Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
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5	
6	
7	
TOTAL	

General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC1

Unit Pure Core 1

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

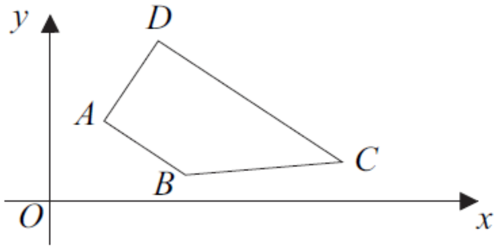
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The trapezium $ABCD$ is shown below.



The line AB has equation $2x + 3y = 14$ and DC is parallel to AB.

(a) Find the gradient of AB. *(2 marks)*

(b) The point D has coordinates $(3, 7)$.

(i) Find an equation of the line DC. *(2 marks)*

(ii) The angle BAD is a right angle. Find an equation of the line AD, giving your answer in the form $mx + ny + p = 0$, where m, n and p are integers. *(4 marks)*

(c) The line BC has equation $5y - x = 6$. Find the coordinates of B. *(3 marks)*

2 (a) Express $\left(3 - \sqrt{5}\right)^2$ in the form $m + n\sqrt{5}$, where m and n are integers. *(2 marks)*

(b) Hence express $\dfrac{\left(3 - \sqrt{5}\right)^2}{1 + \sqrt{5}}$ in the form $p + q\sqrt{5}$, where p and q are integers. *(4 marks)*

3 The polynomial $\mathrm{p}(x)$ is given by

$$\mathrm{p}(x) = x^3 + 7x^2 + 7x - 15$$

(a) (i) Use the Factor Theorem to show that $x + 3$ is a factor of $\mathrm{p}(x)$. *(2 marks)*

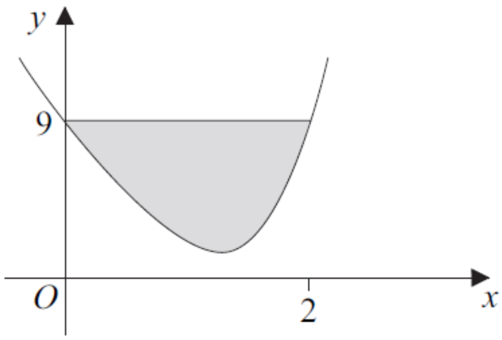
(ii) Express $\mathrm{p}(x)$ as the product of three linear factors. *(3 marks)*

(b) Use the Remainder Theorem to find the remainder when $\mathrm{p}(x)$ is divided by $x - 2$. *(2 marks)*

(c) (i) Verify that $\mathrm{p}(-1) < \mathrm{p}(0)$. *(1 mark)*

(ii) Sketch the curve with equation $y = x^3 + 7x^2 + 7x - 15$, indicating the values where the curve crosses the coordinate axes. *(4 marks)*

4 The curve with equation $y = x^4 - 8x + 9$ is sketched below.



The point $(2, 9)$ lies on the curve.

(a) (i) Find $\int_0^2 (x^4 - 8x + 9)\,\mathrm{d}x$. *(5 marks)*

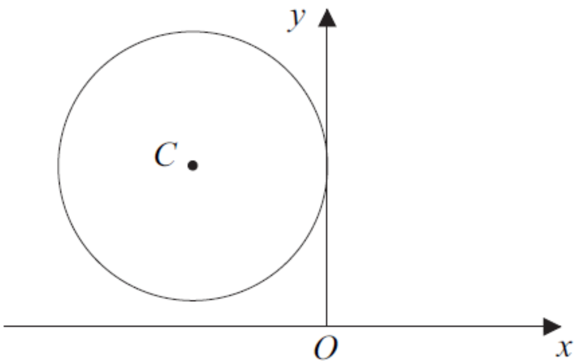
(ii) Hence find the area of the shaded region bounded by the curve and the line $y = 9$. *(2 marks)*

(b) The point $A(1, 2)$ lies on the curve with equation $y = x^4 - 8x + 9$.

(i) Find the gradient of the curve at the point A. *(4 marks)*

(ii) Hence find an equation of the tangent to the curve at the point A. *(1 mark)*

5 A circle with centre $C(-5, 6)$ touches the y-axis, as shown in the diagram.



(a) Find the equation of the circle in the form

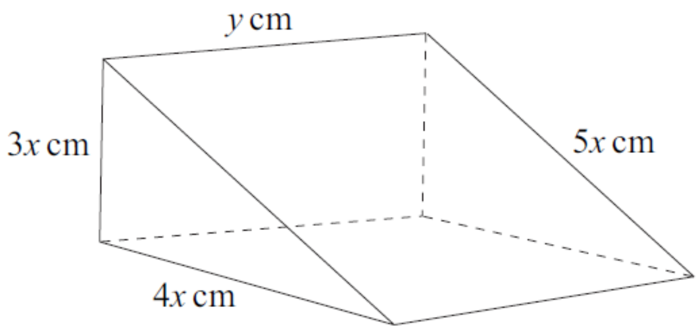
$$(x - a)^2 + (y - b)^2 = r^2$$ *(3 marks)*

(b) (i) Verify that the point $P(-2, 2)$ lies on the circle. *(1 mark)*

(ii) Find an equation of the normal to the circle at the point P. *(3 marks)*

(iii) The mid-point of PC is M. Determine whether the point P is closer to the point M or to the origin O. *(4 marks)*

6 The diagram shows a block of wood in the shape of a prism with triangular cross-section. The end faces are right-angled triangles with sides of lengths $3x$ cm, $4x$ cm and $5x$ cm, and the length of the prism is y cm, as shown in the diagram.



The total surface area of the five faces is $144\ \text{cm}^2$.

(a) (i) Show that $xy + x^2 = 12$. *(3 marks)*

(ii) Hence show that the volume of the block, $V\ \text{cm}^3$, is given by

$$V = 72x - 6x^3$$ *(2 marks)*

(b) (i) Find $\dfrac{\mathrm{d}V}{\mathrm{d}x}$. *(2 marks)*

(ii) Show that V has a stationary value when $x = 2$. *(2 marks)*

(c) Find $\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2}$ and hence determine whether V has a maximum value or a minimum value when $x = 2$. *(2 marks)*

7 (a) (i) Express $2x^2 - 20x + 53$ in the form $2(x - p)^2 + q$, where p and q are integers. *(2 marks)*

(ii) Use your result from part **(a)(i)** to explain why the equation $2x^2 - 20x + 53 = 0$ has no real roots. *(2 marks)*

(b) The quadratic equation $(2k - 1)x^2 + (k + 1)x + k = 0$ has real roots.

(i) Show that $7k^2 - 6k - 1 \leqslant 0$. *(4 marks)*

(ii) Hence find the possible values of k. *(4 marks)*

END OF QUESTIONS