

# Can I make regression with fancy lingo beat Eilmer?

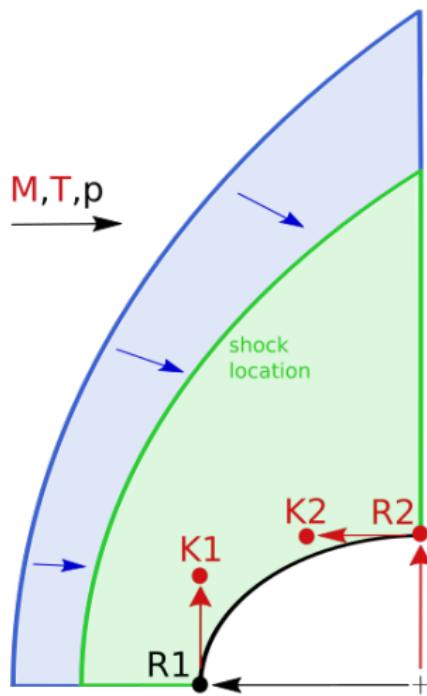
Machine Learning of Bow Shocks

**Jamie Border**

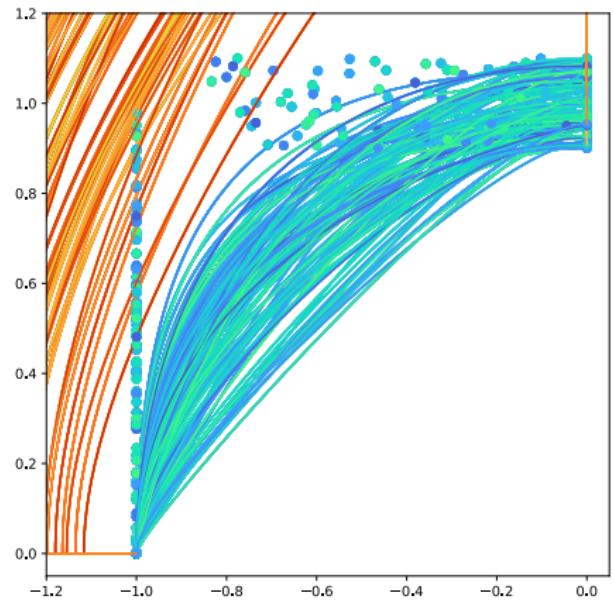
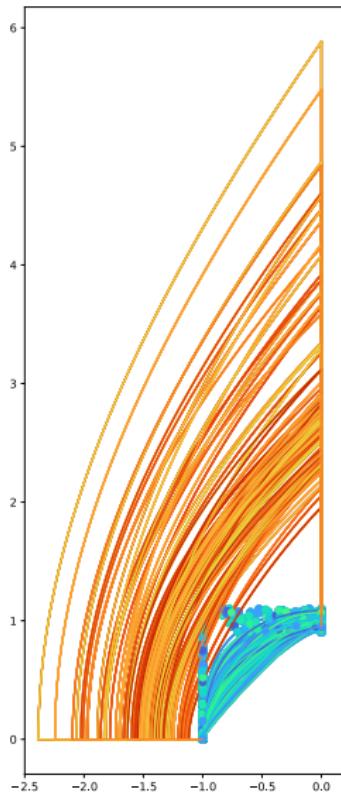
A Lightning Talk

November 18th, 2022

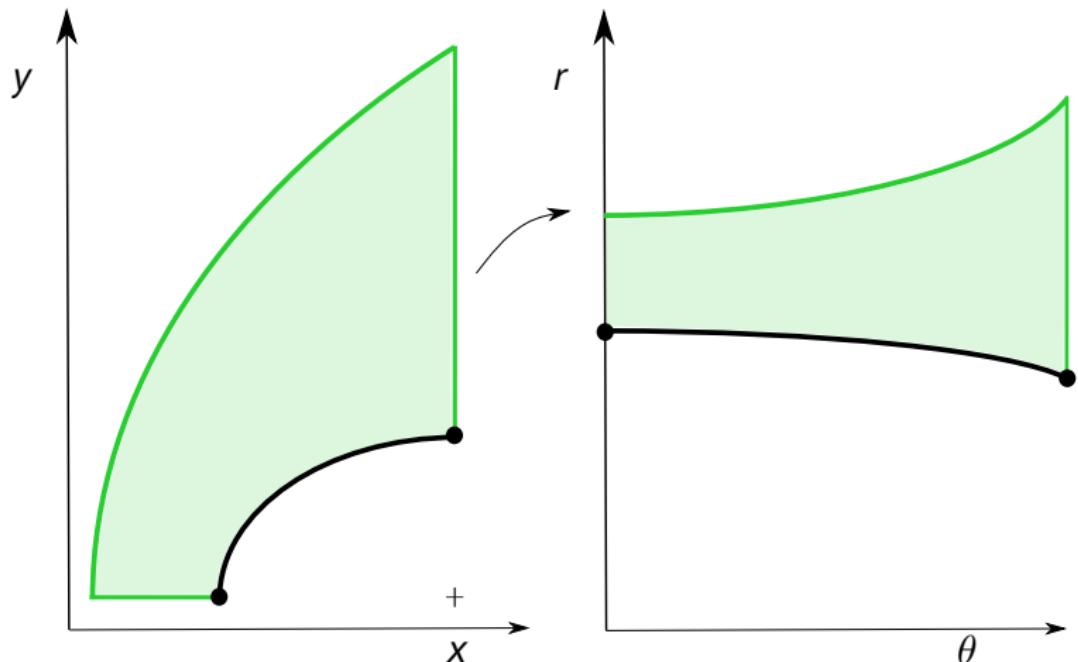
## Problem definition



## Randomly generated bodies and shock-fit bow shocks



## Mapping from cartesian to polar coordinates



## Fitting to shock data

$$r(\theta) = \sum_{k=0}^m a_k \theta^k, \quad \frac{dr(\theta)}{d\theta} = \sum_{k=1}^m k a_k \theta^{k-1}, \quad \theta = [0, \pi/2]$$

Least-squares fit,

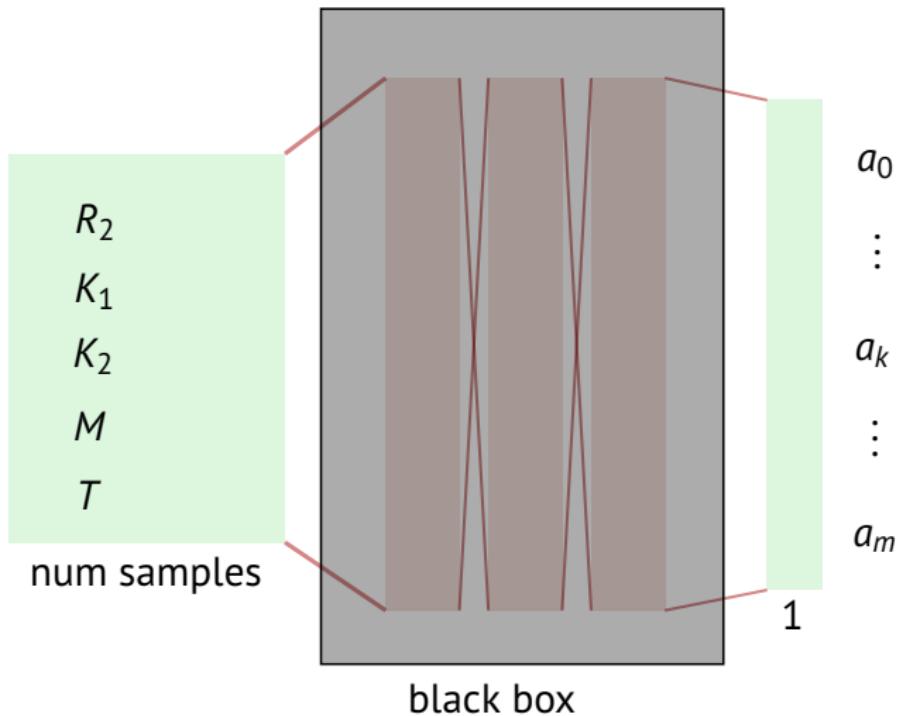
$$I = \sum_{i=1}^N \|r(\theta_i) - r_i\|^2;$$

in matrix form, solving  $I = (\mathbf{Ax} - \mathbf{b})^2$ , giving

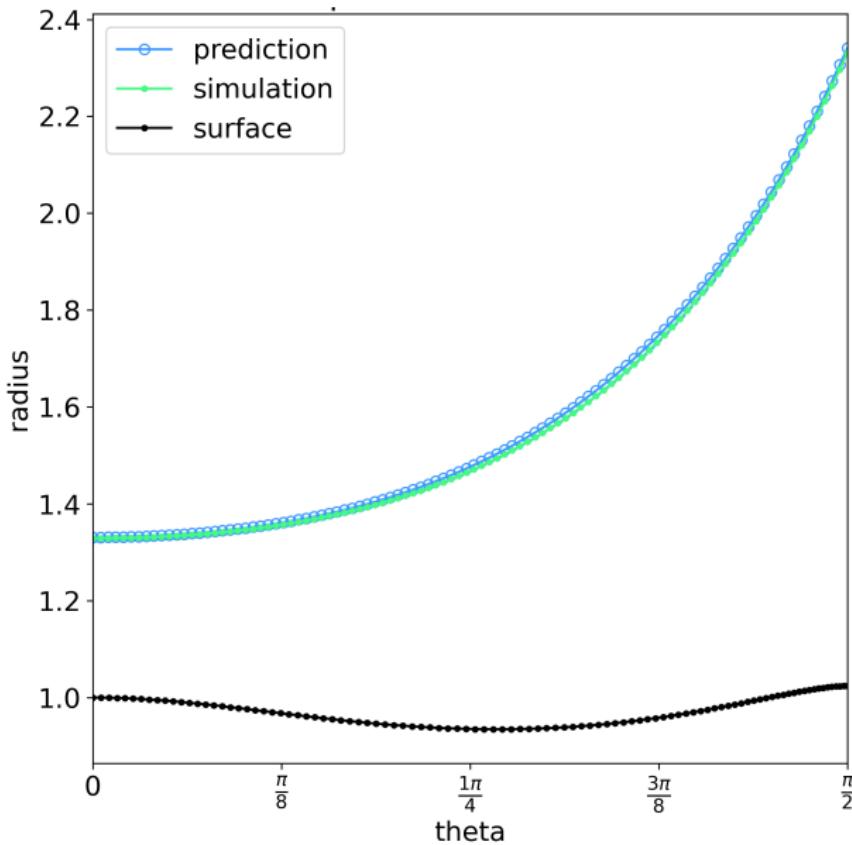
$$\mathbf{A}^T \mathbf{Ax} = \mathbf{A}^T \mathbf{b},$$

or if including gradient terms as constraints

$$\begin{bmatrix} \mathbf{A}^T \mathbf{A} & \mathbf{C}^T \\ \mathbf{C} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T \mathbf{b} \\ \mathbf{d} \end{bmatrix}.$$



## Example fit in polar coordinates



## Example fit in cartesian coordinates

