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Homework Assignment (Problem Set) 1:

Note, Problem Set 1 directly focuses on Modules 1 and 2; Introduction to Decision Analysis and Formulation and Solving Linear Programs.

5 questions

Rubric:

All questions worth 30 points
30 Points: Answer and solution are fully correct and detailed professionally.
26-29 Points: Answer and solution are deficient in some manner but mostly correct.
21-25 Points: Answer and solution are missing a key element or two.
1-20 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

ALL CODING IS IN JUPYTER NOTEBOOK PDF BELOW

Question 1:

Clocks in Sox is a small company that manufactures wristwatches in two separate workshops, each with a single watch maker (or horologist, as they are called). Each watchmaker works a different number of hours per month to make the three models sold by Clocks in Sox: Model A, Model B, and Model C. Watchmaker 1 works a maximum of 350 hours per month while Watchmaker 2 works a maximum of 250 hours per month and the time (in hours) and cost of materials for each watch differ by watchmaker due to their experience and equipment (shown below). Each month, Clocks in Sox must produce at least 60 Model A watches, 80 Model B watches, and 50 Model C watches. Clearly formulate a linear program (LP) to minimize the cost of manufacturing the desired amount of watches.

Table 1

Workshop	Model A		Model B		Model C	
	Cost ($)	Time	Cost ($)	Time	Cost ($)	Time
Watchmaker 1	10	2	11	4	12	3
Watchmaker 2	9	9	10	4	13	7

Answer:

In order to minimize manufacturing cost while reaching production goals without exceeding watchmaker work hour maximums, Worker 1 should complete 60, 17.5, and 50 hours for Models A, B, and C respectively. Worker 2 should only work on Model B for a total of 62.5 hours. With this schedule they will be able to meet an optimal minimization of $2,017.50 as cost for production.

Question 2:

Consider the following linear program:

Min $Z = -9x_1 + 18x_2$
Subject To
$-x_1 + 5x_2 \geq 5$
$x_1 + 4x_2 \geq 12$
$x_1 + x_2 \geq 5$
$x_1 \leq 5$ and $x_1, x_2 \geq 0$

Part A: Write the LP in standard equality form.

Answer:

Objective Function: $-9x_1+18x_2$
Subject To:
$-x_1 + 5x_2 - S_1 = 5$
$x_1 + 4x_2 - S_2 = 12$
$x_1 + x_2 - S_3 = 5$
$x_1 + S_4 = 5$
$x_1, x_2 \geq 0$
$S_1, S_2, S_3, S_4 \geq 0$

Part B: Solve the original LP graphically (<u>to scale</u>). Clearly identify the <u>feasible region</u> and, if one or more exist, the <u>optimal solution(s)</u> (provide exact values for x_1, x_2, and Z).

Answer:

The optimal minimum for the linear programming model is 9, with values x1 and x2 being 5, and 2 respectively. The feasible region that satisfies the constrains provided is represented on the graph.

Question 3:

InvestCo currently has $500 in cash. InvestCo receives revenues at the start of months 1 – 4, after which it pays bills (see Table 2 below). Any money left over should be invested and interest for one month is 0.5%, two months is 2%, three months is 4%, and four months is 8% (total - no compounding). Use linear programming to determine an investment strategy that maximizes cash on hand at the beginning of month 5. Formulate an LP to maximize InvestCo's profit. Do not solve.

Hint: What is coming in and what is going out each month?

Table 2

Month	Revenues ($)	Bills ($)
1 (x_1)	600	700
2 (x_2)	900	400
3 (x_3)	300	700
4 (x_4)	500	350

Answer:

Maximize: Z=C5

M1: 500+600-(700+X1,1 + X1,2 + X1,3 +X1,4)
M2: (M1+.005M1,1+900) –(400+X2,1 + X2,2 + X2,3)
M3: (M2 + 1.02X1,2 + 1.005X2,1 + 300) – (700+X3,1 +X3,2)
M4: (M4 + 1.04X1,3 +1.02X2,2 +.1.005X3,1 +500) – (350-+X4,1)
M5: (M4 + 1.08X1,4 + 1.04X2,3 + 1.02X3,2 + 1.005X4,1)

M1, M2,M3,M4,X1,1, X1,2,X1,3,X1,4,X2,1,X2,2,X2,3,X3,1,X3,2,X4,1>=0

Subject To:
X1,1: Cash investment M1
X1,2: Cash invest M1 for 2 months
X1,3: Cash investment M1 for 3 months
X1,4: Cash investment M1 for 4 months
X2,1: Cash investments M2 for 1 months
X2,2: Cash investments M2 for 2 months
X2,3: Cash investments M2 for 3 months
X3,1: Cash investments M3 for 1 months
X3,2: Cash investments M3 for 2 months
X4,1: Cash investments M4 for 1 month

Question 4:

Floor is Java sells premium coffee to restaurants. They sell two roasts which they call (cleverly) Roast 1 and Roast 2, each of which is a blend of Columbian and Arabica coffee beans. Columbian beans cost $20 for a 5 pound box while Arabica beans cost $15 for a 6 pound box. Roast 1 sells for $6 per pound and must be at least 75% Columbian beans, while Roast 2 sells for $5 per pound and must be at least 60% Columbian beans. At most, 40 pounds of Roast 1 and 60 pounds of Roast 2 can be sold each month.

Part A: Formulate an LP to maximize Floor is Java's profit.

Answer:

Objective Function: 6(CB1+AB1) + 5(CB2+AB2) - (20/5 *(CB1+CB2) + (15/6 *(AB1+AB2)

Subject To:
CB1 >= 0.75 * (CB1 + AB1)
CB2 >= 0.60 * (CB2 + AB2)
CB1 + AB1 <= 40
CB2 + AB2 <= 60

Part B: Solve the LP (provide **exact** values (do not restrict to integer) for all variables and the optimal objective function).

Answer:

The optimal solution to maximize profit and satisfy constraints is for roast 1 to consist of 30 pounds of Columbian beans and 10 pounds of Arabica beans (total of 40 pounds of R1 sold). Roast 2 should contain 36 pounds of Columbian coffee beans and 24 Arabica coffee beans (total of 60 pounds of R2 sold). This will lead to a maximized profit of $191.00.

Question 5:

Food Beach, a local grocery store, is building a work schedule for its stockers and has specific requirements over each 24 hour period (shown in the table below). Each stocker must work two consecutive shifts.

Shift	Workers
Midnight - 4 am	8
4 am - 8 am	7
8 am - Noon	5
Noon - 4 pm	4
4 pm - 8 pm	4
8 pm - Midnight	7

Part A: Formulate an LP model to minimize the number of workers required to meet requirements.

Answer:

Objective Function: x1 + x2 + x3 + x4 + x5 + x6

x1 = LpVariable("x1", 0, None) #12am-4am
x2 = LpVariable("x2", 0, None) #4am-8am
x3 = LpVariable("x3", 0, None) #8am-12pm
x4 = LpVariable("x4", 0, None) #12pm-4pm
x5 = LpVariable("x5", 0, None) #4pm-8pm
x6 = LpVariable("x6", 0, None) #8pm-12am

Subject To:
x6 + x1 >= 8
x1 + x2 >= 7
x2 + x3 >= 5
x3 + x4 >= 4
x4 + x5 >= 4
x5 + x6 >= 7

Part B: Solve the LP (provide exact values for all variables and the optimal objective function).

Answer:

To minimize the number of workers each shift has to factor in the previous employees who are eligible to work the next shift. The optimal minimum value of employees needed to cover shifts is 18. Starting 8pm-12pm the shift will need all 7 people, 12am-4am will need to add 1 employee, 4am-8am will add 6 new as only one from the previous eligible to work another consecutive shift. 8am-12pm will be covered by those already there completing their second shift. 12pm-4pm will require 4 new employees, and 4pm-8pm will not need any new employees to cover.