Homework Assignment (Problem Set) 3:

Note, Problem Set 3 directly focuses on Modules 5 and 6: Integer Programs, Nonlinear and Multi-objective Programming.

5 questions

SEE ATTACHED PYTHON CODE

1. An engineer at Fertilizer Company has synthesized a sensational new fertilizer made of just two interchangeable basic raw materials. The company wants to take advantage of this opportunity and produce as much as possible of the new fertilizer. The company currently has $180 to buy raw materials at a unit price of $8 and $5 per unit, respectively. When amounts x_1 and x_2 of the basic raw materials are combined, a quantity q of fertilizer results given by: $q = 6x_1 + 4x_2 - 0.25x_1^2 - 0.125x_2^2$

Part A: Formulate as a constrained nonlinear program. Clearly indicate the variables, objective function, and constraints.

Answer:

Obj Function: 6x1 +4x2 -.25x2/1 -.125x2/2 >= 180

Variables:
Cost of Raw Material x1: $8
Cost of Raw Material x2: $5
Amount to Spend: $180

Constraints:
8x1+5x2 <= 180

Part B: Solve the Program (provide exact values for all variables and the optimal objective function).

Answer:

The optimal amounts of raw material 1, and raw material 2 are 12, and 16 units respectively for a maximum quantity of 68 units of fertilizer.

2. A neighbor is looking to build a rectangular fenced enclosure for his chickens and wants to build using fencing he found in his local farm supply store. He can buy at most 120 feet of fencing, and the price of the fencing is the square root of the length purchased. So, he can purchase 100 feet of fencing for $10, 64 feet for $8, etc. He must also purchase fence posts to reinforce the fencing, and due to wind conditions in his yard, he must purchase more posts for east-west fencing than north-south fencing. The cost for fence and posts in the east-west direction is $3 per foot while it is $2 per foot in the north-south direction. You do not need to consider the number of posts, but simply the cost of the fence and reinforcing posts. He has $25 to spend on fencing and $150 to spend on fence posts.

Part A: Formulate the problem as a constrained nonlinear program that will enable us to maximize the area of the fenced area, with constraints. Clearly indicate the variables, objective function, and constraints.

Answer:

Area: (x3/2)*(x2/2)

Variables:
x1: length of fencing purchased
x2: length of fence East-West
x3: length of fence North-South

Constraints:
x1 <= 120
√x1<= 25
(3x2-√ (x1) +(2x3-√(x1) <= 150
x1,x2,x3 >= 0

Part B: Solve the program (provide exact values for all variables and the optimal objective function).

The maximized area of the fenced are is 289.38 feet, with North/South fencing length at 42.10 feet and East/West fencing length at 27.48 feet.

3. Toy-Vey makes three types of new toys: tanks, trucks, and turtles. It takes two hours of labor to make one tank, two hours for one truck, and one hour for a turtle. The cost of manufacturing one tank is \$7, 1 truck is \$5 and 1 turtle is \$4; a target budget of \$164,000 is initially used as a guideline for the company to follow. Material requirements for the toys are shown below.

Toy	Plastic	Rubber	Metal
Tank	2	1	2
Truck	3	1	1
Turtle	4	2	0
Available	16,000	5,000	9000

Management has ranked three goals it wishes to achieve, arranged from highest to lowest priorities.

a) Minimize labor hours over 10,000 hours a week for production (40 hours for each of the 250 employees)
b) Minimize over-utilization of the weekly available supply of materials used in making the toys and place twice as much emphasis on the plastic
c) Minimize the under and over-utilization of the budget. Maximize available labor hour usage

Formulate the above decision problem as a single linear goal program. Do not solve.

*Bonus (5 points): Solve the problem and give the number of each toy to produce as well as any violations of the goals (weights don't have to add up to 1; use simple weights – 1, 2, 3, etc.).

Answer:

Budget Obj: 7(x1+x11+x21) +5(x2+x12+x22) +4(x3+x13+x23) <= 164,000

Variables:

x1 = plastic tank
x2 = plastic truck
x3 = plastic turtle

x21: metal tank
x22: metal truck
x23: metal turtle

x11: rubber tank
x12: rubber truck
x13: rubber turtle

Constraints

2(x1+x11+x21) + 2 (x2+x12+x22) + (x3+x13+x23) <= 10,000 labor constraint
2x1 + 3x2 + 4x3 <= 16,000 plastic constraint
x11 + x12 + 2x13 <= 5,000 rubber constraint
2x21 + x22 <= 9,000 metal constraint

Goal Programming Form

$7(x1+x11+x21) +5(x2+x12+x22) +4(x3+x13+x23) + d1^+ – d1^- = 164000$
$2x1 + 3x2 + 4x3 + d2^+ – d2^- = 16000$
$x11 + x12 + 2x13 + d3^+ – d3^- = 5000$
$2x21 + x22 + d4^+ – d4^- = 9000$
$2(x1+x11+x21) + 2 (x2+x12+x22) + (x3+x13+x23) + d5^+ – d5^- = 10,000$

Minimize Obj Function:

$\mathbf{(d1^+ + d1^-) + 5(d2^+) + 4(d3^+) + 2(d4^+) + 2(d5^-) + (d2^-)}$

4. Breaking Ad is planning its advertising campaign for a customer's new product and is going to leverage podcasts and YouTube for its advertisements. The total number of exposures per $1,000 is estimated to be 10,000 for podcasts and 7,500 for YouTube. The customer sees the campaign as successful if 750,000 people are reached and consider the campaign to be superbly successful if the exposures exceed 1 million people. The customer also wants to target its two largest age groups: 18 – 21 and 25 – 30. The total number of exposures per $1,000 for these age groups are shown below.

Age Group	Podcasts	YouTube
18 - 21	2,500	3,000
25 - 30	3,000	1,500
Total Exposures	10,000	7,500

Management has ranked five goals it wishes to achieve, arranged from highest to lowest priorities.

a) Successful campaign – at least 750,000 exposures
b) Limit advertising costs to $100,000.
c) Limit podcast advertising costs to $70,000
d) Superbly successful campaign – at least 1 million exposures
e) Achieve at least 250,000 exposures for each of the two age groups. However, as the 25 – 30 age group has more buying power, double the emphasis on this age group over the 18 – 21 age group

Formulate the above decision problem as a single linear goal program. Do not solve.

*Bonus (5 points): Solve the problem and give the expenditures for each media advertising campaign as well as any violations of the goals (weights don't have to add up to 1; use simple weights – 1, 2, 3, etc.).

Variables:
x1 = podcast advertisement money (thousandths), x2 = YouTube advertisement money (thousandths)

Contraints:
$10x1 + 7.5x2 >= 750$
$x1 + x2 <= 100$
$10x1 <= 70$
$10x1 + 7.5x2 >= 1000$
18-21 years: $2.5x1 + 3x2 >= 250$
25-30 years: $3x1 + 1.5x2 >= 250$
$x1, x2 \geq 0$

Goal Programming Form
Priority 1 - Minimum Total Advertising Exposures:
$10.x1 + 7.5x2 + d1^{+} – d1^{-} = 750$
Priority 2 - Total Advertising:
$10x1 + 7.5x2 + d2^{+} – d2^{-}= 100$
Priority 3 - Podcast Advertising:
$10x1 + d3^{+} – d3^{-}= 70$
Priority 4 - Successful Total Advertising:
$10x1 + 7.5x2 + d4^{+} – d4^{-} = 1000$
Priority 5 - Age Group Advertising:
25-30 years: $3x1 + 1.5x2 + d5^{+} – d5^{-} = 250$
18-21 years: $2.5x1 + 3x2 + d6^{+} – d6^{-} = 250$
$x1, x2 \geq 0$

Maximize: $5(d1^{-})+4(d2^{+})+3(d3^{+})+2(d4^{-})+2(d5^{+})+(d6^{+})$

5. A local farmer's market sells, among other things, fresh apples during the harvest season. The market has $750 to purchase bushels of apples from orchard 1 at $5 per bushel, orchard 2 at $6 per bushel, or orchard 3 at $8 per bushel. However, the quality of the apples varies by orchard and the market can earn (in profit) $10 per bushel from orchard 1, $11 per bushel from orchard 2, and $20 per bushel from orchard 3. Orchard 3 is selective with its sales and will only sell between 20 and 40 bushels to the market. That is, it will not sell to the farmer's market if they order fewer than 20 bushels and will not sell more than 40 bushels to the market. Further, orchard 1 only has 50 bushels available to sell.

Part A: Formulate this as a linear or nonlinear program by using an indicator variable to maximize the market's profit

Obj Function: 10x1 + 11x2 + 20x3

Variables :
x1 = orchard 1
x2 = orchard 2
x3 = orchard 3
y = binary indicator variable (0,1)

Contraints:
5x1 + 6x2 + 8x3 <= 750
20 <= x3 <= 40
x1 <= 50

Part B: Solve the problem and give the solution (decision variables and objective function).

Answer: The optimal solution for a maximized profit is for 50,30, and 40 bushels to be purchased for orchard 1,2 and 3 respectively for a maximized profit of $1630.