General Certificate of Education January 2009 Advanced Subsidiary Examination



# MATHEMATICS Unit Pure Core 1

MPC1

Friday 9 January 2009 9.00 am to 10.30 am

#### For this paper you must have:

- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.



Time allowed: 1 hour 30 minutes

## **Instructions**

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC1.
- Answer all questions.
- Show all necessary working; otherwise marks for method may be lost.
- The use of calculators (scientific and graphics) is **not** permitted.

## **Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

#### **Advice**

• Unless stated otherwise, you may quote formulae, without proof, from the booklet.

P10450/Jan09/MPC1 6/6/6/ MPC1

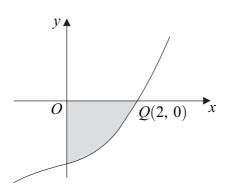
# Answer all questions.

- 1 The points A and B have coordinates (1, 6) and (5, -2) respectively. The mid-point of AB is M.
  - (a) Find the coordinates of M. (2 marks)
  - (b) Find the gradient of AB, giving your answer in its simplest form. (2 marks)
  - (c) A straight line passes through M and is perpendicular to AB.
    - (i) Show that this line has equation x 2y + 1 = 0. (3 marks)
    - (ii) Given that this line passes through the point (k, k+5), find the value of the constant k. (2 marks)
- 2 (a) Factorise  $2x^2 5x + 3$ . (1 mark)
  - (b) Hence, or otherwise, solve the inequality  $2x^2 5x + 3 < 0$ . (3 marks)
- 3 (a) Express  $\frac{7+\sqrt{5}}{3+\sqrt{5}}$  in the form  $m+n\sqrt{5}$ , where m and n are integers. (4 marks)
  - (b) Express  $\sqrt{45} + \frac{20}{\sqrt{5}}$  in the form  $k\sqrt{5}$ , where k is an integer. (3 marks)
- 4 (a) (i) Express  $x^2 + 2x + 5$  in the form  $(x + p)^2 + q$ , where p and q are integers. (2 marks)
  - (ii) Hence show that  $x^2 + 2x + 5$  is always positive. (1 mark)
  - (b) A curve has equation  $y = x^2 + 2x + 5$ .
    - (i) Write down the coordinates of the minimum point of the curve. (2 marks)
    - (ii) Sketch the curve, showing the value of the intercept on the y-axis. (2 marks)
  - (c) Describe the geometrical transformation that maps the graph of  $y = x^2$  onto the graph of  $y = x^2 + 2x + 5$ . (3 marks)

5 A model car moves so that its distance, x centimetres, from a fixed point O after time t seconds is given by

$$x = \frac{1}{2}t^4 - 20t^2 + 66t$$
,  $0 \le t \le 4$ 

- (a) Find:
  - (i)  $\frac{\mathrm{d}x}{\mathrm{d}t}$ ; (3 marks)
  - (ii)  $\frac{d^2x}{dt^2}$ . (2 marks)
- (b) Verify that x has a stationary value when t = 3, and determine whether this stationary value is a maximum value or a minimum value. (4 marks)
- (c) Find the rate of change of x with respect to t when t = 1. (2 marks)
- (d) Determine whether the distance of the car from O is increasing or decreasing at the instant when t = 2. (2 marks)
- 6 (a) The polynomial p(x) is given by  $p(x) = x^3 + x 10$ .
  - (i) Use the Factor Theorem to show that x 2 is a factor of p(x). (2 marks)
  - (ii) Express p(x) in the form  $(x-2)(x^2+ax+b)$ , where a and b are constants.
  - (b) The curve C with equation  $y = x^3 + x 10$ , sketched below, crosses the x-axis at the point Q(2, 0).



- (i) Find the gradient of the curve C at the point Q. (4 marks)
- (ii) Hence find an equation of the tangent to the curve C at the point Q. (2 marks)
- (iii) Find  $\int (x^3 + x 10) \, dx$ . (3 marks)
- (iv) Hence find the area of the shaded region bounded by the curve C and the coordinate axes. (2 marks)

## Turn over for the next question

- 7 A circle with centre C has equation  $x^2 + y^2 6x + 10y + 9 = 0$ .
  - (a) Express this equation in the form

$$(x-a)^2 + (y-b)^2 = r^2$$
 (3 marks)

- (b) Write down:
  - (i) the coordinates of C;
  - (ii) the radius of the circle.

(2 marks)

- (c) The point D has coordinates (7, -2).
  - (i) Verify that the point D lies on the circle.

(1 mark)

- (ii) Find an equation of the normal to the circle at the point D, giving your answer in the form mx + ny = p, where m, n and p are integers. (3 marks)
- (d) (i) A line has equation y = kx. Show that the x-coordinates of any points of intersection of the line and the circle satisfy the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$
 (2 marks)

(ii) Find the values of k for which the equation

$$(k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$$

has equal roots.

(5 marks)

(iii) Describe the geometrical relationship between the line and the circle when k takes either of the values found in part (d)(ii). (1 mark)

# END OF QUESTIONS