

# Proofs

Proofs should work in a general case, ie instead of a specific vector in a proof it should be an algebraic vector that is specific. Instead of being stuck in a specific dimension, we should expand to multiple dimensions if the proof is tackling that.

If  $n$  is an even integer, then  $n^2$  is even

## Direct proof

Let  $n$  be an even integer. Then  $n$  can be written  $n = 2k$  for some integer  $k$ . Then  $n^2 = 4k^2 = 2(2k^2)$  which must be even.

#Direct\_proof

## Proof by contradiction

Let  $n$  be an even integer and assume, for the sake of contradiction, that  $n^2$  is odd. Since  $n$  is even,  $n = 2k$  for some integer  $k$ . Then  $n^2 = 4k^2 = 2(2k^2)$ . However, this requires that

#Proof\_by\_contradiction

the magnitude of a vector is

$$\sqrt{n_1^2 + n_2^2 + \dots + n_n^2}$$

for any positive set of  $n_i \in \mathbb{R}$ . The sum of positive numbers can never be zero. Thus,  $a$ ,  $b$  and  $c$  must be zero for the magnitude to be zero.

## if and only if

requires every statement to have  $\Leftrightarrow$ .

I.E.

# Proof by induction

In a proof by mathematical induction, we prove the statement is true for a base case and then all the cases

suppose  $u_1, u_2, \dots, u_k$  are a set of vectors in  $R^n$  that are all orthogonal to  $\vec{v} \in R^n$  with  $k \leq n$ . Then  $v$  is orthogonal to any linear combination of

Base case ( $k=1$ ): any linear combination of one vector can be written  $c_1 \vec{u}_1$ . then  $(c_1 \vec{u}_1) \cdot \vec{v} = c_1 \cdot (\vec{u}_1 \cdot \vec{v}) = 0$  so they are orthogonal.

Inductive hypothesis: Assume the statement is true for  $k = j, j \leq n - 1$

inductive step ( $j \rightarrow j + 1$ ): suppose that all vectors  $u_j$  are orthogonal to  $\vec{v}$  consider any linear combination, which we can write.

$$\vec{v} \cdot (c_1 \vec{u}_1 + \dots + c_j \vec{u}_j + c_{j+1} \vec{u}_{j+1})$$

#Proofs

see next: [Systems of linear equations](#)

see previous: [Vectors](#)