1 Likilihood

Where Ω are the movies seen by the individual
Where j is the movie
Where T are the number of individuals
Where i is a hidden variable

$$\begin{aligned} P(\{R_j = r_j\}_{j\in\Omega}) &= \sum_{i=1}^{k} P(Z = i, \{R_j = r_j\}_{j\in\Omega}) \\ &= \sum_{i=1}^{k} P(\{R_j = r_j\}_{j\in\Omega} | Z = i) P(Z = i) \\ &= \sum_{i=1}^{k} P(Z = i) \prod_{j\in\Omega} P(\{R_j = r_j\}_{j\in\Omega} | Z = i) \end{aligned}$$

2 Expectation (E-Step)

$$\begin{aligned} P(Z = i | \{R_j = r_j\}_{j\in\Omega}) &= \frac{P(Z = i) P(\{R_j = r_j\}_{j\in\Omega} | Z = i)}{P(\{R_j = r_j\}_{j\in\Omega})} \\ &= \frac{P(Z = i) \prod_{j\in\Omega} P(\{R_j = r_j\}_{j\in\Omega} | Z = i)}{\sum_{i'=1}^{k} P(Z = i') \prod_{j\in\Omega} P(\{R_j = r_j\}_{j\in\Omega} | Z = i')} \end{aligned}$$

3 Maximization (M-Step)

3.1 Hidden Variable Updates

$$P(Z = i) \leftarrow \frac{1}{T} \sum_{t=1}^{T} P(\{R_j = r_j^t\}_{j\in\Omega}$$

3.2 Observed Rating Probability Updates

$$P(R_l = 1|Z = i) \leftarrow \frac{\sum_t P(Z = i, R_l = 1|\{R_j = r_j\}_{j\in\Omega})}{\sum_{t=1}^{T} P(\{R_j = r_j^t\}_{j\in\Omega}}$$

$$\begin{aligned} P(Z = i, R_l = 1|\{R_j = r_j\}_{j\in\Omega}) =& I(R_l, 1)P(Z = i, R_l = 1|\{R_j = r_j\}_{j\in\Omega}) \\ =& I(R_l, 1)P(Z = i|\{R_j = r_j\}_{j\in\Omega})P(Z = i|R_l = 1, \{R_j = r_j\}_{j\in\Omega}) \\ =& I(R_l, 1)P(Z = i|\{R_j = r_j\}_{j\in\Omega})P(R_l = 1|Z = i) \end{aligned}$$

$$\begin{aligned} \rho_{it} =& P(Z = i|\{R_j = r_j\}_{j\in\Omega}) \\ P(R_l = 1|Z = i) \leftarrow& \frac{\sum_{t|l\in\Omega} I(R_l, 1)\rho_{it} + \sum_{t|l\notin\Omega} P(R_l = 1|Z = i)\rho_{it}}{\sum_{t=1}^{T} P(\{R_j = r_j^t\}_{j\in\Omega}} \end{aligned}$$