Linear independence, basis, and dimension

Linear independence and bases:

The #power of the concept of abstract vector spaces is that we can use the concepts / strategies / ideas we've learned for $\mathbb{R}^n$ in those new contexts (e.g. polynomials!)

A subset of V is a #basis if it is linearly independent and spans V.

Unique representations: if B is a basis for V, then for every $\vec{v} \in V$, there is exactly one way to write $\vec{v}$ as a linear combination of the vectors in B.

Basis theorem: If V has a basis with n vectors, then every basis of V has exactly n vectors. Any fewer doesn't span V, any more is linearly dependent.