

General Certificate of Education (A-level) January 2012

Mathematics

MPC1

(Specification 6360)

Pure Core 1

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC1

Q	Solution	Marks	Total	Comments
1(a)	$(OA^2 =) 6^2 + (-4)^2 ; (OB^2 =) (-2)^2 + 7^2$	M1		either correct PI by 52 or 53 seen
	$(OA^2 =) 52$ and $(OB^2 =) 53$ or $(OA =) \sqrt{52}$ and $(OB =) \sqrt{53}$	A1		both correct values 52 or $\sqrt{52}$ and 53 or $\sqrt{53}$ seen
	$OA = \sqrt{52}$ and $OB = \sqrt{53}$ $\Rightarrow OA < OB$	A1	3	or $OA^2 = 52$ and $OB^2 = 53$ correct working + concluding statement involving OA and/or OB
(b)(i)	$\operatorname{grad} AB = \frac{7+4}{-2-6}$	M1		condone one sign error
	$=-\frac{11}{8}$	A1	2	
(ii)	y4 = 'their grad $AB'(x - 6)or y - 7 = 'their grad AB'(x2)$	M1		or $y =$ 'their grad AB ' $x + c$ and attempt to find c using $x = 6$, $y = -4$ or $x = -2$, $y = 7$
	$y + 4 = -\frac{11}{8}(x - 6)$ OE	A1		any correct form eg $y = -\frac{11}{8}x + \frac{34}{8}$ but must simplify $$ to $+$
	$\Rightarrow 11x + 8y = 34$	A1	3	condone $8y + 11x = 34$ or any multiple of these equations
(c)	$(\operatorname{grad} AC =) \frac{8}{11}$	B1√		FT -1 / 'their grad AB '
	$\frac{4}{k-6} = 'their \frac{8}{11}' \text{ OE}$ $\Rightarrow 2k-12=11$	M1		equating gradients; LHS must be correct and RHS is "attempt" at perp grad to AB
	$\Rightarrow k = \frac{23}{2}$	A1cso	3	k = 11.5 OE
	Total		11	

(c) Alternative: Eqn AC: $(y+4) = 'their \frac{8}{11}' (x-6)$ B1 $\sqrt{(11y-8x-92)}$ **AND** must sub y=0 for M1 or $(y-0) = 'their \frac{8}{11}' (x-k)$ B1 $\sqrt{$ **AND** must sub x=6, y=-4 for M1

MPC1 (cont))			
Q	Solution	Marks	Total	Comments
2(a)	(x-6)(x+2)	B1	1	ISW for $x = 6$, $x = -2$ etc
(b)	$ \begin{array}{c c} x = -2 \\ x = 6 \end{array} $	B1√		correct x values or FT 'their' factors (x-intercepts stated or marked on sketch)
	,			may be seen in (a)
	y = -12	B1		(stated <i>or</i> –12 marked on sketch)
	∪ – shaped curve	M1		approximately
	"correct" shape in all 4 quadrants with minimum to right of y-axis	A1	4	
(c)(i)	$(x-2)^2$	M1		p=2
	$(x-2)^2 - 16$	A1	2	p=2 and $q=16$
(ii)	(Minimum value is) -16	B1√	1	FT 'their $-q$ '
(d)	Replacing each x by $x + 3$ OR adding 2 to their quadratic	M1		in original equation or 'their' completed square or factorised form or replacing y by $y-2$
	$y = [(x+3)^{2} - 4(x+3) - 12] + 2$ or $y = (x+1)^{2} - 14$ or $y = x^{2} + 2x - 13$ or $y - 2 = (x-3)(x+5)$	A1	2	OE any correct equation in x and y unsimplified
	Total		10	1

Q Q	Solution	Marks	Total	Comments
3(a)(i)	$\left(3\sqrt{2}\right)^2 = 18$	B1	1	
(ii)	$(3\sqrt{2} - 1)^{2} = 'their 18' - 3\sqrt{2} - 3\sqrt{2} + 1$ $= 18 - 3\sqrt{2} - 3\sqrt{2} + 1$	M1 A1		FT their $(3\sqrt{2})^2$ $(=19-6\sqrt{2})$
	$(3+\sqrt{2})^2 = 9+3\sqrt{2}+3\sqrt{2}+2$	B1		$ (=19-6\sqrt{2}) $ $ (=11+6\sqrt{2}) $
	\Rightarrow Sum = 30	Alcso	4	
(b)	$\frac{4\sqrt{5} - 7\sqrt{2}}{2\sqrt{5} + \sqrt{2}} \times \frac{2\sqrt{5} - \sqrt{2}}{2\sqrt{5} - \sqrt{2}}$	M1		
	Numerator = $8\left(\sqrt{5}\right)^2 - 4\sqrt{5}\sqrt{2} - 14\sqrt{5}\sqrt{2} + 7\left(\sqrt{2}\right)^2$	m1		correct unsimplified $(=54-18\sqrt{10})$
	Denominator = $(2\sqrt{5})^2 - (\sqrt{2})^2$ = 18	B1		must be seen as denominator
	\Rightarrow Answer = $3 - \sqrt{10}$	A1cso	4	
	Total		9	

Q Q	Solution	Marks	Total	Comments
		M1		one term correct
4(a)(i)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \int 5x^4 - 6x + 1$	A 1		another term correct
	(dx)	A 1	3	all correct (no + c etc)
(ii)	$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right) = 20x^3 - 6$	B1√	1	FT 'their' $\frac{dy}{dx}$
(b)	$x = -1 \Rightarrow \frac{dy}{dx} = 5(-1)^4 - 6(-1) + 1 (= 12)$	M1		must sub $x = -1$ into 'their' $\frac{dy}{dx}$
	dx			d.A
	$\Rightarrow y = 12(x+1)$	A1cso	2	any correct form with $(x-1)$ simplified
				condone $y = 12x + c$, $c = 12$
(c)	$x = 1 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 5 - 6 + 1$	M1		sub $x = 1$ into their $\frac{dy}{dx}$
				dx
	$\frac{dy}{dx} = 0 \implies \text{stationary point}$	A1cso		shown $= 0$ plus correct statement
				-2
	when $x = 1$, $\frac{d^2 y}{dx^2} = 14$			or $\frac{d^2y}{dx^2} = 20 - 6 > 0$
		F14	2	
	\Rightarrow (<i>B</i> is a) minimum (point)	E1	3	\Rightarrow (B is a) minimum (point)
				must have correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for E1
		M1		one term correct
(i)(b)	$\frac{x^6}{6} - \frac{3x^3}{3} + \frac{x^2}{2} + 5x$	A1		another term correct
(4)(1)	6 3 2	A1		all correct (may have $+c$)
	$\left[\frac{1}{6} - 1 + \frac{1}{2} + 5\right] - \left[\frac{1}{6} + 1 + \frac{1}{2} - 5\right]$	m1		'their' $F(1) - F(-1)$ with powers of 1 and -1 evaluated correctly
	= 8	A1cso	5	
(ii)	'their answer to part (i)' -2	M1		
	\Rightarrow Area = 6	A1cso	2	
	Total		16	
			-	<u> </u>

MPC1 (cont)		3.6	TD 4 3	
Q	Solution	Marks	Total	Comments
5(a)	$p(-2) = (-2)^3 + (-2)^2 c + (-2)d - 12$	M1		p(-2) attempted or
				long division by $x+2$ as far as remainder
	'their' $-8 + 4c - 2d - 12 = -150$	m1		putting expression for remainder = -150
	$\Rightarrow 2c - d + 65 = 0$	A1cso	3	AG terms all on one side in any order (check that there are no errors in working)
(b)	$p(3) = 3^3 + 3^2 c + 3d - 12$	M1		p(3) attempted or long division by x -3 as far as remainder
	9c + 3d + 15 = 0	A1	2	any correct equation with terms collected eg $3c+d=-5$
(c)	$ 2c - d + 65 = 0 $ $3c + d + 5 = 0 $ $\Rightarrow 5c = -70$	M1		Elimination of c or d
	$\Rightarrow c = -14$, $d = 37$ OE	A1		value of <i>c</i> or <i>d</i> correct unsimplified
	→ C 11 , W 37 OE	A1	3	both c and d correct unsimplified
	Total		8	•
6(a)	Sides are x and $x + 4$			
	$\Rightarrow x + x + x + 4 + x + 4 > 30$			
	or 2x + 2x + 8 > 30			must see this line OF
	or 2(2x+4) > 30			must see this line OE
	or $4x + 8 > 30$			
	$(\Rightarrow 4x > 22)$			
	$\Rightarrow 2x > 11$	B1	1	AG (be convinced) condone $11 < 2x$
(b)	$x\left(x+4\right) < 96$			must see this line OE
(2)	$\Rightarrow x^2 + 4x - 96 < 0$	B1	1	AG
	→ A TA = 70 \ 0	Di	1	
(c)	(x+12)(x-8)	M1		correct factors or correct quadratic equation formula
	Critical values 8, -12	A1		
	$ \begin{array}{c c} & y & \text{or} \\ \hline -12 & 8 & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & & & \\ & & & \\ \hline & &$	M1		sketch or sign diagram
	$\Rightarrow -12 < x < 8$	A1cso	4	accept $x < 8$ AND $x > -12$ but not $x < 8$ OR $x > -12$ nor $x < 8$, $x > -12$
(d)	$5\frac{1}{2} < x < 8$	B1	1	
	Total		7	
				ı

Q Solution Marks Total Comments 7(a) $(x+7)^2 + (y-5)^2$ MI A1 one term correct; condone $(x-7)^2$ both terms correct with squares and plus sign between terms $(x+7)^2 + (y-5)^2 = 5^2$ A1 cao 3 condone 25 for 5^2 correct or FT 'their' circle equation (ii) $r=5$ B1 $\sqrt{}$ 2 correct or FT 'their' circle equation (c) must draw axes M1	MPC1 (cont)				T
the first containing the same and plus sign between terms condone 25 for 5^2 . (b)(i) $C(-7,5)$ B1 $\sqrt{}$ (c) $C(-7,5)$ B1 $\sqrt{}$ (d)(i) $C(-7,5)$ B1 $\sqrt{}$ (e) must draw axes M1 -7 M2 Clear attempt to sub $y = kx + 6$ into original or 'their' circle equation A1 -7 M1 -7 M1 -7 M2 -7 M1 -7 M3 A2 A3 A4 Condone $x^2(1+k^2) + 2x(7+k) +$ Cut allow statement alone if discriminant in terms of k attempted discriminant (condone one slip) M1 -7 -7 M1 -7 M1 -7 M2 AG AI AG AII AG AG	_		Marks	Total	_
$(x+7)^2 + (y-5)^2 = 5^2$ A1cao $(x+7)^2 + (y-7)^2 = 5^2$ A1c	7(a)	$(x+7)^2 + (y-5)^2$	M1		one term correct; condone $(x-7)^2$
(b)(i) $C(-7,5)$ $B1$ correct or FT 'their' circle equation $r=5$ $B1$ correct or FT 'their' circle equation $\sqrt{25}$ etc but not $\pm\sqrt{25}$ freehand circle with C correct or FT 'their C ' for quadrant of centre C circle touching C axis at C with C correct or FT 'their C ' for quadrant of centre C into C i			A1		•
(b)(i) $C(-7,5)$ $B1$ correct or FT 'their' circle equation (ii) $r=5$ $B1$ 2 correct or FT 'their' $r^2>0$ condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ must draw axes -7 with -7 marked (need not show 5 on y-axis) but circle must not touch y-axis -7 with -7 marked (need not show 5 on y-axis) but circle must not touch y-axis -7 but circle must not not -7 but circle must not not -7 but circle must not not not -7 but circle must not n					
(ii) $r=5$ B1 $\sqrt{}$ 2 correct or FT 'their' $r^2>0$ condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ freehand circle with C correct or FT 'their C ' for quadrant of centre or FT 'their C ' for quadrant		$(x+7)^2 + (y-5)^2 = 5^2$	A1cao	3	condone 25 for 5 ²
(ii) $r=5$ B1 $\sqrt{}$ 2 correct or FT 'their' $r^2>0$ condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ freehand circle with C correct or FT 'their C ' for quadrant of centre or FT 'their C ' for quadrant	G		^		
must draw axes M1 A1 2 condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ freehand circle with C correct or FT 'their C ' for quadrant of centre circle touching x -axis at -7 with -7 marked (need not show 5 on y -axis) but circle must not touch y -axis clear attempt to sub $y = kx + 6$ into original or 'their' circle equation $x^2 + k^2x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ A1cso AG condone $x^2(1+k^2) + 2x(7+k) +$ etc allow statement alone if discriminant in terms of k attempted $[2(k+7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but $= 0$ must appear before last line correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}$, $k = \frac{4}{3}$ OE are values of k for which line is a tangent Total 14	(b)(i)	C(-7,5)	B1√`		correct or FT 'their' circle equation
must draw axes M1 A1 2 condone $\sqrt{25}$ etc but not $\pm\sqrt{25}$ freehand circle with C correct or FT 'their C ' for quadrant of centre circle touching x -axis at -7 with -7 marked (need not show 5 on y -axis) but circle must not touch y -axis clear attempt to sub $y = kx + 6$ into original or 'their' circle equation $x^2 + k^2x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ A1cso AG condone $x^2(1+k^2) + 2x(7+k) +$ etc allow statement alone if discriminant in terms of k attempted $[2(k+7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but $= 0$ must appear before last line correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}$, $k = \frac{4}{3}$ OE are values of k for which line is a tangent Total 14	(ii)	r = 5	B1√	2	correct or FT 'their' $r^2 > 0$
(d)(i) $x^2 + (kx+6)^2 + 14x - 10(kx+6) + 49 = 0$ $x^2 + k^2x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k+7)x + 25 = 0$ Half (ii) Equal roots ' $b^2 - 4ac = 0$ ' $[2(k+7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ Al 3 AG all working correct but = 0 must appear before last line (iii) $(4k+3)(3k-4)$ M1 freehand circle with C correct or FT 'their C ' for quadrant of centre (ircle touching x -axis at -7 with -7 marked (need not show 5 on y-axis) but circle must not touch y-axis clear attempt to sub $y = kx + 6$ into original or 'their' circle equation and attempt to multiply out AG condone $x^2(1+k^2) + 2x(7+k) +$ etc allow statement alone if discriminant in terms of k attempted discriminant (condone one slip) AG all working correct but = 0 must appear before last line correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3}$ OE are values of k for which line is a tangent Total					
(d)(i) $x^2 + (kx+6)^2 + 14x - 10(kx+6) + 49 = 0$	(c)	must draw axes			
(d)(i) $x^2 + (kx+6)^2 + 14x - 10(kx+6) + 49 = 0$			M1		
(d)(i) $x^2 + (kx + 6)^2 + 14x - 10(kx + 6) + 49 = 0$ $x^2 + (kx + 6)^2 + 14x - 10(kx + 6) + 49 = 0$ $x^2 + (k^2 + 12kx + 36 + 14x - 10kx - 60 + 49 = 0)$ $(1 + k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso 2 AG condone $x^2(1 + k^2) + 2x(7 + k) +$ etc allow statement alone if discriminant in terms of k attempted $[2(k + 7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but = 0 must appear before last line (iii) $(4k + 3)(3k - 4)$ M1 $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3}$ OE are values of k for which line is a tangent Total 14					then e for quadrant or centre
$(\mathbf{d})(\mathbf{i}) x^2 + (kx + 6)^2 + 14x - 10(kx + 6) + 49 = 0$ $x^2 + k^2x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1 + k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso $(\mathbf{i}) \text{Equal roots } \cdot b^2 - 4ac = 0 \cdot $ B1 $[2(k + 7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 $3 \text{AG} \text{all working correct but } = 0 \text{must appear before last line}$ $(\mathbf{iii}) (4k + 3)(3k - 4) \text{M1} \text{correct factors or correct use of formula as far as } k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent Total 14			A1	2	_
(d)(i) $x^2 + (kx+6)^2 + 14x - 10(kx+6) + 49 = 0$ clear attempt to sub $y = kx + 6$ into original or 'their' circle equation $x^2 + k^2x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso 2 AG condone $x^2(1+k^2) + 2x(7+k) +$ etc (ii) Equal roots ' $b^2 - 4ac = 0$ ' B1 $[2(k+7)]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but = 0 must appear before last line (iii) $(4k+3)(3k-4)$ M1 correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3}$ OE are values of k for which line is a tangent Total		7			· · · · · · · · · · · · · · · · · · ·
original or 'their' circle equation $x^2 + k^2 x^2 + 12kx + 36 + 14x$ $-10kx - 60 + 49 = 0$ $(1 + k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso $2 \textbf{AG} \text{ condone } x^2(1 + k^2) + 2x(7 + k) + \dots \text{ etc}$ allow statement alone if discriminant in terms of k attempted $\left[2(k+7)\right]^2 - 4 \times 25(k^2 + 1)$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ Al $3 \textbf{AG} \text{all working correct but } = 0 \text{ must appear before last line}$ (iii) $(4k+3)(3k-4) \textbf{M1} \text{correct factors or correct use of}$ $5k = -\frac{3}{4}, k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent $\textbf{Total} \textbf{14}$		_/			but circle must not touch y-axis
$x^2 + k^2 x^2 + 12kx + 36 + 14x \\ -10kx - 60 + 49 = 0$ $(1+k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso $2 \textbf{AG} \text{condone} x^2(1+k^2) + 2x(7+k) + \dots \text{ etc}$ $allow \text{statement alone if discriminant in terms of } k \text{ attempted}$ $\left[2(k+7) \right]^2 - 4 \times 25(k^2 + 1) \textbf{M1} \text{discriminant (condone one slip)}$ $4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ Al $3 \textbf{AG} \text{all working correct but } = 0 \text{ must appear before last line}$ $(iii) (4k+3)(3k-4) \textbf{M1} \text{correct factors or correct use of } formula \text{ as far as } k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{OE} \text{are values of } k \text{ for which line is a tangent}$	(d)(i)	$x^{2} + (kx+6)^{2} + 14x - 10(kx+6) + 49 = 0$			clear attempt to sub $y = kx + 6$ into
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$					original or 'their' circle equation
$-10kx - 60 + 49 = 0$ $(1 + k^2)x^2 + 2kx + 14x + 25 = 0$ $\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$ Alcso $2 \qquad \textbf{AG} \text{condone} x^2(1 + k^2) + 2x(7 + k) + \dots \text{ etc}$ allow statement alone if discriminant in terms of k attempted $\left[2(k + 7)\right]^2 - 4 \times 25(k^2 + 1)$ $4\left\{k^2 + 14k + 49 - 25k^2 - 25\right\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ Al $3 \qquad \textbf{AG} \text{all working correct}$ but = 0 must appear before last line $(4k + 3)(3k - 4)$ M1 $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent $Total$ Al 2			M1		and attempt to multiply out
(ii) Equal roots ' $b^2 - 4ac = 0$ ' Equal roots ' $b^2 - 4ac = 0$ ' B1 allow statement alone if discriminant in terms of k attempted $\begin{bmatrix} 2(k+7) \end{bmatrix}^2 - 4 \times 25(k^2+1) & \text{M1} \\ 4\{k^2+14k+49-25k^2-25\} = 0 \\ -24k^2+14k+24 = 0 \\ \Rightarrow 12k^2-7k-12 = 0 & \text{A1} & 3 & \text{AG all working correct but } = 0 \text{ must appear before last line} \\ \text{(iii)} & (4k+3)(3k-4) & \text{M1} & \text{correct factors or correct use of } \\ \Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3} \text{ OE} \\ \text{are values of } k \text{ for which line is a tangent} & \text{Total} & \text{14} \\ \end{bmatrix}$					
(ii) Equal roots ' $b^2 - 4ac = 0$ ' $\begin{bmatrix} 2(k+7) \end{bmatrix}^2 - 4 \times 25(k^2 + 1) \\ 4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0 \\ -24k^2 + 14k + 24 = 0 \\ \Rightarrow 12k^2 - 7k - 12 = 0 \end{bmatrix}$ All 3 AG all working correct but = 0 must appear before last line (iii) $(4k+3)(3k-4)$ M1 correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{ OE}$ are values of k for which line is a tangent Total					AG 1 27 12 2 7 12
terms of k attempted $\begin{bmatrix} 2(k+7) \end{bmatrix}^2 - 4 \times 25(k^2 + 1) & \text{M1} \\ 4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0 \\ -24k^2 + 14k + 24 = 0 \\ \Rightarrow 12k^2 - 7k - 12 = 0 & \text{A1} \end{bmatrix}$ $(iii) (4k+3)(3k-4)$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{OE} \\ \text{are values of } k \text{ attempted}$ $\text{discriminant (condone one slip)}$ $AG \text{all working correct but } = 0 \text{ must appear before last line}$ $\text{correct factors or correct use of}$ $\text{formula as far as } k = \frac{7 \pm \sqrt{49 + 576}}{24}$		$\Rightarrow (k^2 + 1)x^2 + 2(k + 7)x + 25 = 0$	Alcso	2	AG condone $x^2(1+k^2)+2x(7+k)+$ etc
terms of k attempted $\begin{bmatrix} 2(k+7) \end{bmatrix}^2 - 4 \times 25(k^2+1) & \text{M1} \\ 4\{k^2+14k+49-25k^2-25\} = 0 \\ -24k^2+14k+24 = 0 \\ \Rightarrow 12k^2-7k-12 = 0 & \text{A1} \end{bmatrix} \qquad \begin{array}{l} \textbf{AG} \text{ all working correct} \\ \text{but} = 0 \text{ must appear before last line} \\ \text{(iii)} & (4k+3)(3k-4) & \text{M1} \\ \Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3} \text{ OE} \\ \text{are values of } k \text{ for which line is a tangent} \end{array} \qquad \begin{array}{l} \textbf{A1} \qquad 2 \\ \textbf{A1} \qquad 2 \\ \textbf{A2} \end{array}$	(**)	E 1 442 4 0	D.1		allow statement alone if discriminant in
$4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but = 0 must appear before last line $(4k + 3)(3k - 4)$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{ OE}$ are values of k for which line is a tangent $A1$ 2 $A2$ $A3$ $A4$ $A4$ 2 $A4$ 2 4 3 4 4 4 4 4 4 4 4 4 4	(11)	Equal roots $b - 4ac = 0$	ы		terms of k attempted
$4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$ $-24k^2 + 14k + 24 = 0$ $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but = 0 must appear before last line $(4k + 3)(3k - 4)$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{ OE}$ are values of k for which line is a tangent $A1$ 2 $A2$ $A3$ $A4$ $A4$ 2 $A4$ 2 4 3 4 4 4 4 4 4 4 4 4 4		$[a(1, 3)]^2$	3.54		
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$\Rightarrow 12k^2 - 7k - 12 = 0$ A1 3 AG all working correct but = 0 must appear before last line (iii) $(4k+3)(3k-4)$ M1 $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3} \text{ OE}$ are values of k for which line is a tangent Total $\Rightarrow 12k^2 - 7k - 12 = 0$ A1 $\Rightarrow AG$ all working correct but = 0 must appear before last line correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$		$4\{k^2 + 14k + 49 - 25k^2 - 25\} = 0$			
(iii) $(4k+3)(3k-4)$ Al $but = 0$ must appear before last line correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, k = \frac{4}{3}$ OE Al 2 are values of k for which line is a tangent 2 Total 2		$-24k^2 + 14k + 24 = 0$			
(iii) $(4k+3)(3k-4)$ M1 correct factors or correct use of formula as far as $k = \frac{7 \pm \sqrt{49 + 576}}{24}$ $\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3}$ OE are values of k for which line is a tangent Total 14		$\Rightarrow 12k^2 - 7k - 12 = 0$	A1	3	
$\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent Total are values of k for which line is a tangent					out – 0 must appear before fast fine
$\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent Total 2	(iii)	(4k+3)(3k-4)	M1		correct factors or correct use of
$\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3} \text{OE}$ are values of k for which line is a tangent Total 2					$7 \pm \sqrt{49 + 576}$
are values of k for which line is a tangent Total 14					formula as far as $k = {24}$
are values of k for which line is a tangent Total 14		$\Rightarrow k = -\frac{3}{4}, \ k = \frac{4}{3}$ OE	A1	2	
Total 14		1 3			
TOTAL 75				14	
IUIAL 15		TOTAL		75	