

What follows is my submission for Project 4 in Machine Learning. The goal was to answer Problem 2.24 in *Learning From Data*.^[1]

PROBLEM 2.24^[1]

Consider a simplified learning scenario. Assume that the input dimension is one. Assume that the input variable x is uniformly distributed in the interval $[-1, 1]$. The data consists of 2 points $\{x_1, x_2\}$ and assume that the target function is $f(x) = x^2$. Thus, the full data set is $\mathcal{D} = \{(x_1, x_1^2), (x_2, x_2^2)\}$. The learning algorithm returns the line fitting these two points as g (\mathcal{H} consists of functions of the form $h(x) = ax + b$). We are interested in the test performance ($\mathbb{E}[E_{\text{out}}]$) of our learning system with respect to the squared error measure, the **bias** and the **var**.

1. Give an analytic expression for the average function $\bar{g}(x)$.
2. Describe an experiment that you could run to determine (numerically) $\bar{g}(x)$, $\mathbb{E}[E_{\text{out}}]$, **bias**, and **var**.
3. Run your experiment and report the results. Compare $\mathbb{E}[E_{\text{out}}]$ (expectation is with respect to data sets) with **bias** + **var**. Provide a plot of your $\bar{g}(x)$ and $f(x)$ (on the same plot).
4. Compute analytically what $\bar{g}(x)$, **bias**, and **var** should be.

The solution begins on the following page and includes all of my data, code, and plots.

SOLUTION 2.24

1. Since $\bar{g}(x)$ depends on \mathcal{D} , it is the expected value, or mean, of hypotheses over all possible data sets \mathcal{D} . Since the data sets are drawn at random from $[-1, 1]$, both x_1 and x_2 have mean 0. Intuitively $\bar{g}(x) = 0$ because everything is uniform over $[-1, 1]$.

$$\left. \begin{aligned} \bar{g}(x) &= \mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(x)] \\ g(x) &= y = ax + b \end{aligned} \right\} g^{(\mathcal{D})}(x) = a^{(\mathcal{D})}x + b^{(\mathcal{D})}$$

$$\left. \begin{aligned} a^{(\mathcal{D})} &= \frac{x_2^2 - x_1^2}{x_2 - x_1} = x_2 + x_1 \\ b^{(\mathcal{D})} &= x_1^2 - a^{(\mathcal{D})}x_1 = x_1^2 - (x_2 + x_1)x_1 \\ &= -x_2x_1 \end{aligned} \right\} \begin{aligned} g^{(\mathcal{D})}(x) &= (x_2 + x_1)x + x_1^2 - (x_2 + x_1)x_1 \\ &= x_2x + x_1x - x_2x_1 \end{aligned}$$

Fact 1. If $h(x)$ is a function then $Y = h(X)$ is a random variable and

$$\mathbb{E}[Y] = \mathbb{E}[h(X)] = \int h(x)F_X dx$$

The \mathcal{D} are continuous, $\bar{g}(x)$ is as well. Also, x, x_1, x_2 are all drawn uniformly randomly from $[-1, 1]$ so their probability density functions are

$$F_{x_1} = F_{x_2} = \frac{1}{b-a} = \frac{1}{1-(-1)} = \frac{1}{2}$$

So finally,

$$\begin{aligned} \bar{g}(x) &= \mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(x)] \\ &= \iint_{\mathcal{D}} g^{(\mathcal{D})}(x) F_{\mathcal{D}} F_X d\mathcal{D} = \iint_{\mathcal{D}} x_2x + x_1x - x_2x_1 F_{\mathcal{D}} d\mathcal{D} \\ &= \int_{x_2} \int_{-1}^1 x_2x + x_1x - x_2x_1 F_{X_1} F_{X_2} dx_1 dx_2 \\ &= \int_{x_2} \frac{1}{2} \left[x_2xx_1 + x \frac{1}{2}x^2 - x_2 \frac{1}{2}x_1^2 \right]_{-1}^1 F_{X_2} dx_2 \\ &= \int_{-1}^1 x_2x F_{X_2} dx_2 = \frac{x}{4} x_2^2 \Big|_{-1}^1 \\ &= 0 \end{aligned}$$

2. The experiment is essentially to run the learning algorithm on a large number of datasets, find the hypothesis for each data set: $g^{\mathcal{D}_i}(x) = \mathcal{A}(\mathcal{D}_i)$ and then average all those hypotheses to get $\hat{g}(x)$ (write \hat{g} not \bar{g} because this is a sample average). Then calculate **bias** and **var**, and thus $E_{\text{test}} \approx \mathbb{E}[E_{\text{out}}]$.

1 NOTATION

- (a) Each \mathcal{D}_i determines a trial which returns $(a^{\mathcal{D}_i}, b^{\mathcal{D}_i})$
- (b) An experiment set \mathcal{E} is a set of trials \mathcal{D}_i
- (c) The sample average function $\hat{g}(x) \approx \bar{g}(x)$
- (d) $\hat{\mathcal{D}}$ denotes a set of x values drawn independently from the same distribution as the \mathcal{D}_i 's (but is not necessarily identical to, or disjoint from, their union)
- (e) A numerical experiment delivers values for \hat{g} , **bias**, **var**, and E_{test}
- (f) Many numerical experiments are run and plotted

2 DESCRIPTION OF A NUMERICAL EXPERIMENT

- (a) For $g^{\mathcal{D}_i}(x) = x_2x + x_1x - x_2x_1$, generate two random x -values to define \mathcal{D}_i and find the line that joins them.
- (b) For $\hat{g}(x) \approx \bar{g}(x)$,

$$\hat{g}(x) = \frac{1}{|\mathcal{E}|} \sum_{\mathcal{D} \in \mathcal{E}} g^{\mathcal{D}}(x)$$

generate many $g^{\mathcal{D}_i}$ and take their mean.

- (c) For **bias** $\approx \mathbb{E}_x[\text{bias}(x)]$, after finding $\hat{g}(x) = ax + b$, take the mean of $(\hat{g}(x) - x^2)^2$ over many samples of x :

$$\text{bias} = \frac{1}{|\hat{\mathcal{D}}|} \sum_{x \in \hat{\mathcal{D}}} (\hat{g}(x) - x^2)^2$$

```

1 # Take the bias over a new data set
2 def getBias(x_values, averageFunction):
3     evaluation = evaluateLineAtX(x_values, averageFunction)
4     with np.errstate(over=overflow_err_state):
5         try:
6             deviation = evaluation - np.square(x_values)
7             bias = np.mean(np.square(deviation))
8         except FloatingPointError:
9             print("Catching overflow in getBias")
10            raise FloatingPointError
11    return bias

```

Listing 1: Finding **bias**

- (d) For $\mathbf{var} \approx \mathbb{E}_x[\mathbf{var}(x)]$, save $g^{\mathcal{D}}(x)$ from each trial, sum all $(g^{\mathcal{D}}(x) - \hat{g}(x))^2$, divide by $(|\mathcal{E}| - 1)$, and then take the mean of the results of evaluating this new function for many x .

$$\mathbf{var} = \frac{1}{|\hat{\mathcal{D}}|} \sum_{x \in \hat{\mathcal{D}}} \left(\frac{1}{|\mathcal{E}| - 1} \sum_{\mathcal{D} \in \mathcal{E}} (g^{\mathcal{D}}(x) - \hat{g}(x))^2 \right)$$

```

1 # Find the sample variance over test data
2 def getVar(x_values, hypothesis_set, averageFunction):
3     num_hyps = len(hypothesis_set)
4     deviation = hypothesis_set - averageFunction
5     error_matrix = np.tensordot(deviation[:, 0], x_values, 0) +
6     deviation[:, 1:]
7     with np.errstate(over=overflow_err_state):
8         try:
9             error_matrix = np.square(error_matrix)
10            except FloatingPointError:
11                print("Catching overflow in getVar")
12    mean_over_x = np.mean(error_matrix, 1)
13    return np.sum(mean_over_x) / (num_hyps - 1)

```

Listing 2: Finding \mathbf{var}

- (e) For $E_{\text{test}} \approx \mathbb{E}[E_{\text{out}}]$, evaluate all $(g^{\mathcal{D}_i}(x) - f(x)^2)$ at many x , then take the mean.

$$E_{\text{test}} = \frac{1}{|\hat{\mathcal{D}}|} \sum_{x \in \hat{\mathcal{D}}} \left(\frac{1}{|\mathcal{E}|} \sum_{\mathcal{D} \in \mathcal{E}} (g^{\mathcal{D}}(x) - x^2)^2 \right)$$

```

1 # E[E_out] as mean approximation of expectation over data sets and
2   test data
3 def getEEout(x_values, hypothesis_set):
4     evaluation = np.tensordot(hypothesis_set[:, 0], x_values, 0) +
5     hypothesis_set[:, 1:]
6     with np.errstate(over=overflow_err_state):
7         try:
8             deviation = evaluation - np.square(x_values)
9             error_matrix = np.square(deviation)
10            except FloatingPointError:
11                print("Catching overflow in getEEout")
12    return np.mean(error_matrix)

```

Listing 3: Finding E_{test}

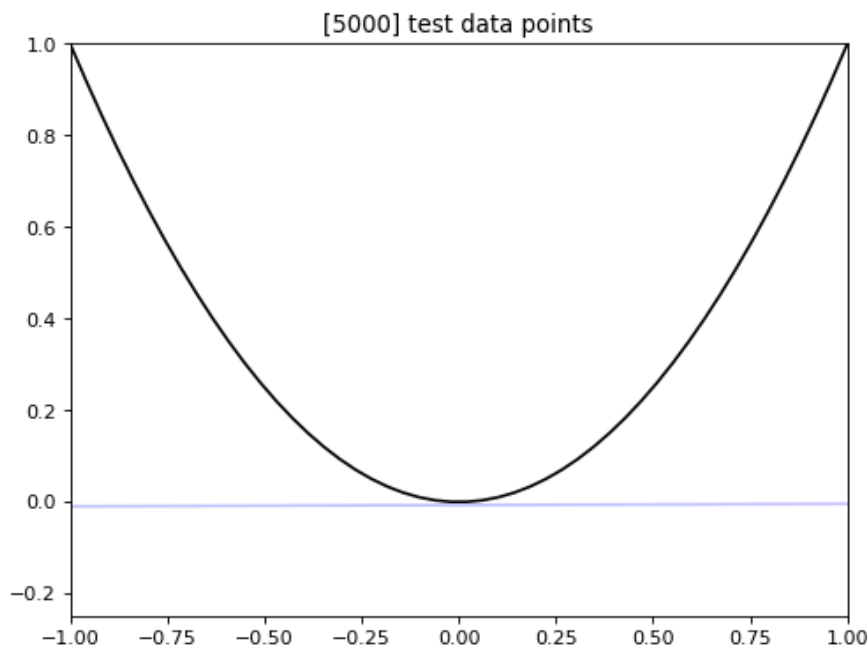
My experiment has three parts:

- (a) Running the experiment for 5,000 training sets and 5,000 test points.
- (b) Running the experiment for increasing numbers of trials (training data sets) with a fixed test data set and seeing if there is a trend. I did this millions of times before settling on the exact sequence of trial sizes used for the final data.
- (c) Running the experiment for increasing numbers of test data with a fixed number of trials (training data sets) and seeing if there is a trend. I did this millions of times before settling on the exact sequence of trial sizes used for the final data.

Parts (b) and (c) were ran three times for different fixed test data and trials, respectively, and each of those runs were repeated to weed out atypical cases. The first part was done for 2500, 5000, and 10,000 points test points. The second part was done for 25, 50, and 100 trials.

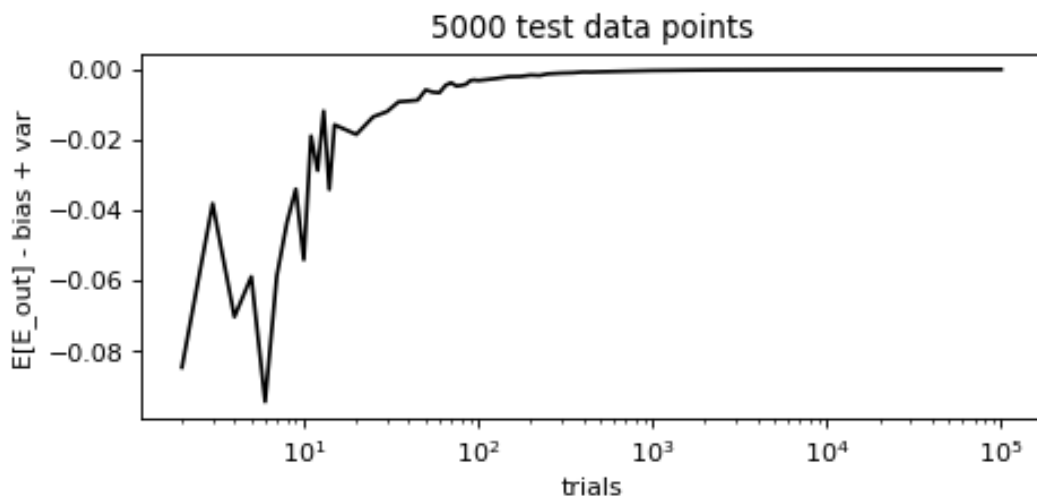
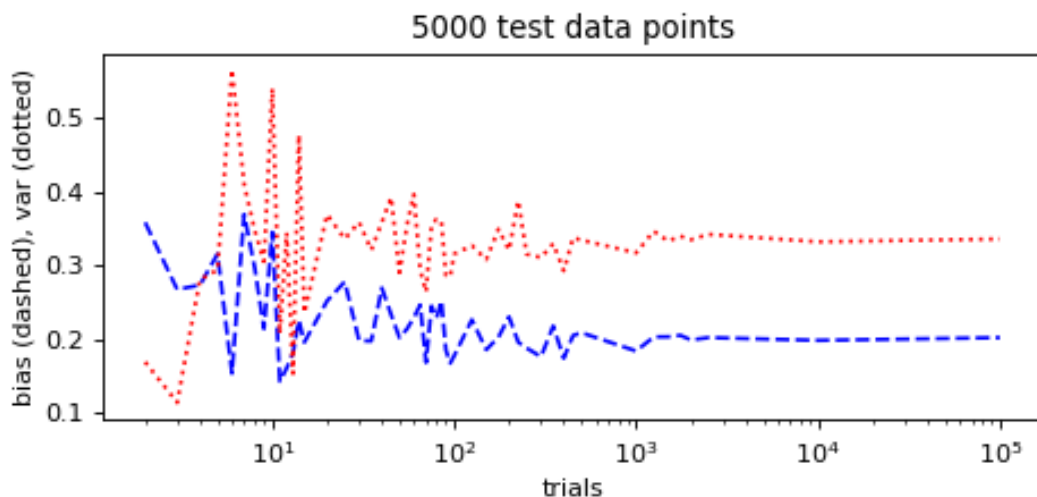
Training data was created randomly and differently with every trial. Test data was created randomly in a fixed sequence. The algorithm caused floating point overflow in some cases. Those exceptions were caught and the trials were restarted with new training or test data for up to fifteen attempts. In the final runs every case completed.

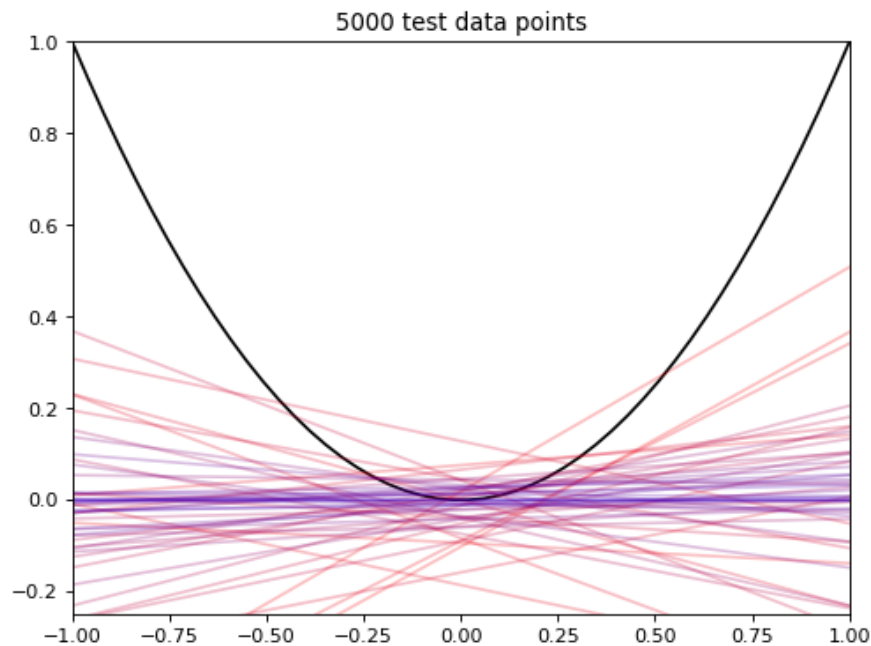
- (a) This resulted in a line $\hat{g}(x) = 0.0027828607607221405x + -0.006683991780068334$ which is very close to the expected value $\bar{g}(x) = 0$. The other values are:
 - i. **bias** = 0.2069807421246095
 - ii. **var** = 0.3437476483550565
 - iii. $\mathbb{E}[E_{test}] = 0.5506596409499945$**bias** + **var** and $\mathbb{E}[E_{test}]$ differ by only $6.874952967145243 \times 10^{-5}$.



- (b) Running these trials showed that the main indicator of result quality was, obviously, the number of training sets. With less than a hundred sets the average function, **bias**, and **var** varied wildly. Around a thousand sets it stabilized significantly near the numbers reported in (a) above. This is easily seen in the following plots. The results for each of the three runs (2500, 5000, and 10,000 test points) were not substantially different so only the plots for 5000 points are shown here. The others are attached at the end.

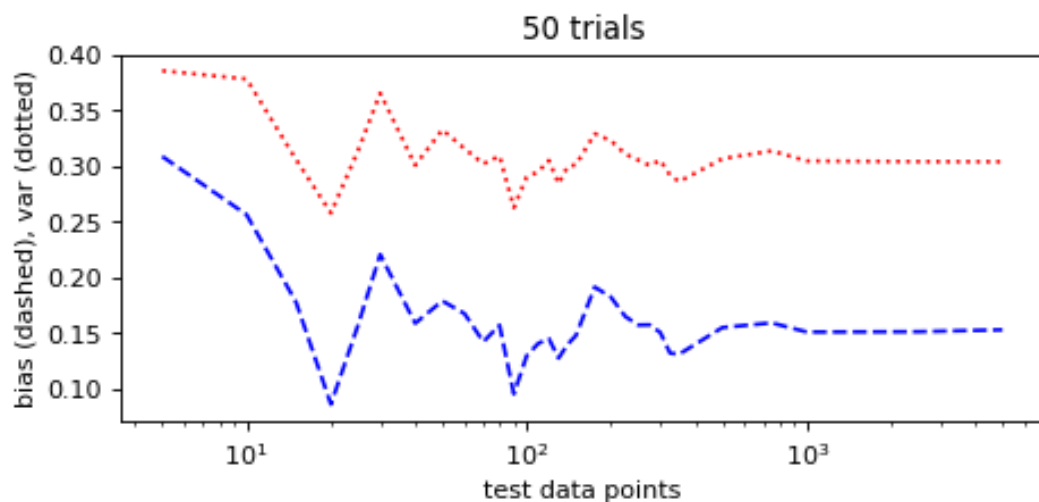
The first plot shows **bias** as a blue dashed line, and **var** as a red dotted line. The second plot shows $\mathbb{E}[E_{test}] - (\mathbf{bias} + \mathbf{var})$ (it is slightly mislabeled). On the third plot, a shift in color from red to blue indicates increasing number of data sets used to generate the average function, hence the thick purple band centered near zero.

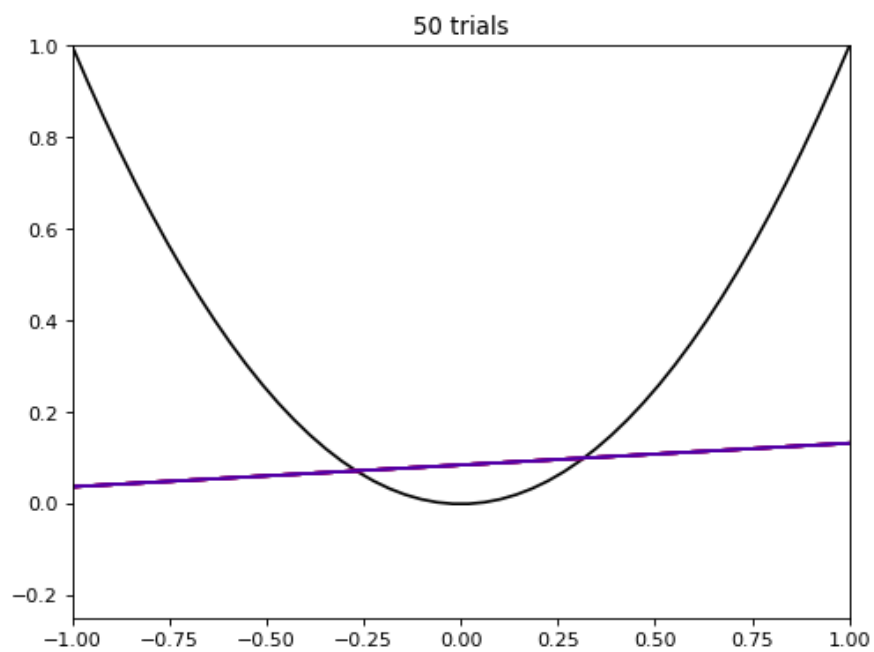
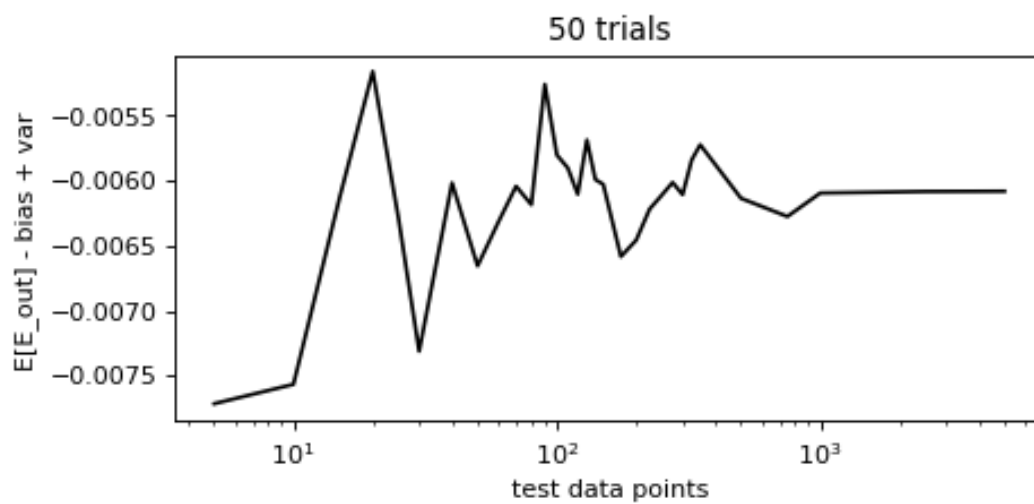




- (c) Running these trials showed, unsurprisingly, that having a low number of trials (average functions) produced poor results. This can be seen in part (b) as well, but is especially clear here. I ran this part to see if there were any interesting trends in **bias** and **var**. **var** was always greater than **bias** and it's shape matched **bias** more closely than in part (b).

In the typical case, **bias** and **var** were more unstable than in part (b). Interestingly, they stabilized around 1,000 test points. In part (b) they stabilized around 1,000 trials (average functions).





3. Analytic Solutions

(a) The value of **bias** is:

$$\mathbb{E}_x[\mathbf{bias}(x)] = \int_x (\bar{g}(x) - x^2)^2 \frac{1}{2} dx = \int_x (-x^2)^2 \frac{1}{2} dx = \frac{1}{2} \int_{-1}^1 x^4 dx = \frac{1}{2} \frac{x^5}{5} \Big|_{-1}^1 = \frac{1}{5}$$

(b) The value of **var** is:

$$\begin{aligned} \mathbf{var} &= \mathbb{E}_x \mathbb{E}_{\mathcal{D}}[(g^{\mathcal{D}}(x) - \bar{g}(x))^2] = \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[g^{\mathcal{D}}(x)] = \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[(x_2 x + x_1 x - x_2 x_1)^2] \\ &= \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1^2 x^2] + \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_2^2 x^2] \\ &\quad - 2\mathbb{E}_{\mathcal{D}}[x_1^2 x_2] \mathbb{E}_x[x] - 2\mathbb{E}_{\mathcal{D}}[x_1 x_2^2] \mathbb{E}_x[x] + \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1 x_2 x^2] \\ &= \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1^2 x^2] + \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_2^2 x^2] + \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1^2 x_2^2] \\ &\quad - 2\mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1^2 x_2 x] - 2\mathbb{E}_{\mathcal{D}}[x_1 x_2^2] \mathbb{E}_x[x] + \mathbb{E}_{x_2} \mathbb{E}_x[\mathbb{E}_{x_1}[x_1]] \\ &= \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_1^2 x^2] + \mathbb{E}_{\mathcal{D}} \mathbb{E}_x[x_2^2 x^2] \\ &= \mathbb{E}_{x_1}[x_1^2] \mathbb{E}_x[x^2] + \mathbb{E}_{x_2}[x_2^2] \mathbb{E}_x[x^2] + \mathbb{E}_{x_1}[x_1^2] \mathbb{E}_{x_2}[x_2^2] \\ &= \frac{1}{9} + \frac{1}{9} + \frac{1}{9} \\ &= \frac{1}{3} \end{aligned}$$

Because $\mathbb{E}_u[u] = 0$ and $\mathbb{E}_u[u^2] = \frac{1}{9}$ when u is uniformly distributed over $[-1, 1]$.

(c) The expected out of sample error is:

$$\mathbb{E}[E_{\text{out}}] = \mathbf{bias} + \mathbf{var} = \frac{8}{15}$$

PROBLEM 2.22

1. Show that if $y(\mathbf{x}) = f(\mathbf{x}) + \epsilon$, where ϵ is a zero-mean noise random variable with variance σ^2 , then the bias-variance decomposition becomes $\mathbb{E}_{\mathcal{D}}[E_{\text{out}}(g^{(\mathcal{D})})] = \sigma^2 + \mathbf{bias} + \mathbf{var}$.

$$\begin{aligned}
\mathbb{E}_{\mathcal{D}}[E_{\text{out}}(g^{(\mathcal{D})})] &= \mathbb{E}_{\mathcal{D}}[\mathbb{E}_{\mathbf{x}, y}[(y(\mathbf{x}) - g^{(\mathcal{D})}(\mathbf{x}))^2]] \\
&= \mathbb{E}_{\mathcal{D}}[\mathbb{E}_{\mathbf{x}, \epsilon}[(f(\mathbf{x}) + \epsilon - g^{(\mathcal{D})}(\mathbf{x}))^2]] \\
&= \mathbb{E}_{x, \mathcal{D}}[\mathbb{E}_{\epsilon}[(f(\mathbf{x}) + \epsilon - g^{(\mathcal{D})}(\mathbf{x}))^2]] \\
&= \mathbb{E}_{x, \mathcal{D}}[\mathbb{E}_{\epsilon}[f(\mathbf{x})^2 + \epsilon^2 + g^{(\mathcal{D})}(\mathbf{x})^2 + 2\epsilon f(\mathbf{x}) + 2\epsilon g^{(\mathcal{D})}(\mathbf{x}) - 2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})]] \\
&= \mathbb{E}_{x, \mathcal{D}}[\mathbb{E}_{\epsilon}[f(\mathbf{x})^2 + \epsilon^2 + g^{(\mathcal{D})}(\mathbf{x})^2 - 2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})] + \underbrace{\mathbb{E}_{\epsilon}[2\epsilon f(\mathbf{x})]}_0 - \underbrace{\mathbb{E}_{\epsilon}[2\epsilon g^{(\mathcal{D})}(\mathbf{x})]}_0] \\
&= \mathbb{E}_{x, \mathcal{D}}[\underbrace{\mathbb{E}_{\epsilon}[f(\mathbf{x})^2]}_{f(\mathbf{x})^2} + \underbrace{\mathbb{E}_{\epsilon}[\epsilon^2]}_{\mathbf{var}(\epsilon) + \mathbb{E}_{\epsilon}[\epsilon]} + \underbrace{\mathbb{E}_{\epsilon}[g^{(\mathcal{D})}(\mathbf{x})^2]}_{g^{(\mathcal{D})}(\mathbf{x})^2} - \underbrace{\mathbb{E}_{\epsilon}[2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})]}_{2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})}] \\
&= \mathbb{E}_x \mathbb{E}_{\mathcal{D}}[f(\mathbf{x})^2 + (\underbrace{\mathbf{var}(\epsilon)}_{\sigma^2} + \underbrace{\mathbb{E}_{\epsilon}[\epsilon]}_0) + g^{(\mathcal{D})}(\mathbf{x})^2 - 2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})] \\
&= \mathbb{E}_x \mathbb{E}_{\mathcal{D}}[f(\mathbf{x})^2 + \sigma^2 + g^{(\mathcal{D})}(\mathbf{x})^2 - 2f(\mathbf{x})g^{(\mathcal{D})}(\mathbf{x})] \\
&= \mathbb{E}_{\mathbf{x}}[\underbrace{\mathbb{E}_{\mathcal{D}}[f(\mathbf{x})^2]}_{f(\mathbf{x})^2} + \underbrace{\mathbb{E}_{\mathcal{D}}[\sigma^2]}_{\sigma^2} + \underbrace{\mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})^2]}_{\mathbf{var}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})] + \mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})]^2} - \underbrace{\mathbb{E}_{\mathcal{D}}[2f(\mathbf{x})]}_{2f(\mathbf{x})} \cdot \mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})]] \\
&= \mathbb{E}_{\mathbf{x}}[\sigma^2 + \mathbf{var}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})] + f(\mathbf{x})^2 - 2f(\mathbf{x})\underbrace{\mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})]}_{\bar{g}(\mathbf{x})} + \underbrace{\mathbb{E}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})]^2}_{\bar{g}(\mathbf{x})^2}] \\
&= \mathbb{E}_{\mathbf{x}}[\sigma^2 + \underbrace{\mathbf{var}_{\mathcal{D}}[g^{(\mathcal{D})}(\mathbf{x})]}_{\mathbf{var}(\mathbf{x})} + \underbrace{f(\mathbf{x})^2 - 2f(\mathbf{x})\bar{g}(\mathbf{x}) + \bar{g}(\mathbf{x})^2}_{(f(\mathbf{x}) - \bar{g}(\mathbf{x}))^2}] \\
&= \mathbb{E}_{\mathbf{x}}[\sigma^2 + \mathbf{var}(\mathbf{x}) + (f(\mathbf{x}) - \bar{g}(\mathbf{x}))^2] \\
&= \underbrace{\mathbb{E}_{\mathbf{x}}[\sigma^2]}_{\sigma^2} + \underbrace{\mathbb{E}_{\mathbf{x}}[\mathbf{var}(\mathbf{x})]}_{\mathbf{var}} + \underbrace{\mathbb{E}_{\mathbf{x}}[(f(\mathbf{x}) - \bar{g}(\mathbf{x}))^2]}_{\mathbf{bias}} \\
&= \sigma^2 + \mathbf{bias} + \mathbf{var}
\end{aligned}$$

Because $\mathbb{E}_{\epsilon}[\epsilon] = 0$ (since it has zero-mean) and $\mathbb{E}_u[f] = f$ when f does not depend on u .

ALL CODE

Listing 4: experiement.funcs.py

```

1 import numpy as np
2 from numpy.random import default_rng
3 import warnings
4
5 # ##### DEFINE EXPERIMENT ##### DEFINE EXPERIMENT
6 # ##### DEFINE EXPERIMENT ##### #
7 # Seed a rng so all test sets have the same sequence, for result comparability.
8 test_set_rng = default_rng(0)
9
10 # Configure exception handling for testing
11 overflow_err_state = "raise"
12 max_attempts = 15
13
14 # This is a top-level experiment function. Do a number of experiments of
15 # varying trial counts
16 def doExperimentSetTrials(trial_count_set, test_size):
17     sz_list = []
18     hyp_list = []
19     stat_list = []
20     for trial_count in trial_count_set:
21         test_data = test_set_rng.uniform(-1, 1, test_size)
22
23         for attempt in range(0, max_attempts):
24             try:
25                 hypothesis_set = findHypothesisSet(trial_count)
26                 experiment_results = calcExperimentOutputs(test_data,
27 hypothesis_set)
28             except FloatingPointError:
29                 if attempt == max_attempts - 1:
30                     print("Attempt ", attempt + 1, " failed, aborting trial
31 count: ", trial_count)
32                     break
33                 else:
34                     print("Caught in doExperimentSetTrials, retrying with
35 attempt:", attempt + 2, " of ", max_attempts)
36                 else:
37                     exp_size = np.array([trial_count, test_size])
38
39                     sz_list.append(exp_size)
40                     hyp_list.append(experiment_results[0])
41                     stat_list.append(experiment_results[1])
42                     break
43
44     return np.array(sz_list), np.array(hyp_list), np.array(stat_list)

```

```

43 # This is a top-level experiment function. Do a number of experiments of
    varying test data sizes
44 def doExperimentSetTests(trial_count, test_sizes_set):
45     sz_list = []
46     hyp_list = []
47     stat_list = []
48     hypothesis_set = findHypothesisSet(trial_count)
49     for test_size in test_sizes_set:
50         for attempt in range(0, max_attempts):
51             try:
52                 test_data = test_set_rng.uniform(-1, 1, test_size)
53                 experiment_results = calcExperimentOutputs(test_data,
hypothesis_set)
54             except FloatingPointError:
55                 if attempt == max_attempts - 1:
56                     print("Attempt ", attempt + 1, " failed, aborting test size
: ", test_size)
57                     break
58                 else:
59                     print("Caught in deExperimentSetTests, retrying with
attempt:", attempt + 2, " of ", max_attempts)
60             else:
61                 exp_size = np.array([trial_count, test_size])
62
63                 sz_list.append(exp_size)
64                 hyp_list.append(experiment_results[0])
65                 stat_list.append(experiment_results[1])
66                 break
67     return np.array(sz_list), np.array(hyp_list), np.array(stat_list)
68
69
70 # Given a number of trials t, generate t training-sets and find the hypothesis
    for each one
71 def findHypothesisSet(trials):
72     hypothesis_set = np.empty((trials, 2))
73     for i in range(0, trials - 1):
74         data_set = getDataSet()
75         hypothesis_set[i] = getHypothesis(data_set)
76
77     return hypothesis_set
78
79
80 # Given a list of hypotheses, get the average function, bias, var, and two
    versions of E[E_out]
81 def calcExperimentOutputs(x_values, hypothesis_set):
82     averageHypothesis = findExpectedHypothesis(hypothesis_set)
83     try:
84         bias = getBias(x_values, averageHypothesis)
85         var = getVar(x_values, hypothesis_set, averageHypothesis)
86         EEout = getEEout(x_values, hypothesis_set)
87     except FloatingPointError:
88         print("Caught exception in calcExperimentOutputs")

```

```

89         raise FloatingPointError
90
91     return averageHypothesis, np.array([bias, var, EEout])
92
93
94 # ##### GET EXPERIMENT INPUT ##### GET EXPERIMENT INPUT
95 # ##### GET EXPERIMENT INPUT ##### #
96
97 # Get data set
98 def getDataSet():
99     x_one = np.random.uniform(-1, 1)
100    x_two = np.random.uniform(-1, 1)
101    return np.array([[x_one, x_one ** 2], [x_two, x_two ** 2]])
102
103
104 # ##### CALCULATION FUNCS ##### CALCULATION FUNCS
105 # ##### CALCULATION FUNCS ##### #
106
107 # Get the hypothesis:
108 def getHypothesis(data_set):
109     return data_set[1][0] + data_set[0][0], -(data_set[0][0]*data_set[1][0])
110
111
112 # Return the average function, the mean of many hypotheses
113 def findExpectedHypothesis(manyTrialsResults):
114     return np.mean(manyTrialsResults, 0)
115
116
117 # Evaluate a  $g(x) = ax + b$  at  $x$ 
118 def evaluateLineAtX(x, coefficients):
119     return coefficients[0] * x + coefficients[1]
120
121
122 # Take the bias over a new data set
123 def getBias(x_values, averageFunction):
124     evaluation = evaluateLineAtX(x_values, averageFunction)
125     with np.errstate(over=overflow_err_state):
126         try:
127             deviation = evaluation - np.square(x_values)
128             bias = np.mean(np.square(deviation))
129         except FloatingPointError:
130             print("Catching overflow in getBias")
131             raise FloatingPointError
132     return bias
133
134
135 # Find the sample variance over test data
136 def getVar(x_values, hypothesis_set, averageFunction):
137     num_hyps = len(hypothesis_set)
138     deviation = hypothesis_set - averageFunction

```

```

139 error_matrix = np.tensordot(deviation[:, 0], x_values, 0) + deviation[:,
140 1:]
141 with np.errstate(over=overflow_err_state):
142     try:
143         error_matrix = np.square(error_matrix)
144     except FloatingPointError:
145         print("Catching overflow in getVar")
146 mean_over_x = np.mean(error_matrix, 1)
147 return np.sum(mean_over_x) / (num_hyps - 1)
148
149 # E[E_out] as mean approximation of expectation over data sets and test data
150 def getEEout(x_values, hypothesis_set):
151     evaluation = np.tensordot(hypothesis_set[:, 0], x_values, 0) +
152     hypothesis_set[:, 1:]
153     with np.errstate(over=overflow_err_state):
154         try:
155             deviation = evaluation - np.square(x_values)
156             error_matrix = np.square(deviation)
157         except FloatingPointError:
158             print("Catching overflow in getEEout")
159     return np.mean(error_matrix)

```

Listing 5: experiment_printer.py

```

1 import matplotlib.pyplot as plt
2 import numpy as np
3 import experiment_funcs as expf
4
5
6 def resultPrinterGeneral(result_row, filename):
7
8     num_trials = result_row[0][0]
9     test_size = result_row[0][1]
10    avg_func = getLineEquationAsString(result_row[1])
11    bias = result_row[2][0]
12    var = result_row[2][1]
13    eeout = result_row[2][2]
14
15    print("# of Trials: ", num_trials, "; test data set size: ", test_size,
16    file=filename)
17    print("-----", file
18    =filename)
19    print("g_hat(x) = ", avg_func, file=filename)
20    print("bias = ", bias, file=filename)
21    print("var = ", var, file=filename)
22    print("E[E_out] = ", eeout, file=filename)
23    print("", file=filename)
24    print("bias + var = ", bias + var, file=filename)
25    print("E[E_out] - (bias + var) = ", eeout - bias - var, file=filename)
26    print("", file=filename)

```

```
25
26
27 def printLineEquation(coefficients, filename):
28     print(coefficients[0], "x + ", coefficients[1], file=filename)
29
30
31 def getLineEquationAsString(coefficients):
32     return "%sx + %s" % tuple(coefficients)
33
34
35 output_prefix = "results/final/"
36
37
38 def plotQuestionAnswer(results, title):
39     fig, ax = plt.subplots()
40     ax.set_title(title)
41     ax.set_xlim(-1, 1)
42     ax.set_ylim(-0.25, 1)
43     target_x_range = np.linspace(-1, 1)
44     target_y = np.square(target_x_range)
45     ax.plot(target_x_range, target_y, c='k')
46
47     num_hyps = len(results[1])
48     alpha_bias = 0.75
49     hyp_alpha = alpha_bias/num_hyps
50     hyp_col_bias = 1
51     hyp_col_shift = hyp_col_bias/num_hyps
52     for hyp in results[1]:
53         hyp_y = expf.evaluateLineAtX(target_x_range, hyp)
54         hyp_alpha = 0.25
55         ax.plot(target_x_range, hyp_y, alpha=hyp_alpha, c=(1 - hyp_col_shift,
120         0, hyp_col_shift))
56         if hyp_alpha >= 1:
57             hyp_alpha = 1
58         else:
59             hyp_alpha += alpha_bias/num_hyps
60         if hyp_col_shift >= 1:
61             hyp_col_shift = 1
62         else:
63             hyp_col_shift += alpha_bias/num_hyps
64
65     fig.show()
66     fig.savefig(output_prefix + title + '.png', transparent=False, dpi=80,
120     bbox_inches="tight")
67     plt.close(fig)
68
69
70 def initPlot_trials(sup_title, x_label, y_label, x_ticks, y_ticks, x_scale,
120     y_scale):
71     trials_fig, trials_ax = plt.subplots()
72     trials_fig.set_size_inches(6, 3)
73
```

```

74     super_title = sup_title
75     trials_ax.set_title(sup_title)
76
77     trials_ax.set_xlabel(x_label)
78     trials_ax.set_ylabel(y_label)
79
80     trials_ax.set_xscale(x_scale)
81     trials_ax.set_yscale(y_scale)
82
83     return trials_fig, trials_ax
84
85
86 def trials_plotter(results):
87
88     # print(results)
89     title = str(results[0][0][1]) + " test data points"
90     file_title = str(results[0][0][1]) + "_data_points"
91     x_trials_range = results[0][:, 0]
92     y_bias_range = results[2][:, 0]
93     y_var_range = results[2][:, 1]
94     y_ticks = np.concatenate((results[2][:, 0], results[2][:, 1]))
95     fig_bias_var, ax_bias_var = initPlot_trials(title, "trials", "bias (dashed)",
96     , var (dotted)", x_trials_range, y_ticks, "log", "linear")
97
98     ax_bias_var.plot(x_trials_range, y_bias_range, ls='dashed', c='b')
99     ax_bias_var.plot(x_trials_range, y_var_range, ls='dotted', c='r')
100
101     fig_bias_var.show()
102     fig_bias_var.savefig(output_prefix + file_title + '_bias-var' + '.png',
103     transparent=False, dpi=80, bbox_inches="tight")
104     plt.close(fig_bias_var)
105
106     # #####
107
108     y_bias_var_range = y_bias_range + y_var_range
109     y_eeout_range = results[2][:, 2]
110
111     y_diff_range = y_eeout_range - y_bias_var_range
112     fig_bias_var_diff, ax_bias_var_diff = initPlot_trials(title, "trials", "E[
113     E_out] - bias + var ", x_trials_range, y_diff_range, "log", "linear")
114
115     ax_bias_var_diff.plot(x_trials_range, y_diff_range, c='k')
116
117     fig_bias_var_diff.show()
118     fig_bias_var_diff.savefig(output_prefix + file_title + '_diff' + '.png',
119     transparent=False, dpi=80, bbox_inches="tight")
120     plt.close(fig_bias_var_diff)
121
122 def tests_plotter(results):
123
124     # print(results)

```



```

122     title = str(results[0][0][0]) + " trials"
123     file_title = str(results[0][0][0]) + "_trials"
124     x_trials_range = results[0][:, 1]
125     y_bias_range = results[2][:, 0]
126     y_var_range = results[2][:, 1]
127     y_ticks = np.concatenate((results[2][:, 0], results[2][:, 1]))
128     fig_bias_var, ax_bias_var = initPlot_trials(title, "test data points", "
bias (dashed), var (dotted)", x_trials_range, y_ticks, "log", "linear")
129
130     # ax_bias_var.set_yscale("symlog", linthresh=.0000000000000001)
131     ax_bias_var.plot(x_trials_range, y_bias_range, ls='dashed', c='b')
132     ax_bias_var.plot(x_trials_range, y_var_range, ls='dotted', c='r')
133     fig_bias_var.show()
134     fig_bias_var.savefig(output_prefix + file_title + '_bias-var' + '.png',
transparent=False, dpi=80, bbox_inches="tight")
135     plt.close(fig_bias_var)
136
137     # #####
138
139     y_bias_var_range = y_bias_range + y_var_range
140     y_eeout_range = results[2][:, 2]
141
142     y_diff_range = y_eeout_range - y_bias_var_range
143     fig_bias_var_diff, ax_bias_var_diff = initPlot_trials(title, "test data
points", "E[E_out] - bias + var ", x_trials_range, y_diff_range, "log", "
linear")
144
145     ax_bias_var_diff.plot(x_trials_range, y_diff_range, c='k')
146
147     fig_bias_var_diff.show()
148     fig_bias_var_diff.savefig(output_prefix + file_title + '_diff' + '.png',
transparent=False, dpi=80, bbox_inches="tight")
149     plt.close(fig_bias_var_diff)

```

Listing 6: new_meta_experiment.py

```

1  import experiment_funcs as expf
2  import experiment_printer as expp
3  import numpy as np
4  import matplotlib.pyplot as plt
5
6
7  # ##### DO THE EXPERIMENT ##### DO THE EXPERIMENT
   ##### DO THE EXPERIMENT #####
8  seq_tiny = (2, 3, 4, 5)
9  seq_small = (5, 10, 25, 50, 100)
10 seq_medium = (10, 25, 75, 150, 500)
11 seq_big = (10, 15, 20, 25, 50, 75, 100, 150, 175, 200, 250, 500, 1000)
12 seq_massive = (10, 100, 1000, 10000, 100000, 1000000, 10000000)
13
14

```

```

15 # Final sequence to use for both varying trial count, and varying test size
16 final_trial_count_seq = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 20,
    25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 125, 150,
    175, 200, 225, 250, 300, 350, 400, 450, 500, 1000, 1250, 1500, 1750, 2000,
    2500, 10000, 100000)
17 final_test_count_seq = (5, 10, 15, 20, 25, 30, 40, 50, 60, 70, 80, 90, 100,
    110, 120, 130, 140, 150, 175, 200, 225, 250, 275, 300, 325, 350, 500, 750,
    1000, 2500, 5000)
18
19 # Alias this so as to not change following code during program debug/testing.
20 alias_trial_count_seq = final_trial_count_seq
21 alias_test_count_seq = final_test_count_seq
22
23
24 # Generate results for reports, varying number of trials on fixed test size
25 def metaExperiment_trials(trial_count_set, test_size, output_file):
26     print("===== DOING TRIALS (", trial_count_set[0], ", ..., ",
    trial_count_set[-1],
27         ") with ", test_size, " test points =====", file=output_file)
28     results = expf.doExperimentSetTrials(trial_count_set, test_size)
29
30     for i in range(0, len(results[0])):
31         result_row = results[0][i], results[1][i], results[2][i]
32         expf.resultPrinterGeneral(result_row, output_file)
33
34     expf.trials_plotter(results)
35     expf.plotQuestionAnswer(results, str(results[0][0][1]) + " test data points
    ")
36
37
38 # Generate results for reports, varying number of tests on fixed trial size
39 def metaExperiment_tests(trial_count, test_data_sizes, output_file):
40     print("===== DOING TESTS (", test_data_sizes[0], ", ..., ",
    test_data_sizes[-1],
41         ") with ", trial_count, " trial size =====", file=output_file)
42     results = expf.doExperimentSetTests(trial_count, test_data_sizes)
43
44     for i in range(0, len(results[0])):
45         result_row = results[0][i], results[1][i], results[2][i]
46         expf.resultPrinterGeneral(result_row, output_file)
47
48     expf.tests_plotter(results)
49     expf.plotQuestionAnswer(results, str(results[0][0][0]) + " trials")
50
51
52 # ##### DO THE EXPERIMENT ##### DO THE EXPERIMENT
    ##### DO THE EXPERIMENT #####
53 # In every innermost function call, test data is generated by a seeded
    generator, so the sequence is fixed
54 # In every innermost function call, training data is generated by an un-seeded
    generator, so the sequence is different
55

```

```
56 # In this section metaExperiment_trials increments trial count
57 output_prefix = "results/final/"
58 print("Doing 2500 test data")
59 results_2500_test = open(output_prefix + "results_test_points_2500.dat", 'w')
60 metaExperiment_trials(alias_trial_count_seq, 2500, results_2500_test)
61 results_2500_test.close()
62 # # # #####
63 print("Doing 5000 test")
64 results_5000_test = open(output_prefix + "results_test_points_5000.dat", 'w')
65 metaExperiment_trials(alias_trial_count_seq, 5000, results_5000_test)
66 results_5000_test.close()
67 # # # #####
68 print("Doing 10000 test data")
69 results_10000_test = open(output_prefix + "results_test_points_10000.dat", 'w')
70 metaExperiment_trials(alias_trial_count_seq, 10000, results_10000_test)
71 results_10000_test.close()
72 # # # #####
73 # # # #####
74
75 # # In this section metaExperiment_tests increments test data size
76 print("Doing 25 trials")
77 results_25_trials = open(output_prefix + "results_trials_25.dat", 'w')
78 metaExperiment_tests(25, alias_trial_count_seq, results_25_trials)
79 results_25_trials.close()
80 # # # #####
81 print("Doing 50 trials")
82 results_50_trials = open(output_prefix + "results_trials_50.dat", 'w')
83 metaExperiment_tests(50, alias_trial_count_seq, results_50_trials)
84 results_50_trials.close()
85 # # # #####
86 print("Doing 100 trials")
87 results_100_trials = open(output_prefix + "results_trials_100.dat", 'w')
88 metaExperiment_tests(100, alias_trial_count_seq, results_100_trials)
89 results_100_trials.close()
90 # # # #####
91 single_name = "results/final/single_exp_5000_5000.dat"
92 single_file = open(single_name, "w")
93 metaExperiment_trials(np.array([5000]), np.array([5000]), single_file)
94 single_file.close()
95
96 # #####
97 print("All done!")
```

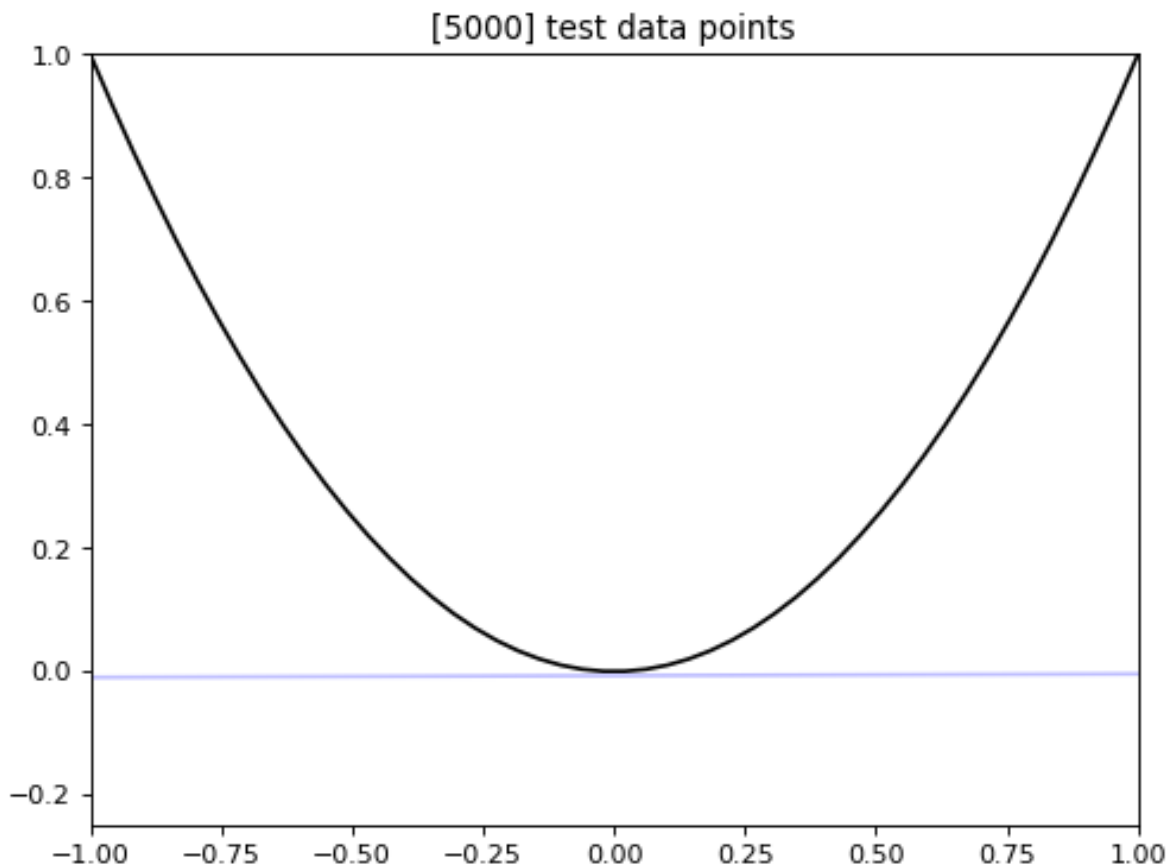
ALL DATA AND PLOTS

This is the single experiment shown in part (a):

single_exp_5000_5000.dat

```
===== DOING TRIALS 5000 , ..., 5000 with [5000] test points =====
# of Trials: 5000 ; test data set size: [5000]
-----
g_hatx = 0.0027828607607221405x + -0.006683991780068334
bias    = 0.2069807421246095
var     = 0.3437476483550565
E[E_out] = 0.5506596409499945

bias + var = 0.550728390479666
E[E_out] - bias + var = -6.874952967145243e-05
```



On the following pages are all data and plots for the other experiments.

results_test_points_2500.dat

===== DOING TRIALS 2 , ..., 100000 with 2500 test points =====

of Trials: 2 ; test data set size: 2500

g_hatx = 0.30522134134489265x + -0.043142279942284534
bias = 0.2695735330428795
var = 0.06692868420569062
E[E_out] = 0.3030378751457249

bias + var = 0.3365022172485701
E[E_out] - bias + var = -0.03346434210284524

of Trials: 3 ; test data set size: 2500

g_hatx = -0.853592673691368x + -0.2614200900771848
bias = 0.6825852829495351
var = 0.28067041369291257
E[E_out] = 0.8696988920781435

bias + var = 0.9632556966424477
E[E_out] - bias + var = -0.09355680456430415

of Trials: 4 ; test data set size: 2500

g_hatx = -0.29131839887491173x + 0.27429911617897507
bias = 0.11468622544680018
var = 0.23094273700092097
E[E_out] = 0.2878932781974909

bias + var = 0.3456289624477211
E[E_out] - bias + var = -0.05773568425023026

of Trials: 5 ; test data set size: 2500

g_hatx = 0.02356969958487687x + 0.10198774428122097
bias = 0.14269098134679142
var = 0.4075133509249999
E[E_out] = 0.4687016620867913

bias + var = 0.5502043322717913
E[E_out] - bias + var = -0.08150267018500001

of Trials: 6 ; test data set size: 2500

g_hatx = -0.22828277615331924x + 0.0354025257156687
bias = 0.1994213950995241
var = 0.20170520692507915
E[E_out] = 0.36750906753709006

bias + var = 0.40112660202460326

```
E[E_out] - bias + var = -0.0336175344875132

# of Trials: 7 ; test data set size: 2500
-----
g_hatx = -0.10550639828401162x + 0.19558852639141053
bias    = 0.11429903578403827
var     = 0.1033682532320474
E[E_out] = 0.20290039569722182

bias + var = 0.21766728901608567
E[E_out] - bias + var = -0.014766893318863855

# of Trials: 8 ; test data set size: 2500
-----
g_hatx = -0.15739761268168212x + 0.19043206599737966
bias    = 0.11312866091775604
var     = 0.1819442668040258
E[E_out] = 0.27232989437127864

bias + var = 0.29507292772178184
E[E_out] - bias + var = -0.022743033350503206

# of Trials: 9 ; test data set size: 2500
-----
g_hatx = -0.1583444742611642x + -0.02514650594645713
bias    = 0.21975671849875617
var     = 0.6734604604822543
E[E_out] = 0.8183882389274266

bias + var = 0.8932171789810104
E[E_out] - bias + var = -0.07482894005358376

# of Trials: 10 ; test data set size: 2500
-----
g_hatx = 0.1954012368266885x + -0.008934152843597354
bias    = 0.21524239460262215
var     = 0.35330707066783607
E[E_out] = 0.5332187582036746

bias + var = 0.5685494652704582
E[E_out] - bias + var = -0.03533070706678365

# of Trials: 11 ; test data set size: 2500
-----
g_hatx = -0.12957619487879615x + -0.04943200465938211
bias    = 0.236986971313133
var     = 0.4224405170875151
E[E_out] = 0.6210238050290559

bias + var = 0.6594274884006481
E[E_out] - bias + var = -0.03840368337159228
```

```
# of Trials: 12 ; test data set size: 2500
-----
g_hatx = -0.29265317786880646x + 0.09757576408376566
bias    = 0.16548924146330088
var     = 0.26582813017776796
E[E_out] = 0.40916502745958816

bias + var = 0.43131737164106887
E[E_out] - bias + var = -0.022152344181480682

# of Trials: 13 ; test data set size: 2500
-----
g_hatx = 0.08623994842527827x + -0.03245202870841363
bias    = 0.2301234136375482
var     = 0.28349597755694295
E[E_out] = 0.49181200830549554

bias + var = 0.5136193911944912
E[E_out] - bias + var = -0.021807382888995586

# of Trials: 14 ; test data set size: 2500
-----
g_hatx = 0.057628483726491084x + -0.09492884248038481
bias    = 0.2747006884028817
var     = 0.2911563670244235
E[E_out] = 0.5450601720684178

bias + var = 0.5658570554273052
E[E_out] - bias + var = -0.020796883358887353

# of Trials: 15 ; test data set size: 2500
-----
g_hatx = 0.11212324641904182x + 0.013875198448762192
bias    = 0.20402887642136816
var     = 0.30304800865403525
E[E_out] = 0.4868736844984677

bias + var = 0.5070768850754034
E[E_out] - bias + var = -0.020203200576935698

# of Trials: 20 ; test data set size: 2500
-----
g_hatx = 0.2755907893698711x + -0.004362223657266573
bias    = 0.22586095799928854
var     = 0.3236701072831541
E[E_out] = 0.533347559918285

bias + var = 0.5495310652824427
E[E_out] - bias + var = -0.016183505364157635

# of Trials: 25 ; test data set size: 2500
-----
```

```
g_hatx = 0.23491045585668757x + 0.07809529745891658
bias    = 0.16401288308233525
var     = 0.30303267675894685
E[E_out] = 0.4549242527709242
```

```
bias + var = 0.4670455598412821
E[E_out] - bias + var = -0.012121307070357912
```

```
# of Trials: 30 ; test data set size: 2500
```

```
-----
g_hatx = -0.1741024951193094x + -0.047806754822846395
bias    = 0.2510949062715534
var     = 0.2779482527486905
E[E_out] = 0.5197782172619543
```

```
bias + var = 0.5290431590202439
E[E_out] - bias + var = -0.009264941758289624
```

```
# of Trials: 35 ; test data set size: 2500
```

```
-----
g_hatx = -0.310421463996251x + -0.015283891514226136
bias    = 0.2370065960642168
var     = 0.23959432094717623
E[E_out] = 0.4697553649843308
```

```
bias + var = 0.47660091701139307
E[E_out] - bias + var = -0.006845552027062246
```

```
# of Trials: 40 ; test data set size: 2500
```

```
-----
g_hatx = 0.024608322487902935x + 0.013106601643574245
bias    = 0.19406248341705407
var     = 0.31488790479052253
E[E_out] = 0.5010781905878136
```

```
bias + var = 0.5089503882075765
E[E_out] - bias + var = -0.007872197619763044
```

```
# of Trials: 45 ; test data set size: 2500
```

```
-----
g_hatx = -0.02785453905725288x + 0.13060691438746705
bias    = 0.13109363728443105
var     = 0.2777826819156411
E[E_out] = 0.4027033707130579
```

```
bias + var = 0.40887631920007217
E[E_out] - bias + var = -0.006172948487014274
```

```
# of Trials: 50 ; test data set size: 2500
```

```
-----
g_hatx = -0.030811308750965164x + 0.05141319274160115
bias    = 0.16769976344807555
```



```
var          = 0.2811708379908185
E[E_out]     = 0.4432471846790776

bias + var   = 0.448870601438894
E[E_out] - bias + var = -0.005623416759816424

# of Trials: 55 ; test data set size: 2500
-----
g_hatx       = 0.15234323666711633x + 0.03420492530205023
bias         = 0.18190327579383994
var          = 0.37696700268318634
E[E_out]     = 0.5520163329736955

bias + var   = 0.5588702784770263
E[E_out] - bias + var = -0.006853945503330794

# of Trials: 60 ; test data set size: 2500
-----
g_hatx       = 0.1480413315848006x + 0.030364178304230535
bias         = 0.18377172993285462
var          = 0.3327603359394099
E[E_out]     = 0.5109860602732745

bias + var   = 0.5165320658722645
E[E_out] - bias + var = -0.005546005598989989

# of Trials: 65 ; test data set size: 2500
-----
g_hatx       = 0.07072126900399134x + 0.00836652124480698
bias         = 0.20244680861745365
var          = 0.38597448477816915
E[E_out]     = 0.5824832243990357

bias + var   = 0.5884212933956228
E[E_out] - bias + var = -0.005938068996587165

# of Trials: 70 ; test data set size: 2500
-----
g_hatx       = 0.015582991527246431x + 0.01981203888062174
bias         = 0.1937588425184167
var          = 0.28806912555164715
E[E_out]     = 0.4777126948478975

bias + var   = 0.4818279680700639
E[E_out] - bias + var = -0.004115273222166371

# of Trials: 75 ; test data set size: 2500
-----
g_hatx       = 0.22212934277251728x + -0.0134176842686325
bias         = 0.22073421272336144
var          = 0.2846224880660939
E[E_out]     = 0.5015617342819074
```

```
bias + var = 0.5053567007894554
E[E_out] - bias + var = -0.003794966507547959

# of Trials: 80 ; test data set size: 2500
-----
g_hatx = -0.0472468553446768x + -0.05131060789891932
bias    = 0.2280335662823789
var     = 0.33714967512108895
E[E_out] = 0.5609688704644542

bias + var = 0.5651832414034679
E[E_out] - bias + var = -0.0042143709390136674

# of Trials: 85 ; test data set size: 2500
-----
g_hatx = 0.03867371424309164x + -0.011806627444401564
bias    = 0.19432098192565303
var     = 0.3648895530382966
E[E_out] = 0.5549177166929108

bias + var = 0.5592105349639497
E[E_out] - bias + var = -0.00429281827103889

# of Trials: 90 ; test data set size: 2500
-----
g_hatx = -0.01002501683885764x + 0.05926364704866311
bias    = 0.16838069220412133
var     = 0.2997167980285899
E[E_out] = 0.4647673035879491

bias + var = 0.46809749023271124
E[E_out] - bias + var = -0.0033301866447621453

# of Trials: 95 ; test data set size: 2500
-----
g_hatx = 0.023543211871933246x + 0.040368067416035845
bias    = 0.17503856502017362
var     = 0.2578033818396594
E[E_out] = 0.4301282270509945

bias + var = 0.432841946859833
E[E_out] - bias + var = -0.0027137198088384995

# of Trials: 100 ; test data set size: 2500
-----
g_hatx = -0.06541253352084021x + 0.01094965098048514
bias    = 0.1964937689523962
var     = 0.3031896671239578
E[E_out] = 0.4966515394051144

bias + var = 0.499683436076354
```

```
E[E_out] - bias + var = -0.003031896671239598

# of Trials: 125 ; test data set size: 2500
-----
g_hatx = 0.19570506552150024x + -0.014121435981425217
bias    = 0.22124929866275683
var     = 0.31102379239895617
E[E_out] = 0.5297849007225214

bias + var = 0.532273091061713
E[E_out] - bias + var = -0.0024881903391916205

# of Trials: 150 ; test data set size: 2500
-----
g_hatx = -0.09413029523825109x + 0.017613038947189682
bias    = 0.19275065797275692
var     = 0.3028665208002216
E[E_out] = 0.4935980686343105

bias + var = 0.4956171787729785
E[E_out] - bias + var = -0.002019110138668012

# of Trials: 175 ; test data set size: 2500
-----
g_hatx = 0.02025218924176626x + 0.020828382723337553
bias    = 0.18233713615241012
var     = 0.3262150348477109
E[E_out] = 0.5066880850867055

bias + var = 0.5085521710001211
E[E_out] - bias + var = -0.0018640859134155252

# of Trials: 200 ; test data set size: 2500
-----
g_hatx = 0.08589449671566023x + -0.034520820976173844
bias    = 0.23519054392482686
var     = 0.3771422811721187
E[E_out] = 0.6104471136910847

bias + var = 0.6123328250969455
E[E_out] - bias + var = -0.0018857114058608015

# of Trials: 225 ; test data set size: 2500
-----
g_hatx = -0.09519062231131883x + -0.0027364285525583484
bias    = 0.204472864444037388
var     = 0.3377465383891489
E[E_out] = 0.5407183071033488

bias + var = 0.5422194028295229
E[E_out] - bias + var = -0.0015010957261740177
```

```
# of Trials: 250 ; test data set size: 2500
-----
g_hatx = 0.061198156858459814x + 0.014034460714489359
bias    = 0.19919628878367981
var     = 0.31639021393004507
E[E_out] = 0.5143209418580048

bias + var = 0.5155865027137249
E[E_out] - bias + var = -0.0012655608557200493

# of Trials: 300 ; test data set size: 2500
-----
g_hatx = -0.01997354478803994x + 0.01973754998427445
bias    = 0.1925862962285812
var     = 0.31938661902295407
E[E_out] = 0.5109082931881254

bias + var = 0.5119729152515353
E[E_out] - bias + var = -0.0010646220634099035

# of Trials: 350 ; test data set size: 2500
-----
g_hatx = -0.03127233783460913x + 0.016759298754701016
bias    = 0.18956721572264545
var     = 0.29533840717552856
E[E_out] = 0.4840617988776724

bias + var = 0.484905622898174
E[E_out] - bias + var = -0.0008438240205015934

# of Trials: 400 ; test data set size: 2500
-----
g_hatx = 0.03606976376777864x + 0.018102863620160604
bias    = 0.1984265692206784
var     = 0.3274887811897504
E[E_out] = 0.5250966284574544

bias + var = 0.5259153504104288
E[E_out] - bias + var = -0.0008187219529743328

# of Trials: 450 ; test data set size: 2500
-----
g_hatx = -0.0016468355190760496x + -0.019687734451211688
bias    = 0.21842809334992436
var     = 0.3451729351975843
E[E_out] = 0.5628339775804029

bias + var = 0.5636010285475086
E[E_out] - bias + var = -0.0007670509671057757

# of Trials: 500 ; test data set size: 2500
-----
```

```
g_hatx = 0.025750451402855545x + -0.013106326214980264
bias    = 0.20198552369981657
var     = 0.3246229768513163
E[E_out] = 0.5259592545974299
```

```
bias + var = 0.5266085005511328
E[E_out] - bias + var = -0.000649245953703026
```

```
# of Trials: 1000 ; test data set size: 2500
```

```
-----
g_hatx = -0.0011591890492891866x + -0.0011883308475108697
bias    = 0.20439798678237475
var     = 0.3313817287772515
E[E_out] = 0.5354483338308494
```

```
bias + var = 0.5357797155596262
E[E_out] - bias + var = -0.00033138172877683614
```

```
# of Trials: 1250 ; test data set size: 2500
```

```
-----
g_hatx = 0.012384161132222198x + 0.00783867815427398
bias    = 0.19027023804214196
var     = 0.31065083580664843
E[E_out] = 0.5006725531801454
```

```
bias + var = 0.5009210738487904
E[E_out] - bias + var = -0.00024852066864500255
```

```
# of Trials: 1500 ; test data set size: 2500
```

```
-----
g_hatx = -0.02214338818896033x + 0.0021691572590899597
bias    = 0.1937522622753467
var     = 0.3330543716103107
E[E_out] = 0.5265845976379169
```

```
bias + var = 0.5268066338856574
E[E_out] - bias + var = -0.00022203624774053132
```

```
# of Trials: 1750 ; test data set size: 2500
```

```
-----
g_hatx = 0.030635517482591725x + -0.004759734073855929
bias    = 0.20972408242233032
var     = 0.3369685060482125
E[E_out] = 0.5465000350385153
```

```
bias + var = 0.5466925884705428
E[E_out] - bias + var = -0.00019255343202750907
```

```
# of Trials: 2000 ; test data set size: 2500
```

```
-----
g_hatx = -0.0049582696356247095x + -0.005718953752877741
bias    = 0.20470205408793377
```

```
var      = 0.34112098452964595
E[E_out] = 0.5456524781253153
```

```
bias + var = 0.5458230386175797
E[E_out] - bias + var = -0.0001705604922644377
```

```
# of Trials: 2500 ; test data set size: 2500
```

```
-----
g_hatx = -0.005233906109558439x + -0.0022428163397903056
bias    = 0.20857143826775665
var     = 0.33721475337116286
E[E_out] = 0.5456513057375707
```

```
bias + var = 0.5457861916389195
E[E_out] - bias + var = -0.0001348859013488024
```

```
# of Trials: 10000 ; test data set size: 2500
```

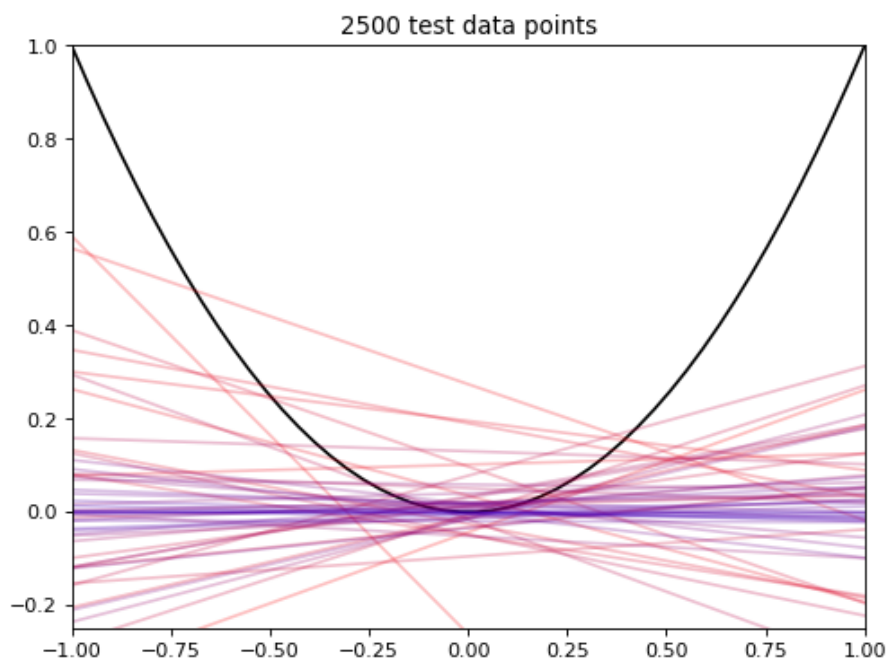
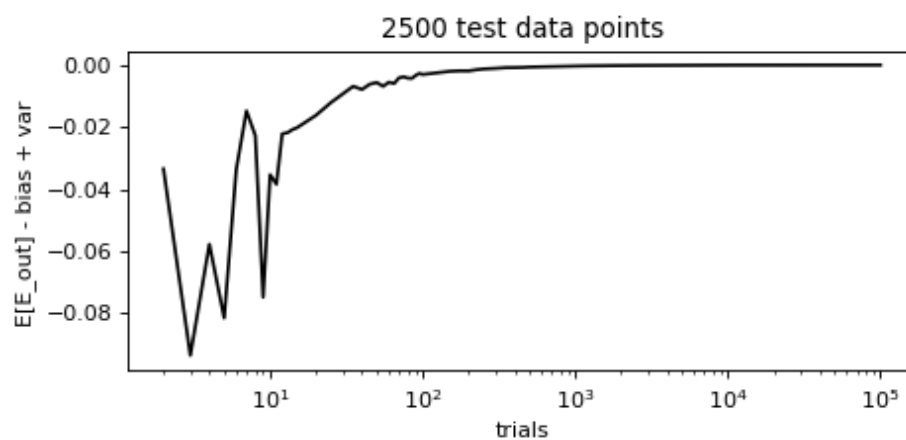
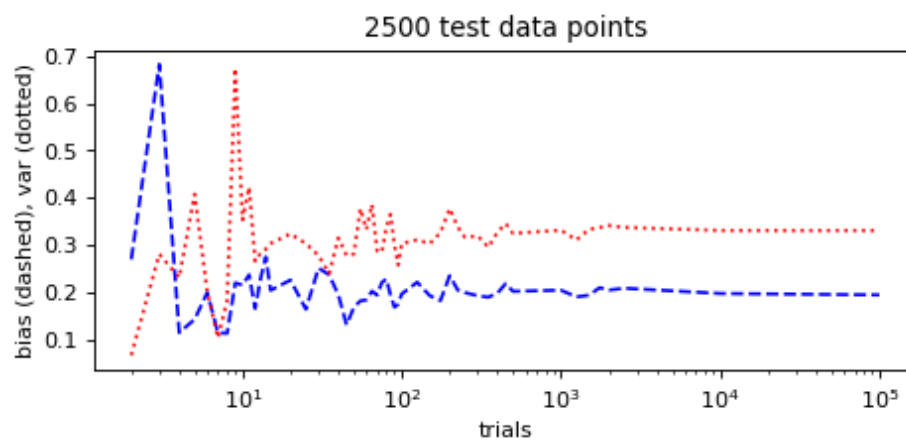
```
-----
g_hatx = -0.014959702606452715x + -0.001239368527455353
bias    = 0.19746143029751873
var     = 0.3308005296163134
E[E_out] = 0.5282288798608709
```

```
bias + var = 0.5282619599138321
E[E_out] - bias + var = -3.308005296126515e-05
```

```
# of Trials: 100000 ; test data set size: 2500
```

```
-----
g_hatx = 0.00021459005844962423x + 0.00010866704397793948
bias    = 0.19443493737141826
var     = 0.3307216538498999
E[E_out] = 0.5251532840047788
```

```
bias + var = 0.5251565912213181
E[E_out] - bias + var = -3.3072165393499553e-06
```



results_test_points_5000.dat

===== DOING TRIALS 2 , ..., 100000 with 5000 test points =====

of Trials: 2 ; test data set size: 5000

g_hatx = 0.4686156569709927x + -0.10197360016460971
bias = 0.3581423588013922
var = 0.16899711803633763
E[E_out] = 0.4426409178195611

bias + var = 0.5271394768377299
E[E_out] - bias + var = -0.08449855901816875

of Trials: 3 ; test data set size: 5000

g_hatx = 0.48969114169999234x + 0.01864399139375907
bias = 0.2673924458595933
var = 0.11410496189332447
E[E_out] = 0.3434624204551429

bias + var = 0.3814974077529178
E[E_out] - bias + var = -0.03803498729777488

of Trials: 4 ; test data set size: 5000

g_hatx = -0.04417881383336586x + -0.0938043221383141
bias = 0.2732225355013841
var = 0.2806951536833631
E[E_out] = 0.48374390076390644

bias + var = 0.5539176891847473
E[E_out] - bias + var = -0.07017378842084077

of Trials: 5 ; test data set size: 5000

g_hatx = 0.4306589687500427x + -0.08906767789587958
bias = 0.31545349060331745
var = 0.2938751084177354
E[E_out] = 0.5505535773375057

bias + var = 0.6093285990210529
E[E_out] - bias + var = -0.0587750216835472

of Trials: 6 ; test data set size: 5000

g_hatx = 0.06568173705119236x + 0.07607402987973404
bias = 0.15359956742198994
var = 0.5654787416482459
E[E_out] = 0.6248318521288615

bias + var = 0.7190783090702357


```
E[E_out] - bias + var = -0.09424645694137429

# of Trials: 7 ; test data set size: 5000
-----
g_hatx = -0.17729478878903768x + -0.18563563408856879
bias    = 0.3692546613108778
var     = 0.4102284255016004
E[E_out] = 0.7208790260265353

bias + var = 0.7794830868124782
E[E_out] - bias + var = -0.05860406078594288

# of Trials: 8 ; test data set size: 5000
-----
g_hatx = -0.3137522942069004x + -0.08330104341532718
bias    = 0.29708132007189864
var     = 0.3478252077727943
E[E_out] = 0.6014283768730937

bias + var = 0.6449065278446929
E[E_out] - bias + var = -0.04347815097159924

# of Trials: 9 ; test data set size: 5000
-----
g_hatx = -0.23120639538636376x + 0.00020664321585217582
bias    = 0.21409896413499965
var     = 0.305350732667523
E[E_out] = 0.4855218376172424

bias + var = 0.5194496968025226
E[E_out] - bias + var = -0.033927859185280296

# of Trials: 10 ; test data set size: 5000
-----
g_hatx = 0.16519133629393334x + -0.15810079812108785
bias    = 0.34495072922540804
var     = 0.5398627939673718
E[E_out] = 0.8308272437960424

bias + var = 0.8848135231927798
E[E_out] - bias + var = -0.05398627939673739

# of Trials: 11 ; test data set size: 5000
-----
g_hatx = -0.17982133508168166x + 0.1286467171172187
bias    = 0.14268951356051635
var     = 0.2075188659651208
E[E_out] = 0.3313430280742626

bias + var = 0.35020837952563716
E[E_out] - bias + var = -0.01886535145137458
```

```
# of Trials: 12 ; test data set size: 5000
-----
g_hatx = 0.0947586809678363x + 0.06622608412712806
bias    = 0.16084318641705914
var     = 0.3442129153053987
E[E_out] = 0.4763716921136745

bias + var = 0.5050561017224579
E[E_out] - bias + var = -0.028684409608783323

# of Trials: 13 ; test data set size: 5000
-----
g_hatx = -0.1502669455495155x + 0.045345538133099286
bias    = 0.18104013397532975
var     = 0.15333596463836363
E[E_out] = 0.32258102441074243

bias + var = 0.3343760986136934
E[E_out] - bias + var = -0.01179507420295095

# of Trials: 14 ; test data set size: 5000
-----
g_hatx = -0.05283248727285147x + -0.037993626768359524
bias    = 0.2256997492631501
var     = 0.47680483423973163
E[E_out] = 0.6684470953429008

bias + var = 0.7025045835028817
E[E_out] - bias + var = -0.034057488159980887

# of Trials: 15 ; test data set size: 5000
-----
g_hatx = -0.32595260072683724x + 0.04308274360484514
bias    = 0.1957830311580211
var     = 0.23541873877073366
E[E_out] = 0.4155071873440392

bias + var = 0.43120176992875475
E[E_out] - bias + var = -0.015694582584715566

# of Trials: 20 ; test data set size: 5000
-----
g_hatx = 0.20949331971498025x + -0.05125581760214868
bias    = 0.2518191046234424
var     = 0.36896673415905
E[E_out] = 0.60233750207454

bias + var = 0.6207858387824925
E[E_out] - bias + var = -0.018448336707952484

# of Trials: 25 ; test data set size: 5000
-----
```

```
g_hatx = 0.17336023397588607x + -0.08918359277393993
bias    = 0.27787544460177405
var     = 0.33598023474910427
E[E_out] = 0.6004164699609141
```

```
bias + var = 0.6138556793508783
E[E_out] - bias + var = -0.01343920938996418
```

```
# of Trials: 30 ; test data set size: 5000
```

```
-----
g_hatx = 0.16426760823580336x + 0.016944598243373462
bias    = 0.1979739007500084
var     = 0.3584733192875802
E[E_out] = 0.5444981093946692
```

```
bias + var = 0.5564472200375886
E[E_out] - bias + var = -0.01194911064291937
```

```
# of Trials: 35 ; test data set size: 5000
```

```
-----
g_hatx = 0.09608701191973558x + 0.00885022685060541
bias    = 0.19744459728734445
var     = 0.3208425701847939
E[E_out] = 0.50912023689543
```

```
bias + var = 0.5182871674721383
E[E_out] - bias + var = -0.009166930576708365
```

```
# of Trials: 40 ; test data set size: 5000
```

```
-----
g_hatx = -0.1622967400161208x + -0.07355137324954521
bias    = 0.2693519472828847
var     = 0.35828896302099217
E[E_out] = 0.6186836862283521
```

```
bias + var = 0.6276409103038769
E[E_out] - bias + var = -0.008957224075524739
```

```
# of Trials: 45 ; test data set size: 5000
```

```
-----
g_hatx = 0.21817401033154862x + -0.012451102147790946
bias    = 0.23307033853670228
var     = 0.3922562219476548
E[E_out] = 0.6166097555521869
```

```
bias + var = 0.6253265604843571
E[E_out] - bias + var = -0.008716804932170197
```

```
# of Trials: 50 ; test data set size: 5000
```

```
-----
g_hatx = 0.12873186566634295x + 0.004689569571220267
bias    = 0.202062918766103
```

```
var          = 0.2903475107157881
E[E_out]    = 0.4866034792675752

bias + var = 0.4924104294818911
E[E_out] - bias + var = -0.005806950214315931

# of Trials: 55 ; test data set size: 5000
-----
g_hatx      = 0.019798923995840795x + -0.020829622662307867
bias        = 0.21386192962823197
var         = 0.3545256991776298
E[E_out]    = 0.5619417070026322

bias + var = 0.5683876288058618
E[E_out] - bias + var = -0.0064459218032295595

# of Trials: 60 ; test data set size: 5000
-----
g_hatx      = 0.06491131910094067x + -0.037242998815063454
bias        = 0.22933549099136408
var         = 0.3962583834162325
E[E_out]    = 0.6189895680173262

bias + var = 0.6255938744075966
E[E_out] - bias + var = -0.006604306390270376

# of Trials: 65 ; test data set size: 5000
-----
g_hatx      = -0.19154698316945193x + -0.03868040757108025
bias        = 0.24636142792020474
var         = 0.293845182601627
E[E_out]    = 0.5356859154048839

bias + var = 0.5402066105218317
E[E_out] - bias + var = -0.004520695116947915

# of Trials: 70 ; test data set size: 5000
-----
g_hatx      = 0.056730886790188596x + 0.04582142749964057
bias        = 0.16821674070466788
var         = 0.26395520679705076
E[E_out]    = 0.42840115883318924

bias + var = 0.4321719475017186
E[E_out] - bias + var = -0.0037707886685294234

# of Trials: 75 ; test data set size: 5000
-----
g_hatx      = 0.017780847020150644x + -0.05902000972023411
bias        = 0.2448585418527458
var         = 0.35350432436891827
E[E_out]    = 0.5936494752300786
```

```
bias + var = 0.598362866221664
E[E_out] - bias + var = -0.004713390991585464

# of Trials: 80 ; test data set size: 5000
-----
g_hatx = 0.14303650149155678x + -0.041788172620778434
bias    = 0.2288075041997471
var     = 0.36274456402295774
E[E_out] = 0.5870177611724178

bias + var = 0.5915520682227049
E[E_out] - bias + var = -0.0045343070502870675

# of Trials: 85 ; test data set size: 5000
-----
g_hatx = 0.05437557117904315x + -0.05842564533705546
bias    = 0.251892408513221
var     = 0.3596650109527834
E[E_out] = 0.6073260663959715

bias + var = 0.6115574194660044
E[E_out] - bias + var = -0.004231353070032862

# of Trials: 90 ; test data set size: 5000
-----
g_hatx = 0.1274762549645074x + 0.02189637017944969
bias    = 0.18521942987361403
var     = 0.2865087326364432
E[E_out] = 0.46854473214743014

bias + var = 0.4717281625100572
E[E_out] - bias + var = -0.003183430362627082

# of Trials: 95 ; test data set size: 5000
-----
g_hatx = -0.009968772848717456x + 0.04426322876260917
bias    = 0.16729003696525122
var     = 0.2845675051815074
E[E_out] = 0.44886209472379546

bias + var = 0.45185754214675866
E[E_out] - bias + var = -0.0029954474229632044

# of Trials: 100 ; test data set size: 5000
-----
g_hatx = -0.11446805124562685x + 0.023118768515339626
bias    = 0.17748901015778276
var     = 0.31538246616524507
E[E_out] = 0.4897176516613753

bias + var = 0.49287147632302786
```

```
E[E_out] - bias + var = -0.003153824661652538

# of Trials: 125 ; test data set size: 5000
-----
g_hatx = -0.11254515460127199x + -0.03497472694654636
bias    = 0.22628549294534997
var     = 0.32679027151922574
E[E_out] = 0.5504614422924217

bias + var = 0.5530757644645757
E[E_out] - bias + var = -0.0026143221721540155

# of Trials: 150 ; test data set size: 5000
-----
g_hatx = 0.056332585238775885x + 0.02816354709608856
bias    = 0.1855667011356123
var     = 0.3079238672913211
E[E_out] = 0.49143774264499124

bias + var = 0.4934905684269334
E[E_out] - bias + var = -0.002052825781942158

# of Trials: 175 ; test data set size: 5000
-----
g_hatx = -0.0202965502773532x + -0.006379980137365953
bias    = 0.20270859073906836
var     = 0.3490394413029918
E[E_out] = 0.5497535209489001

bias + var = 0.5517480320420601
E[E_out] - bias + var = -0.0019945110931600674

# of Trials: 200 ; test data set size: 5000
-----
g_hatx = 0.0216831772943578x + -0.041866626710600624
bias    = 0.23023107184189817
var     = 0.320666453304976
E[E_out] = 0.5492941928803492

bias + var = 0.5508975251468742
E[E_out] - bias + var = -0.0016033322665249816

# of Trials: 225 ; test data set size: 5000
-----
g_hatx = 0.02525761964189235x + 0.013913588375591014
bias    = 0.19574760909468805
var     = 0.3878054643252996
E[E_out] = 0.5818294935785419

bias + var = 0.5835530734199876
E[E_out] - bias + var = -0.001723579841445777
```

```
# of Trials: 250 ; test data set size: 5000
-----
g_hatx = 0.05005804874960797x + 0.022314253208809563
bias    = 0.1878796750846569
var     = 0.31537469067748203
E[E_out] = 0.5019928669994287

bias + var = 0.5032543657621389
E[E_out] - bias + var = -0.0012614987627102003

# of Trials: 300 ; test data set size: 5000
-----
g_hatx = -0.0673803574079747x + 0.032507732809367376
bias    = 0.17617271964019934
var     = 0.3105891767386301
E[E_out] = 0.48572659912303406

bias + var = 0.4867618963788295
E[E_out] - bias + var = -0.0010352972557953755

# of Trials: 350 ; test data set size: 5000
-----
g_hatx = 0.04202220643133549x + -0.020579462430135243
bias    = 0.21781985895084913
var     = 0.32955766893116745
E[E_out] = 0.5464359345422134

bias + var = 0.5473775278820165
E[E_out] - bias + var = -0.0009415933398031484

# of Trials: 400 ; test data set size: 5000
-----
g_hatx = 0.015328292528685767x + 0.029010170005219566
bias    = 0.17404988654151593
var     = 0.2928090583672804
E[E_out] = 0.4661269222628783

bias + var = 0.4668589449087963
E[E_out] - bias + var = -0.0007320226459180401

# of Trials: 450 ; test data set size: 5000
-----
g_hatx = 0.06503205172577722x + -0.01119162533298988
bias    = 0.20494583655934517
var     = 0.3373260437368694
E[E_out] = 0.5415222668656883

bias + var = 0.5422718802962145
E[E_out] - bias + var = -0.0007496134305263191

# of Trials: 500 ; test data set size: 5000
-----
```

```
g_hatx = -0.010364907197048124x + -0.009136537639305579
bias    = 0.2087411455166829
var     = 0.334375306776422
E[E_out] = 0.5424477016795519
```

```
bias + var = 0.5431164522931049
E[E_out] - bias + var = -0.0006687506135529819
```

```
# of Trials: 1000 ; test data set size: 5000
```

```
-----
g_hatx = 0.02993306392194553x + 0.02558199407408325
bias    = 0.18388331853686662
var     = 0.3170870388520029
E[E_out] = 0.5006532703500172
```

```
bias + var = 0.5009703573888695
E[E_out] - bias + var = -0.00031708703885235945
```

```
# of Trials: 1250 ; test data set size: 5000
```

```
-----
g_hatx = -0.004228735515822221x + 0.01185598000089526
bias    = 0.20306259904118357
var     = 0.3462970814512844
E[E_out] = 0.5490826428273067
```

```
bias + var = 0.549359680492468
E[E_out] - bias + var = -0.0002770376651612949
```

```
# of Trials: 1500 ; test data set size: 5000
```

```
-----
g_hatx = 0.011548750553803902x + -0.011513193024172589
bias    = 0.20321655270570282
var     = 0.3330114056238332
E[E_out] = 0.5360059507257875
```

```
bias + var = 0.536227958329536
E[E_out] - bias + var = -0.0002220076037485197
```

```
# of Trials: 1750 ; test data set size: 5000
```

```
-----
g_hatx = 0.01582160640214007x + -0.00811361961818299
bias    = 0.20517321603737232
var     = 0.3396117642319453
E[E_out] = 0.5445909164040423
```

```
bias + var = 0.5447849802693177
E[E_out] - bias + var = -0.00019406386527526243
```

```
# of Trials: 2000 ; test data set size: 5000
```

```
-----
g_hatx = 0.01783803162415005x + 0.014131731920587408
bias    = 0.19892188733977265
```



```
var      = 0.3349122274586549
E[E_out] = 0.5336666586846991
```

```
bias + var = 0.5338341147984276
E[E_out] - bias + var = -0.00016745611372848135
```

```
# of Trials: 2500 ; test data set size: 5000
```

```
-----
g_hatx = -0.0020531385032844882x + 0.00015075457967050142
bias    = 0.20208700292702128
var     = 0.3421114215300357
E[E_out] = 0.5440615798884458
```

```
bias + var = 0.544198424457057
E[E_out] - bias + var = -0.000136844568611183
```

```
# of Trials: 10000 ; test data set size: 5000
```

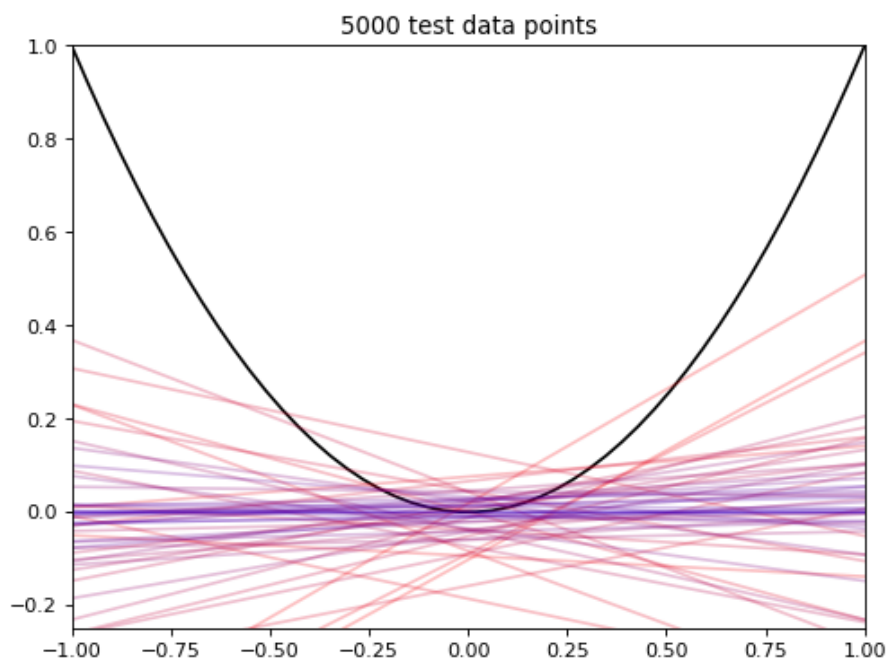
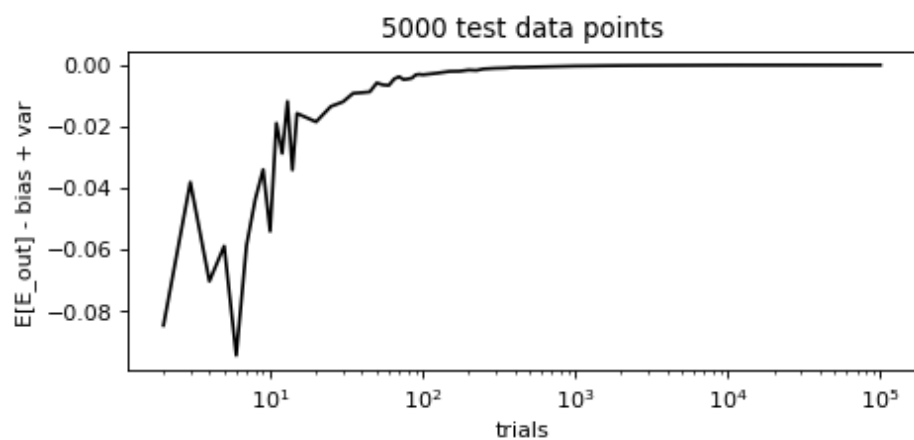
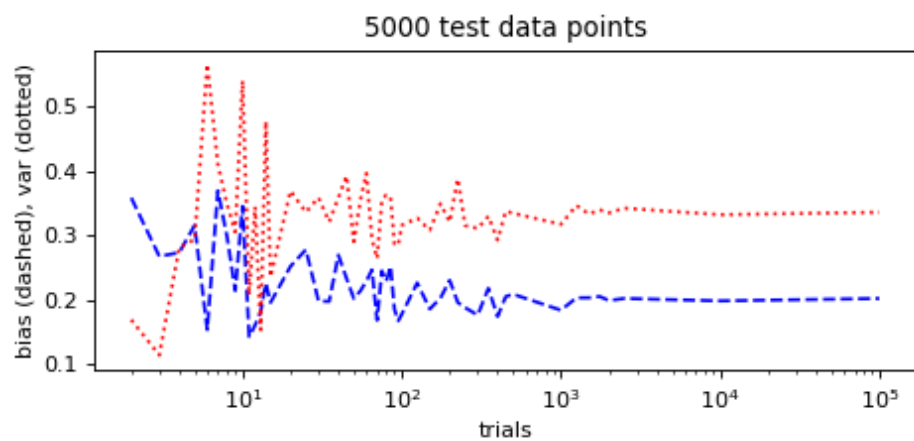
```
-----
g_hatx = 0.0013655123364512975x + -0.0005196850819330593
bias    = 0.1983273738834829
var     = 0.3320325953439668
E[E_out] = 0.5303267659679141
```

```
bias + var = 0.5303599692274497
E[E_out] - bias + var = -3.3203259535596885e-05
```

```
# of Trials: 100000 ; test data set size: 5000
```

```
-----
g_hatx = 0.00012541676753944974x + -1.2480379730067261e-05
bias    = 0.2020482719074776
var     = 0.33578702341859484
E[E_out] = 0.5378319374558374
```

```
bias + var = 0.5378352953260724
E[E_out] - bias + var = -3.357870235087823e-06
```



results_test_points_10000.dat

===== DOING TRIALS 2 , ..., 100000 with 10000 test points =====

of Trials: 2 ; test data set size: 10000

g_hatx = 0.3253441926750037x + 0.04120521977334852
bias = 0.21129370954663176
var = 0.07408232568280206
E[E_out] = 0.24833487238803278

bias + var = 0.28537603522943383
E[E_out] - bias + var = -0.037041162841401035

of Trials: 3 ; test data set size: 10000

g_hatx = 0.22143198752493007x + 0.18214027584715123
bias = 0.1256667036984032
var = 0.2580673477965717
E[E_out] = 0.297711602229451

bias + var = 0.38373405149497486
E[E_out] - bias + var = -0.08602244926552391

of Trials: 4 ; test data set size: 10000

g_hatx = 0.15793201528960904x + -0.01636546806584213
bias = 0.21825069375906797
var = 0.28242001974912545
E[E_out] = 0.4300657085709121

bias + var = 0.5006707135081934
E[E_out] - bias + var = -0.07060500493728133

of Trials: 5 ; test data set size: 10000

g_hatx = 0.4572929126184735x + -0.06389288834703152
bias = 0.3171963793982079
var = 0.30151527526336264
E[E_out] = 0.5584085996088981

bias + var = 0.6187116546615705
E[E_out] - bias + var = -0.060303055052672416

of Trials: 6 ; test data set size: 10000

g_hatx = 0.11589561739561749x + -0.0678374102610182
bias = 0.25798655310379515
var = 0.15589177537494606
E[E_out] = 0.38789636591625015

bias + var = 0.4138783284787412

```
E[E_out] - bias + var = -0.02598196256249105

# of Trials: 7 ; test data set size: 10000
-----
g_hatx = -0.11412514287045905x + 0.15380955701177065
bias    = 0.12637353363867493
var     = 0.10523438408893783
E[E_out] = 0.2165744342863359

bias + var = 0.23160791772761274
E[E_out] - bias + var = -0.015033483441276849

# of Trials: 8 ; test data set size: 10000
-----
g_hatx = 0.0627839778764609x + -0.19413781211780579
bias    = 0.36122495112896
var     = 0.42720943733158523
E[E_out] = 0.735033208794097

bias + var = 0.7884343884605453
E[E_out] - bias + var = -0.053401179666448306

# of Trials: 9 ; test data set size: 10000
-----
g_hatx = -0.06954984457342112x + -0.15176585213406418
bias    = 0.3260464584170248
var     = 0.36891328052631756
E[E_out] = 0.6539693744404181

bias + var = 0.6949597389433424
E[E_out] - bias + var = -0.040990364502924204

# of Trials: 10 ; test data set size: 10000
-----
g_hatx = 0.5235005503022954x + -0.0405562416536414
bias    = 0.3196811973429745
var     = 0.41615504314969115
E[E_out] = 0.6942207361776965

bias + var = 0.7358362404926657
E[E_out] - bias + var = -0.04161550431496913

# of Trials: 11 ; test data set size: 10000
-----
g_hatx = 0.006692962016885709x + 0.05440768128291895
bias    = 0.17161323041232932
var     = 0.38340560768187193
E[E_out] = 0.5201637828503947

bias + var = 0.5550188380942013
E[E_out] - bias + var = -0.03485505524380661
```

```
# of Trials: 12 ; test data set size: 10000
-----
g_hatx = -0.23545049111606006x + -0.03125496293263837
bias    = 0.2381830683344091
var     = 0.32734345564241835
E[E_out] = 0.5382479026732927

bias + var = 0.5655265239768275
E[E_out] - bias + var = -0.027278621303534756

# of Trials: 13 ; test data set size: 10000
-----
g_hatx = -0.3598436961222515x + -0.04140255642884224
bias    = 0.2752294475593554
var     = 0.2987771437195412
E[E_out] = 0.5510237340697013

bias + var = 0.5740065912788966
E[E_out] - bias + var = -0.02298285720919535

# of Trials: 14 ; test data set size: 10000
-----
g_hatx = 0.11880976428812565x + 0.03456381208864908
bias    = 0.18194619279785978
var     = 0.30266001442482815
E[E_out] = 0.4629876347637716

bias + var = 0.48460620722268793
E[E_out] - bias + var = -0.021618572458916352

# of Trials: 15 ; test data set size: 10000
-----
g_hatx = -0.19566223685408268x + -0.05971614169020372
bias    = 0.2561948529232603
var     = 0.5263567123712889
E[E_out] = 0.7474611178031297

bias + var = 0.7825515652945492
E[E_out] - bias + var = -0.03509044749141943

# of Trials: 20 ; test data set size: 10000
-----
g_hatx = -0.09219346099465134x + 0.04287022390136539
bias    = 0.1714100932904922
var     = 0.2999183921207993
E[E_out] = 0.45633256580525167

bias + var = 0.47132848541129146
E[E_out] - bias + var = -0.014995919606039854

# of Trials: 25 ; test data set size: 10000
-----
```

```
g_hatx = 0.2831625226416845x + -0.02205260774857779
bias    = 0.23745394428130676
var     = 0.24554805314003572
E[E_out] = 0.4731800752957411
```

```
bias + var = 0.4830019974213425
E[E_out] - bias + var = -0.009821922125601401
```

```
# of Trials: 30 ; test data set size: 10000
```

```
-----
g_hatx = -0.02165426920178264x + 0.011452207933566817
bias    = 0.1926234641357546
var     = 0.3174548454862452
E[E_out] = 0.4994964814391249
```

```
bias + var = 0.5100783096219998
E[E_out] - bias + var = -0.010581828182874897
```

```
# of Trials: 35 ; test data set size: 10000
```

```
-----
g_hatx = -0.03633938546789343x + 0.0025712610296676174
bias    = 0.20113959226284406
var     = 0.2991886791601588
E[E_out] = 0.49178002344699817
```

```
bias + var = 0.5003282714230028
E[E_out] - bias + var = -0.008548247976004675
```

```
# of Trials: 40 ; test data set size: 10000
```

```
-----
g_hatx = 0.10799646082484242x + 0.026053009967173423
bias    = 0.18765211842935842
var     = 0.3003138111658865
E[E_out] = 0.4804580843160976
```

```
bias + var = 0.48796592959524493
E[E_out] - bias + var = -0.007507845279147329
```

```
# of Trials: 45 ; test data set size: 10000
```

```
-----
g_hatx = -0.17490682267181187x + 0.04907803071142521
bias    = 0.17768299050441685
var     = 0.3586651534136474
E[E_out] = 0.5283778071755387
```

```
bias + var = 0.5363481439180643
E[E_out] - bias + var = -0.007970336742525508
```

```
# of Trials: 50 ; test data set size: 10000
```

```
-----
g_hatx = 0.026375630075233955x + -0.012122415869094003
bias    = 0.20794080643053134
```

```
var          = 0.3961763115276254
E[E_out]     = 0.596193591727604

bias + var   = 0.6041171179581567
E[E_out] - bias + var = -0.007923526230552758

# of Trials: 55 ; test data set size: 10000
-----
g_hatx       = -0.12057246139469005x + 0.06546132761784502
bias         = 0.1708598994073289
var          = 0.2773780122299118
E[E_out]     = 0.44319467505124227

bias + var   = 0.4482379116372407
E[E_out] - bias + var = -0.005043236585998423

# of Trials: 60 ; test data set size: 10000
-----
g_hatx       = -0.10159346207325948x + -0.05731787104632042
bias         = 0.24150009748065177
var          = 0.35013534794941825
E[E_out]     = 0.5857998562975797

bias + var   = 0.5916354454300701
E[E_out] - bias + var = -0.005835589132490326

# of Trials: 65 ; test data set size: 10000
-----
g_hatx       = 0.2037925108436385x + -0.041425947039148606
bias         = 0.24548505348533936
var          = 0.34255261959752553
E[E_out]     = 0.5827676327813647

bias + var   = 0.5880376730828649
E[E_out] - bias + var = -0.0052700403015001895

# of Trials: 70 ; test data set size: 10000
-----
g_hatx       = 0.10092541276116236x + -0.034171131122520876
bias         = 0.22627836008286623
var          = 0.3262052101360098
E[E_out]     = 0.5478234957883616

bias + var   = 0.552483570218876
E[E_out] - bias + var = -0.00466007443051436

# of Trials: 75 ; test data set size: 10000
-----
g_hatx       = -0.049377032081664704x + -0.0067297948406893435
bias         = 0.20486924619114943
var          = 0.29895653291195323
E[E_out]     = 0.4998396919976098
```

```
bias + var = 0.5038257791031027
E[E_out] - bias + var = -0.003986087105492864

# of Trials: 80 ; test data set size: 10000
-----
g_hatx = 0.020534209402326205x + -0.03511566438888134
bias    = 0.22597325962775794
var     = 0.331030743132507
E[E_out] = 0.5528661184711088

bias + var = 0.557004002760265
E[E_out] - bias + var = -0.0041378842891561796

# of Trials: 85 ; test data set size: 10000
-----
g_hatx = -0.030316588706194428x + -0.014715542600431952
bias    = 0.2112072237334512
var     = 0.3830966821455898
E[E_out] = 0.5897968860890926

bias + var = 0.5943039058790409
E[E_out] - bias + var = -0.00450701978994833

# of Trials: 90 ; test data set size: 10000
-----
g_hatx = 0.00835945210215429x + -0.062121283817557445
bias    = 0.24322900398811503
var     = 0.38638786905341627
E[E_out] = 0.6253236744964932

bias + var = 0.6296168730415312
E[E_out] - bias + var = -0.004293198545038124

# of Trials: 95 ; test data set size: 10000
-----
g_hatx = 0.051332182901941165x + -0.03835838718165295
bias    = 0.22502955350079473
var     = 0.3898944258245749
E[E_out] = 0.6108198274745845

bias + var = 0.6149239793253696
E[E_out] - bias + var = -0.004104151850785109

# of Trials: 100 ; test data set size: 10000
-----
g_hatx = -0.046445755198045335x + 0.018347521485838382
bias    = 0.18883130630866543
var     = 0.3215397117398488
E[E_out] = 0.5071556209311158

bias + var = 0.5103710180485143
```



```
E[E_out] - bias + var = -0.003215397117398433

# of Trials: 125 ; test data set size: 10000
-----
g_hatx = 0.08548622964552725x + 0.04851017637687247
bias    = 0.17068630993744563
var     = 0.30242800979890416
E[E_out] = 0.47069489565795886

bias + var = 0.47311431973634976
E[E_out] - bias + var = -0.0024194240783909082

# of Trials: 150 ; test data set size: 10000
-----
g_hatx = -0.0437192228080012x + 0.03441242739741545
bias    = 0.17663381672112127
var     = 0.2820227589466278
E[E_out] = 0.45677642394143814

bias + var = 0.4586565756677491
E[E_out] - bias + var = -0.0018801517263109524

# of Trials: 175 ; test data set size: 10000
-----
g_hatx = 0.1227821732055049x + -0.02731884942547583
bias    = 0.22431340014334727
var     = 0.3514396770599407
E[E_out] = 0.5737448504772312

bias + var = 0.5757530772032879
E[E_out] - bias + var = -0.002008226726056772

# of Trials: 200 ; test data set size: 10000
-----
g_hatx = -0.07984144189796717x + -0.007573662288449752
bias    = 0.21386650896139994
var     = 0.32836855379099944
E[E_out] = 0.5405932199834447

bias + var = 0.5422350627523994
E[E_out] - bias + var = -0.0016418427689546822

# of Trials: 225 ; test data set size: 10000
-----
g_hatx = 0.05513828457806916x + 0.021079101857519558
bias    = 0.18986869164963827
var     = 0.3335545965468823
E[E_out] = 0.5219408233229789

bias + var = 0.5234232881965206
E[E_out] - bias + var = -0.0014824648735417
```

```
# of Trials: 250 ; test data set size: 10000
-----
g_hatx = -0.0026715667995391387x + -0.02600948382072581
bias    = 0.21646910985842274
var     = 0.33358533703614923
E[E_out] = 0.5487201055464276

bias + var = 0.550054446894572
E[E_out] - bias + var = -0.0013343413481443633

# of Trials: 300 ; test data set size: 10000
-----
g_hatx = 0.015858507667375673x + -0.007539182854029279
bias    = 0.20987728725397584
var     = 0.31959659557595455
E[E_out] = 0.5284085608446774

bias + var = 0.5294738828299304
E[E_out] - bias + var = -0.0010653219852529983

# of Trials: 350 ; test data set size: 10000
-----
g_hatx = -0.024686753483228498x + 0.004023130090547186
bias    = 0.20371373152372074
var     = 0.3312633867371052
E[E_out] = 0.5340306514415775

bias + var = 0.534977118260826
E[E_out] - bias + var = -0.0009464668192484815

# of Trials: 400 ; test data set size: 10000
-----
g_hatx = 0.042344601891510104x + -0.03245925841048945
bias    = 0.2170452524591235
var     = 0.3412424963440494
E[E_out] = 0.5574346425623133

bias + var = 0.5582877488031729
E[E_out] - bias + var = -0.0008531062408596024

# of Trials: 450 ; test data set size: 10000
-----
g_hatx = 0.004223659193843887x + -0.024085675027613884
bias    = 0.2189037192871824
var     = 0.36859116938491104
E[E_out] = 0.5866757971845712

bias + var = 0.5874948886720934
E[E_out] - bias + var = -0.0008190914875222943

# of Trials: 500 ; test data set size: 10000
-----
```

```
g_hatx = 0.0030636265898100635x + 0.006356590861649642
bias    = 0.1949353581465581
var     = 0.3329744656044348
E[E_out] = 0.5272438748197834
```

```
bias + var = 0.5279098237509929
E[E_out] - bias + var = -0.0006659489312094702
```

```
# of Trials: 1000 ; test data set size: 10000
```

```
-----
g_hatx = -0.040381751806959985x + -0.009968219851117587
bias    = 0.20591601714680802
var     = 0.3272078426043867
E[E_out] = 0.5327966519085904
```

```
bias + var = 0.5331238597511947
E[E_out] - bias + var = -0.0003272078426043157
```

```
# of Trials: 1250 ; test data set size: 10000
```

```
-----
g_hatx = 0.027479153629139882x + -0.007733063504701714
bias    = 0.20646665897540842
var     = 0.3368933822673576
E[E_out] = 0.5430905265369524
```

```
bias + var = 0.543360041242766
E[E_out] - bias + var = -0.000269514705813545
```

```
# of Trials: 1500 ; test data set size: 10000
```

```
-----
g_hatx = -0.017790135483496144x + 2.474378474120283e-05
bias    = 0.2020670114677688
var     = 0.33740755467735817
E[E_out] = 0.5392496277753424
```

```
bias + var = 0.539474566145127
E[E_out] - bias + var = -0.0002249383697845997
```

```
# of Trials: 1750 ; test data set size: 10000
```

```
-----
g_hatx = -0.026896035250723507x + -0.006174480205611782
bias    = 0.20330632458757683
var     = 0.33121153737738757
E[E_out] = 0.5343285982293204
```

```
bias + var = 0.5345178619649644
E[E_out] - bias + var = -0.00018926373564392973
```

```
# of Trials: 2000 ; test data set size: 10000
```

```
-----
g_hatx = -0.01527874072348723x + -0.00244323555178742
bias    = 0.2029499949152771
```

```
var          = 0.33561524603265247
E[E_out]    = 0.5383974333249131

bias + var = 0.5385652409479296
E[E_out] - bias + var = -0.00016780762301649155

# of Trials: 2500 ; test data set size: 10000
-----
g_hatx      = 0.00017492234337228069x + 0.008795459392959363
bias        = 0.19660108064606088
var         = 0.31957158246538453
E[E_out]    = 0.5160448344784592

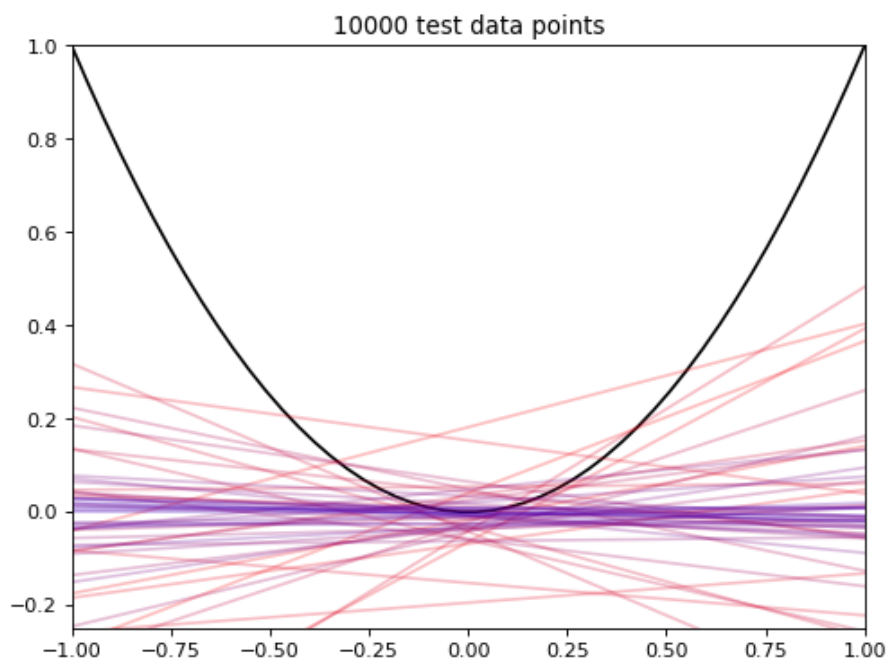
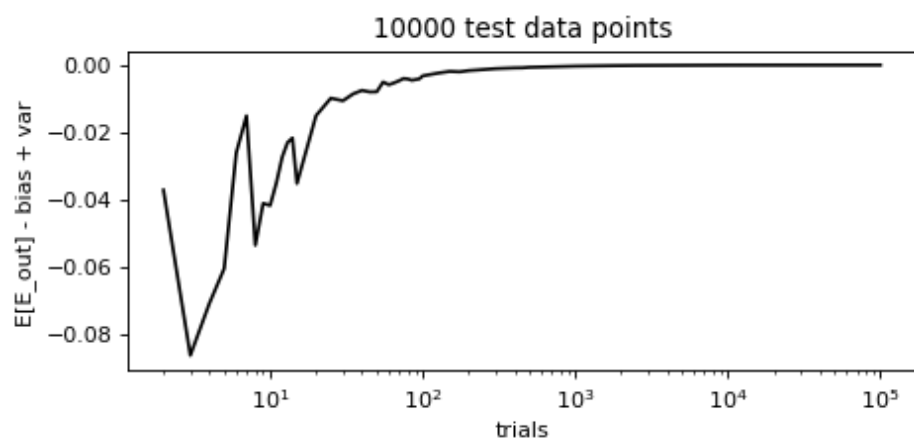
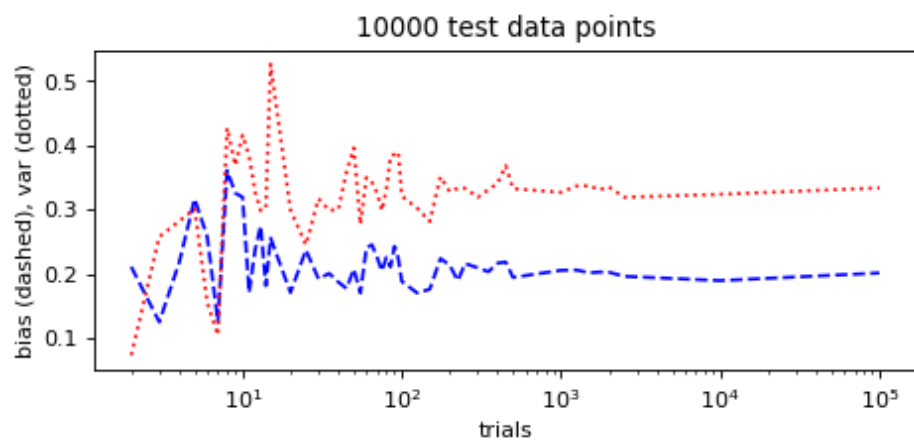
bias + var = 0.5161726631114454
E[E_out] - bias + var = -0.0001278286329862155

# of Trials: 10000 ; test data set size: 10000
-----
g_hatx      = -0.019681917964858308x + 0.008873755771705957
bias        = 0.18998543079419236
var         = 0.3241424641460453
E[E_out]    = 0.5140954806938245

bias + var = 0.5141278949402377
E[E_out] - bias + var = -3.2414246413203784e-05

# of Trials: 100000 ; test data set size: 10000
-----
g_hatx      = -0.004133610840487063x + -0.002527418476091182
bias        = 0.20181060090388522
var         = 0.3340926825799743
E[E_out]    = 0.5358999425570393

bias + var = 0.5359032834838595
E[E_out] - bias + var = -3.340926820250001e-06
```



results_trial2_25.dat

===== DOING TESTS 5 , ..., 5000 with 25 trial size =====

of Trials: 25 ; test data set size: 5

g_hatx = -0.24061677679417215x + -0.061796101243319954
bias = 0.2970337152102858
var = 0.3622805853444551
E[E_out] = 0.6448230771409625

bias + var = 0.6593143005547409
E[E_out] - bias + var = -0.014491223413778398

of Trials: 25 ; test data set size: 10

g_hatx = -0.24061677679417215x + -0.061796101243319954
bias = 0.436091083045029
var = 0.37422404054162506
E[E_out] = 0.7953461619649891

bias + var = 0.8103151235866541
E[E_out] - bias + var = -0.014968961621665

of Trials: 25 ; test data set size: 15

g_hatx = -0.24061677679417215x + -0.061796101243319954
bias = 0.324693071100075
var = 0.2788478130779813
E[E_out] = 0.592386971654937

bias + var = 0.6035408841780563
E[E_out] - bias + var = -0.011153912523119258

of Trials: 25 ; test data set size: 20

g_hatx = -0.24061677679417215x + -0.061796101243319954
bias = 0.19173100667124734
var = 0.21739242857478847
E[E_out] = 0.4004277381030443

bias + var = 0.4091234352460358
E[E_out] - bias + var = -0.008695697142991538

of Trials: 25 ; test data set size: 25

g_hatx = -0.24061677679417215x + -0.061796101243319954
bias = 0.2594172698303383
var = 0.28131363264038894
E[E_out] = 0.5294783571651117

bias + var = 0.5407309024707272

```
E[E_out] - bias + var = -0.011252545305615536

# of Trials: 25 ; test data set size: 30
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.5112650924891708
var     = 0.3623074340909262
E[E_out] = 0.8590802292164599

bias + var = 0.8735725265800971
E[E_out] - bias + var = -0.014492297363637163

# of Trials: 25 ; test data set size: 40
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.2645292835695911
var     = 0.269295677725714
E[E_out] = 0.5230531341862765

bias + var = 0.5338249612953051
E[E_out] - bias + var = -0.010771827109028576

# of Trials: 25 ; test data set size: 50
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.3550381660279706
var     = 0.312352097700806
E[E_out] = 0.6548961798207443

bias + var = 0.6673902637287766
E[E_out] - bias + var = -0.012494083908032305

# of Trials: 25 ; test data set size: 60
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.3052712441083864
var     = 0.29012853427113355
E[E_out] = 0.5837946370086747

bias + var = 0.59539977837952
E[E_out] - bias + var = -0.011605141370845229

# of Trials: 25 ; test data set size: 70
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.28643455170024246
var     = 0.275053181138374
E[E_out] = 0.5504856055930815

bias + var = 0.5614877328386165
E[E_out] - bias + var = -0.01100212724553501
```

```
# of Trials: 25 ; test data set size: 80
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.253922966391532
var     = 0.27756445100535876
E[E_out] = 0.5203848393566765

bias + var = 0.5314874173968908
E[E_out] - bias + var = -0.011102578040214306

# of Trials: 25 ; test data set size: 90
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.20591158347859176
var     = 0.22523809000861503
E[E_out] = 0.42214014988686216

bias + var = 0.4311496734872068
E[E_out] - bias + var = -0.009009523600344627

# of Trials: 25 ; test data set size: 100
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.2437261701698468
var     = 0.25612978401062886
E[E_out] = 0.48961076282005045

bias + var = 0.4998559541804757
E[E_out] - bias + var = -0.010245191360425204

# of Trials: 25 ; test data set size: 110
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.22262182928863225
var     = 0.25965481066895835
E[E_out] = 0.4718904475308323

bias + var = 0.4822766399575906
E[E_out] - bias + var = -0.010386192426758284

# of Trials: 25 ; test data set size: 120
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.2867334856635655
var     = 0.2759450686354268
E[E_out] = 0.5516407515535753

bias + var = 0.5626785542989923
E[E_out] - bias + var = -0.011037802745416991

# of Trials: 25 ; test data set size: 130
-----
```



```
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.21588181749725377
var     = 0.24942537553027888
E[E_out] = 0.4553301780063214
```

```
bias + var = 0.46530719302753265
E[E_out] - bias + var = -0.009977015021211233
```

```
# of Trials: 25 ; test data set size: 140
```

```
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.23904160351432577
var     = 0.2669914421253921
E[E_out] = 0.4953533879547022
```

```
bias + var = 0.5060330456397178
E[E_out] - bias + var = -0.01067965768501572
```

```
# of Trials: 25 ; test data set size: 150
```

```
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.2435175000456305
var     = 0.26882734115661516
E[E_out] = 0.501591747555981
```

```
bias + var = 0.5123448412022457
E[E_out] - bias + var = -0.010753093646264644
```

```
# of Trials: 25 ; test data set size: 175
```

```
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.3186591080717051
var     = 0.30421524485964707
E[E_out] = 0.6107057431369662
```

```
bias + var = 0.6228743529313522
E[E_out] - bias + var = -0.012168609794385943
```

```
# of Trials: 25 ; test data set size: 200
```

```
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.31162582551650025
var     = 0.2965065414011517
E[E_out] = 0.596272105261606
```

```
bias + var = 0.6081323669176519
E[E_out] - bias + var = -0.011860261656045923
```

```
# of Trials: 25 ; test data set size: 225
```

```
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.25942481383333377
```

```
var          = 0.2784385826250067
E[E_out]     = 0.5267258531533402

bias + var   = 0.5378633964583405
E[E_out] - bias + var = -0.011137543305000275

# of Trials: 25 ; test data set size: 250
-----
g_hatx      = -0.24061677679417215x + -0.061796101243319954
bias        = 0.24725161421566227
var         = 0.27117135683086896
E[E_out]    = 0.5075761167732965

bias + var   = 0.5184229710465312
E[E_out] - bias + var = -0.01084685427323473

# of Trials: 25 ; test data set size: 275
-----
g_hatx      = -0.24061677679417215x + -0.061796101243319954
bias        = 0.2628368153182596
var         = 0.26805556787197987
E[E_out]    = 0.5201701604753604

bias + var   = 0.5308923831902395
E[E_out] - bias + var = -0.01072222714879104

# of Trials: 25 ; test data set size: 300
-----
g_hatx      = -0.24061677679417215x + -0.061796101243319954
bias        = 0.2691879164625797
var         = 0.27551485823316135
E[E_out]    = 0.5336821803664147

bias + var   = 0.5447027746957411
E[E_out] - bias + var = -0.011020594329326394

# of Trials: 25 ; test data set size: 325
-----
g_hatx      = -0.24061677679417215x + -0.061796101243319954
bias        = 0.24009950456038873
var         = 0.2581585535098911
E[E_out]    = 0.4879317159298842

bias + var   = 0.4982580580702798
E[E_out] - bias + var = -0.0103263421403956

# of Trials: 25 ; test data set size: 350
-----
g_hatx      = -0.24061677679417215x + -0.061796101243319954
bias        = 0.22886242663845086
var         = 0.25034443991368555
E[E_out]    = 0.4691930889555889
```

```
bias + var = 0.4792068665521364
E[E_out] - bias + var = -0.01001377759654748

# of Trials: 25 ; test data set size: 500
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.26148612082390593
var     = 0.2757217562420115
E[E_out] = 0.526179006816237

bias + var = 0.5372078770659174
E[E_out] - bias + var = -0.011028870249680467

# of Trials: 25 ; test data set size: 750
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.28411433838426026
var     = 0.28592113076392733
E[E_out] = 0.5585986239176305

bias + var = 0.5700354691481876
E[E_out] - bias + var = -0.011436845230557102

# of Trials: 25 ; test data set size: 1000
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.26532621965297015
var     = 0.2741457625007432
E[E_out] = 0.5285061516536836

bias + var = 0.5394719821537133
E[E_out] - bias + var = -0.010965830500029694

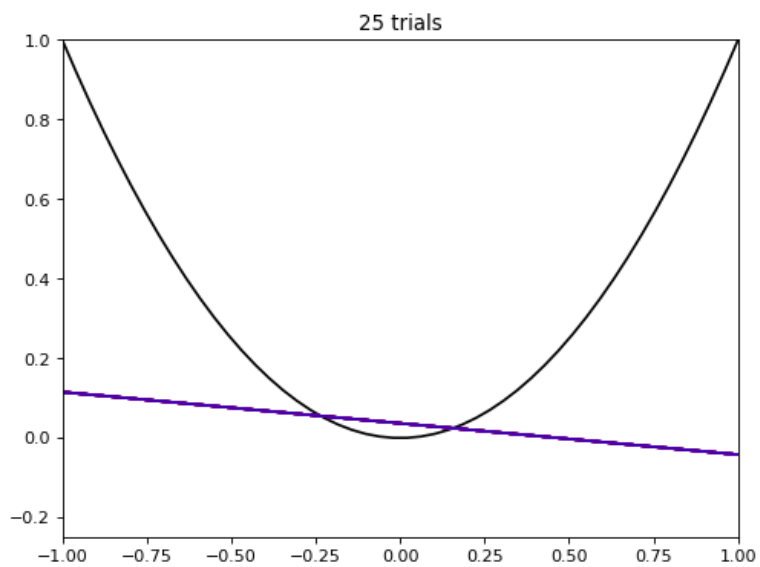
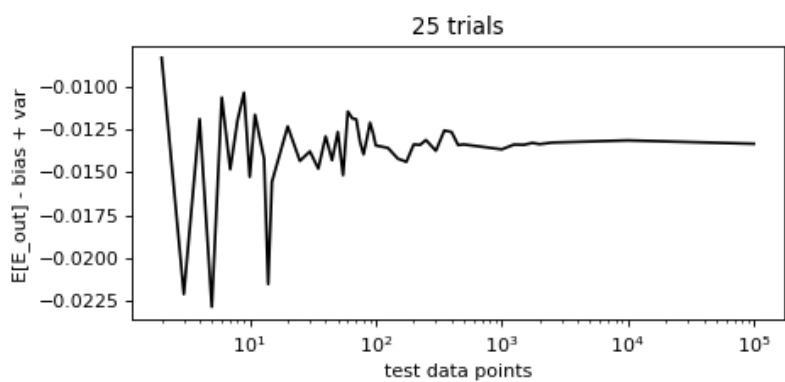
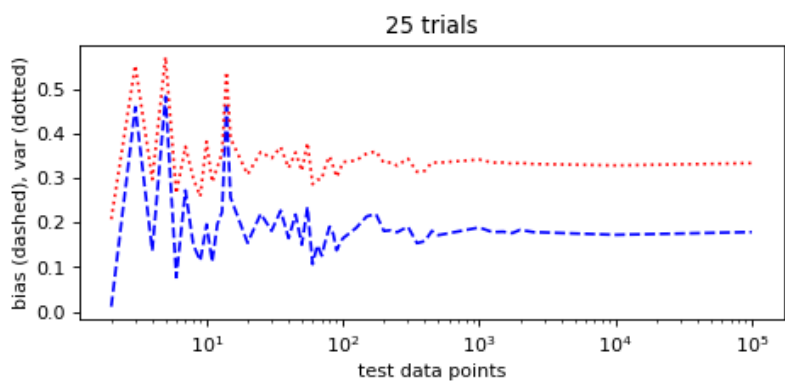
# of Trials: 25 ; test data set size: 2500
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.2611870909249068
var     = 0.2727525701293731
E[E_out] = 0.5230295582491049

bias + var = 0.5339396610542799
E[E_out] - bias + var = -0.01091010280517496

# of Trials: 25 ; test data set size: 5000
-----
g_hatx = -0.24061677679417215x + -0.061796101243319954
bias    = 0.26868411828637934
var     = 0.27340818326034716
E[E_out] = 0.5311559742163127

bias + var = 0.5420923015467265
```

$$E[E_{\text{out}}] - \text{bias} + \text{var} = -0.010936327330413831$$



results_pathological_50.dat

```
===== DOING TESTS 5 , ..., 5000 with 50 trial size =====
```

```
# of Trials: 50 ; test data set size: 5
```

```
-----  
g_hatx = 0.047610780339347986x + 0.08523701152871307  
bias    = 0.3088280515485921  
var     = 0.38580132639133585  
E[E_out] = 0.6869133514121012
```

```
bias + var = 0.694629377939928  
E[E_out] - bias + var = -0.007716026527826769
```

```
# of Trials: 50 ; test data set size: 10
```

```
-----  
g_hatx = 0.047610780339347986x + 0.08523701152871307  
bias    = 0.2564875756379502  
var     = 0.3783474328461328  
E[E_out] = 0.6272680598271603
```

```
bias + var = 0.634835008484083  
E[E_out] - bias + var = -0.007566948656922756
```

```
# of Trials: 50 ; test data set size: 15
```

```
-----  
g_hatx = 0.047610780339347986x + 0.08523701152871307  
bias    = 0.178243149604129  
var     = 0.3062138675132507  
E[E_out] = 0.4783327397671147
```

```
bias + var = 0.48445701711737965  
E[E_out] - bias + var = -0.0061242773502649395
```

```
# of Trials: 50 ; test data set size: 20
```

```
-----  
g_hatx = 0.047610780339347986x + 0.08523701152871307  
bias    = 0.08620183265849454  
var     = 0.257998590344859  
E[E_out] = 0.33904045119645637
```

```
bias + var = 0.3442004230033535  
E[E_out] - bias + var = -0.005159971806897146
```

```
# of Trials: 50 ; test data set size: 25
```

```
-----  
g_hatx = 0.047610780339347986x + 0.08523701152871307  
bias    = 0.15731287658646942  
var     = 0.3139579019604222  
E[E_out] = 0.46499162050768317
```

```
bias + var = 0.47127077854689164
```

```
E[E_out] - bias + var = -0.006279158039208477

# of Trials: 50 ; test data set size: 30
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.22077025026700986
var     = 0.3656278988110064
E[E_out] = 0.579085591101796

bias + var = 0.5863981490780162
E[E_out] - bias + var = -0.007312557976220213

# of Trials: 50 ; test data set size: 40
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.1589140525387191
var     = 0.3008799941534975
E[E_out] = 0.4537764468091466

bias + var = 0.4597940466922166
E[E_out] - bias + var = -0.006017599883069991

# of Trials: 50 ; test data set size: 50
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.17906084717681478
var     = 0.3327882523131788
E[E_out] = 0.5051933344437299

bias + var = 0.5118490994899936
E[E_out] - bias + var = -0.006655765046263695

# of Trials: 50 ; test data set size: 60
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.16760575643244316
var     = 0.3158779320763586
E[E_out] = 0.4771661298672745

bias + var = 0.48348368850880175
E[E_out] - bias + var = -0.006317558641527277

# of Trials: 50 ; test data set size: 70
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.14230606757610328
var     = 0.3021489979793724
E[E_out] = 0.43841208559588823

bias + var = 0.44445506555547565
E[E_out] - bias + var = -0.006042979959587413
```

```
# of Trials: 50 ; test data set size: 80
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15718125290847457
var     = 0.30923241659442674
E[E_out] = 0.4602290211710127

bias + var = 0.4664136695029013
E[E_out] - bias + var = -0.006184648331888587

# of Trials: 50 ; test data set size: 90
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.095436736942027
var     = 0.2629400606788521
E[E_out] = 0.35311799640730207

bias + var = 0.35837679762087915
E[E_out] - bias + var = -0.005258801213577025

# of Trials: 50 ; test data set size: 100
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.12976832474357067
var     = 0.29013818857268353
E[E_out] = 0.4141037495448005

bias + var = 0.41990651331625417
E[E_out] - bias + var = -0.005802763771453667

# of Trials: 50 ; test data set size: 110
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.1408528276603388
var     = 0.2950998125598493
E[E_out] = 0.430050643968991

bias + var = 0.4359526402201881
E[E_out] - bias + var = -0.005901996251197084

# of Trials: 50 ; test data set size: 120
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.14514453666628585
var     = 0.30523265161788227
E[E_out] = 0.4442725352518105

bias + var = 0.4503771882841681
E[E_out] - bias + var = -0.006104653032357632

# of Trials: 50 ; test data set size: 130
-----
```

```
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.12761372367718074
var     = 0.284349324356404
E[E_out] = 0.4062760615464567
```

```
bias + var = 0.41196304803358474
E[E_out] - bias + var = -0.00568698648712801
```

```
# of Trials: 50 ; test data set size: 140
```

```
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.14003794095456165
var     = 0.29957712901875055
E[E_out] = 0.433623527392937
```

```
bias + var = 0.43961506997331223
E[E_out] - bias + var = -0.005991542580375153
```

```
# of Trials: 50 ; test data set size: 150
```

```
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.14796927286106998
var     = 0.30142546347874055
E[E_out] = 0.44336622707023576
```

```
bias + var = 0.44939473633981053
E[E_out] - bias + var = -0.006028509269574767
```

```
# of Trials: 50 ; test data set size: 175
```

```
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.19152648133489822
var     = 0.3291977206957336
E[E_out] = 0.5141402476167172
```

```
bias + var = 0.5207242020306319
E[E_out] - bias + var = -0.006583954413914683
```

```
# of Trials: 50 ; test data set size: 200
```

```
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.18238316360630272
var     = 0.3229063093874826
E[E_out] = 0.4988313468060357
```

```
bias + var = 0.5052894729937853
E[E_out] - bias + var = -0.006458126187749602
```

```
# of Trials: 50 ; test data set size: 225
```

```
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.16503961136164058
```



```
var          = 0.3109046851732561
E[E_out]    = 0.46972620283143157

bias + var = 0.47594429653489667
E[E_out] - bias + var = -0.0062180937034651

# of Trials: 50 ; test data set size: 250
-----
g_hatx      = 0.047610780339347986x + 0.08523701152871307
bias        = 0.1573862654097443
var         = 0.30561833884309836
E[E_out]    = 0.4568922374759807

bias + var = 0.4630046042528426
E[E_out] - bias + var = -0.006112366776861944

# of Trials: 50 ; test data set size: 275
-----
g_hatx      = 0.047610780339347986x + 0.08523701152871307
bias        = 0.15757092998143768
var         = 0.30069134678716264
E[E_out]    = 0.45224844983285706

bias + var = 0.4582622767686003
E[E_out] - bias + var = -0.0060138269357432605

# of Trials: 50 ; test data set size: 300
-----
g_hatx      = 0.047610780339347986x + 0.08523701152871307
bias        = 0.15068453831800288
var         = 0.3053818091815825
E[E_out]    = 0.4499587113159538

bias + var = 0.45606634749958536
E[E_out] - bias + var = -0.006107636183631582

# of Trials: 50 ; test data set size: 325
-----
g_hatx      = 0.047610780339347986x + 0.08523701152871307
bias        = 0.1321446327737991
var         = 0.2920127372165405
E[E_out]    = 0.41831711524600873

bias + var = 0.4241573699903396
E[E_out] - bias + var = -0.005840254744330864

# of Trials: 50 ; test data set size: 350
-----
g_hatx      = 0.047610780339347986x + 0.08523701152871307
bias        = 0.1310786343659875
var         = 0.2861877487958512
E[E_out]    = 0.4115426281859217
```

```
bias + var = 0.41726638316183867
E[E_out] - bias + var = -0.005723754975916984

# of Trials: 50 ; test data set size: 500
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15514819346314335
var     = 0.30679193429341717
E[E_out] = 0.4558042890706921

bias + var = 0.4619401277565605
E[E_out] - bias + var = -0.006135838685868422

# of Trials: 50 ; test data set size: 750
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15929053204564123
var     = 0.3137963747891628
E[E_out] = 0.46681097933902077

bias + var = 0.47308690683480403
E[E_out] - bias + var = -0.006275927495783207

# of Trials: 50 ; test data set size: 1000
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15111981754764073
var     = 0.30476520060685275
E[E_out] = 0.4497897141423565

bias + var = 0.45588501815449345
E[E_out] - bias + var = -0.006095304012136982

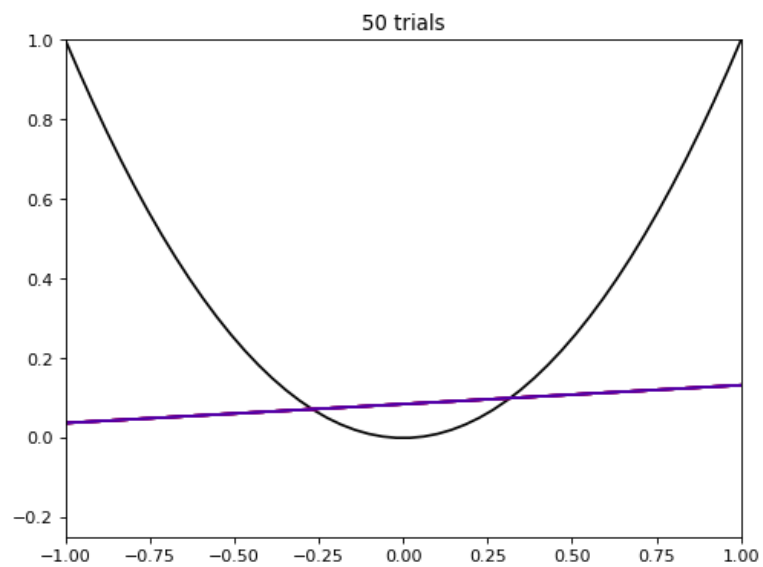
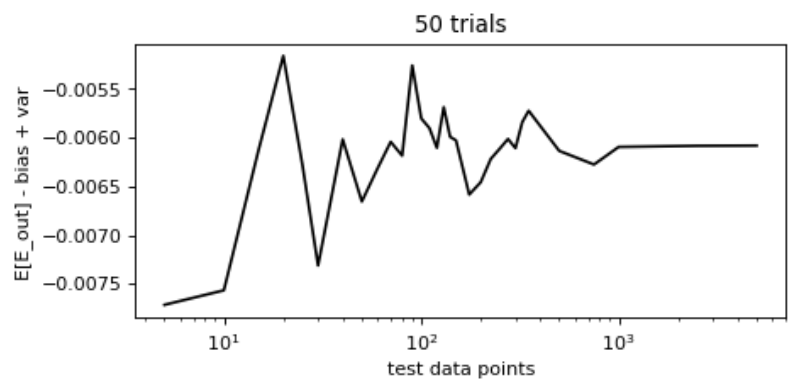
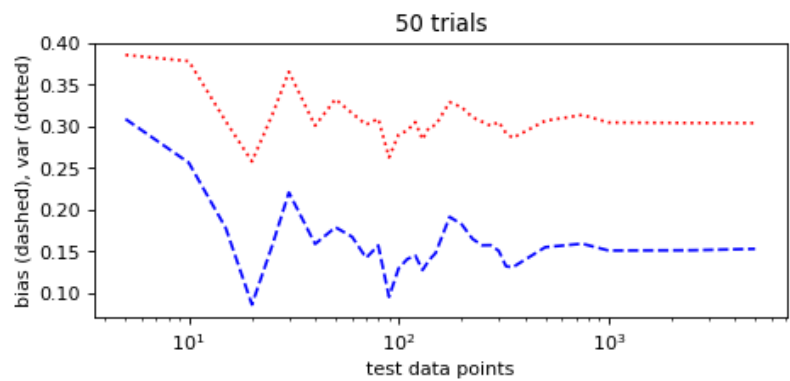
# of Trials: 50 ; test data set size: 2500
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15141982468994397
var     = 0.3041812954632408
E[E_out] = 0.44951749424392

bias + var = 0.45560112015318477
E[E_out] - bias + var = -0.006083625909264756

# of Trials: 50 ; test data set size: 5000
-----
g_hatx = 0.047610780339347986x + 0.08523701152871307
bias    = 0.15308618766872192
var     = 0.304070467514253
E[E_out] = 0.45107524583268976

bias + var = 0.4571566551829749
```

$E[E_{\text{out}}] - \text{bias} + \text{var} = -0.006081409350285172$



results_trial2_100.dat

===== DOING TESTS 5 , ..., 5000 with 100 trial size =====

of Trials: 100 ; test data set size: 5

g_hatx = 0.08594654919953842x + -0.02039296273331749
bias = 0.43482133103859644
var = 0.4862309471102464
E[E_out] = 0.9161899686777404

bias + var = 0.9210522781488428
E[E_out] - bias + var = -0.004862309471102444

of Trials: 100 ; test data set size: 10

g_hatx = 0.08594654919953842x + -0.02039296273331749
bias = 0.34635224681481025
var = 0.5036241883485465
E[E_out] = 0.8449401932798714

bias + var = 0.8499764351633567
E[E_out] - bias + var = -0.005036241883485393

of Trials: 100 ; test data set size: 15

g_hatx = 0.08594654919953842x + -0.02039296273331749
bias = 0.23922101469680296
var = 0.3948941170651385
E[E_out] = 0.6301661905912902

bias + var = 0.6341151317619415
E[E_out] - bias + var = -0.0039489411706512545

of Trials: 100 ; test data set size: 20

g_hatx = 0.08594654919953842x + -0.02039296273331749
bias = 0.12960499539357717
var = 0.3252320396874078
E[E_out] = 0.4515847146841109

bias + var = 0.454837035080985
E[E_out] - bias + var = -0.00325232039687412

of Trials: 100 ; test data set size: 25

g_hatx = 0.08594654919953842x + -0.02039296273331749
bias = 0.23034207982561386
var = 0.39635812124061315
E[E_out] = 0.6227366198538209

bias + var = 0.626700201066227

```
E[E_out] - bias + var = -0.0039635812124060865

# of Trials: 100 ; test data set size: 30
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2883886924930223
var     = 0.49088846719731205
E[E_out] = 0.7743682750183611

bias + var = 0.7792771596903343
E[E_out] - bias + var = -0.0049088846719732016

# of Trials: 100 ; test data set size: 40
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.22369020759551642
var     = 0.38357141178355253
E[E_out] = 0.6034259052612333

bias + var = 0.607261619379069
E[E_out] - bias + var = -0.0038357141178356202

# of Trials: 100 ; test data set size: 50
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.24891961081604552
var     = 0.4328064630513045
E[E_out] = 0.677398009236837

bias + var = 0.6817260738673501
E[E_out] - bias + var = -0.0043280646305129955

# of Trials: 100 ; test data set size: 60
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.23470953120861412
var     = 0.40749481698493106
E[E_out] = 0.6381294000236959

bias + var = 0.6422043481935452
E[E_out] - bias + var = -0.004074948169849324

# of Trials: 100 ; test data set size: 70
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2011922701885561
var     = 0.39084198855213953
E[E_out] = 0.5881258388551743

bias + var = 0.5920342587406956
E[E_out] - bias + var = -0.003908419885521319
```

```
# of Trials: 100 ; test data set size: 80
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2279135394034812
var     = 0.3925170470428624
E[E_out] = 0.6165054159759148

bias + var = 0.6204305864463435
E[E_out] - bias + var = -0.003925170470428785

# of Trials: 100 ; test data set size: 90
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.14002785708601256
var     = 0.3344035397512418
E[E_out] = 0.471087361439742

bias + var = 0.4744313968372544
E[E_out] - bias + var = -0.0033440353975123793

# of Trials: 100 ; test data set size: 100
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.18813249800930051
var     = 0.36874185032408596
E[E_out] = 0.5531869298301456

bias + var = 0.5568743483333864
E[E_out] - bias + var = -0.0036874185032408913

# of Trials: 100 ; test data set size: 110
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2067953904120371
var     = 0.37223443589099814
E[E_out] = 0.5753074819441251

bias + var = 0.5790298263030352
E[E_out] - bias + var = -0.003722344358910168

# of Trials: 100 ; test data set size: 120
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.20684853152063568
var     = 0.39130676021717914
E[E_out] = 0.594242224135643

bias + var = 0.5981552917378148
E[E_out] - bias + var = -0.003913067602171805

# of Trials: 100 ; test data set size: 130
-----
```

```
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.18640762872621858
var     = 0.3612633197330466
E[E_out] = 0.5440583152619347
```

```
bias + var = 0.5476709484592652
E[E_out] - bias + var = -0.0036126331973304615
```

```
# of Trials: 100 ; test data set size: 140
```

```
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.20528498668129308
var     = 0.38084377583256473
E[E_out] = 0.5823203247555322
```

```
bias + var = 0.5861287625138578
E[E_out] - bias + var = -0.0038084377583256224
```

```
# of Trials: 100 ; test data set size: 150
```

```
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.21473077009312647
var     = 0.3828313633150837
E[E_out] = 0.5937338197750595
```

```
bias + var = 0.5975621334082102
E[E_out] - bias + var = -0.003828313633150726
```

```
# of Trials: 100 ; test data set size: 175
```

```
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2673487777193795
var     = 0.4229430054595658
E[E_out] = 0.6860623531243497
```

```
bias + var = 0.6902917831789452
E[E_out] - bias + var = -0.004229430054595584
```

```
# of Trials: 100 ; test data set size: 200
```

```
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.25481842433809954
var     = 0.41426067929447624
E[E_out] = 0.6649364968396311
```

```
bias + var = 0.6690791036325758
E[E_out] - bias + var = -0.004142606792944703
```

```
# of Trials: 100 ; test data set size: 225
```

```
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.23730072376273764
```

```
var          = 0.39328188509507805
E[E_out]     = 0.6266497900068648

bias + var   = 0.6305826088578157
E[E_out] - bias + var = -0.00393281885095087

# of Trials: 100 ; test data set size: 250
-----
g_hatx       = 0.08594654919953842x + -0.02039296273331749
bias         = 0.22790045683503785
var          = 0.38494906561536557
E[E_out]     = 0.6090000317942497

bias + var   = 0.6128495224504034
E[E_out] - bias + var = -0.0038494906561537534

# of Trials: 100 ; test data set size: 275
-----
g_hatx       = 0.08594654919953842x + -0.02039296273331749
bias         = 0.22263509067552564
var          = 0.38198599751256834
E[E_out]     = 0.6008012282129682

bias + var   = 0.604621088188094
E[E_out] - bias + var = -0.0038198599751257944

# of Trials: 100 ; test data set size: 300
-----
g_hatx       = 0.08594654919953842x + -0.02039296273331749
bias         = 0.21573601037642273
var          = 0.39070758829042196
E[E_out]     = 0.6025365227839402

bias + var   = 0.6064435986668447
E[E_out] - bias + var = -0.003907075882904509

# of Trials: 100 ; test data set size: 325
-----
g_hatx       = 0.08594654919953842x + -0.02039296273331749
bias         = 0.19232171372801127
var          = 0.37097672855524993
E[E_out]     = 0.5595886749977088

bias + var   = 0.5632984422832612
E[E_out] - bias + var = -0.00370976728555239

# of Trials: 100 ; test data set size: 350
-----
g_hatx       = 0.08594654919953842x + -0.02039296273331749
bias         = 0.1899569664365571
var          = 0.36204909740700963
E[E_out]     = 0.5483855728694967
```



```
bias + var = 0.5520060638435668
E[E_out] - bias + var = -0.003620490974070023

# of Trials: 100 ; test data set size: 500
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.2230184822505276
var     = 0.3906562112630383
E[E_out] = 0.6097681314009356

bias + var = 0.6136746935135658
E[E_out] - bias + var = -0.003906562112630296

# of Trials: 100 ; test data set size: 750
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.22762416832856847
var     = 0.40244616621351836
E[E_out] = 0.6260458728799518

bias + var = 0.6300703345420868
E[E_out] - bias + var = -0.004024461662134993

# of Trials: 100 ; test data set size: 1000
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.21664189093219255
var     = 0.3890508337520601
E[E_out] = 0.601802216346732

bias + var = 0.6056927246842527
E[E_out] - bias + var = -0.0038905083375206373

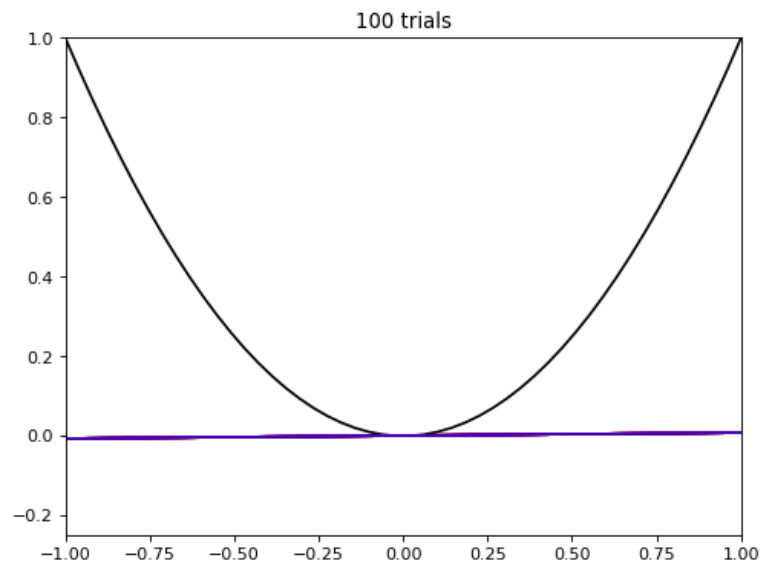
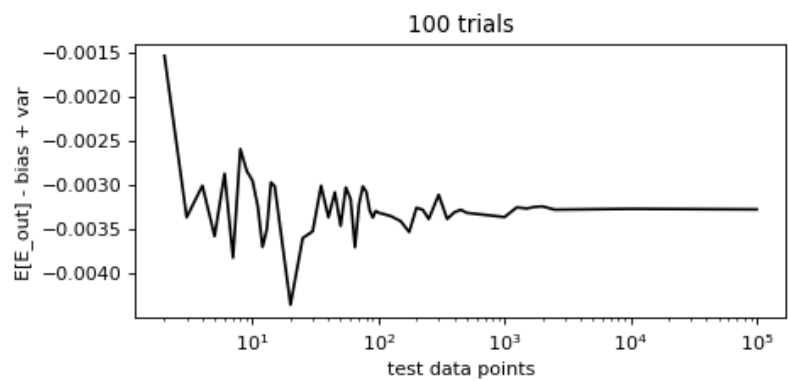
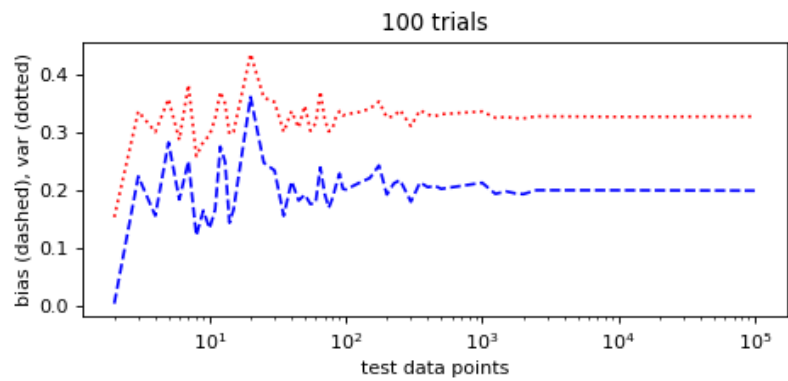
# of Trials: 100 ; test data set size: 2500
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.21746883980158035
var     = 0.3873549398471439
E[E_out] = 0.6009502302502528

bias + var = 0.6048237796487242
E[E_out] - bias + var = -0.003873549398471454

# of Trials: 100 ; test data set size: 5000
-----
g_hatx = 0.08594654919953842x + -0.02039296273331749
bias    = 0.21804245330313315
var     = 0.3882413512321753
E[E_out] = 0.6024013910229866

bias + var = 0.6062838045353085
```

$E[E_{\text{out}}] - \text{bias} + \text{var} = -0.003882413512321825$



AML BOOK

- [1] Malik Magdon-Ismail Yaser S. Abu-Mostafa and Hsuan-Tien Lin. 2012. *Learning From Data A Short Course*. This book was typset by the authors, was printed, and bound in the United States of America, <http://amlbook.com/>. ISBN: 10:1-60049-006-9.