

Implement a computer program (either in C or Java) following the specifications of the system given below.

Applicable Sets

- $\mathbf{A} : \{x \in \mathbf{Z}^+ \mid x \leq 8\}$
- $\mathbf{P} : \mathbf{A} \times \mathbf{A}$
- $\mathbf{S} : \{(x, y) \in \mathbf{P} \mid (x \bmod 2 = 1 \wedge y \bmod 2 = 0) \vee (x \bmod 2 = 0 \wedge y \bmod 2 = 1)\}$
- $\mathbf{B} : \{\text{true}, \text{false}\}$

System Variables

- $\text{Alpha}, \text{Beta}, \text{Free} \subseteq \mathbf{S}$
- $a\text{Turn} \in \mathbf{B}$
- $\text{over} \in \mathbf{B}$
- $\text{ok} \in \mathbf{B}$

System Facts

- $\text{Free} = \mathbf{S} - (\text{Alpha} \cup \text{Beta})$
- $\text{over} \leftrightarrow (|\text{Alpha}| = 0 \oplus |\text{Beta}| = 0)$

System Initialization

- $\text{over} = \text{false}$
- $\text{ok} = \text{false}$
- $a\text{Turn} = \text{true}$
- $\text{Alpha} = \{(x, y) \in \mathbf{S} \mid x \leq 3\}$
- $\text{Beta} = \{(x, y) \in \mathbf{S} \mid x \geq 6\}$

System States and Behavior

NextPlayerMove($\text{prev}, \text{next} \in \mathbf{S}$)

$(a, b) = \text{prev}$

$(c, d) = \text{next}$

$a\text{Turn} \wedge \text{prev} \in \text{Alpha} \wedge c = a + 1 \wedge (d = b + 1 \vee b = d + 1) \rightarrow \text{ok} = \neg \text{ok}$

$\neg a\text{Turn} \wedge \text{prev} \in \text{Beta} \wedge a = c + 1 \wedge (d = b + 1 \vee b = d + 1) \rightarrow \text{ok} = \neg \text{ok}$

$\text{ok} \wedge a\text{Turn} \wedge \text{next} \in \text{Free} \rightarrow \begin{aligned} &\text{Alpha} = \text{Alpha} - \{\text{prev}\} \cup \{\text{next}\} \\ &\wedge a\text{Turn} = \neg a\text{Turn} \\ &\wedge \text{ok} = \neg \text{ok} \end{aligned}$

$\text{ok} \wedge \neg a\text{Turn} \wedge \text{next} \in \text{Free} \rightarrow \begin{aligned} &\text{Beta} = \text{Beta} - \{\text{prev}\} \cup \{\text{next}\} \\ &\wedge a\text{Turn} = \neg a\text{Turn} \\ &\wedge \text{ok} = \neg \text{ok} \end{aligned}$

$\text{ok} \wedge a\text{Turn} \wedge \text{next} \in \text{Beta} \rightarrow \begin{aligned} &\text{Beta} = \text{Beta} - \{\text{next}\} \\ &\wedge \text{Alpha} = \text{Alpha} - \{\text{prev}\} \cup \{\text{next}\} \\ &\wedge \text{ok} = \neg \text{ok} \end{aligned}$

$\text{ok} \wedge \neg a\text{Turn} \wedge \text{next} \in \text{Alpha} \rightarrow \begin{aligned} &\text{Alpha} = \text{Alpha} - \{\text{next}\} \\ &\wedge \text{Beta} = \text{Beta} - \{\text{prev}\} \cup \{\text{next}\} \\ &\wedge \text{ok} = \neg \text{ok} \end{aligned}$

GameOver(over)

$\text{result} \in \{\text{Beta Wins}, \text{Alpha Wins}\}$

$|\text{Alpha}| = 0 \rightarrow \text{result} = \text{Beta Wins}$

$|\text{Beta}| = 0 \rightarrow \text{result} = \text{Alpha Wins}$