

General Certificate of Education (A-level) January 2011

Mathematics

MPC1

(Specification 6360)

Pure Core 1

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from: aqa.org.uk

Copyright © 2011 AQA and its licensors. All rights reserved.

Copyright

AQA retains the copyright on all its publications. However, registered centres for AQA are permitted to copy material from this booklet for their own internal use, with the following important exception: AQA cannot give permission to centres to photocopy any material that is acknowledged to a third party even for internal use within the centre.

Set and published by the Assessment and Qualifications Alliance.

Key to mark scheme abbreviations

| M | mark is for method |
|-------------|--|
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| В | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| √or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| –x EE | deduct x marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

| Q | Solution | Marks | Total | Comments |
|--------|--|----------|-------|---|
| 1(a) | $\frac{dy}{dx} = 18 + 6x - 12x^2$ | M1 A1 | | one of these terms correct another term correct |
| | | A1 | 3 | all correct (no + c etc) (penalise + c once only in question) |
| (b) | $18 + 6x - 12x^2 = 0$ | M1 | | putting their $\frac{dy}{dx} = 0$, PI by attempt to solve or factorise |
| | 6 $(3-2x)(x+1)$ (= 0) | m1 | | attempt at factors of their quadratic or use of quadratic equation formula |
| | $x = -1, \ x = \frac{3}{2}$ OE | A1 | 3 | must see both values unless $x = -1$ is verified separately |
| | | | | If M1 not scored, award SC B1 for |
| | | | | verifying that $x = -1$ leads to $\frac{dy}{dx} = 0$ and |
| | | | | a further SC B2 for finding $x = \frac{3}{2}$ as other |
| | | | | value |
| (c)(i) | $\frac{d^2 y}{dx^2} = 6 - 24x$ When $x = -1$, $\frac{d^2 y}{dx^2} = 6 - (24 \times -1)$ | B1√ | | FT their $\frac{dy}{dx}$ but $\frac{d^2y}{dx^2}$ must be correct if 3 |
| | .2 | | | marks earned in part (a) |
| | When $x = -1$, $\frac{d^2 y}{dx^2} = 6 - (24 \times -1)$ | M1 | | Sub $x = -1$ into 'their' $\frac{d^2 y}{dx^2}$ |
| | $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 30$ | Alcso | 3 | |
| (ii) | Minimum point | E1√ | 1 | must have a value in (c)(i) |
| | | | | FT "maximum" if their value of $\frac{d^2 y}{dx^2} < 0$ |
| | Total | | 10 | |

| MPC1 (cont | | Manda | T-4-1 | Community |
|-------------|---|----------|-------|--|
| Q 2(a) | Solution | Marks | Total | Comments |
| 2(a) (b) | $\frac{4\sqrt{3} + 3\sqrt{7}}{3\sqrt{3} + \sqrt{7}} \times \frac{3\sqrt{3} - \sqrt{7}}{3\sqrt{3} - \sqrt{7}}$ | B1 M1 | 1 | |
| | (Numerator =) $36 + 9\sqrt{21} - 4\sqrt{21} - 21$ | m1 | | expanding numerator condone one slip or omission |
| | (Denominator =) 20 $\frac{15 + 5\sqrt{21}}{20}$ | B1 | | must be seen as denominator |
| | $=\frac{3+\sqrt{21}}{4}$ | A1cso | 4 | $m = 3$, $n = 4$ condone $\frac{3}{4} + \frac{\sqrt{21}}{4}$ |
| | Total | | 5 | |
| 3(a)(i) | $y = \frac{1}{2} \left(7 - 3x \right)$ | M1 | | attempt at $y =$ or use of 2 correct points using $\frac{\Delta y}{\Delta x}$ |
| | \Rightarrow gradient = $-\frac{3}{2}$ | A1 | 2 | condone slip in rearranging if gradient is correct |
| (ii) | y = 'their grad' $x + cand substitution of x = 2, y = -7$ | M1 | | or using $3x + 2y = k$ with $x = 2$, $y = -7$ and attempt to find k or $y7 = $ 'their grad' $(x - 2)$ |
| | $y = -\frac{3}{2}x + c, c = -4$ | A1 | | correct equation in any form $y+7=-\frac{3}{2}(x-2)$, $3x+2y+8=0$, etc |
| | $(x=0 \Rightarrow) y=-4$ | A1cso | 3 | or y-intercept = -4 or $D(0, -4)$ |
| (b) | $3x+2(1-4x)=7$, $y=1-\frac{4}{3}(7-2y)$ | M1 | | elimination of y (or x) (condone one slip) |
| | x = -1 $y = 5$ | A1 A1 | 3 | one coordinate correct other coordinate correct coordinates of $A(-1, 5)$ |
| (c) | $(5-2)^2 + (k+7)^2 = 5^2$ (or $k+7=4$ or $k+7=-4$) | M1 | | condone one sign slip within one bracket |
| | k = -3 | A1 | | one correct value of k |
| | or $k = -11$ | A1 | 3 | both correct (and no other values) |
| | Total | | 11 | |

| MPC1 (cont | | | | |
|------------|--|----------------|-------|---|
| Q | Solution | Marks | Total | Comments |
| 4(a)(i) | $\frac{\mathrm{d}y}{\mathrm{d}x} = -1 - 4x^3$ | M1 A1 | | one of these terms correct all correct (no $+ c$) |
| | (When $x = 1$, grad =) -5 | A1cso | 3 | (Check that $\frac{dy}{dx}$ is actually correct!) |
| (ii) | y-12 = 'their grad' $(x-1)$ | M1 | | any form of equation through $(1, 12)$ and attempt at c if using $y = mx + c$ |
| | y = -5x + 17 (or $y = 17 - 5x$) | A1√ | 2 | FT their gradient Condone $y = -5x + c$, $c = 17$ etc |
| (b)(i) | $14x - \frac{x^2}{2} - \frac{x^5}{5}$ $\left[\right]_{-2}^{1} =$ | M1 A1 A1 | | one of these terms correct another term correct all correct (may have $+ c$) |
| | $\begin{bmatrix} 1 \\ -2 \end{bmatrix} = \begin{bmatrix} 14 - \frac{1}{2} - \frac{1}{5} \end{bmatrix} - \left(-28 - 2 + \frac{32}{5} \right)$ | m1 | | F(1) and F(-2) attempted |
| | = 36.9 OE | A1 | 5 | Condone recovery to this value |
| (ii) | Area $\Delta = \frac{1}{2} \times 3 \times 12$ $= 18$ | M1 | | Correct area of triangle unsimplified |
| | \Rightarrow shaded area = 18.9 | A1cso | 2 | |
| | Total | | 12 | |

| Q | G 1 4° | | | |
|---------|--|-------|-------|--|
| | Solution | Marks | Total | Comments |
| 5(a)(i) | <i>y</i> † | M1 | | cubic curve with one max and one min (either way up) |
| | | A1 | | curve touching positive <i>x</i> -axis (either way up) |
| | | A1 | 3 | correct graph passing through <i>O</i> and touching <i>x</i> -axis at 2 |
| (ii) | $x\left(x^2 - 4x + 4\right) = 3$ | | | |
| | $\Rightarrow x^3 - 4x^2 + 4x - 3 = 0$ | B1 | 1 | AG (must have $= 0$) |
| (b)(i) | $p(-1) = (-1)^{3} - 4(-1)^{2} + 4(-1) - 3$ $(= -1 - 4 - 4 - 3)$ | M1 | | p(-1) attempted (condone one slip) or full long division to remainder |
| | = -12 | A1 | 2 | must indicate remainder $=-12$ if long division used |
| | $p(3) = 3^3 - 4 \times 3^2 + 4 \times 3 - 3$ | M1 | | p(3) attempted (condone one slip) NOT long division |
| | p(3) = 27 - 36 + 12 - 3 | | | |
| | $p(3) = 0 \Rightarrow x - 3$ is factor | A1 | 2 | shown = 0 plus statement |
| ` ′ | Either $b = -1$ (coefficient of x correct) or $c = 1$ (constant term correct) | M1 | | allow M1 for full attempt at long division or comparing coefficients if neither <i>b</i> nor <i>c</i> is correct |
| | $p(x) = (x-3)(x^2 - x + 1)$ | A1 | 2 | |
| (c) | Discriminant of 'their quadratic' $= (-1)^2 - 4$ | M1 | | numerical expression must be seen |
| | Discriminant = -3 (or < 0) \Rightarrow no real roots | A1cso | | must have correct quadratic and statement and all working correct |
| | (Only real root is $x = 3$) | В1 | 3 | |
| | Total | | 13 | |

| MPC1 (cont | Solution | Marks | Total | Comments |
|------------|--|----------|-------|--|
| 6(a)(i) | $(x+3)^2 + (y-1)^2$ | B1 | | condone $(x3)^2$ |
| | = 13 | B1 | 2 | condone $(\sqrt{13})^2$ |
| (ii) | $x^{2} + 6x + 9 + y^{2} - 2y + 1$ $x^{2} + y^{2} + 6x - 2y$ | M1 | | attempt to multiply out both of 'their' brackets; must have <i>x</i> and <i>y</i> terms |
| | $x^{2} + y^{2} + 6x - 2y$ | A1 | 2 | both $m = 6$ and $n = -2$ |
| | -3 = 0 | A1 | 3 | All correct, $p = -3$ and = 0 |
| (b) | $x = 0 \implies y^2 - 2y - 3 = 0$ $\Rightarrow (y - 3)(y + 1) = 0$ y = 3, y = -1 | M1 A1 | | putting $x = 0$ PI and attempt to solve or factorise |
| | $\Rightarrow \text{Distance } AB = 3 + 1 = 4$ | Alcso | 3 | OR Pythagoras $d^2 = 13-3^2$ M1 d = 2 A1 distance = $2 \times 2 = 4$ A1 |
| (c)(i) | $(-5+3)^2 + (-2-1)^2 = 4+9$ = 13 | | | Substitution $x = -5$, $y = -2$ into any correct circle equation |
| | $\Rightarrow D$ lies on circle | B1 | 1 | convincing verification plus statement |
| (ii) | $\operatorname{grad} CD = \frac{1+2}{-3+5}$ | M1 | | condone one sign slip |
| | $=\frac{3}{2}$ (or 1.5) | A1 | 2 | $\cot \frac{-3}{-2}$ |
| (iii) | Perpendicular gradient $=-\frac{2}{3}$ | M1 | | ft their grad CD or $m_1m_2 = -1$ stated |
| | Tangent has equation $y+2=-\frac{2}{3}(x+5)$ | A1 | 2 | any form of correct equation eg $2x + 3y + 16 = 0$ $y = -\frac{2}{3}x + c$, $c = -\frac{16}{3}$ |
| | Total | | 13 | |
| | Total | | 13 | |

| MPC1 (cont | Solution | Marks | Total | Comments |
|-----------------|--|-------|-------|---|
| 7(a)(i) | $(-) (x+5)^2$ | M1 | 10001 | $q = 5$; condone $(-x-5)^2$ |
| | $29 - (x+5)^2$ | A1 | 2 | p = 29 and q = 5 |
| | ` ' | | | |
| (ii) | x = -5 is line of symmetry | B1√ | 1 | FT $x = -$ 'their q ' or correct |
| (b)(i) | $4 - 10x - x^2 = k(4x - 13)$ | | | |
| | $\Rightarrow x^2 + 4kx + 10x - 13k - 4 = 0$ | | | Must see both these lines OE |
| | $\Rightarrow x^2 + 2(2k+5)x - (13k+4) = 0$ | B1 | 1 | AG all correct working and = 0 |
| | 2 11 12 1 | | | |
| (ii) | 2 distinct roots $\Rightarrow b^2 - 4ac > 0$ | B1 | | stated or used (must be > 0) |
| | Discriminant = $4(2k+5)^2 + 4(13k+4)$ | M1 | | condone one slip (may be within formula) |
| | $4(4k^2 + 20k + 25 + 13k + 4) > 0$ | | | or $16k^2 + 132k + 116 > 0$ |
| | $\Rightarrow 4k^2 + 33k + 29 > 0$ | A1 | 3 | AG > 0 must appear before final line |
| (iii) | (4k+29)(k+1) | M1 | | correct factors or correct unsimplified quadratic equation formula $\frac{-33 \pm \sqrt{33^2 - 4 \times 4 \times 29}}{8}$ |
| | $k = -\frac{29}{4}$, $k = -1$ | A1 | | condone $k = -\frac{58}{8}$, -7.25 etc but not left |
| | 4 | | | with square roots etc as above |
| $-\frac{29}{4}$ | y -1 O | M1 | | sketch or sign diagram including values + - + -29/4 -1 |
| | $k < -\frac{29}{4}, k > -1$ | A1 | 4 | condone use of OR but not AND |
| | Take their final line as their answer Total | | 11 | |
| | TOTAL | | 75 | |