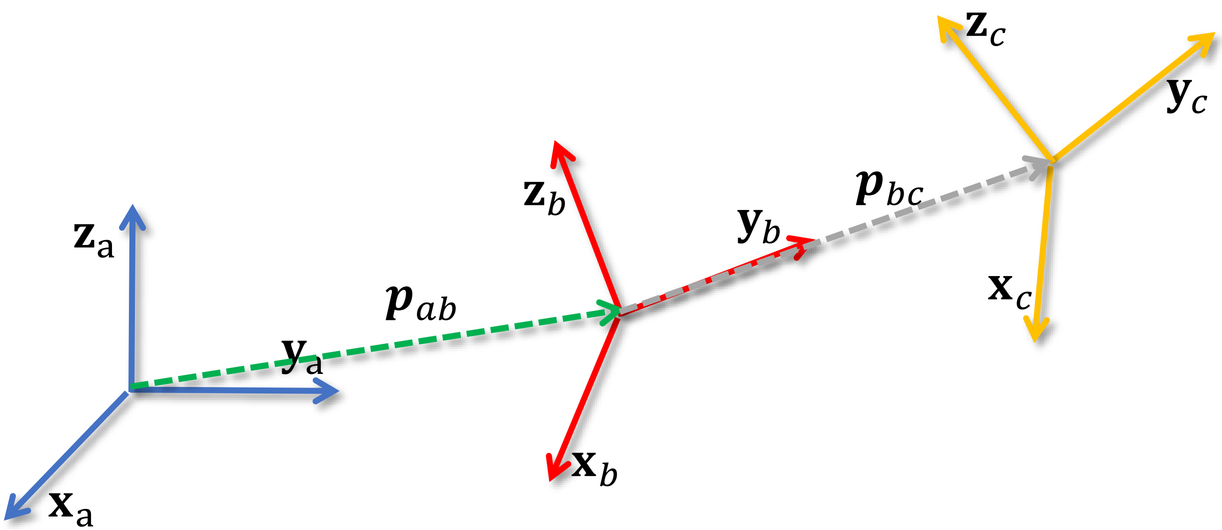
Rigid Body Motion

- Homogeneous coordinates of a point:
  - $\bar{\boldsymbol{p}} = \begin{bmatrix} p_x \\ p_y \\ p_z \\ 1 \end{bmatrix}$
- Homogeneous coordinates of a vector:
  - $\bar{\boldsymbol{v}} = \begin{bmatrix} v_x \\ v_y \\ v_z \\ 0 \end{bmatrix}$
- Homogeneous representation of rigid body motion:
  - $\bar{\boldsymbol{p}}^a = \begin{bmatrix} \boldsymbol{p}^a \\ 1 \end{bmatrix} = \begin{bmatrix} \boldsymbol{R}_{ab} & \boldsymbol{p}_{ab} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{p}^b \\ 1 \end{bmatrix} = \bar{g}_{ab} \bar{\boldsymbol{p}}^b$

Courtesy: Shaojie Shen

Rigid Body Motion

- Homogeneous representation of rigid body motion:
  - $\bar{g}_{ab} = \begin{bmatrix} \boldsymbol{R}_{ab} & \boldsymbol{p}_{ab} \\ 0 & 1 \end{bmatrix}$
- Composition rule for rigid body motions:
  - $\bar{g}_{ac} = \bar{g}_{ab} \cdot \bar{g}_{bc} = \begin{bmatrix} \boldsymbol{R}_{ab}\boldsymbol{R}_{bc} & \boldsymbol{R}_{ab}\boldsymbol{p}_{bc} + \boldsymbol{p}_{ab} \\ 0 & 1 \end{bmatrix}$
  - Compare with composition of rotational motion: $\boldsymbol{R}_{ac} = \boldsymbol{R}_{ab} \cdot \boldsymbol{R}_{bc}$



Courtesy: Shaojie Shen

Properties of Rigid Body Motion

- $SE(3)=\{(\boldsymbol{p},\boldsymbol{R}):\boldsymbol{p}\in\mathbb{R}$^3, $\boldsymbol{R}\in SO(3)\}=\mathbb{R}^{3}\times SO(3)$

- $SE(3)$ is a group under the operation of matrix multiplication
  - Closure
  - Identity
  - Inverse
  - Associativity

- $g \in SE(3)$ is a rigid body transformation
  - Lengths are preserved
  - Cross products are preserved

Courtesy: Shaojie Shen

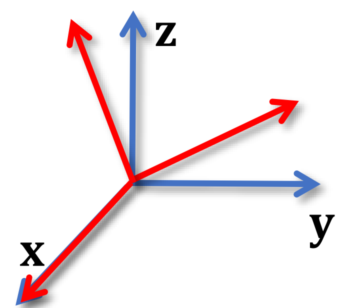
Rotation Representations

Rotation Representations

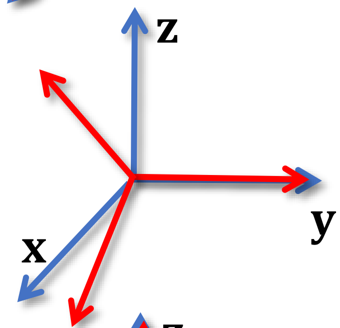
- Rotation matrices
- Euler angles
- Exponential coordinates
- Quaternions

Euler Angles

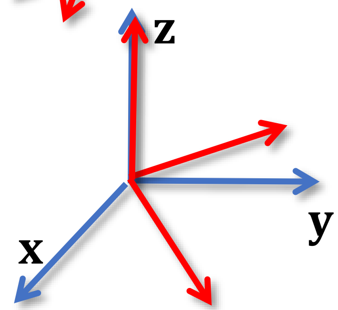
- Elementary rotations:
  - $\boldsymbol{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$



  - $\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$



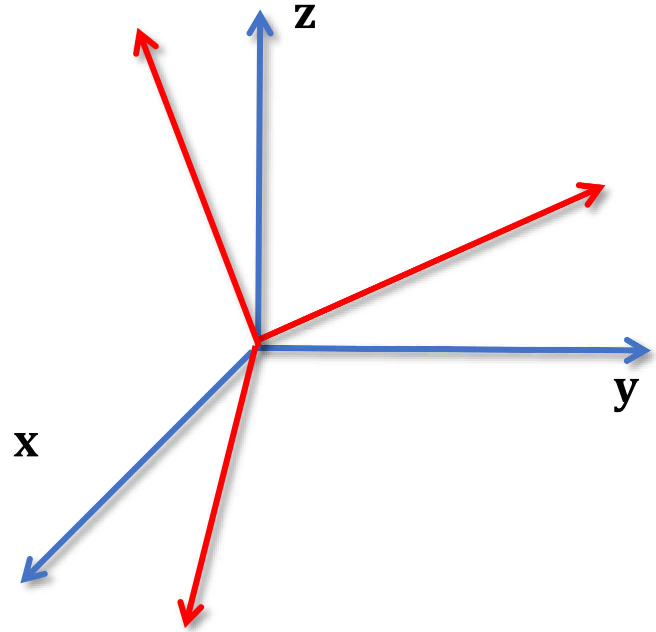
  - $\boldsymbol{R}_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Courtesy: Shaojie Shen

Euler Angles

- Any rotation can be described by three successive rotations about linear independent axes
- However, this is an almost 1-1 transform with singularities:
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \Rightarrow \boldsymbol{R}$
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \nLeftarrow \boldsymbol{R}$



Courtesy: Shaojie Shen

Euler Angles

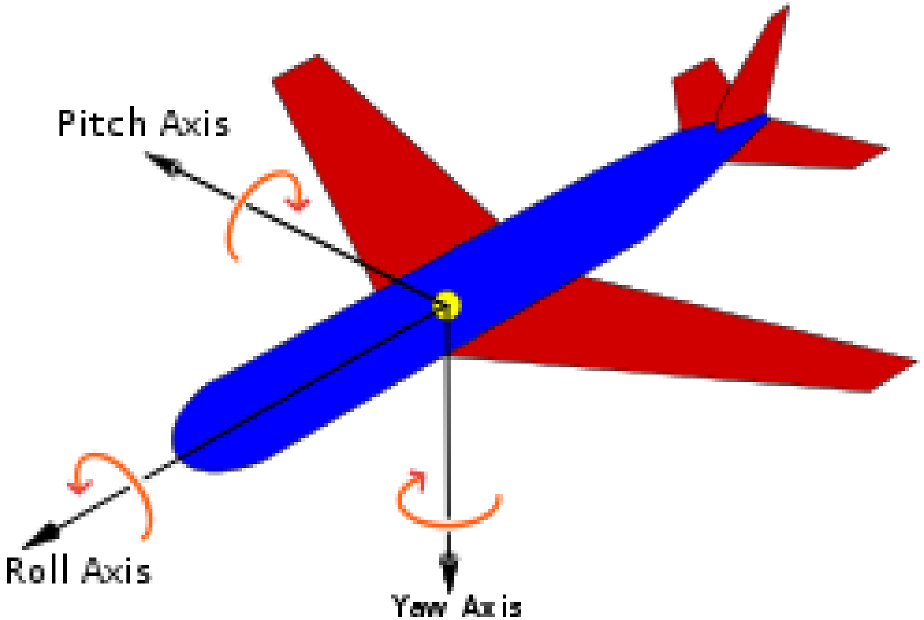
- Different Euler angle conversions:

Proper Euler angles	Tait-Bryan angles
$X_1Z_2X_3 = \begin{bmatrix} c_2 & -c_3 s_2 & s_2 s_3 \\ c_1 s_2 & c_1 c_2 c_3 - s_1 s_3 & -c_3 s_1 - c_1 c_2 s_3 \\ s_1 s_2 & c_1 s_3 + c_2 c_3 s_1 & c_1 c_3 - c_2 s_1 s_3 \end{bmatrix}$	$X_1Z_2Y_3 = \begin{bmatrix} c_2 c_3 & -s_2 & c_2 s_3 \\ s_1 s_3 + c_1 c_3 s_2 & c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 \\ c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 \end{bmatrix}$
$X_1Y_2X_3 = \begin{bmatrix} c_2 & s_2 s_3 & c_3 s_2 \\ s_1 s_2 & c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 \\ -c_1 s_2 & c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 \end{bmatrix}$	$X_1Y_2Z_3 = \begin{bmatrix} c_2 c_3 & -c_2 s_3 & s_2 \\ c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 & -c_2 s_1 \\ s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 & c_1 c_2 \end{bmatrix}$
$Y_1X_2Y_3 = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & s_1 s_2 & c_1 s_3 + c_2 c_3 s_1 \\ s_2 s_3 & c_2 & -c_3 s_2 \\ -c_3 s_1 - c_1 c_2 s_3 & c_1 s_2 & c_1 c_2 c_3 - s_1 s_3 \end{bmatrix}$	$Y_1X_2Z_3 = \begin{bmatrix} c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 & c_2 s_1 \\ c_2 s_3 & c_2 c_3 & -s_2 \\ c_1 s_2 s_3 - c_3 s_1 & c_1 c_3 s_2 + s_1 s_3 & c_1 c_2 \end{bmatrix}$
$Y_1Z_2Y_3 = \begin{bmatrix} c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 & c_3 s_1 + c_1 c_2 s_3 \\ c_3 s_2 & c_2 & s_2 s_3 \\ -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 & c_1 c_3 - c_2 s_1 s_3 \end{bmatrix}$	$Y_1Z_2X_3 = \begin{bmatrix} c_1 c_2 & s_1 s_3 - c_1 c_3 s_2 & c_3 s_1 + c_1 s_2 s_3 \\ s_2 & c_2 c_3 & -c_2 s_3 \\ -c_2 s_1 & c_1 s_3 + c_3 s_1 s_2 & c_1 c_3 - s_1 s_2 s_3 \end{bmatrix}$
$Z_1Y_2Z_3 = \begin{bmatrix} c_1 c_2 c_3 - s_1 s_3 & -c_3 s_1 - c_1 c_2 s_3 & c_1 s_2 \\ c_1 s_3 + c_2 c_3 s_1 & c_1 c_3 - c_2 s_1 s_3 & s_1 s_2 \\ -c_3 s_2 & s_2 s_3 & c_2 \end{bmatrix}$	$Z_1Y_2X_3 = \begin{bmatrix} c_1 c_2 & c_1 s_2 s_3 - c_3 s_1 & s_1 s_3 + c_1 c_3 s_2 \\ c_2 s_1 & c_1 c_3 + s_1 s_2 s_3 & c_3 s_1 s_2 - c_1 s_3 \\ -s_2 & c_2 s_3 & c_2 c_3 \end{bmatrix}$
$Z_1X_2Z_3 = \begin{bmatrix} c_1 c_3 - c_2 s_1 s_3 & -c_1 s_3 - c_2 c_3 s_1 & s_1 s_2 \\ c_3 s_1 + c_1 c_2 s_3 & c_1 c_2 c_3 - s_1 s_3 & -c_1 s_2 \\ s_2 s_3 & c_3 s_2 & c_2 \end{bmatrix}$	$Z_1X_2Y_3 = \begin{bmatrix} c_1 c_3 - s_1 s_2 s_3 & -c_2 s_1 & c_1 s_3 + c_3 s_1 s_2 \\ c_3 s_1 + c_1 s_2 s_3 & c_1 c_2 & s_1 s_3 - c_1 c_3 s_2 \\ -c_2 s_3 & s_2 & c_2 c_3 \end{bmatrix}$

Courtesy: Shaojie Shen

Aircraft

- Aviation Community



Euler Angles

- Example: Z-Y-Z Euler angles:
  - Sequence of three rotations about body-fixed axes
  - $\boldsymbol{R} = \boldsymbol{R}_z(\phi) \cdot \boldsymbol{R}_y(\theta) \cdot \boldsymbol{R}_z(\psi)$
  - $\boldsymbol{R} = \begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi s\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi c\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

- If $\sin\theta \neq 0$:
  - $\theta = \mathrm{acos}(r_{33})$
  - $\psi = \mathrm{atan}\,2(\frac{r_{32}}{\sin\theta}, -\frac{r_{31}}{\sin\theta})$
  - $\phi = \mathrm{atan}\,2(\frac{r_{23}}{\sin\theta}, \frac{r_{13}}{\sin\theta})$

Courtesy: Shaojie Shen

Euler Angles

- Example: Z-Y-Z Euler angles:
  - Sequence of three rotations about body-fixed axes
  - $\boldsymbol{R} = \boldsymbol{R}_z(\phi) \cdot \boldsymbol{R}_y(\theta) \cdot \boldsymbol{R}_z(\psi)$
  - $\boldsymbol{R} = \begin{bmatrix} c\phi c\theta c\psi - s\phi s\psi & -c\phi c\theta s\psi - s\phi c\psi & c\phi s\theta \\ s\phi c\theta c\psi + c\phi s\psi & -s\phi c\theta s\psi + c\phi c\psi & s\phi c\theta \\ -s\theta c\psi & s\theta s\psi & c\theta \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$

- If sinθ=0:
  - $\boldsymbol{R} = \begin{bmatrix} c\phi c\psi - s\phi s\psi & -c\phi s\psi - s\phi c\psi & 0 \\ c\phi s\psi + s\phi c\psi & -s\phi s\psi + c\phi c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} = \boldsymbol{R}_z(\phi + \psi)$
  - As long as $\phi + \psi$ is preserved, we have infinite set of Euler angles!

Courtesy: Shaojie Shen

Rotation Representations

- Rotation matrices ☑
- Euler angles ☑

We will not introduce others in ELEC 3210 for less confusion!

- Exponential coordinates
  - 3 numbers with singularity, often for robotic arms
- Quaternions
  - Good properties

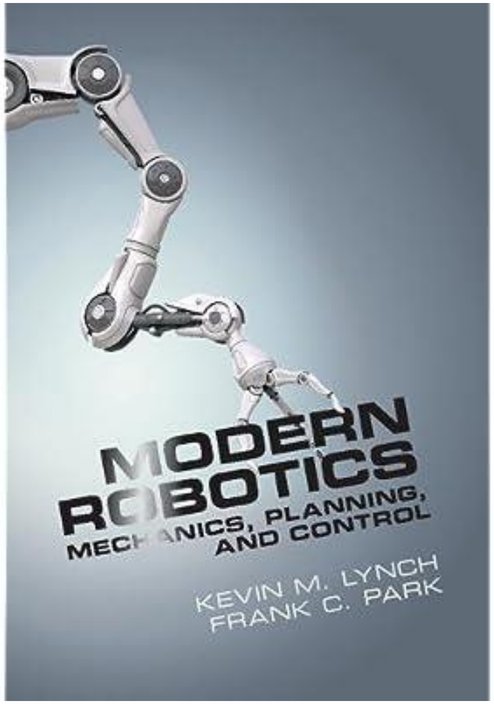
Quaternion

- Rotation matrix $\boldsymbol{R} \in SO(3)$
  - No singularity
  - Redundant parameters
- ZYX Euler angle
  - Singular at roll angle of 90 degrees
  - Minimum number of parameters
- Is there a singularity **free parameterization** that with **reduced** parameters?
  - **YES, quaternion**

Courtesy: Shaojie Shen

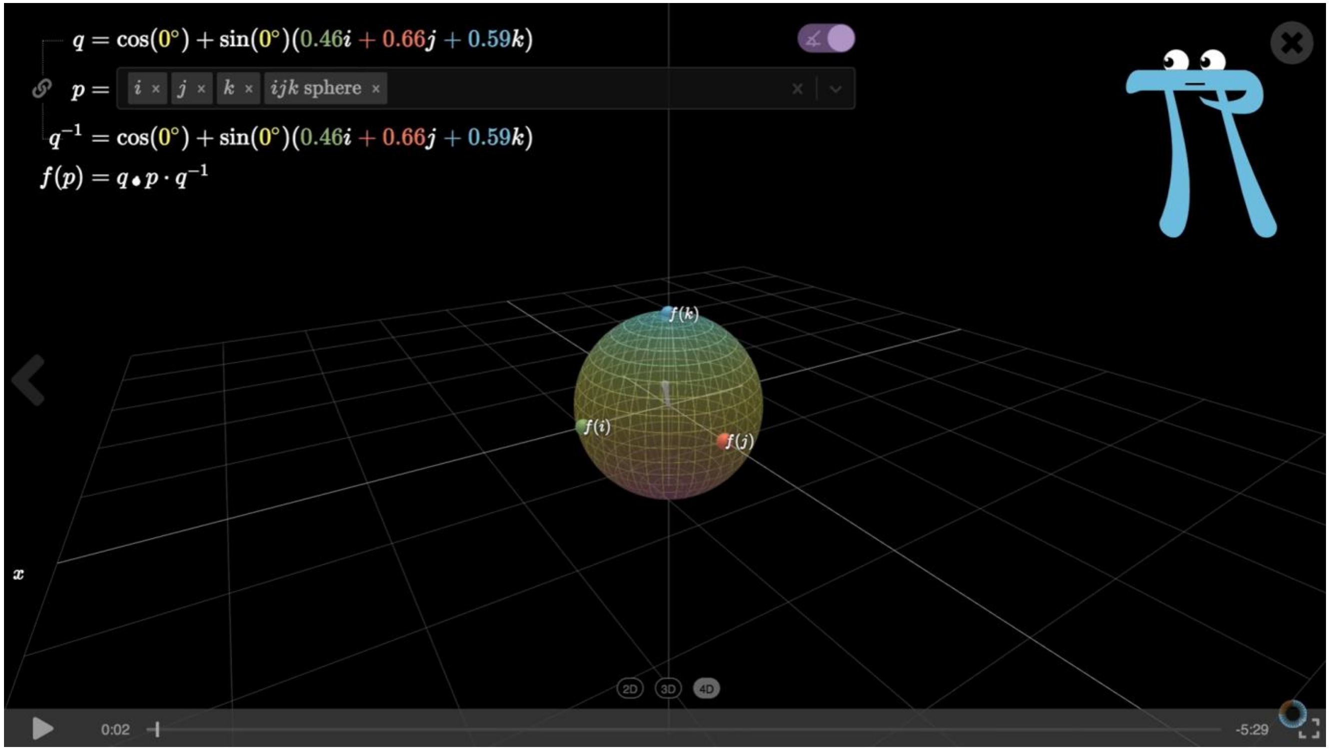
Resources - Books

- Lynch, Kevin M., and Frank C. Park. **Modern robotics.** Cambridge University Press, 2017.
  - Chapter 3.2.1 Rotation Matrices
  - Chapter B.3 Other Representations of Rotations



Resources - Videos

- 3Blue1Brown
  - Visualizing quaternions (4d numbers) with stereographic projection
  - Quaternions and 3d rotation, explained interactively



Courtesy: 3Blue1Brown

Summary

- Rigid Body & Displacement
- Rotation Matrix
- Rigid Body Motion
  - Homogeneous Representation
- Other Represenations
  - Eular Angles

Next Lecture

- ROS
- Mobile Robot Localization
- Locomotion and Kinematics