Homework Assignment (Problem Set) 4:

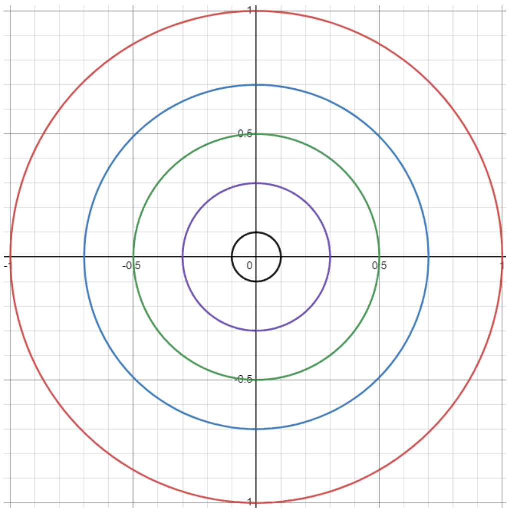
Note, Problem Set 4 directly focuses on Modules 7 and 8: Metaheuristic Algorithms and Monte Carlo Simulation

5 questions

Rubric:

All questions worth 30 points
30 Points: Answer and solution are fully correct and detailed professionally.
26-29 Points: Answer and solution are deficient in some manner but mostly correct.
21-25 Points: Answer and solution are missing a key element or two.
1-20 Points: Answer and solution are missing multiple elements are significantly deficient/incomprehensible.
0 Points: No answer provided.

1. Perform a Monte Carlo simulation to estimate the probability of hitting each section of a dartboard (shown below) and the long-term average score of the player. Assume that any dart thrown at the dartboard will hit somewhere on the space. Generate N pairs of random numbers (x,y) and use the equation of a circle ($x^2 + y^2 = r^2$) to determine which space a given dart hits. For example, if your random number is (0.4,0.6), we know that $r = \sqrt{0.4^2 + 0.6^2} = 0.721$, which equates to a 1 point shot between the blue and red circles. The radius and point value for each section is also shown below.



Circle Color	Radius	Points (if within)
Red	1	1
Blue	0.7	2
Green	0.5	3
Purple	0.3	4
Black	0.1	5

Part A: Determine the probability of hitting each section of the dartboard, to include hitting the section outside of the red circle.

Answer:

Red: 40.22%
Blue: 18.7%
Green: 12.63%
Purple: 6.3%
Black: .8%
Outside: 21.07 %

Part B: Determine the long-term average score per shot of a player.

Note: Ensure you use a large enough value for N in your simulation – use anywhere between 400 and 20,000.

Answer: The long term average score is 4.16.

2. A bicycle shop, Take a Bike, offers bonuses to its sales team for selling more than 4 bicycles in a day. Each salesperson can sell between 0 and 8 bikes per day and has a 40% probability of selling more than 4 bicycles in any given day (60% probability of selling 4 or fewer). If the salesperson sells more than 4 bikes, the probability of selling 5, 6, 7, or 8 bikes is shown below. The bonus that is paid is dependent on the model of each bike sold, each of which has a different probability of sale and bonus payout (also shown below). If the salesperson sells more than 4 bikes, the bonus is paid for each of the bikes sold, not just the number above 4. So if a salesperson sells 6 bikes, the bonus is paid for each of those 6 bikes, not just the 2 above the threshold. Develop a simulation model to calculate the bonus a salesperson can expect in a day. Do not simply calculate expected values for this – use random numbers to determine the number and types of bikes sold. Ensure you iterate your simulation multiple times (between 400 and 20,000).

Number sold	Probability
5	40%
6	35%
7	20%
8	5%

Model	Portion of total sales	Bonus
A	45%	$ 10
B	35%	$ 15
C	15%	$ 25
D	5%	$ 30

Answer:

Expected daily bonus is $35.27, or around $35.

3. Jim is investing in his company's 401(k) retirement plan, funding 6% of his salary to get a 3% match (thus effectively investing 9% of his annual salary). He invests in each of the three available funds. 50% of his contributions go into investment A, which has an average return of 6.91% with a standard deviation of 12.89%. The rest of his contributions are equally divided between investment B, which has an average return of 8.94% with a standard deviation of 15.21%, and investment C, which has an average return of 9.88% with a standard deviation of 17.14%. Jim is currently 24 years old and earns $55,000 this year, but anticipates a pay raise of, on average, 2.83% with a standard deviation of 0.72%. Develop a simulation model to predict his 401(k) balance at age 60. Ensure you iterate your simulation multiple times (between 400 and 20,000).

Answer:

Expected 401K balance at 60 is $327,484.67

4. Develop a simple heuristic (I recommend a construction or destruction heuristic) in either R or Python to solve the following knapsack problem: (Note, this problem can be solved to optimality using integer programming; however, the focus of this question is on developing a heuristic and metaheuristic). Then, develop a metaheuristic using your heuristic as a subroutine. You can iterate the heuristic *n* times and return the best solution, prevent previous solutions from being selected, etc. I am evaluating your ability to find a solution quickly with a heuristic and your approach to *improve* the heuristic with a metaheuristic.

Maximize	$12x_1 + 16x_2 + 22x_3 + 8x_4$
S.T.	$4x_1 + 5x_2 + 7x_3 + 3x_4 \leq 140$
	$0 \leq x_i \leq 10$ and x_i is integer

The best value is 516. Best solutions for x1, x2, x3, and x4 are 10,10,2, and 10 respectively.
-OR-
The best value is 438. Best solutions for x1, x2, x3, and x4 are 6,10,9, and 1 respectively.

5. Develop a simulated annealing procedure in either R or Python to solve the same knapsack problem: (Note, this problem can be solved to optimality using integer programming; however, the focus of this question is on developing the simulated annealing method). Do not simply return a shell code from a web search, but try to implement the simulated annealing metaheuristic for this specific problem.

Maximize	$12x_1 + 16x_2 + 22x_3 + 8x_4$
S.T.	$4x_1 + 5x_2 + 7x_3 + 3x_4 \leq 140$
	$0 \leq x_i \leq 10$ and x_i is integer

Answer: The best value is 156, and best solution is 1,7,0,4.