Box-Jack System Simulation

Introduction

The default one is chosen for my final project: a system simulating a box executed by external forces, causing the jack to bounce inside.

Modeling

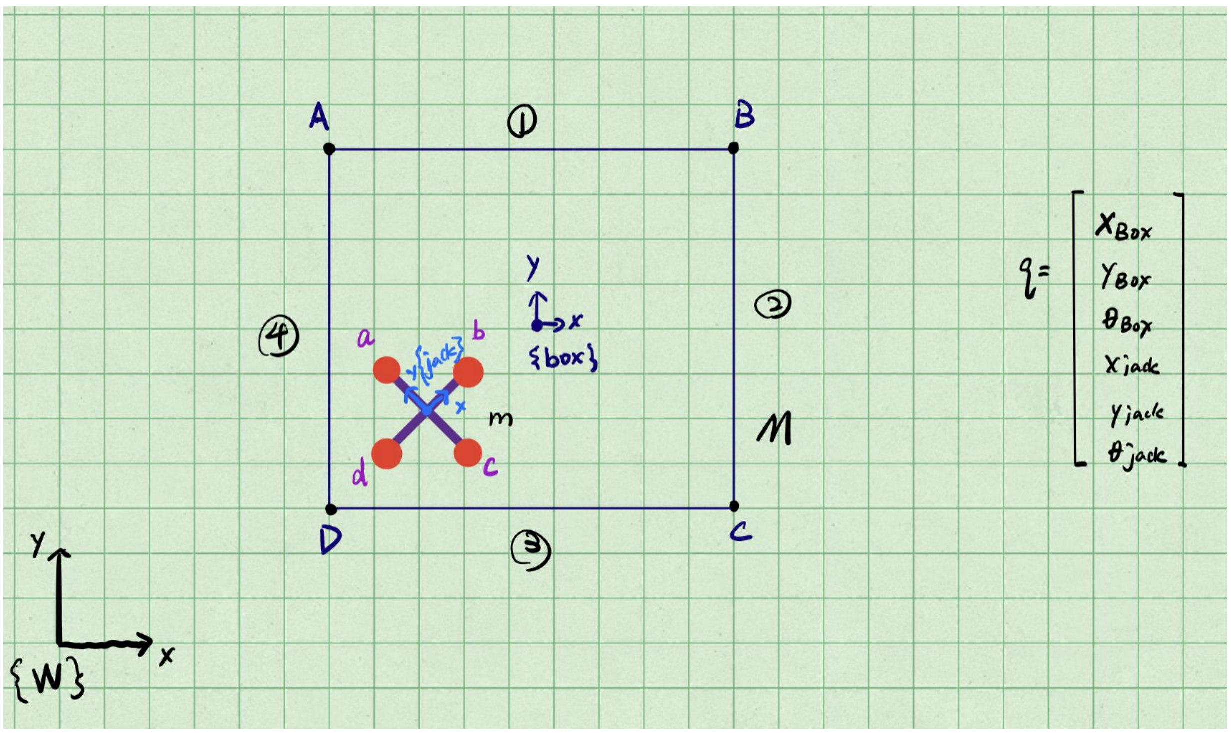

Figure 1: System Modeling

In this system, there is a world frame {W}; a box has a frame in its center of mass {box}, four vertices [A, B, C, D] and four edges [1, 2, 3, 4]; Inside the box, a jack has a frame in its center of mass {jack} and four vertices [a, b, c, d]. The configuration of the system is defined as
$q = [x_{box}, y_{box}, \theta_{box}, x_{jack}, y_{jack}, \theta_{jack}]$

Rigid Body Transformation:

1. World to box: g_{w2b}
2. Box to vertices ABCD: g_{b2A} , g_{b2B} , g_{b2C} , g_{b2D}
3. World to vertices ABCD:
   a. $g_{w2A} = g_{w2b} * g_{b2A}$

   b. $g_{w2B} = g_{w2b} * g_{b2B}$
   c. $g_{w2C} = g_{w2b} * g_{b2C}$
   d. $g_{w2D} = g_{w2b} * g_{b2D}$
4. Box to edges 1234: g_{b1}, g_{b2}, g_{b3}, g_{b4}
5. World to edges 1234:
   a. $g_{w1} = g_{w2b} * g_{b1}$
   b. $g_{w2} = g_{w2b} * g_{b2}$
   c. $g_{w3} = g_{w2b} * g_{b3}$
   d. $g_{w4} = g_{w2b} * g_{b4}$
6. World to jack: g_{w2j}
7. Jack to vertices abcd: g_{j2a}, g_{j2b}, g_{j2c}, g_{j2d}
8. World to jack vertices abcd:
   a. $g_{w2a} = g_{w2j} * g_{j2a}$
   b. $g_{w2b} = g_{w2j} * g_{j2b}$
   c. $g_{w2c} = g_{w2j} * g_{j2c}$
   d. $g_{w2d} = g_{w2j} * g_{j2d}$
9. Box to jack: $g_{b2j} = g_{w2b}^{-1} * g_{w2j}$
10. Box edges 1234 to jack:
    a. $g_{1j} = g_{b1}^{-1} * g_{b2j}$
    b. $g_{2j} = g_{b2}^{-1} * g_{b2j}$
    c. $g_{3j} = g_{b3}^{-1} * g_{b2j}$
    d. $g_{4j} = g_{b4}^{-1} * g_{b2j}$
11. Box edges 1234 to jack vertices abcd:
    a. $g_{1a} = g_{1j} * g_{j2a}$
    b. $g_{1b} = g_{1j} * g_{j2b}$
    c. $g_{1c} = g_{1j} * g_{j2c}$
    d. $g_{1d} = g_{1j} * g_{j2d}$
    e. $g_{2a} = g_{2j} * g_{j2a}$
    f. $g_{2b} = g_{2j} * g_{j2b}$
    g. $g_{2c} = g_{2j} * g_{j2c}$
    h. $g_{2d} = g_{2j} * g_{j2d}$
    i. $g_{3a} = g_{3j} * g_{j2a}$

j. $g_{3b} = g_{3j} * g_{j2b}$
k. $g_{3c} = g_{3j} * g_{j2c}$
l. $g_{3d} = g_{3j} * g_{j2d}$
m. $g_{4a} = g_{4j} * g_{j2a}$
n. $g_{4b} = g_{4j} * g_{j2b}$
o. $g_{4c} = g_{4j} * g_{j2c}$
p. $g_{4d} = g_{4j} * g_{j2d}$

Properties

M: mass of the box
L: length of each edge of the box
I_b: inertia of the box
V_b: velocity of the box
m: mass of the jack
l: length of each diagonal of the jack
I_j: inertia of the jack
V_j: velocity of the jack
$grav$: $gravitational\ acceleration\ 9.8\ m/s^2$

Calculation

$Euler - Lagrange\ Equations$:

$L = KE - PE$
Where:

$$\nu_b = g_{w2b}^{-1} * \frac{d}{dt}(g_{w2b})$$
$$\nu_j = g_{w2j}^{-1} * \frac{d}{dt}(g_{w2j})$$
$$KE = 1/2 * V_b^T * I_b * V_b + 1/2 * V_j^T * I_j * V_j$$
$$PE = m * grav * y_j$$

Note:
1. Assume there is no gravitational force for the box.
2. We have to unhat the $each\ \nu\ to\ get\ the\ corresponding\ V$ using helper function in code

$Constraint$:

There are constraints that exist to ensure the jack stays within the boundaries of the box. They are defined by:

$$\Phi_{list} = [g_{1a}[7],\ g_{1b}[7],\ g_{1c}[7],\ g_{1d}[7],\ g_{2a}[3],\ g_{2b}[3],\ g_{2c}[3],\ g_{2d}[3],$$
$$g_{3a}[7],\ g_{3b}[7],\ g_{3c}[7],\ g_{3d}[7],\ g_{4a}[3],\ g_{4b}[3],\ g_{4c}[3],\ g_{4d}[3]\ ]$$

Where:

$g_{ij}[7]$ $represents\ x\ component,$ $g_{ij}[3]$ $represents\ y\ component$

Note:

The constraint list defines the boundary condition when the vertices of the jack hits the edges of the box. Elements of the list are extracted from previous rigid body transformation.

$External\ force$:

$$F = [f_{bx}, f_{by}, f_{b\theta}, f_{jx}, f_{jy}, f_{j\theta}]$$

Where:

$$f_{bx} = 50000 * cos(\pi * t),\ f_{by} = 0,\ f_{b\theta} = 50000 * cos(\pi * t)$$
$$f_{jx} = 0,\ f_{jy} = 0,\ f_{j\theta} = 0$$

Note:

1. Applying a x-direction force and a torque on the box and using the cosine enable the force to change the direction in a certain period. Therefore, the applied external force and torque can act like shaking the box.
2. No force applied on y-direction because the jack will initially fall and hit the box due to the gravity.

$Impact\ Update\ Laws$:

Configuration updates:

$$q^{-} = [\ x_b^{-},\ y_b^{-},\ \theta_b^{-}\ , x_j^{-},\ y_j^{-},\ \theta_j^{-}\ \ ]$$

$$q^{+} = [\ x_b^{+},\ y_b^{+},\ \theta_b^{+}\ , x_j^{+},\ y_j^{+},\ \theta_j^{+}\ \ ]$$

Hamiltonian equations:

$$p = \frac{dL}{dqdot}$$
$$H = p * qdot - L$$
$$H^{-} = H(q^{-}\ )$$

$$H^{+} \;=\; H(q^{+})$$

Impact update equation:

$$\frac{dL}{dqdot^{+}} - \frac{dL}{dqdot^{-}} = \lambda \triangledown \phi(q^{-}) \qquad (1)$$

$$H^{+} - H^{-} = 0 \qquad (2)$$

Note: (1) will solve for two unknowns and (2) will solve for one unknown

Result and Discussion

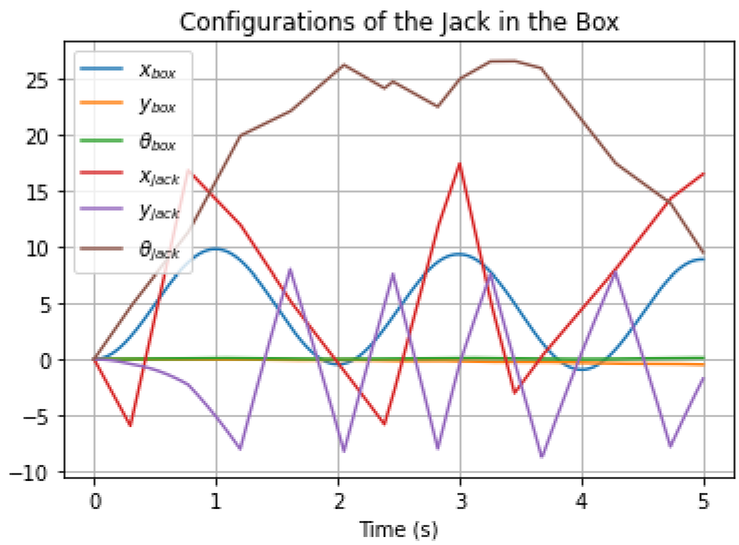

Figure 2: Simulation Result

From the simulation, we can tell the jack had an initial linear velocity in negative x direction and angular velocity in counterclockwise direction. The box moved in positive x direction initially and moved backward in x direction after a certain period of time, that is because we execute a cosine force on the box in x direction, which will change its sign periodically.

In the plot, we can see the x direction trajectory of jack changed the direction of movement and the difference range of it is nearly 20; the range of y direction of jack movement is also near to 20. This performance makes sense because the edge of the box is 20, the jack cannot move beyond this range. In addition, the x direction plot pattern of jack matches the cosine wave of the box movement, with similar period and similar time when changing the direction (when hitting).