Coordinates

$Ag: \quad (0,0,0), (0,\frac{1}{2},\frac{1}{4}), (\frac{1}{2},0,\frac{3}{4}), (\frac{1}{2},\frac{1}{2},\frac{1}{2})$

$Al: \quad (\frac{1}{2},\frac{1}{2},0), (\frac{1}{2},0,\frac{1}{4}), (0,0,\frac{1}{2}), (0,\frac{1}{2},\frac{3}{4})$

$Te: \quad (\frac{1}{4},\frac{1}{4},\frac{1}{8}), (\frac{3}{4},\frac{3}{4},\frac{1}{8}), (\frac{3}{4},\frac{1}{4},\frac{3}{8}), (\frac{1}{4},\frac{3}{4},\frac{3}{8}), (\frac{3}{4},\frac{3}{4},\frac{5}{8}), (\frac{1}{4},\frac{1}{4},\frac{5}{8}),$
$\qquad (\frac{3}{4},\frac{1}{4},\frac{7}{8}), (\frac{1}{4},\frac{3}{4},\frac{7}{8})$

$Reflections: \quad (0,0,2), (0,1,1), (1,1,0), (1,1,2), (0,1,3), (0,2,0),$
$\qquad (0,0,4), (0,2,2), (2,1,1), (1,1,4), (1,2,3), (2,2,0),$
$\qquad (0,1,5), (0,2,4), (2,2,2), (0,3,1), (3,1,0), (3,1,2),$
$\qquad (1,2,5), (3,2,1), (1,3,4), (2,3,3), (0,4,0), (0,4,2),$
$\qquad (4,1,1), (3,3,2), (4,1,3), (4,2,0), (2,3,5), (3,3,4),$
$\qquad (1,4,5), (2,4,4), (4,3,1), (0,5,1), (5,1,0), (5,1,2),$
$\qquad (0,0,0)$

$$\begin{aligned}
F_{hkl} = \ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{k}{2}+\frac{l}{4})} + e^{2\pi i(\frac{h}{2}+0+\frac{3\cdot l}{4})} + e^{2\pi i(\frac{h}{2}+\frac{k}{2}+\frac{l}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{h}{2}+\frac{k}{2}+0)} + e^{2\pi i(\frac{h}{2}+0+\frac{l}{4})} + e^{2\pi i(0+0+\frac{l}{2})} + e^{2\pi i(0+\frac{k}{2}+\frac{3\cdot l}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{h}{4}+\frac{k}{4}+\frac{l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{3\cdot k}{4}+\frac{l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{k}{4}+\frac{3\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{3\cdot k}{4}+\frac{3\cdot l}{8})}\right. \\
& \qquad \left. + e^{2\pi i(\frac{3\cdot h}{4}+\frac{3\cdot k}{4}+\frac{5\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{k}{4}+\frac{5\cdot l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{k}{4}+\frac{7\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{3\cdot k}{4}+\frac{7\cdot l}{8})}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{k}{2}+\frac{l}{4})} + e^{2\pi i(\frac{h}{2}+\frac{3\cdot l}{4})} + e^{2\pi i(\frac{h}{2}+\frac{k}{2}+\frac{l}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{h}{2}+\frac{k}{2})} + e^{2\pi i(\frac{h}{2}+\frac{l}{4})} + e^{2\pi i(\frac{l}{2})} + e^{2\pi i(\frac{k}{2}+\frac{3\cdot l}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{h}{4}+\frac{k}{4}+\frac{l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{3\cdot k}{4}+\frac{l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{k}{4}+\frac{3\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{3\cdot k}{4}+\frac{3\cdot l}{8})}\right. \\
& \qquad \left. + e^{2\pi i(\frac{3\cdot h}{4}+\frac{3\cdot k}{4}+\frac{5\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{k}{4}+\frac{5\cdot l}{8})} + e^{2\pi i(\frac{3\cdot h}{4}+\frac{k}{4}+\frac{7\cdot l}{8})} + e^{2\pi i(\frac{h}{4}+\frac{3\cdot k}{4}+\frac{7\cdot l}{8})}\right)
\end{aligned}$$

$$\begin{aligned}
F_{002} = {} & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{0}{2}+\frac{2}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{2}{4})} + e^{2\pi i(0+0+\frac{2}{2})} + e^{2\pi i(0+\frac{0}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot2}{8})}\right) \\
= {} & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{0}{2}+\frac{2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2})} + e^{2\pi i(\frac{0}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot2}{8})}\right) \\
= {} & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(1)}\right) \\
& + f_{Al}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(1)} + e^{2\pi i(\frac{3}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4})} + e^{2\pi i(\frac{1}{4})} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(\frac{3}{4})}\right. \\
& \left. + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{7}{4})}\right) \\
= {} & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(1i + 1i + -1i + -1i + 1i + 1i + -1i + -1i) \\
= {} & 0 \, (Forbidden Reflection)
\end{aligned}$$

2

$$F_{011} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{1}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{1}{4})} + e^{2\pi i(0+0+\frac{1}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{1}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{1}{2})} + e^{2\pi i(\frac{0}{2}+\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(1)}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{5}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{3}{8})} + e^{2\pi i(\frac{7}{8})} + e^{2\pi i(\frac{5}{8})} + e^{2\pi i(\frac{9}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{7}{8})} + e^{2\pi i(\frac{9}{8})} + e^{2\pi i(\frac{1}{3}8)}\right)$$

$$= f_{Ag}(1 + -1i + -1i + 1)$$

$$+ f_{Al}(-1 + 1i + -1 + 1i)$$

$$+ f_{Te}\big([-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$

$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$

$$+ [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$

$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]\big)$$

$$= (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}$$

$$
\begin{aligned}
F_{110} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{0}{4})} + e^{2\pi i(\frac{1}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{1}{2}+\frac{1}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{1}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8})}\right. \\
& \quad\ \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{0}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{1}{2}+\frac{1}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{1}{2})} + e^{2\pi i(\frac{1}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8})}\right. \\
& \quad\ \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(1)}\right) \\
& + f_{Al}\left(e^{2\pi i(1)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(1)} + e^{2\pi i(1)}\right. \\
& \quad\ \left. + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(1)} + e^{2\pi i(1)}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(-1 + -1 + 1 + 1 + -1 + -1 + 1 + 1) \\
=\ & 0\,(Forbidden Reflection)
\end{aligned}
$$

$$
\begin{aligned}
F_{112} = \ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+0+\frac{3\cdot2}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{1}{2}+0+\frac{2}{4}\right)} + e^{2\pi i\left(0+0+\frac{2}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot2}{8}\right)}\right. \\
& \left. \quad + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot2}{8}\right)}\right) \\[4pt]
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot2}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{2}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot2}{8}\right)}\right. \\
& \left. \quad + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot2}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot2}{8}\right)}\right) \\[4pt]
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(1)} + e^{2\pi i(1)} + e^{2\pi i(1)} + e^{2\pi i(2)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{7}{4}\right)}\right. \\
& \left. \quad + e^{2\pi i\left(\frac{1}{1}4\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{1}{1}4\right)} + e^{2\pi i\left(\frac{1}{1}4\right)}\right) \\[4pt]
= \ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(-1i + -1i + -1i + -1i + -1i + -1i + -1i + -1i) \\[4pt]
= \ & 4f_{Ag} + 4f_{Al} - 8if_{Te}
\end{aligned}
$$

$$\begin{aligned}
F_{013} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{3}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{0}{2}+\frac{1}{2}+\frac{3}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{3}{4})} + e^{2\pi i(0+0+\frac{3}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot3}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot3}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot3}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{3}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{0}{2}+\frac{1}{2}+\frac{3}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{1}{2})} + e^{2\pi i(\frac{0}{2}+\frac{3}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot3}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot3}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot3}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{1}4)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{5}{8})} + e^{2\pi i(\frac{9}{8})} + e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{1}{5}8)}\right. \\
& \left. + e^{2\pi i(\frac{2}{1}8)} + e^{2\pi i(\frac{1}{7}8)} + e^{2\pi i(\frac{2}{3}8)} + e^{2\pi i(\frac{2}{7}8)}\right) \\
=\ & f_{Ag}(1 + 1i + 1i + 1) \\
& + f_{Al}(-1 + -1i + -1 + -1i) \\
& + f_{Te}([-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i] \\
& \quad + [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& \quad + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i] \\
& \quad + [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]) \\
=\ & (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}
\end{aligned}$$

$$F_{020} = f_{Ag}(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{2}{2}+\frac{0}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{0}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot0}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{0}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{2}{2}+\frac{0}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{0}{2}+\frac{2}{2})} + e^{2\pi i(\frac{0}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot0}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{0}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(0)} + e^{2\pi i(1)})$$

$$+ f_{Al}(e^{2\pi i(1)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(1)})$$

$$+ f_{Te}(e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})}$$

$$+ e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})})$$

$$= f_{Ag}(1+1+1+1)$$

$$+ f_{Al}(1+1+1+1)$$

$$+ f_{Te}(-1+-1+-1+-1+-1+-1+-1+-1)$$

$$= 4f_{Ag} + 4f_{Al} - 8f_{Te}$$

$$
\begin{aligned}
F_{004} &= f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{0}{2}+\frac{4}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{4}{2})}\right) \\
&\quad + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{4}{4})} + e^{2\pi i(0+0+\frac{4}{2})} + e^{2\pi i(0+\frac{0}{2}+\frac{3\cdot4}{4})}\right) \\
&\quad + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot4}{8})}\right. \\
&\quad \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot4}{8})}\right) \\
&= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{0}{2}+\frac{4}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{4}{2})}\right) \\
&\quad + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2})} + e^{2\pi i(\frac{0}{2}+\frac{4}{4})} + e^{2\pi i(\frac{4}{2})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot4}{4})}\right) \\
&\quad + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot4}{8})}\right. \\
&\quad \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot4}{8})}\right) \\
&= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(3)} + e^{2\pi i(2)}\right) \\
&\quad + f_{Al}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(3)}\right) \\
&\quad + f_{Te}\left(e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{3}{2})}\right. \\
&\quad \left. + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(\frac{7}{2})}\right) \\
&= f_{Ag}(1+1+1+1) \\
&\quad + f_{Al}(1+1+1+1) \\
&\quad + f_{Te}(-1+-1+-1+-1+-1+-1+-1+-1) \\
&= 4f_{Ag} + 4f_{Al} - 8f_{Te}
\end{aligned}
$$

$$
\begin{aligned}
F_{022} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{3\cdot2}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+0\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{2}{4}\right)} + e^{2\pi i\left(0+0+\frac{2}{2}\right)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{3\cdot2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{3\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{3\cdot2}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{7\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{7\cdot2}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{2}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{3\cdot2}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{2}{2}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{3\cdot2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{3\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{3\cdot2}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot2}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{2}{4}+\frac{7\cdot2}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot2}{4}+\frac{7\cdot2}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(1)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i(1)} + e^{2\pi i\left(\frac{5}{2}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{5}{4}\right)} + e^{2\pi i\left(\frac{9}{4}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{1}{1}4\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{9}{4}\right)} + e^{2\pi i\left(\frac{1}{3}4\right)}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(-1i + -1i + 1i + 1i + -1i + -1i + 1i + 1i) \\
=\ & 0\,(Forbidden\,Reflection)
\end{aligned}
$$

$$F_{211} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{1}{4})} + e^{2\pi i(\frac{2}{2}+0+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{2}{2}+\frac{1}{2}+\frac{1}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{2}{2}+0+\frac{1}{4})} + e^{2\pi i(0+0+\frac{1}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot1}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{1}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{1}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{1}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{2}{2}+\frac{1}{2}+\frac{1}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{1}{2})} + e^{2\pi i(\frac{2}{2}+\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot1}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot1}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{1}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{1}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{1}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(2)}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{5}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{7}{8})} + e^{2\pi i(\frac{1}{9}8)} + e^{2\pi i(\frac{1}{7}8)} + e^{2\pi i(\frac{1}{3}8)}\right.$$
$$\left. + e^{2\pi i(\frac{2}{3}8)} + e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{2}{1}8)} + e^{2\pi i(\frac{1}{7}8)}\right)$$

$$= f_{Ag}(1 + -1i + -1i + 1)$$
$$+ f_{Al}(-1 + 1i + -1 + 1i)$$
$$+ f_{Te}([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]$$
$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]$$
$$+ [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]$$
$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i])$$

$$= (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}$$

$$F_{114} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+0+\frac{3\cdot4}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+\frac{4}{2}\right)}\right)$$

$$+ f_{Al}\left(e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{1}{2}+0+\frac{4}{4}\right)} + e^{2\pi i\left(0+0+\frac{4}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot4}{4}\right)}\right)$$

$$+ f_{Te}\left(e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot4}{8}\right)}\right.$$

$$\left. + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot4}{8}\right)}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot4}{4}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}+\frac{4}{2}\right)}\right)$$

$$+ f_{Al}\left(e^{2\pi i\left(\frac{1}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{4}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot4}{4}\right)}\right)$$

$$+ f_{Te}\left(e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{3\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{3\cdot4}{8}\right)}\right.$$

$$\left. + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{3\cdot1}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{1}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot1}{4}+\frac{1}{4}+\frac{7\cdot4}{8}\right)} + e^{2\pi i\left(\frac{1}{4}+\frac{3\cdot1}{4}+\frac{7\cdot4}{8}\right)}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i\left(\frac{7}{2}\right)} + e^{2\pi i(3)}\right)$$

$$+ f_{Al}\left(e^{2\pi i(1)} + e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i(2)} + e^{2\pi i\left(\frac{7}{2}\right)}\right)$$

$$+ f_{Te}\left(e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i\left(\frac{5}{2}\right)}\right.$$

$$\left. + e^{2\pi i(4)} + e^{2\pi i(3)} + e^{2\pi i\left(\frac{9}{2}\right)} + e^{2\pi i\left(\frac{9}{2}\right)}\right)$$

$$= f_{Ag}(1 + -1 + -1 + 1)$$

$$+ f_{Al}(1 + -1 + 1 + -1)$$

$$+ f_{Te}(1 + 1 + -1 + -1 + 1 + 1 + -1 + -1)$$

$$= 0 \, (Forbidden Reflection)$$

$$
\begin{aligned}
F_{123} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{3}{4})} + e^{2\pi i(\frac{1}{2}+0+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{1}{2}+\frac{2}{2}+\frac{3}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{1}{2}+0+\frac{3}{4})} + e^{2\pi i(0+0+\frac{3}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot3}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{3\cdot3}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{7\cdot3}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{3}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{1}{2}+\frac{2}{2}+\frac{3}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{2}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot3}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{3\cdot3}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{7\cdot3}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{3}4)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{9}{8})} + e^{2\pi i(\frac{2}{1}8)} + e^{2\pi i(\frac{1}{9}8)} + e^{2\pi i(\frac{2}{3}8)}\right. \\
& \left. + e^{2\pi i(\frac{3}{3}8)} + e^{2\pi i(\frac{2}{1}8)} + e^{2\pi i(\frac{3}{1}8)} + e^{2\pi i(\frac{3}{5}8)}\right) \\
=\ & f_{Ag}(1 + -1i + -1i + 1) \\
& + f_{Al}(-1 + 1i + -1 + 1i) \\
& + f_{Te}\big([0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i] \\
& + [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i] \\
& + [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]\big) \\
=\ & (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}
\end{aligned}
$$

$$F_{220} = f_{Ag}(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{2}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{2}{2}+\frac{2}{2}+\frac{0}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{2}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{2}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot0}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{2}{2}+\frac{2}{2}+\frac{0}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{2}{2}+\frac{2}{2})} + e^{2\pi i(\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot0}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(1)} + e^{2\pi i(2)})$$

$$+ f_{Al}(e^{2\pi i(2)} + e^{2\pi i(1)} + e^{2\pi i(0)} + e^{2\pi i(1)})$$

$$+ f_{Te}(e^{2\pi i(1)} + e^{2\pi i(3)} + e^{2\pi i(2)} + e^{2\pi i(2)}$$

$$+ e^{2\pi i(3)} + e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(2)})$$

$$= f_{Ag}(1+1+1+1)$$

$$+ f_{Al}(1+1+1+1)$$

$$+ f_{Te}(1+1+1+1+1+1+1+1)$$

$$= 4f_{Ag} + 4f_{Al} + 8f_{Te}$$

$$\begin{aligned}
F_{015} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{5}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{3\cdot5}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{1}{2}+\frac{5}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{5}{4}\right)} + e^{2\pi i\left(0+0+\frac{5}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot5}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{1}{4}+\frac{5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot5}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot5}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{5}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{3\cdot5}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{1}{2}+\frac{5}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{5}{4}\right)} + e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot5}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{1}{4}+\frac{5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{3\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{3\cdot5}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot1}{4}+\frac{5\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{1}{4}+\frac{5\cdot5}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{1}{4}+\frac{7\cdot5}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot1}{4}+\frac{7\cdot5}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{1}{5}4\right)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{5}{4}\right)} + e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i\left(\frac{1}{7}4\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{7}{8}\right)} + e^{2\pi i\left(\frac{1}{1}8\right)} + e^{2\pi i\left(\frac{1}{7}8\right)} + e^{2\pi i\left(\frac{2}{1}8\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3}{1}8\right)} + e^{2\pi i\left(\frac{2}{7}8\right)} + e^{2\pi i\left(\frac{3}{7}8\right)} + e^{2\pi i\left(\frac{4}{1}8\right)}\right) \\
=\ & f_{Ag}(1 + -1i + -1i + 1) \\
& + f_{Al}(-1 + 1i + -1 + 1i) \\
& + f_{Te}([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i] \\
& + [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]) \\
=\ & (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}
\end{aligned}$$

$$\begin{aligned}
F_{024} = \ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{3\cdot 4}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+\frac{4}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+0\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{4}{4}\right)} + e^{2\pi i\left(0+0+\frac{4}{2}\right)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{3\cdot 4}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{3\cdot 2}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{2}{4}+\frac{3\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot 2}{4}+\frac{3\cdot 4}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{3\cdot 2}{4}+\frac{5\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{2}{4}+\frac{7\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot 2}{4}+\frac{7\cdot 4}{8}\right)}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{2}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{3\cdot 4}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}+\frac{4}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{2}{2}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{4}{2}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{3\cdot 4}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{3\cdot 2}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{2}{4}+\frac{3\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot 2}{4}+\frac{3\cdot 4}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{3\cdot 2}{4}+\frac{5\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{2}{4}+\frac{5\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 0}{4}+\frac{2}{4}+\frac{7\cdot 4}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot 2}{4}+\frac{7\cdot 4}{8}\right)}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(2)} + e^{2\pi i(3)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(1)} + e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(4)}\right) \\
& + f_{Te}\left(e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(2)} + e^{2\pi i(3)}\right. \\
& \left. + e^{2\pi i(4)} + e^{2\pi i(3)} + e^{2\pi i(4)} + e^{2\pi i(5)}\right) \\
= \ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(1+1+1+1+1+1+1+1) \\
= \ & 4f_{Ag} + 4f_{Al} + 8f_{Te}
\end{aligned}$$

$$
\begin{aligned}
F_{222} = \; & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2}+0+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{2}{2}+\frac{2}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{2}{2}+0+\frac{2}{4})} + e^{2\pi i(0+0+\frac{2}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{7\cdot2}{8})}\right) \\
= \; & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{2}{2}+\frac{2}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{2}{2})} + e^{2\pi i(\frac{2}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot2}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{2}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{2}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot2}{4}+\frac{7\cdot2}{8})}\right) \\
= \; & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(1)} + e^{2\pi i(\frac{5}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{1}{3}4)} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{1}{1}4)}\right. \\
& \left. + e^{2\pi i(\frac{1}{7}4)} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{1}{5}4)} + e^{2\pi i(\frac{1}{5}4)}\right) \\
= \; & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(1i + 1i + -1i + -1i + 1i + 1i + -1i + -1i) \\
= \; & 0 \, (Forbidden\,Reflection)
\end{aligned}
$$

$$
\begin{aligned}
F_{031} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3}{2}+\frac{1}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{1}{4})} + e^{2\pi i(0+0+\frac{1}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot1}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot3}{4}+\frac{3\cdot1}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot3}{4}+\frac{7\cdot1}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3}{2}+\frac{1}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{3}{2})} + e^{2\pi i(\frac{0}{2}+\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot1}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot3}{4}+\frac{3\cdot1}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot3}{4}+\frac{7\cdot1}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{9}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{7}{8})} + e^{2\pi i(\frac{1}{9}8)} + e^{2\pi i(\frac{9}{8})} + e^{2\pi i(\frac{2}{1}8)}\right. \\
& \left. + e^{2\pi i(\frac{2}{3}8)} + e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{1}{3}8)} + e^{2\pi i(\frac{2}{5}8)}\right) \\
=\ & f_{Ag}(1 + -1i + -1i + 1) \\
& + f_{Al}(-1 + 1i + -1 + 1i) \\
& + f_{Te}([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& \quad + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i] \\
& \quad + [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& \quad + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]) \\
=\ & (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}
\end{aligned}
$$

$$
\begin{aligned}
F_{310} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{0}{4})} + e^{2\pi i(\frac{3}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{3}{2}+\frac{1}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{3}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{1}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{1}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{0}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{3}{2}+\frac{1}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{1}{2})} + e^{2\pi i(\frac{3}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot1}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{1}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{1}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{1}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(1)} + e^{2\pi i(3)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{3}{2})}\right. \\
& \left. + e^{2\pi i(3)} + e^{2\pi i(1)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{3}{2})}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(1 + 1 + -1 + -1 + 1 + 1 + -1 + -1) \\
=\ & 0\,(Forbidden\,Reflection)
\end{aligned}
$$

$$
\begin{aligned}
F_{312} = \; & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+0+\frac{3\cdot 2}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{1}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{3}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{3}{2}+0+\frac{2}{4}\right)} + e^{2\pi i\left(0+0+\frac{2}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot 2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}+\frac{1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{3\cdot 1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{1}{4}+\frac{3\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot 1}{4}+\frac{3\cdot 2}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{3\cdot 1}{4}+\frac{5\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{1}{4}+\frac{5\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{1}{4}+\frac{7\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot 1}{4}+\frac{7\cdot 2}{8}\right)}\right) \\
= \; & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{3\cdot 2}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{1}{2}+\frac{2}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{3}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{2}{4}\right)} + e^{2\pi i\left(\frac{2}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot 2}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}+\frac{1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{3\cdot 1}{4}+\frac{2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{1}{4}+\frac{3\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot 1}{4}+\frac{3\cdot 2}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{3\cdot 1}{4}+\frac{5\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{1}{4}+\frac{5\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3\cdot 3}{4}+\frac{1}{4}+\frac{7\cdot 2}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot 1}{4}+\frac{7\cdot 2}{8}\right)}\right) \\
= \; & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(3)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(2)} + e^{2\pi i(1)} + e^{2\pi i(2)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{5}{4}\right)} + e^{2\pi i\left(\frac{1}{3}4\right)} + e^{2\pi i\left(\frac{1}{3}4\right)} + e^{2\pi i\left(\frac{9}{4}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{1}{7}4\right)} + e^{2\pi i\left(\frac{9}{4}\right)} + e^{2\pi i\left(\frac{1}{7}4\right)} + e^{2\pi i\left(\frac{1}{3}4\right)}\right) \\
= \; & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(1i+1i+1i+1i+1i+1i+1i+1i) \\
= \; & 4f_{Ag} + 4f_{Al} + 8if_{Te}
\end{aligned}
$$

$$F_{125} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{5}{4})} + e^{2\pi i(\frac{1}{2}+0+\frac{3\cdot5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{2}{2}+\frac{5}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{1}{2}+0+\frac{5}{4})} + e^{2\pi i(0+0+\frac{5}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot5}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{3\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{3\cdot5}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{7\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{7\cdot5}{8})}\right)$$
$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{2}{2}+\frac{5}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{2}{2})} + e^{2\pi i(\frac{1}{2}+\frac{5}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot5}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{3\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{3\cdot5}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot2}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{2}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{2}{4}+\frac{7\cdot5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot2}{4}+\frac{7\cdot5}{8})}\right)$$
$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{1}{7}4)} + e^{2\pi i(4)}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{1}{9}4)}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{2}{3}8)} + e^{2\pi i(\frac{2}{5}8)} + e^{2\pi i(\frac{2}{9}8)}\right.$$
$$\left. + e^{2\pi i(\frac{4}{3}8)} + e^{2\pi i(\frac{3}{1}8)} + e^{2\pi i(\frac{4}{5}8)} + e^{2\pi i(\frac{4}{9}8)}\right)$$
$$= f_{Ag}(1 + 1i + 1i + 1)$$
$$+ f_{Al}(-1 + -1i + -1 + -1i)$$
$$+ f_{Te}([-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$
$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]$$
$$+ [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$
$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i])$$
$$= (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}$$

$$
\begin{aligned}
F_{321} = \ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+0+\frac{3\cdot1}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{2}{2}+\frac{1}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{3}{2}+\frac{2}{2}+0\right)} + e^{2\pi i\left(\frac{3}{2}+0+\frac{1}{4}\right)} + e^{2\pi i\left(0+0+\frac{1}{2}\right)} + e^{2\pi i\left(0+\frac{2}{2}+\frac{3\cdot1}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}+\frac{2}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{3\cdot2}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{2}{4}+\frac{3\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot2}{4}+\frac{3\cdot1}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{3\cdot2}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{2}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{2}{4}+\frac{7\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot2}{4}+\frac{7\cdot1}{8}\right)}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{2}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{3\cdot1}{4}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{2}{2}+\frac{1}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{3}{2}+\frac{2}{2}\right)} + e^{2\pi i\left(\frac{3}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{3\cdot1}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{4}+\frac{2}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{3\cdot2}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{2}{4}+\frac{3\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot2}{4}+\frac{3\cdot1}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{3\cdot2}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{2}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot3}{4}+\frac{2}{4}+\frac{7\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3}{4}+\frac{3\cdot2}{4}+\frac{7\cdot1}{8}\right)}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{5}{4}\right)} + e^{2\pi i\left(\frac{9}{4}\right)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i\left(\frac{7}{4}\right)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{7}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{1}{1}8\right)} + e^{2\pi i\left(\frac{3}{1}8\right)} + e^{2\pi i\left(\frac{2}{5}8\right)} + e^{2\pi i\left(\frac{2}{1}8\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3}{5}8\right)} + e^{2\pi i\left(\frac{1}{5}8\right)} + e^{2\pi i\left(\frac{2}{9}8\right)} + e^{2\pi i\left(\frac{2}{5}8\right)}\right) \\
= \ & f_{Ag}(1 + 1i + 1i + 1) \\
& + f_{Al}(-1 + -1i + -1 + -1i) \\
& + f_{Te}([-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i] \\
& + [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]) \\
= \ & (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}
\end{aligned}
$$

$$
\begin{aligned}
F_{134} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{4}{4})} + e^{2\pi i(\frac{1}{2}+0+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3}{2}+\frac{4}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{1}{2}+0+\frac{4}{4})} + e^{2\pi i(0+0+\frac{4}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot4}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot3}{4}+\frac{3\cdot4}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot3}{4}+\frac{7\cdot4}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{4}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3}{2}+\frac{4}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{3}{2})} + e^{2\pi i(\frac{1}{2}+\frac{4}{4})} + e^{2\pi i(\frac{4}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot4}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot3}{4}+\frac{3\cdot4}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3\cdot3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot1}{4}+\frac{3}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot3}{4}+\frac{7\cdot4}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(4)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(2)} + e^{2\pi i(\frac{9}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(3)} + e^{2\pi i(4)}\right. \\
& \left. + e^{2\pi i(\frac{1}{1}2)} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(5)} + e^{2\pi i(6)}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(-1 + -1 + 1 + 1 + -1 + -1 + 1 + 1) \\
=\ & 0 \, (Forbidden Reflection)
\end{aligned}
$$

$$F_{233} = f_{Ag}(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{3}{4})} + e^{2\pi i(\frac{2}{2}+0+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3}{2}+\frac{3}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{2}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{2}{2}+0+\frac{3}{4})} + e^{2\pi i(0+0+\frac{3}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot3}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{3\cdot3}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{7\cdot3}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{3}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3}{2}+\frac{3}{2})})$$

$$+ f_{Al}(e^{2\pi i(\frac{2}{2}+\frac{3}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot3}{4})})$$

$$+ f_{Te}(e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{3\cdot3}{8})}$$

$$+ e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{7\cdot3}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{1}{3}4)} + e^{2\pi i(4)})$$

$$+ f_{Al}(e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{5}4)})$$

$$+ f_{Te}(e^{2\pi i(\frac{1}{3}8)} + e^{2\pi i(\frac{3}{3}8)} + e^{2\pi i(\frac{2}{7}8)} + e^{2\pi i(\frac{3}{1}8)}$$

$$+ e^{2\pi i(\frac{4}{5}8)} + e^{2\pi i(\frac{2}{5}8)} + e^{2\pi i(\frac{3}{9}8)} + e^{2\pi i(\frac{4}{3}8)})$$

$$= f_{Ag}(1 + 1i + 1i + 1)$$

$$+ f_{Al}(-1 + -1i + -1 + -1i)$$

$$+ f_{Te}([-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$

$$+ [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$

$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$

$$+ [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i])$$

$$= (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}$$

$$
\begin{aligned}
F_{040} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{4}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{3\cdot0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{4}{2}+\frac{0}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{4}{2}+0\right)} + e^{2\pi i\left(\frac{0}{2}+0+\frac{0}{4}\right)} + e^{2\pi i\left(0+0+\frac{0}{2}\right)} + e^{2\pi i\left(0+\frac{4}{2}+\frac{3\cdot0}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{4}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{3\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{3\cdot0}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{4}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{7\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{7\cdot0}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{4}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{3\cdot0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{4}{2}+\frac{0}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{0}{2}+\frac{4}{2}\right)} + e^{2\pi i\left(\frac{0}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{3\cdot0}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{0}{4}+\frac{4}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{3\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{3\cdot0}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{4}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{7\cdot0}{8}\right)} + e^{2\pi i\left(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{7\cdot0}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(2)} + e^{2\pi i(0)} + e^{2\pi i(2)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(2)}\right) \\
& + f_{Te}\left(e^{2\pi i(1)} + e^{2\pi i(3)} + e^{2\pi i(1)} + e^{2\pi i(3)}\right. \\
& + e^{2\pi i(3)} + e^{2\pi i(1)} + e^{2\pi i(1)} + e^{2\pi i(3)}\bigg) \\
=\ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(1+1+1+1+1+1+1+1) \\
=\ & 4f_{Ag} + 4f_{Al} + 8f_{Te}
\end{aligned}
$$

$$
\begin{aligned}
F_{042} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{4}{2}+\frac{2}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{4}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{4}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{2}{4})} + e^{2\pi i(0+0+\frac{2}{2})} + e^{2\pi i(0+\frac{4}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{4}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{4}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{7\cdot2}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{4}{2}+\frac{2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{0}{2}+\frac{4}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{4}{2})} + e^{2\pi i(\frac{0}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2})} + e^{2\pi i(\frac{4}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{4}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot4}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{4}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{4}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot4}{4}+\frac{7\cdot2}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(2)} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(1)} + e^{2\pi i(\frac{7}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{1}{3}4)} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{1}{5}4)}\right. \\
& \left. + e^{2\pi i(\frac{1}{7}4)} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{1}{9}4)}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(1i + 1i + -1i + -1i + 1i + 1i + -1i + -1i) \\
=\ & 0\,(Forbidden\,Reflection)
\end{aligned}
$$

$$
\begin{aligned}
F_{411} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{4}{2}+0+\frac{3\cdot1}{4}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{1}{2}+\frac{1}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{4}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{4}{2}+0+\frac{1}{4}\right)} + e^{2\pi i\left(0+0+\frac{1}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot1}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{4}{4}+\frac{1}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{3\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{1}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{7\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{3\cdot1}{4}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{1}{2}+\frac{1}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{4}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{1}{4}\right)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot1}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{4}{4}+\frac{1}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{3\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{3\cdot1}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{1}{4}+\frac{5\cdot1}{8}\right)} + e^{2\pi i\left(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{7\cdot1}{8}\right)} + e^{2\pi i\left(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{7\cdot1}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{3}{4}\right)} + e^{2\pi i\left(\frac{1}{1}4\right)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i\left(\frac{9}{4}\right)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{5}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{1}{1}8\right)} + e^{2\pi i\left(\frac{3}{1}8\right)} + e^{2\pi i\left(\frac{2}{9}8\right)} + e^{2\pi i\left(\frac{1}{7}8\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3}{5}8\right)} + e^{2\pi i\left(\frac{1}{5}8\right)} + e^{2\pi i\left(\frac{3}{3}8\right)} + e^{2\pi i\left(\frac{2}{1}8\right)}\right) \\
=\ & f_{Ag}(1 + -1i + -1i + 1) \\
& + f_{Al}(-1 + 1i + -1 + 1i) \\
& + f_{Te}([-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i] \\
& + [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i] \\
& + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]) \\
=\ & (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}
\end{aligned}
$$

$$
\begin{aligned}
F_{332} = {}& f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{2}{4})} + e^{2\pi i(\frac{3}{2}+0+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{3}{2}+0+\frac{2}{4})} + e^{2\pi i(0+0+\frac{2}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{7\cdot2}{8})}\right) \\
= {}& f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{2}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot2}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{3}{2})} + e^{2\pi i(\frac{3}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{3\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{3\cdot2}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{5\cdot2}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{7\cdot2}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{7\cdot2}{8})}\right) \\
= {}& f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(2)} + e^{2\pi i(3)} + e^{2\pi i(4)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i(2)} + e^{2\pi i(1)} + e^{2\pi i(3)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{5}{4})}\right. \\
& \left. + e^{2\pi i(\frac{2}{3}4)} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{1}{9}4)} + e^{2\pi i(\frac{1}{9}4)}\right) \\
= {}& f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(-1i + -1i + -1i + -1i + -1i + -1i + -1i + -1i) \\
= {}& 4f_{Ag} + 4f_{Al} - 8if_{Te}
\end{aligned}
$$

$$F_{413} = f_{Ag}(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{3}{4})} + e^{2\pi i(\frac{4}{2}+0+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{4}{2}+\frac{1}{2}+\frac{3}{2})})$$
$$+ f_{Al}(e^{2\pi i(\frac{4}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{4}{2}+0+\frac{3}{4})} + e^{2\pi i(0+0+\frac{3}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot3}{4})})$$
$$+ f_{Te}(e^{2\pi i(\frac{4}{4}+\frac{1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{3\cdot3}{8})}$$
$$+ e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{7\cdot3}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{3}{4})} + e^{2\pi i(\frac{4}{2}+\frac{3\cdot3}{4})} + e^{2\pi i(\frac{4}{2}+\frac{1}{2}+\frac{3}{2})})$$
$$+ f_{Al}(e^{2\pi i(\frac{4}{2}+\frac{1}{2})} + e^{2\pi i(\frac{4}{2}+\frac{3}{4})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot3}{4})})$$
$$+ f_{Te}(e^{2\pi i(\frac{4}{4}+\frac{1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{3\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{3\cdot3}{8})}$$
$$+ e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{1}{4}+\frac{5\cdot3}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{1}{4}+\frac{7\cdot3}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot1}{4}+\frac{7\cdot3}{8})})$$

$$= f_{Ag}(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{4})} + e^{2\pi i(\frac{1}{7}4)} + e^{2\pi i(4)})$$
$$+ f_{Al}(e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{1}{1}4)})$$
$$+ f_{Te}(e^{2\pi i(\frac{1}{3}8)} + e^{2\pi i(\frac{3}{3}8)} + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{2}{3}8)}$$
$$+ e^{2\pi i(\frac{4}{5}8)} + e^{2\pi i(\frac{2}{5}8)} + e^{2\pi i(\frac{4}{7}8)} + e^{2\pi i(\frac{3}{5}8)})$$

$$= f_{Ag}(1 + 1i + 1i + 1)$$
$$+ f_{Al}(-1 + -1i + -1 + -1i)$$
$$+ f_{Te}([-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$
$$+ [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$
$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$
$$+ [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i])$$
$$= (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}$$

$$\begin{aligned}
F_{420} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{4}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{4}{2}+\frac{2}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{4}{2}+\frac{2}{2}+0)} + e^{2\pi i(\frac{4}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{2}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{4}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{2}{2}+\frac{0}{4})} + e^{2\pi i(\frac{4}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{4}{2}+\frac{2}{2}+\frac{0}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{4}{2}+\frac{2}{2})} + e^{2\pi i(\frac{4}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot0}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{4}{4}+\frac{2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot2}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{2}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot2}{4}+\frac{3\cdot0}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{2}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{2}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot2}{4}+\frac{7\cdot0}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(2)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i(2)} + e^{2\pi i(0)} + e^{2\pi i(1)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{9}{2})} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(\frac{5}{2})}\right. \\
& \left. + e^{2\pi i(\frac{9}{2})} + e^{2\pi i(\frac{3}{2})} + e^{2\pi i(\frac{7}{2})} + e^{2\pi i(\frac{5}{2})}\right) \\
=\ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(-1+-1+-1+-1+-1+-1+-1+-1) \\
=\ & 4f_{Ag} + 4f_{Al} - 8f_{Te}
\end{aligned}$$

$$F_{235} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{5}{4})} + e^{2\pi i(\frac{2}{2}+0+\frac{3\cdot5}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3}{2}+\frac{5}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{2}{2}+0+\frac{5}{4})} + e^{2\pi i(0+0+\frac{5}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot5}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{3\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{3\cdot5}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{7\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{7\cdot5}{8})}\right)$$
$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{5}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3\cdot5}{4})} + e^{2\pi i(\frac{2}{2}+\frac{3}{2}+\frac{5}{2})}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{2}{2}+\frac{3}{2})} + e^{2\pi i(\frac{2}{2}+\frac{5}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot5}{4})}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{3\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{3\cdot5}{8})}\right.$$
$$\left. + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3\cdot3}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3}{4}+\frac{5\cdot5}{8})} + e^{2\pi i(\frac{3\cdot2}{4}+\frac{3}{4}+\frac{7\cdot5}{8})} + e^{2\pi i(\frac{2}{4}+\frac{3\cdot3}{4}+\frac{7\cdot5}{8})}\right)$$
$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{1}{9}4)} + e^{2\pi i(5)}\right)$$
$$+ f_{Al}\left(e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{2}{1}4)}\right)$$
$$+ f_{Te}\left(e^{2\pi i(\frac{1}{5}8)} + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{3}{3}8)} + e^{2\pi i(\frac{3}{7}8)}\right.$$
$$\left. + e^{2\pi i(\frac{5}{5}8)} + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{5}{3}8)} + e^{2\pi i(\frac{5}{7}8)}\right)$$
$$= f_{Ag}(1 + -1i + -1i + 1)$$
$$+ f_{Al}(-1 + 1i + -1 + 1i)$$
$$+ f_{Te}([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]$$
$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]$$
$$+ [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]$$
$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i])$$
$$= (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}$$

$$
\begin{aligned}
F_{334} = {} & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{4}{4})} + e^{2\pi i(\frac{3}{2}+0+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3}{2}+\frac{4}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{3}{2}+0+\frac{4}{4})} + e^{2\pi i(0+0+\frac{4}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot4}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{3\cdot4}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{7\cdot4}{8})}\right) \\
= {} & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{4}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot4}{4})} + e^{2\pi i(\frac{3}{2}+\frac{3}{2}+\frac{4}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{3}{2}+\frac{3}{2})} + e^{2\pi i(\frac{3}{2}+\frac{4}{4})} + e^{2\pi i(\frac{4}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot4}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{3\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{3\cdot4}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3\cdot3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3}{4}+\frac{5\cdot4}{8})} + e^{2\pi i(\frac{3\cdot3}{4}+\frac{3}{4}+\frac{7\cdot4}{8})} + e^{2\pi i(\frac{3}{4}+\frac{3\cdot3}{4}+\frac{7\cdot4}{8})}\right) \\
= {} & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{9}{2})} + e^{2\pi i(5)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(2)} + e^{2\pi i(\frac{9}{2})}\right) \\
& + f_{Te}\left(e^{2\pi i(2)} + e^{2\pi i(5)} + e^{2\pi i(\frac{9}{2})} + e^{2\pi i(\frac{9}{2})}\right. \\
& \left. + e^{2\pi i(7)} + e^{2\pi i(4)} + e^{2\pi i(\frac{1}{3}2)} + e^{2\pi i(\frac{1}{3}2)}\right) \\
= {} & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(1 + 1 + -1 + -1 + 1 + 1 + -1 + -1) \\
= {} & 0 (Forbidden Reflection)
\end{aligned}
$$

$$
\begin{aligned}
F_{145} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{4}{2}+\frac{5}{4})} + e^{2\pi i(\frac{1}{2}+0+\frac{3\cdot 5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{4}{2}+\frac{5}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{4}{2}+0)} + e^{2\pi i(\frac{1}{2}+0+\frac{5}{4})} + e^{2\pi i(0+0+\frac{5}{2})} + e^{2\pi i(0+\frac{4}{2}+\frac{3\cdot 5}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{4}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{3\cdot 4}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{4}{4}+\frac{3\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot 4}{4}+\frac{3\cdot 5}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{3\cdot 4}{4}+\frac{5\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{4}{4}+\frac{5\cdot 5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{4}{4}+\frac{7\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot 4}{4}+\frac{7\cdot 5}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{4}{2}+\frac{5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot 5}{4})} + e^{2\pi i(\frac{1}{2}+\frac{4}{2}+\frac{5}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{1}{2}+\frac{4}{2})} + e^{2\pi i(\frac{1}{2}+\frac{5}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{4}{2}+\frac{3\cdot 5}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{4}+\frac{4}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{3\cdot 4}{4}+\frac{5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{4}{4}+\frac{3\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot 4}{4}+\frac{3\cdot 5}{8})}\right. \\
& \left. + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{3\cdot 4}{4}+\frac{5\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{4}{4}+\frac{5\cdot 5}{8})} + e^{2\pi i(\frac{3\cdot 1}{4}+\frac{4}{4}+\frac{7\cdot 5}{8})} + e^{2\pi i(\frac{1}{4}+\frac{3\cdot 4}{4}+\frac{7\cdot 5}{8})}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{3}4)} + e^{2\pi i(\frac{1}{7}4)} + e^{2\pi i(5)}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{2}{3}4)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{1}{5}8)} + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{2}{9}8)} + e^{2\pi i(\frac{4}{1}8)}\right. \\
& \left. + e^{2\pi i(\frac{5}{5}8)} + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{4}{9}8)} + e^{2\pi i(\frac{6}{1}8)}\right) \\
=\ & f_{Ag}(1 + 1i + 1i + 1) \\
& + f_{Al}(-1 + -1i + -1 + -1i) \\
& + f_{Te}\big([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i] \\
& + [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i] \\
& + [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]\big) \\
=\ & (2.0 + 2.0i)f_{Ag} + (-2.0 + -2.0i)f_{Al}
\end{aligned}
$$

$$
\begin{aligned}
F_{244} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{4}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{2}{2}+0+\frac{3\cdot4}{4}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{4}{2}+\frac{4}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{2}{2}+\frac{4}{2}+0\right)} + e^{2\pi i\left(\frac{2}{2}+0+\frac{4}{4}\right)} + e^{2\pi i\left(0+0+\frac{4}{2}\right)} + e^{2\pi i\left(0+\frac{4}{2}+\frac{3\cdot4}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{2}{4}+\frac{4}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{3\cdot4}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{4}{4}+\frac{3\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{3\cdot4}{4}+\frac{3\cdot4}{8}\right)}\right. \\
& \quad \left. + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{3\cdot4}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{4}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{4}{4}+\frac{7\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{3\cdot4}{4}+\frac{7\cdot4}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{4}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{3\cdot4}{4}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{4}{2}+\frac{4}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{2}{2}+\frac{4}{2}\right)} + e^{2\pi i\left(\frac{2}{2}+\frac{4}{4}\right)} + e^{2\pi i\left(\frac{4}{2}\right)} + e^{2\pi i\left(\frac{4}{2}+\frac{3\cdot4}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{2}{4}+\frac{4}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{3\cdot4}{4}+\frac{4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{4}{4}+\frac{3\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{3\cdot4}{4}+\frac{3\cdot4}{8}\right)}\right. \\
& \quad \left. + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{3\cdot4}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{4}{4}+\frac{5\cdot4}{8}\right)} + e^{2\pi i\left(\frac{3\cdot2}{4}+\frac{4}{4}+\frac{7\cdot4}{8}\right)} + e^{2\pi i\left(\frac{2}{4}+\frac{3\cdot4}{4}+\frac{7\cdot4}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(3)} + e^{2\pi i(4)} + e^{2\pi i(5)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i(2)} + e^{2\pi i(2)} + e^{2\pi i(5)}\right) \\
& + f_{Te}\left(e^{2\pi i(2)} + e^{2\pi i(5)} + e^{2\pi i(4)} + e^{2\pi i(5)}\right. \\
& \quad \left. + e^{2\pi i(7)} + e^{2\pi i(4)} + e^{2\pi i(6)} + e^{2\pi i(7)}\right) \\
=\ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(1+1+1+1+1+1+1+1) \\
=\ & 4f_{Ag} + 4f_{Al} + 8f_{Te}
\end{aligned}
$$

$$F_{431} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{3}{2}+\frac{1}{4})} + e^{2\pi i(\frac{4}{2}+0+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{4}{2}+\frac{3}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{4}{2}+\frac{3}{2}+0)} + e^{2\pi i(\frac{4}{2}+0+\frac{1}{4})} + e^{2\pi i(0+0+\frac{1}{2})} + e^{2\pi i(0+\frac{3}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{4}{4}+\frac{3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot3}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot3}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{3}{2}+\frac{1}{4})} + e^{2\pi i(\frac{4}{2}+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{4}{2}+\frac{3}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{4}{2}+\frac{3}{2})} + e^{2\pi i(\frac{4}{2}+\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{3}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{4}{4}+\frac{3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3\cdot3}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3\cdot3}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot4}{4}+\frac{3}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{4}{4}+\frac{3}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(4)}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{7}{2})} + e^{2\pi i(\frac{9}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{9}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{1}{5}8)} + e^{2\pi i(\frac{4}{3}8)} + e^{2\pi i(\frac{3}{3}8)} + e^{2\pi i(\frac{2}{9}8)}\right.$$

$$\left. + e^{2\pi i(\frac{4}{7}8)} + e^{2\pi i(\frac{1}{9}8)} + e^{2\pi i(\frac{3}{7}8)} + e^{2\pi i(\frac{3}{3}8)}\right)$$

$$= f_{Ag}(1 + -1i + -1i + 1)$$

$$+ f_{Al}(-1 + 1i + -1 + 1i)$$

$$+ f_{Te}\left([0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]\right.$$

$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i]$$

$$+ [0.70711 + (-0.70711)i] + [-0.70711 + (0.70711)i]$$

$$\left. + [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]\right)$$

$$= (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}$$

$$F_{051} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{5}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{5}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{5}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{1}{4})} + e^{2\pi i(0+0+\frac{1}{2})} + e^{2\pi i(0+\frac{5}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{5}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot5}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{5}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot5}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot5}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{5}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{5}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot5}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{5}{2}+\frac{1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot1}{4})} + e^{2\pi i(\frac{0}{2}+\frac{5}{2}+\frac{1}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{5}{2})} + e^{2\pi i(\frac{0}{2}+\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{5}{2}+\frac{3\cdot1}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{5}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot5}{4}+\frac{1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{5}{4}+\frac{3\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot5}{4}+\frac{3\cdot1}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{5}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{5}{4}+\frac{5\cdot1}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{5}{4}+\frac{7\cdot1}{8})} + e^{2\pi i(\frac{0}{4}+\frac{5}{4}+\frac{7\cdot1}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{3}{4})} + e^{2\pi i(3)}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{5}{2})} + e^{2\pi i(\frac{1}{4})} + e^{2\pi i(\frac{1}{2})} + e^{2\pi i(\frac{1}{3}4)}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{1}{1}8)} + e^{2\pi i(\frac{3}{1}8)} + e^{2\pi i(\frac{1}{3}8)} + e^{2\pi i(\frac{3}{3}8)}\right.$$

$$\left. + e^{2\pi i(\frac{3}{5}8)} + e^{2\pi i(\frac{1}{5}8)} + e^{2\pi i(\frac{1}{7}8)} + e^{2\pi i(\frac{3}{7}8)}\right)$$

$$= f_{Ag}(1 + -1i + -1i + 1)$$

$$+ f_{Al}(-1 + 1i + -1 + 1i)$$

$$+ f_{Te}([-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$

$$+ [-0.70711 + (-0.70711)i] + [0.70711 + (0.70711)i]$$

$$+ [-0.70711 + (0.70711)i] + [0.70711 + (-0.70711)i]$$

$$+ [0.70711 + (0.70711)i] + [-0.70711 + (-0.70711)i])$$

$$= (2.0 + -2.0i)f_{Ag} + (-2.0 + 2.0i)f_{Al}$$

$$
\begin{aligned}
F_{510} =\ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{5}{2}+0+\frac{3\cdot0}{4}\right)} + e^{2\pi i\left(\frac{5}{2}+\frac{1}{2}+\frac{0}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{5}{2}+\frac{1}{2}+0\right)} + e^{2\pi i\left(\frac{5}{2}+0+\frac{0}{4}\right)} + e^{2\pi i\left(0+0+\frac{0}{2}\right)} + e^{2\pi i\left(0+\frac{1}{2}+\frac{3\cdot0}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{5}{4}+\frac{1}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{3\cdot1}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{1}{4}+\frac{3\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{1}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{1}{4}+\frac{7\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{5}{2}+\frac{3\cdot0}{4}\right)} + e^{2\pi i\left(\frac{5}{2}+\frac{1}{2}+\frac{0}{2}\right)}\right) \\
& + f_{Al}\left(e^{2\pi i\left(\frac{5}{2}+\frac{1}{2}\right)} + e^{2\pi i\left(\frac{5}{2}+\frac{0}{4}\right)} + e^{2\pi i\left(\frac{0}{2}\right)} + e^{2\pi i\left(\frac{1}{2}+\frac{3\cdot0}{4}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{5}{4}+\frac{1}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{3\cdot1}{4}+\frac{0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{1}{4}+\frac{3\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{3\cdot1}{4}+\frac{3\cdot0}{8}\right)}\right. \\
& \left. + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{3\cdot1}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{1}{4}+\frac{5\cdot0}{8}\right)} + e^{2\pi i\left(\frac{3\cdot5}{4}+\frac{1}{4}+\frac{7\cdot0}{8}\right)} + e^{2\pi i\left(\frac{5}{4}+\frac{3\cdot1}{4}+\frac{7\cdot0}{8}\right)}\right) \\
=\ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}\right)} + e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i(3)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i\left(\frac{5}{2}\right)} + e^{2\pi i(0)} + e^{2\pi i\left(\frac{1}{2}\right)}\right) \\
& + f_{Te}\left(e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i\left(\frac{9}{2}\right)} + e^{2\pi i(4)} + e^{2\pi i(2)}\right. \\
& \left. + e^{2\pi i\left(\frac{9}{2}\right)} + e^{2\pi i\left(\frac{3}{2}\right)} + e^{2\pi i(4)} + e^{2\pi i(2)}\right) \\
=\ & f_{Ag}(1 + -1 + -1 + 1) \\
& + f_{Al}(1 + -1 + 1 + -1) \\
& + f_{Te}(-1 + -1 + 1 + 1 + -1 + -1 + 1 + 1) \\
=\ & 0 \,(Forbidden\,Reflection)
\end{aligned}
$$

$$\begin{aligned}
F_{512} = \ & f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{1}{2}+\frac{2}{4})} + e^{2\pi i(\frac{5}{2}+0+\frac{3\cdot 2}{4})} + e^{2\pi i(\frac{5}{2}+\frac{1}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{5}{2}+\frac{1}{2}+0)} + e^{2\pi i(\frac{5}{2}+0+\frac{2}{4})} + e^{2\pi i(0+0+\frac{2}{2})} + e^{2\pi i(0+\frac{1}{2}+\frac{3\cdot 2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{5}{4}+\frac{1}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{3\cdot 1}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{1}{4}+\frac{3\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{3\cdot 1}{4}+\frac{3\cdot 2}{8})}\right. \\
& \left. \quad + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{3\cdot 1}{4}+\frac{5\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{1}{4}+\frac{5\cdot 2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{1}{4}+\frac{7\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{3\cdot 1}{4}+\frac{7\cdot 2}{8})}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{1}{2}+\frac{2}{4})} + e^{2\pi i(\frac{5}{2}+\frac{3\cdot 2}{4})} + e^{2\pi i(\frac{5}{2}+\frac{1}{2}+\frac{2}{2})}\right) \\
& + f_{Al}\left(e^{2\pi i(\frac{5}{2}+\frac{1}{2})} + e^{2\pi i(\frac{5}{2}+\frac{2}{4})} + e^{2\pi i(\frac{2}{2})} + e^{2\pi i(\frac{1}{2}+\frac{3\cdot 2}{4})}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{5}{4}+\frac{1}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{3\cdot 1}{4}+\frac{2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{1}{4}+\frac{3\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{3\cdot 1}{4}+\frac{3\cdot 2}{8})}\right. \\
& \left. \quad + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{3\cdot 1}{4}+\frac{5\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{1}{4}+\frac{5\cdot 2}{8})} + e^{2\pi i(\frac{3\cdot 5}{4}+\frac{1}{4}+\frac{7\cdot 2}{8})} + e^{2\pi i(\frac{5}{4}+\frac{3\cdot 1}{4}+\frac{7\cdot 2}{8})}\right) \\
= \ & f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(1)} + e^{2\pi i(4)} + e^{2\pi i(4)}\right) \\
& + f_{Al}\left(e^{2\pi i(3)} + e^{2\pi i(3)} + e^{2\pi i(1)} + e^{2\pi i(2)}\right) \\
& + f_{Te}\left(e^{2\pi i(\frac{7}{4})} + e^{2\pi i(\frac{1}{9}4)} + e^{2\pi i(\frac{1}{9}4)} + e^{2\pi i(\frac{1}{1}4)}\right. \\
& \left. \quad + e^{2\pi i(\frac{2}{3}4)} + e^{2\pi i(\frac{1}{1}4)} + e^{2\pi i(\frac{2}{3}4)} + e^{2\pi i(\frac{1}{5}4)}\right) \\
= \ & f_{Ag}(1+1+1+1) \\
& + f_{Al}(1+1+1+1) \\
& + f_{Te}(-1i + -1i + -1i + -1i + -1i + -1i + -1i + -1i) \\
= \ & 4f_{Ag} + 4f_{Al} - 8if_{Te}
\end{aligned}$$

$$F_{000} = f_{Ag}\left(e^{2\pi i(0+0+0)} + e^{2\pi i(0+\frac{0}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2}+0+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{0}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2}+0)} + e^{2\pi i(\frac{0}{2}+0+\frac{0}{4})} + e^{2\pi i(0+0+\frac{0}{2})} + e^{2\pi i(0+\frac{0}{2}+\frac{3\cdot0}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot0}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot0}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(\frac{0}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot0}{4})} + e^{2\pi i(\frac{0}{2}+\frac{0}{2}+\frac{0}{2})}\right)$$

$$+ f_{Al}\left(e^{2\pi i(\frac{0}{2}+\frac{0}{2})} + e^{2\pi i(\frac{0}{2}+\frac{0}{4})} + e^{2\pi i(\frac{0}{2})} + e^{2\pi i(\frac{0}{2}+\frac{3\cdot0}{4})}\right)$$

$$+ f_{Te}\left(e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{3\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{3\cdot0}{8})}\right.$$

$$\left. + e^{2\pi i(\frac{3\cdot0}{4}+\frac{3\cdot0}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{0}{4}+\frac{5\cdot0}{8})} + e^{2\pi i(\frac{3\cdot0}{4}+\frac{0}{4}+\frac{7\cdot0}{8})} + e^{2\pi i(\frac{0}{4}+\frac{3\cdot0}{4}+\frac{7\cdot0}{8})}\right)$$

$$= f_{Ag}\left(e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)}\right)$$

$$+ f_{Al}\left(e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)}\right)$$

$$+ f_{Te}\left(e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)}\right.$$

$$\left. + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)} + e^{2\pi i(0)}\right)$$

$$= f_{Ag}(1+1+1+1)$$

$$+ f_{Al}(1+1+1+1)$$

$$+ f_{Te}(1+1+1+1+1+1+1+1)$$

$$= 4f_{Ag} + 4f_{Al} + 8f_{Te}$$