ELEC 3210
Introduction to Mobile Robotics
Lecture 3

(Machine Learning and Infomation Processing for Robotics)

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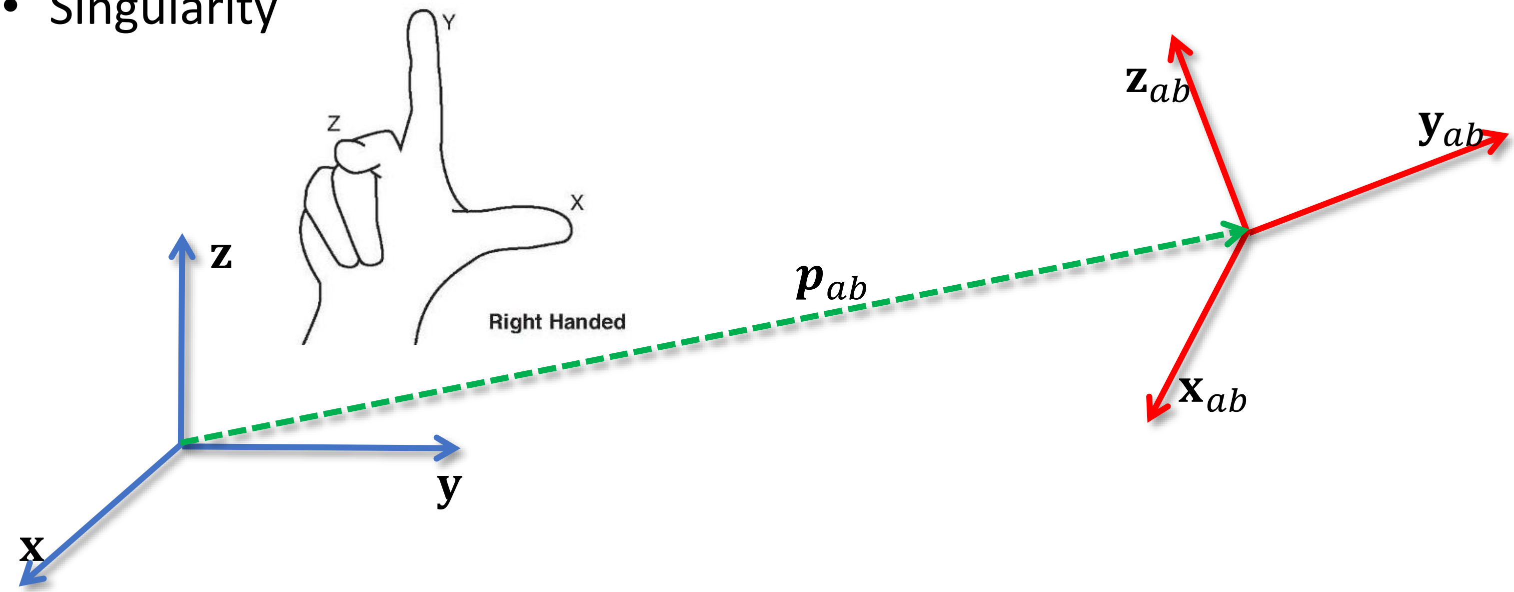
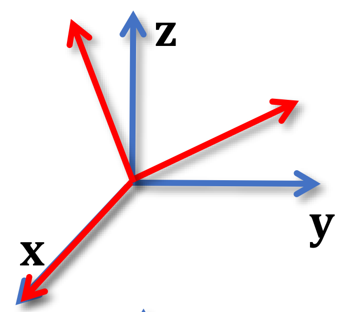
L2

- Rigid Body & Displacement
- Rotation Matrix
- Rigid Body Motion
  - Homogeneous Representation
- Eular Angles
  - Singularity

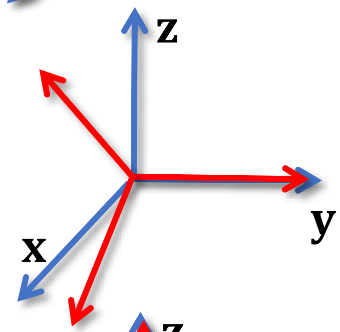


Euler Angles

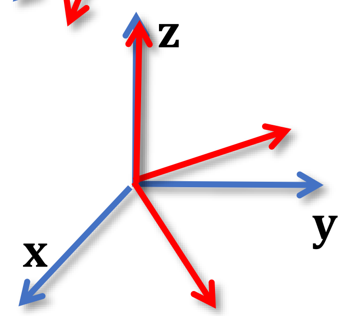
- Elementary rotations:
  - $\boldsymbol{R}_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$



  - $\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$



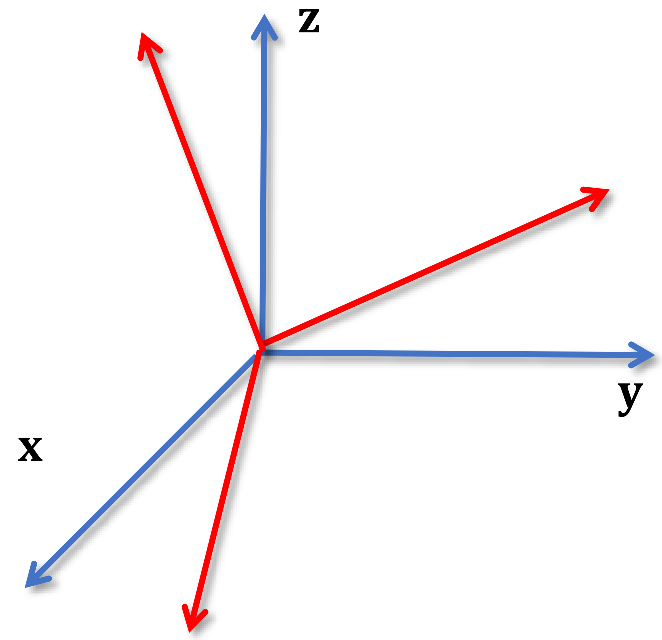
  - $\boldsymbol{R}_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$



Courtesy: Shaojie Shen

Euler Angles

- Any rotation can be described by three successive rotations about linear independent axes
- However, this is an almost 1-1 transform with singularities:
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \Rightarrow \boldsymbol{R}$
  - $\boldsymbol{R}_z(\psi) \cdot \boldsymbol{R}_x(\phi) \cdot \boldsymbol{R}_y(\theta) \nLeftarrow \boldsymbol{R}$



Courtesy: Shaojie Shen

L3/4/5/6 (Tentative)

- Sept 11 - L3
  - Robot Localization
  - Wheels and Kinematics
- Sept 13 - L4
  - Sensors
- Sept 18 - L5
  - Iterative Closest Point, Project 1 Release
- Sept 20 - L6
  - Robot Operating System (ROS)
  - Map Representations

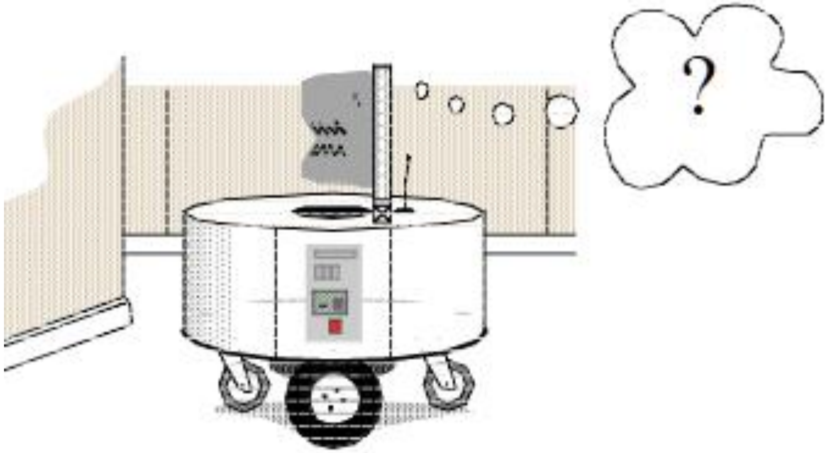
Today’s Outline

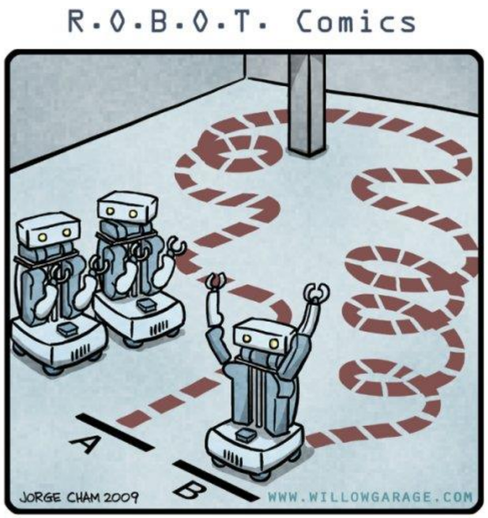
- Robot Localization
  - How to estimate pose
- Wheels and Kinematics
  - Pose estimation via dead reckoning (Forward Kinematics)

Robot Localization

L1 - Three Questions for AMR

- Where am I ? (Sensing/Estimation)
- Where am I going ? (Planning)
- How do I get there ? (Control)

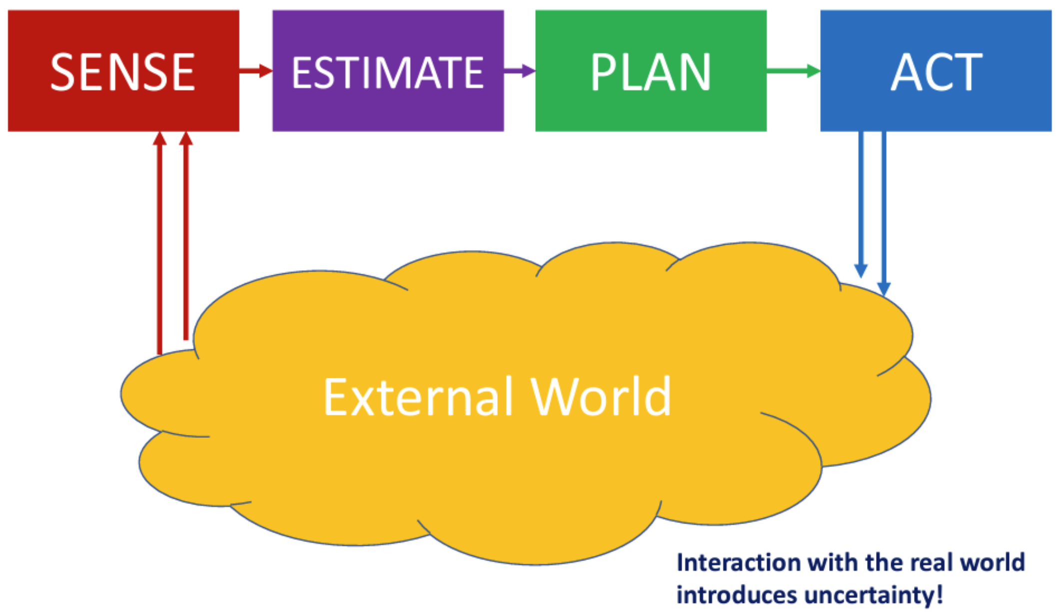




"HIS PATH-PLANNING MAY BE SUB-OPTIMAL, BUT IT'S GOT FLAIR."

L1 - Robot Navigation Paradigm

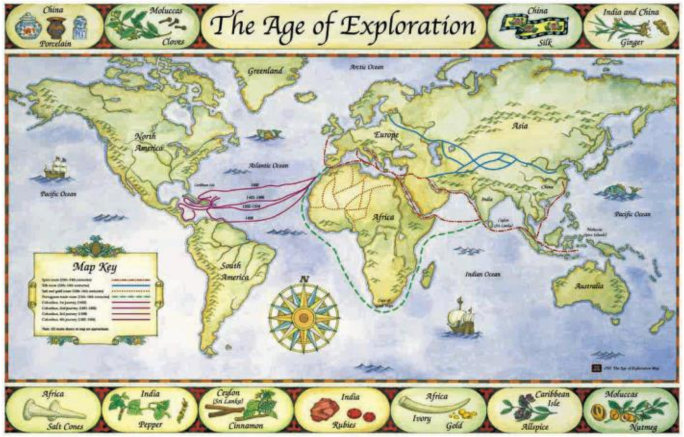
- Sensing&Estimation - Estimate current and past robot pose
- Planning - Generate future robot pose
- Control - Stabilize robot pose



Courtesy: Nikolay Atanasov, PyPose Video

Where am I?

- A question with long history
- Solutions
  - Odometry and Dead Reckoning
  - Simultaneous localization and mapping (SLAM)
  - Map-based localization
  - External infrastructures, like GPS, WiFi

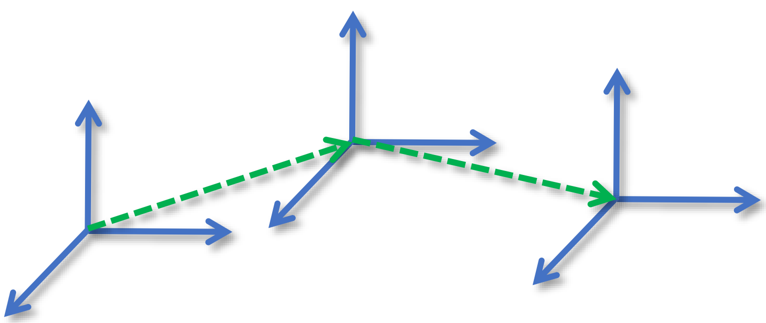


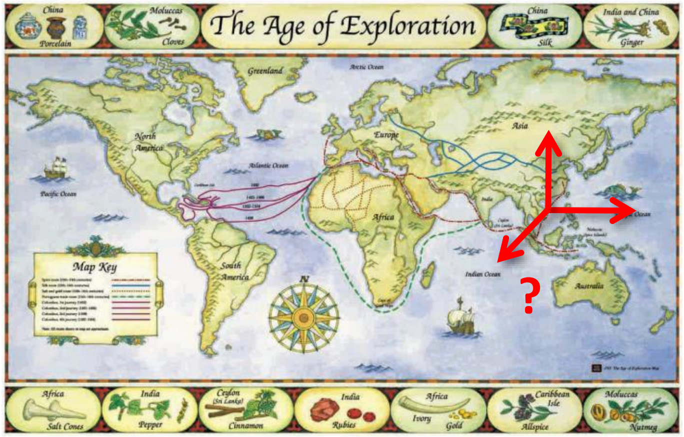
Age of Discovery

Problems

- Pose tracking
  - the initial robot pose is known
  - the pose distribution is bounded, local precision for evaluation
- Global localization (GL)
  - estimate the pose without initial pose
  - with uniform distribution
  - **Kidnapped robot problem:** a variant of the GL problem

  - the robot might believe it knows where it is while it does not

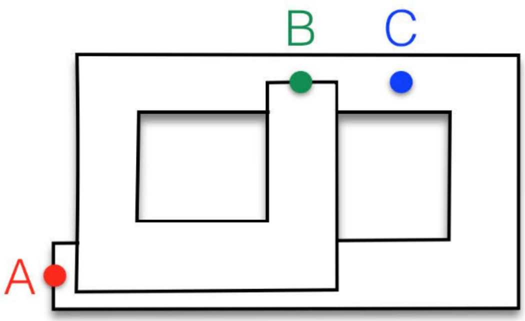




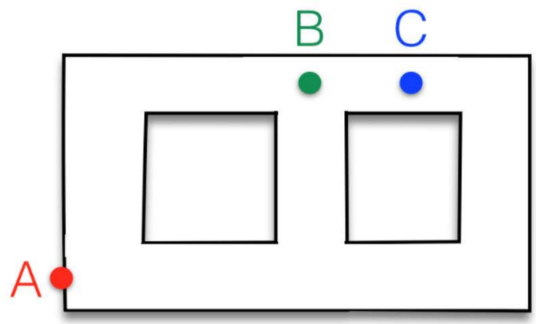
Courtesy: Probabilistic Robotics Chap. 7.2

Solutions

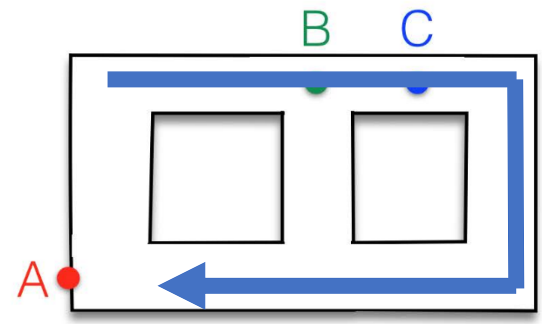
- **Odometry**
  - Wheel Odometry
  - Visual Odometry
  - LiDAR Odometry
  - etc



- **SLAM**
  - Simultaneous localization and mapping

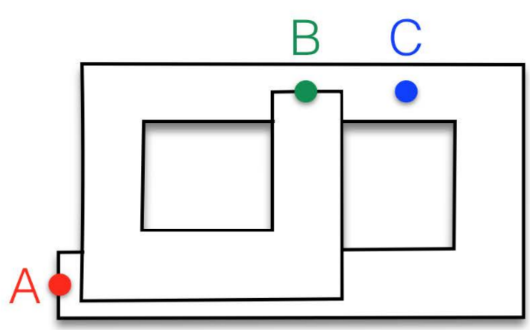


- **Map-based Localization**
  - Localize on a given map



Odometry

- Pose Estimation only
- Pros
  - low computational complexity
  - high frequency
- Cons
  - drift occurs
  - only for pose tracking and no map (may with small-size submap)



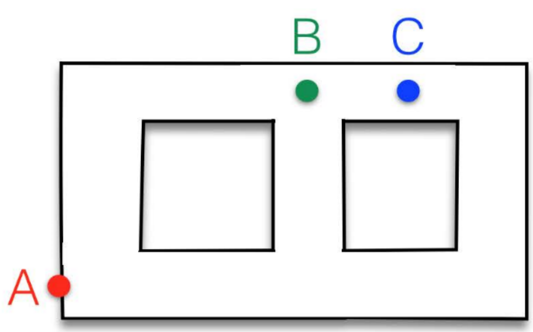
Odometry

- Visual Odometry

Courtesy: WillowGaragevideo

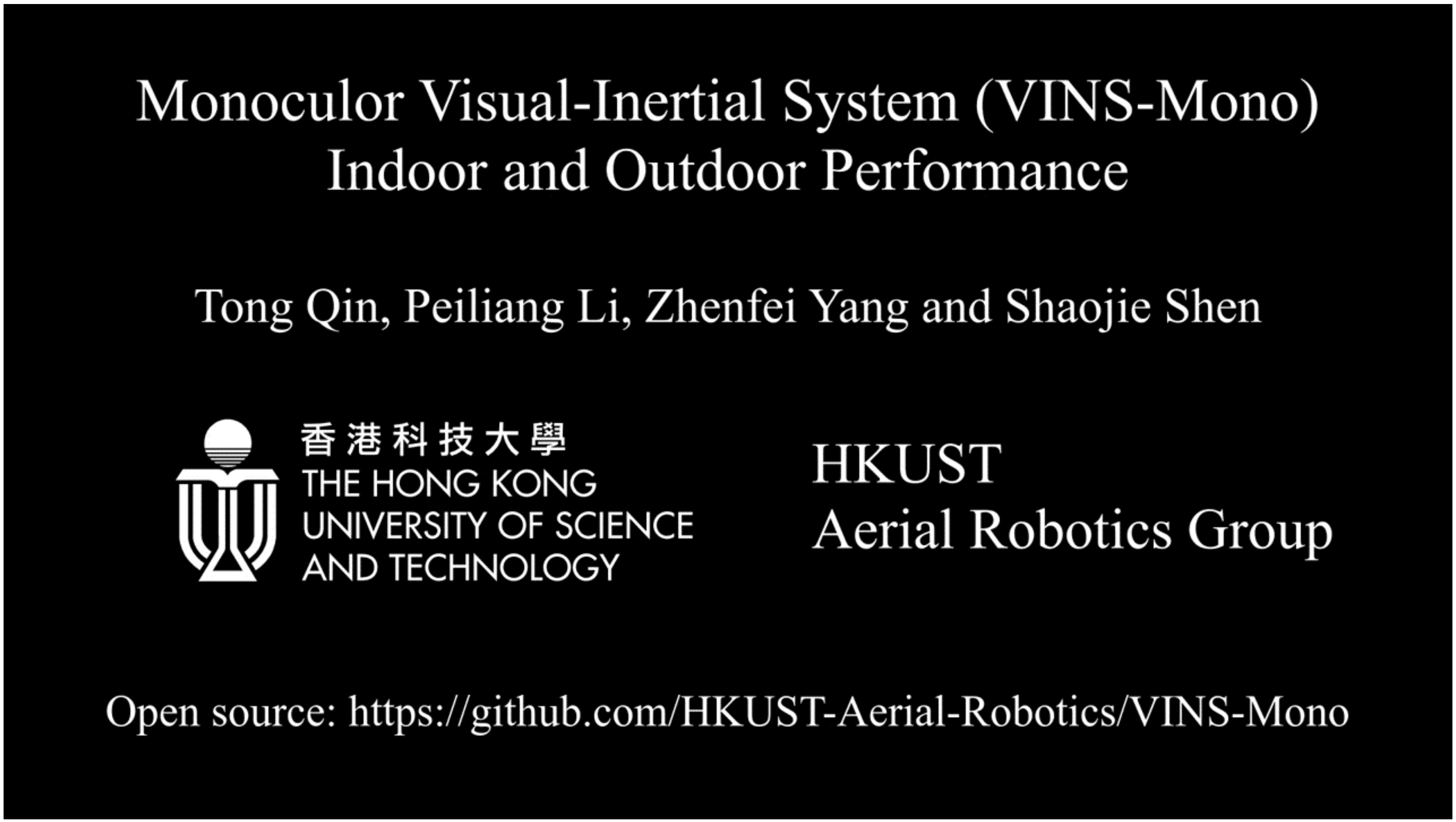
SLAM (Holy Grail of Mobile Robotics)

- Pose Estimation + Mapping
- Pros
  - low drift
  - map built with SLAM
- Cons
  - with higher complexity
  - map redundancy for long-term use
  - no global localization



SLAM

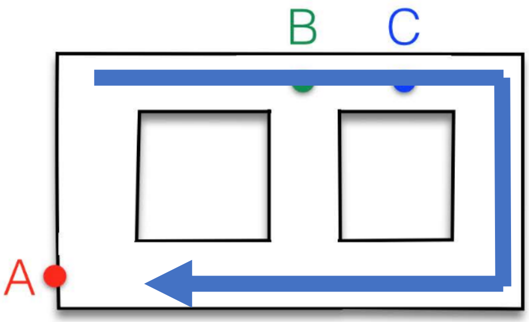
- Visual SLAM



Courtesy: HKUST UAV Group

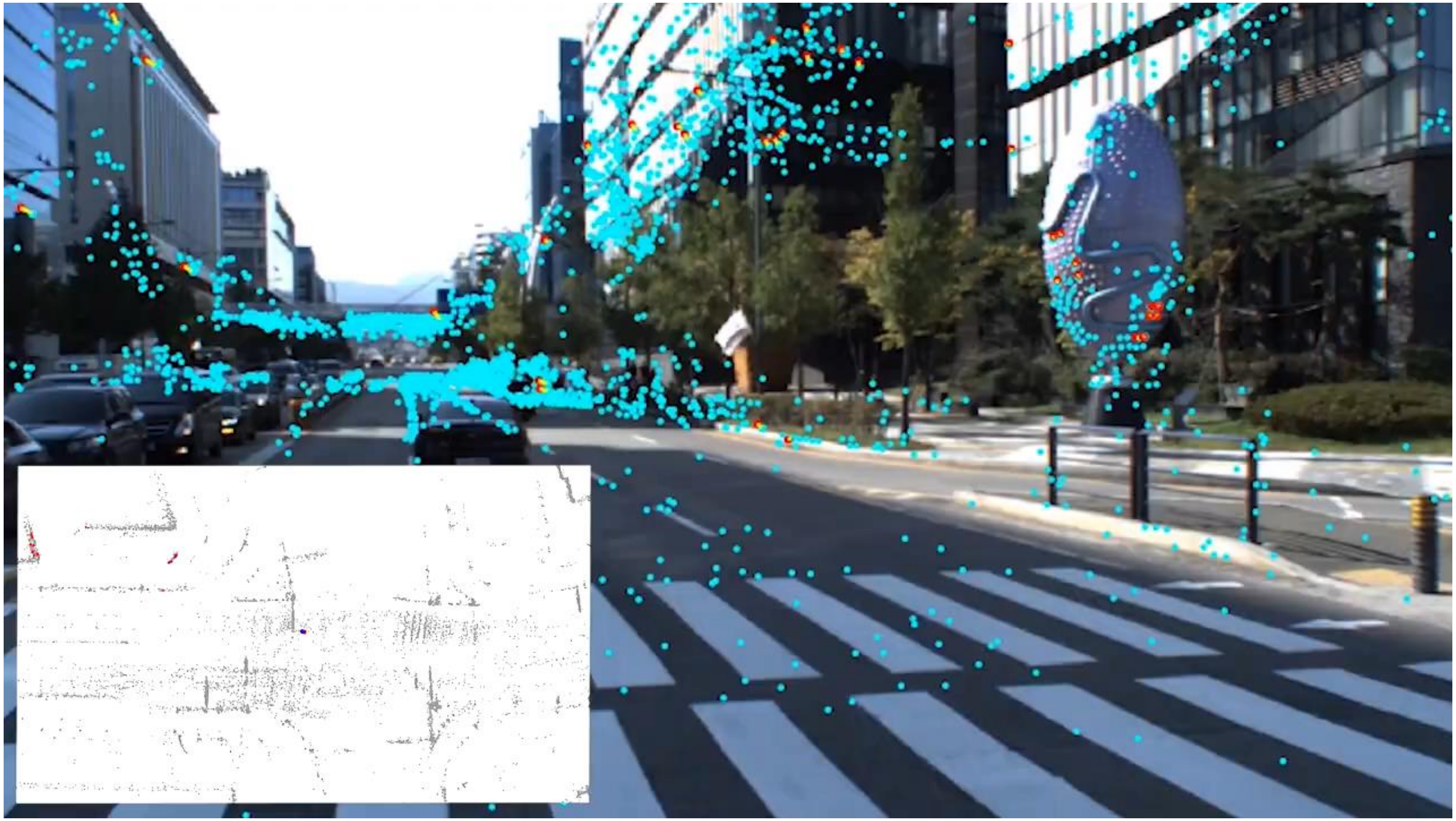
Map-based Localization

- Pose only
- Pros
  - lower drift with a good map
  - suitable for long-term use
  - with global localization capability
  - less computation complexity
- Cons
  - need a good map



Map-based Localization

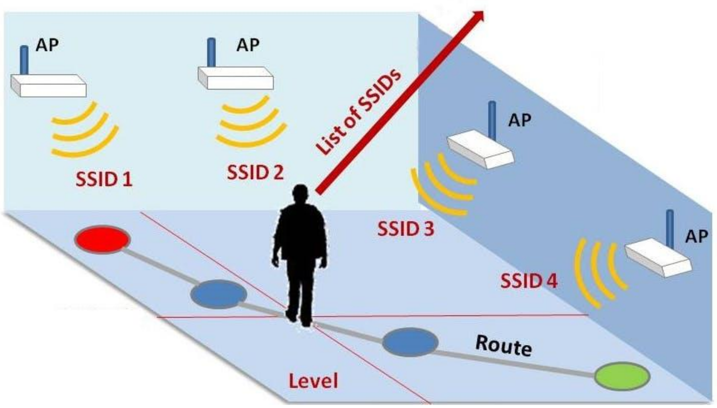
- HD Map-based visual localization

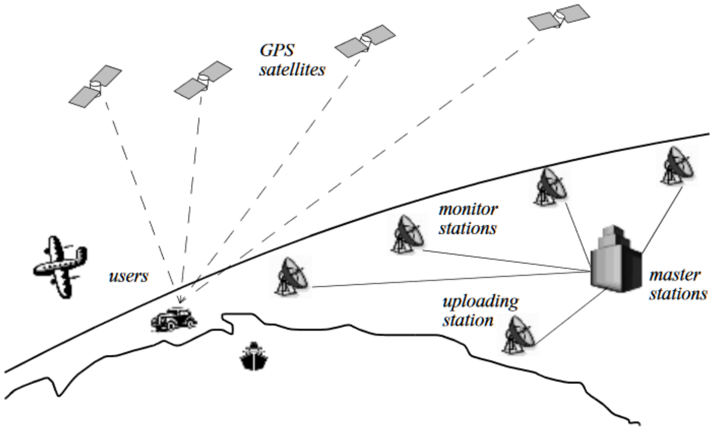


Courtesy: Naver Labs YouTube

External Infrastructures

- WiFi-based, or GPS-based Localization
- GPS is a kind of Global navigation satellite system (GNSS) (Next Lectures)
- Others





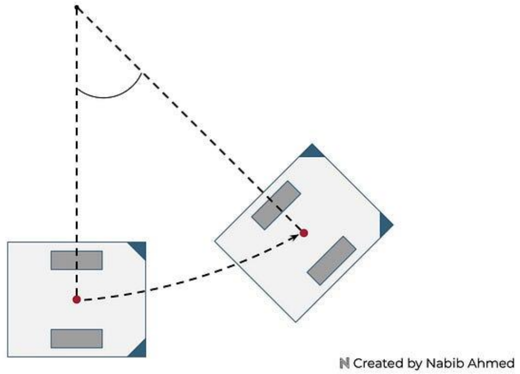
Robot Localization (ELEC 3210)

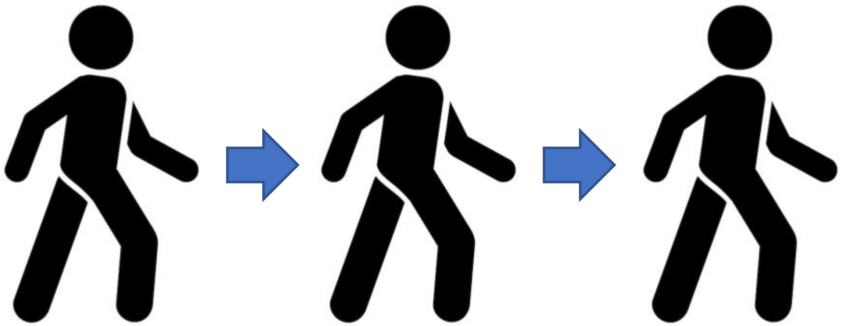
- Two problems
  - Pose tracking
  - Global localization
- Three solutions
  - Odometry
  - SLAM
  - Map-based localization

Courtesy: Huan Yin

Odometry

- If we know the wheeled odometry
  - No sensings, Just “walk”

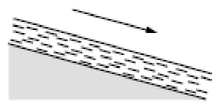
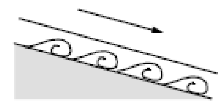
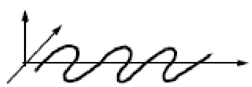
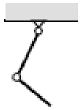
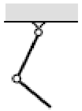
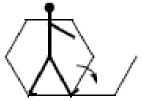




Wheeled Locomotion

Robot Locomotion

- Locomotion: the act or ability of something to transport or move itself from place to place

Type of motion	Resistance to motion	Basic kinematics of motion
Flow in a Channel 	Hydrodynamic forces	Eddies 
Crawl 	Friction forces	 Longitudinal vibration
Sliding 	Friction forces	 Transverse vibration
Running 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Jumping 	Loss of kinetic energy	Oscillatory movement of a multi-link pendulum 
Walking 	Gravitational forces	Rolling of a polygon (see figure 2.2) 

Courtesy: Introduction to Autonomous Mobile Robots

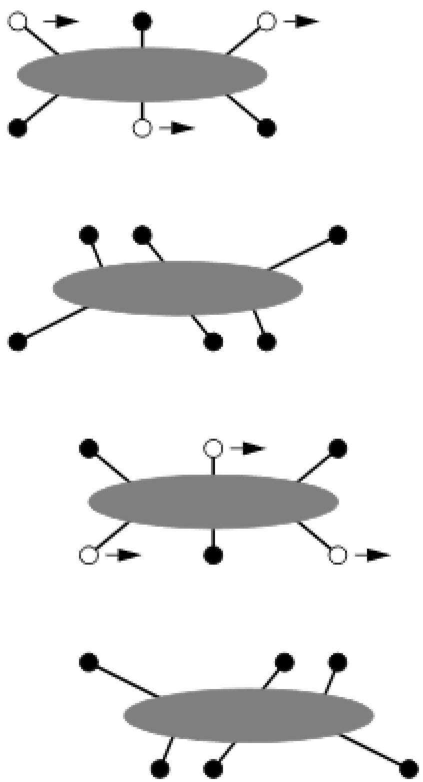
Key Issues of Locomotion

- Stability
  - number and geometry of contact points
  - center of gravity
  - static/dynamic stability
  - inclination of terrain
- Characteristics of contact
  - contact point/path size and shape
  - angle of contact
  - friction
- Type of environment
  - structure
  - medium, (e.g. water, air, soft or hard ground)

Courtesy: Introduction to Autonomous Mobile Robots

Legged Robot

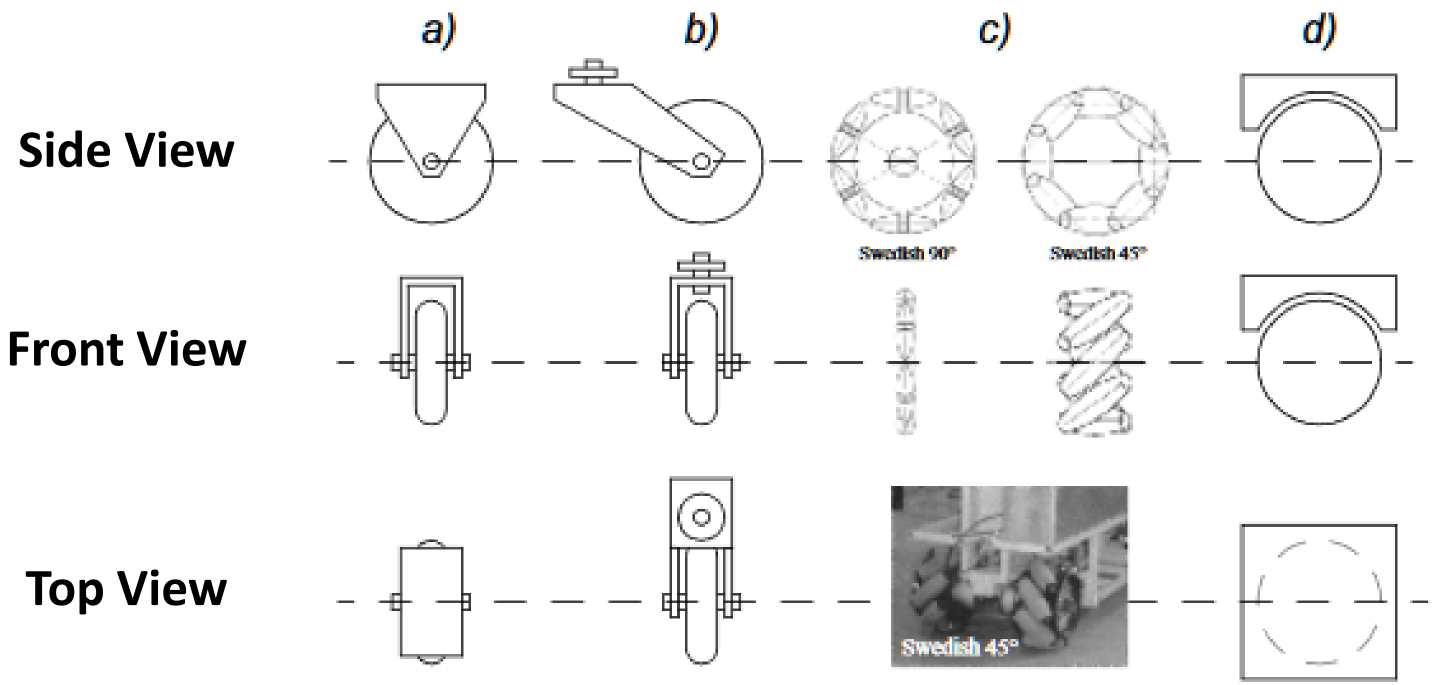
- Static walking with six legs



Courtesy: Introduction to Autonomous Mobile Robots

Wheeled Mobile Robot

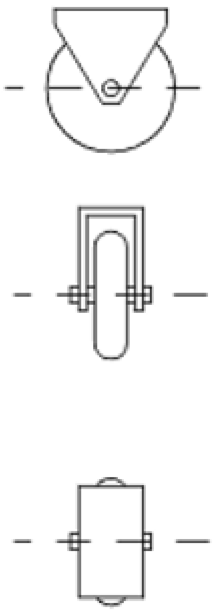
- With different types of wheels
- (a) Standard wheel (b) Castor wheel (c) Swedish wheel (d) Ball or spherical wheel



Courtesy: Introduction to Autonomous Mobile Robots

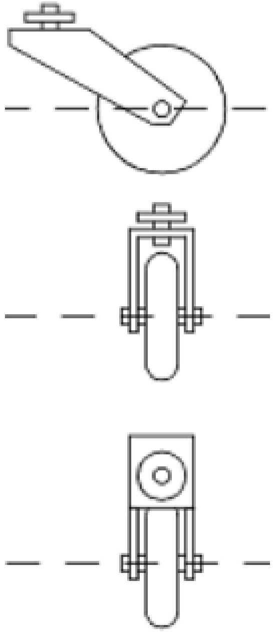
Standard Wheel

- two degrees of freedom
- rotation around the (motorized) wheel axle and the contact point



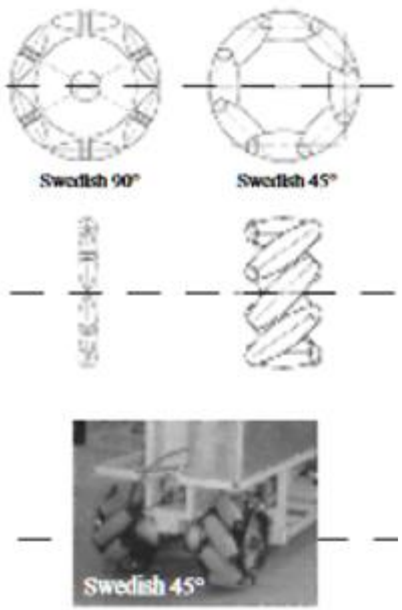
Castor Wheel

- two degrees of freedom
- rotation around an offset steering joint



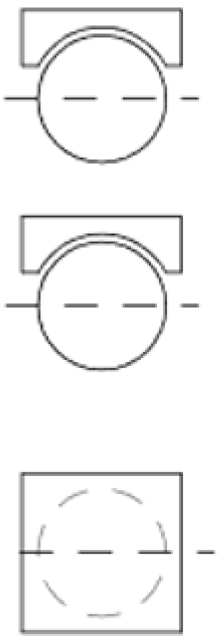
Swedish Wheel

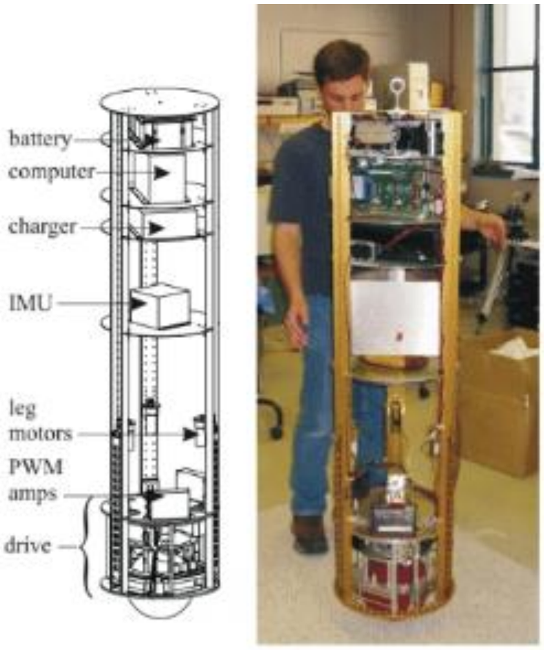
- three degrees of freedom
- rotation around the (motorized) wheel axle, around the rollers, and around the contact point



Ball Wheel

- Real omni wheel
- realization technically difficult





Wheel Geometry

- Chassis Design
- Choices
  - number of wheels
  - type of wheels
  - wheels arrangement
- Three wheels are sufficient to guarantee stable balance

Courtesy: Introduction to Autonomous Mobile Robots

Single Ball Wheel

- Ballbot : A Single-Wheeled Balancing Robot
  - Dynamically balanced

Courtesy: CMU Robotics Institute

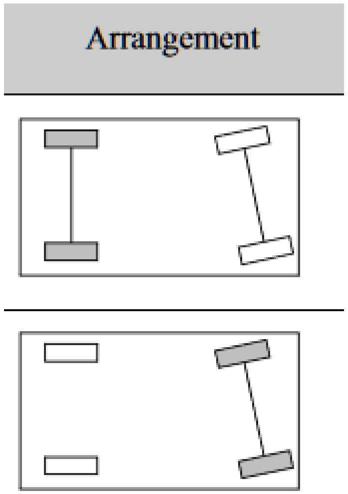
Two Standard Wheels

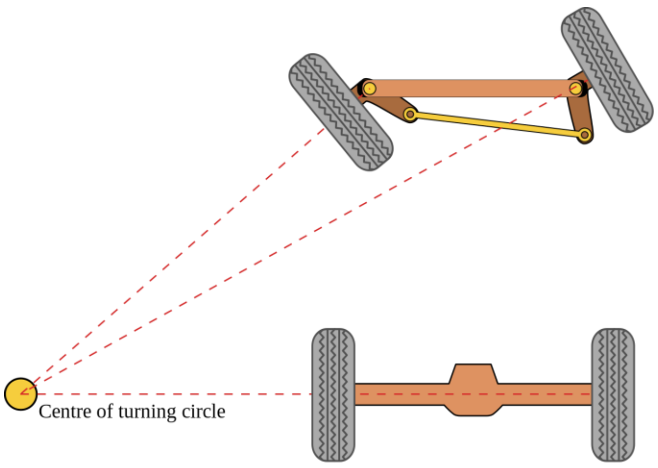
- Need carefully design and tune the control part

Courtesy: Direct Drive Tech

Four standard wheels

- Car with front/rear-wheel drive
- Ackermann steering geometry
- Widely used in the world





Courtesy:

More wheels

- For specific tasks
  - harbor vehicles
  - mars rover
  - rescure robot

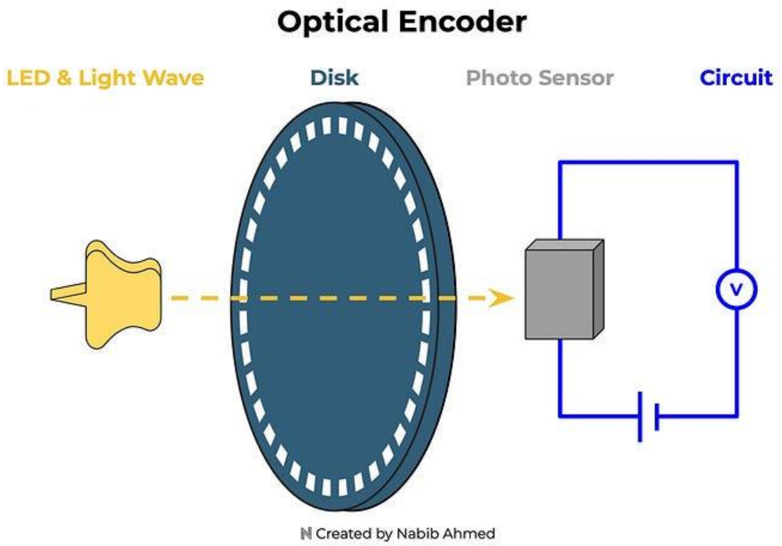
Courtesy: YouTube

Design Space

- Stability
  - can be further improved by adding more wheels
- Controllability
  - the ability to control the behavior of a system
- Maneuverability
  - the quality of being easy to move and direct

From wheels to odometry

- Spinning speed of each wheel is known
  - by control command / voltage etc.
  - by wheel encoder (sensor-based)
- How to estimate the pose?
  - Kinematics



Mobile Robot Kinematics

Kinematics

- Robot kinematics applies geometry to the study of the movement of multi-degree of freedom kinematic chains
  - Manipulator robots are much more complex due to its chain of links
  - mathematical framework for describing and understanding the robot motion

RoboCoaster off-ride

PR2 Robot Fetch Beer

Courtesy: Wikipedia and YouTube

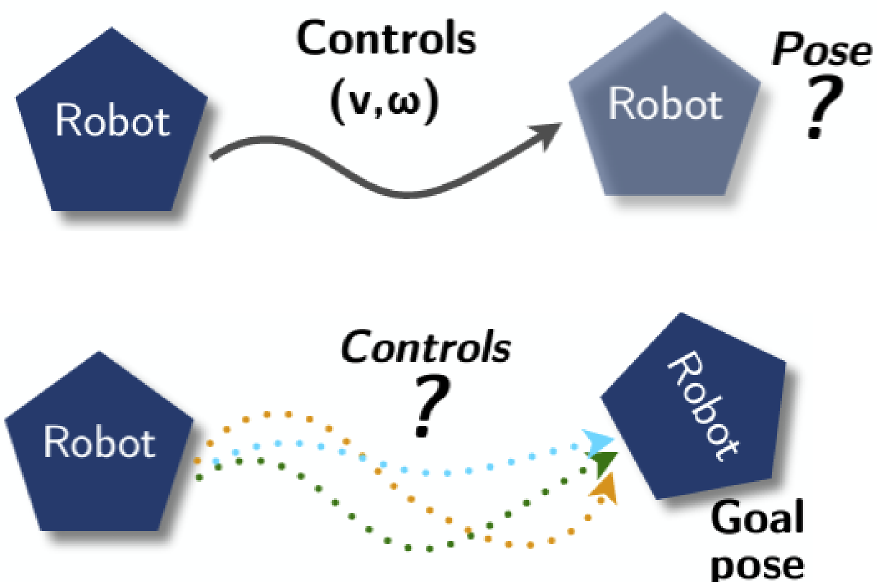
Kinematics and Dynamics

- Kinematics
  - study of motion without regard to forces
  - Geometry
  - Pose
  - Velocity
  - Acceleration
- Dynamics
  - study of motions that result from forces
  - Force
  - Mechanism
  - Acceleration
  - Control

Kinodynamic

Forward vs Inverse Kinematics

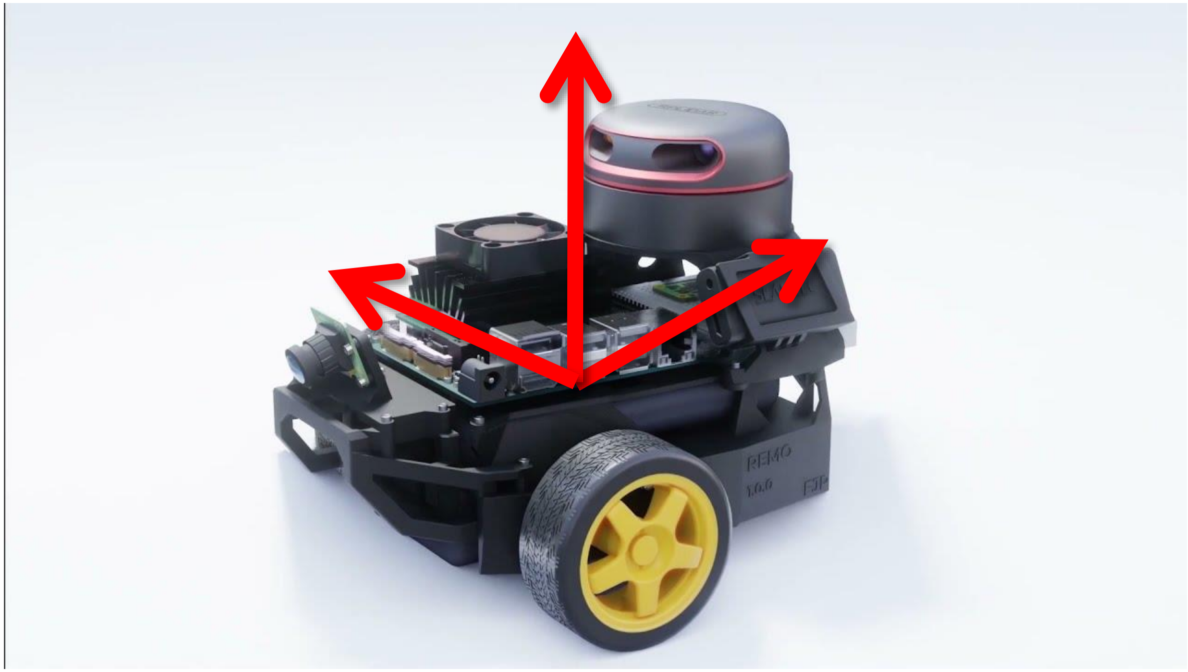
- **Forward and Inverse**
  - Use kinematic equations to determine/predict the final configuration/pose of a robot based on the specific values for the controls
  - Given the desired final configuration (of the effectors/pose), make use of kinematic quations to determine values of the controls that allow to achieve it



Courtesy: Gianni A. Di Caro

FK of Wheeled Mobile Robot

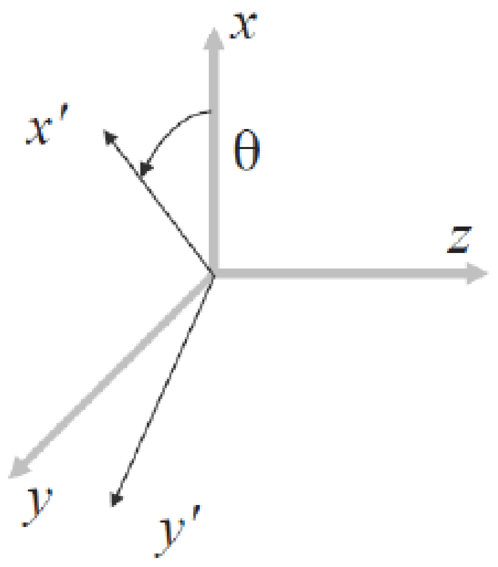
- Start with a two wheel differential drive robot
- A castor/ball wheel is optional



Assumptions

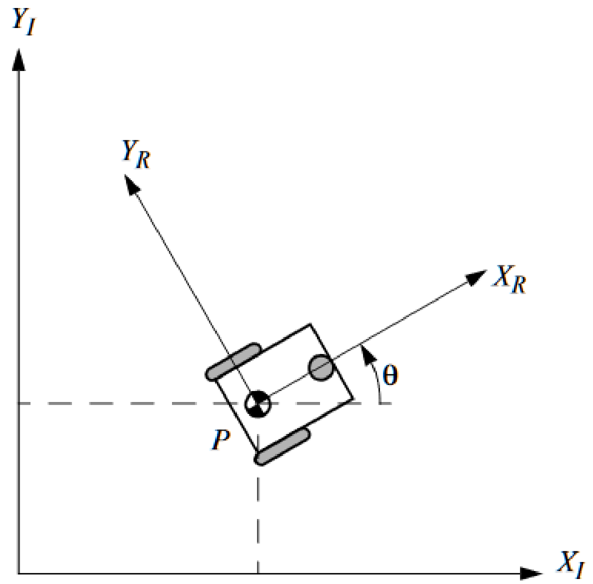
- Rigid motion, ignore the joints inside
  - Can represent the robot with a single point
- Planar motion
  - 3-DoF pose representation
  - L2 Pose and Rotation

$$\mathbf{R}_z(\theta) = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



World-Robot Frame

- Global/World Frame (I)
- Robot Frame (R)
- Motion Transformation



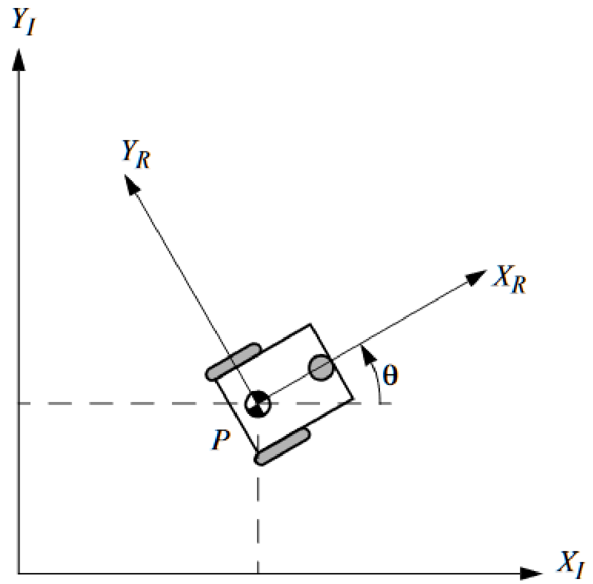
$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{pmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{pmatrix} = R(\theta) \begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{pmatrix}$$

$$\dot{\mathbf{X}}_R = R(\theta)\dot{\mathbf{X}}_I$$

Problem Formulation

- Given
  - wheel diameter
  - distance between wheel and the center
  - spinning speed of wheels
- Robot velocity at the global frame?

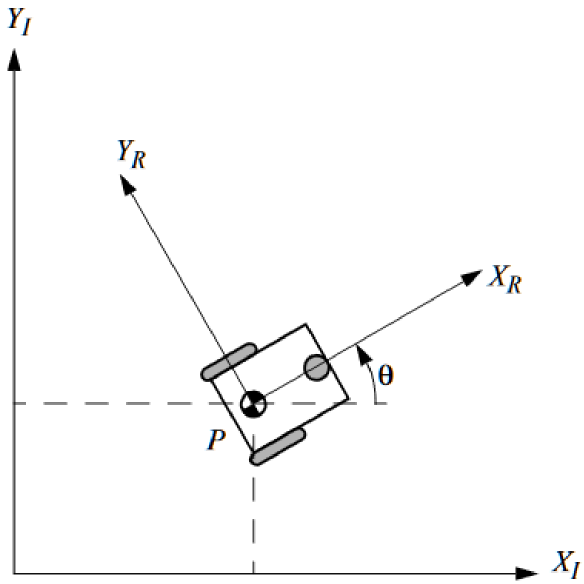


$$\dot{X}_I = \begin{pmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{pmatrix} = f\left(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2\right)$$

$$\dot{\mathbf{X}}_R = R(\theta)\dot{\mathbf{X}}_I$$

Translation Velocity

- Consider the contribution of each wheel's spinning speed to the translation speed at the robot center
- No translation velocity on Yr



- On Xr
  - one spins, one stays (stationary)

$$\dot{x}_R = (1/2) r \dot{\varphi}_1$$

$$\dot{x}_R = (1/2) r \dot{\varphi}_2$$

  - both

$$\dot{x}_R = r\dot{\varphi}_1/2 + r\dot{\varphi}_2/2$$

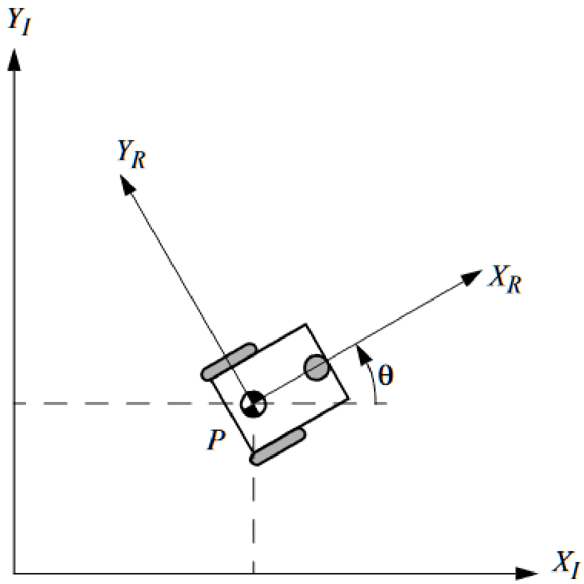
Rotation Velocity

- Also, consider the contribution of each wheel's spinning speed
- Right wheel spins, Left wheel stays (counter-clockwise)

$$\omega_1 = \frac{r\dot{\varphi}_1}{2l}$$

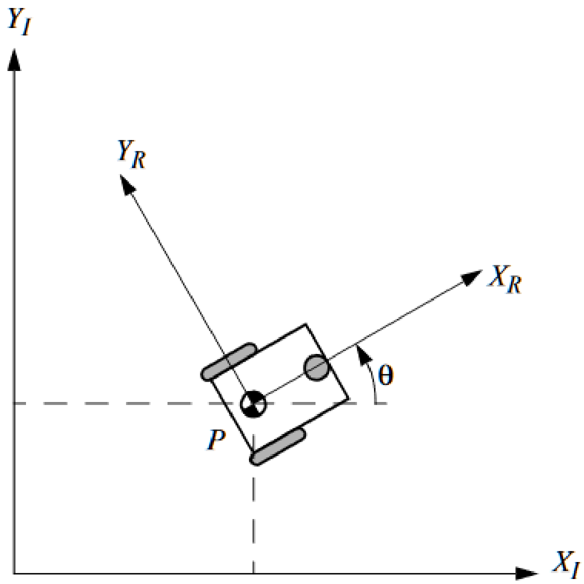
- Left wheel spins, Right wheel stays (clockwise)

$$\omega_2 = -\frac{r\dot{\varphi}_2}{2l}$$



Forward Kinematics

- Combining these individual formulas yields a kinematic model



$$\dot{\mathbf{X}}_I = R(\theta)^{-1}\dot{\mathbf{X}}_R$$

$$= R(\theta)^{-1}\begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

Another method

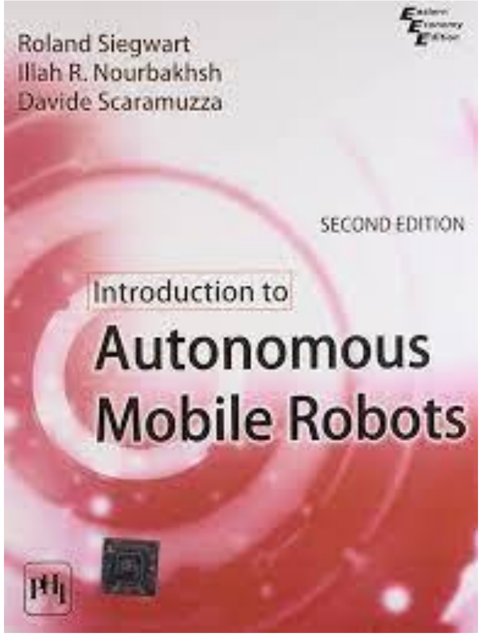
- By contribution of each wheel
- By constraince?
  - Chapter 3.2.5, Introduction to autonomous mobile robots

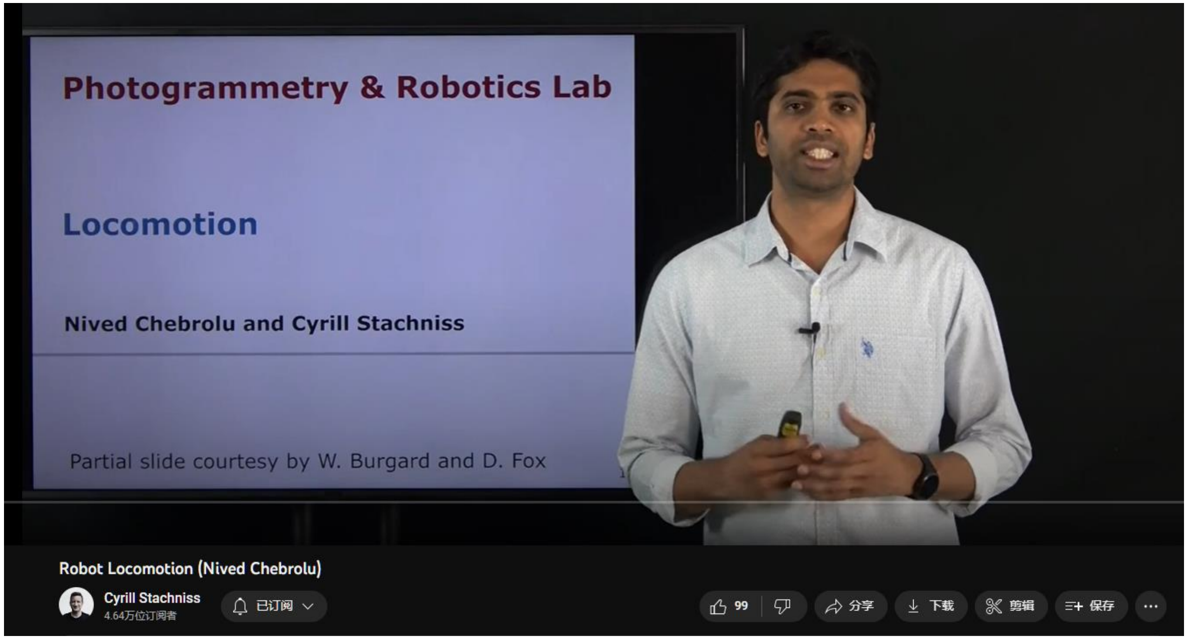
3.2.5 Examples: robot kinematic models and constraints

In section 3.2.2 we presented a forward kinematic solution for $\dot{\xi}_I$ in the case of a simple differential-drive robot by combining each wheel's contribution to robot motion. We can now use the tools presented above to construct the same kinematic expression by direct application of the rolling constraints for every wheel type. We proceed with this technique applied again to the differential drive robot, enabling verification of the method as compared to the results of section 3.2.2. Then we proceed to the case of the three-wheeled omnidirectional robot.

Resources

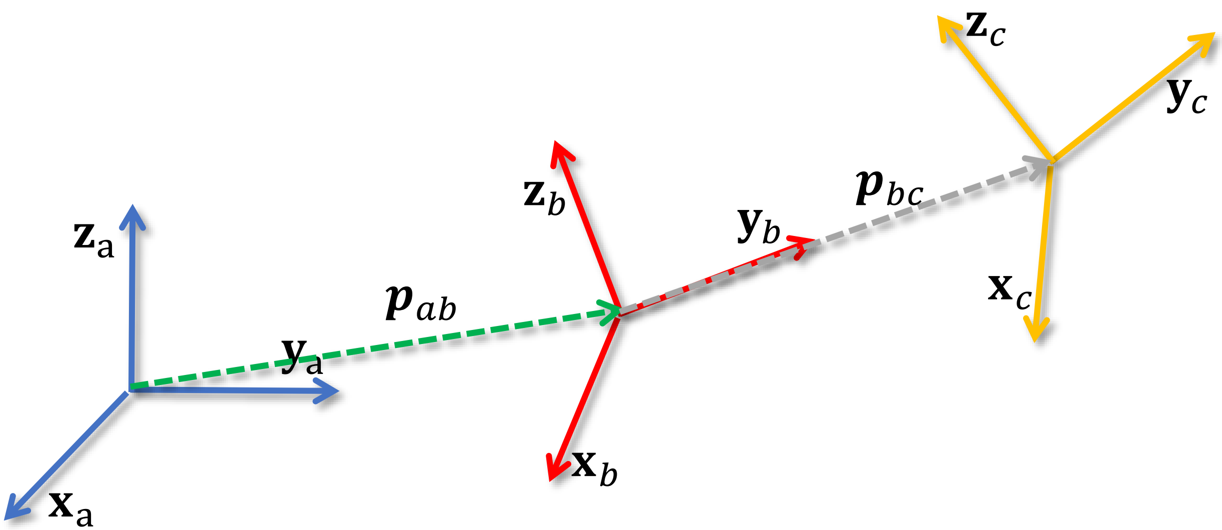
- **Chapter 2 and Chapter 3**. Introduction to autonomous mobile robots. MIT press, 2011.
- Prof Cyrill Stachniss's Lecture on **Locomotion**
  - Kinematics of car model (Ackermann)





L2 - Rigid Body Motion

- If robot velocity is known
- Homogeneous representation of rigid body motion:
  - $\bar{g}_{ab} = \begin{bmatrix} \boldsymbol{R}_{ab} & \boldsymbol{p}_{ab} \\ 0 & 1 \end{bmatrix}$
- Composition rule for rigid body motions:
  - $\bar{g}_{ac} = \bar{g}_{ab} \cdot \bar{g}_{bc} = \begin{bmatrix} \boldsymbol{R}_{ab}\boldsymbol{R}_{bc} & \boldsymbol{R}_{ab}\boldsymbol{p}_{bc} + \boldsymbol{p}_{ab} \\ 0 & 1 \end{bmatrix}$



Drift of wheel odometry

- Drift occurs using wheel odometry
  - wheel slip etc.
  - uncertainties in the external world
  - we need robot perception and mapping

Next Lecture

- Sensors