Tensor Methods and Models for Medical Imaging

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1. Background
2. Data Pipeline
3. Entrywise-l1 NMF
4. Conclusion
5. Future Work

Motivation

- Echocardiograms (ECGs) are widely available ultrasound imaging tools used to view the heart and diagnose various cardiovascular conditions.
- A common type is a stress ECG, which views the heart under increasing cardiovascular stress. This can be used to detect wall motion abnormalities, which is a sign of coronary artery disease.
- The data produced by these ECGs can be naturally formulated as a multi-modal tensor (e.g. pixels by frames by stress level).

Project Goals

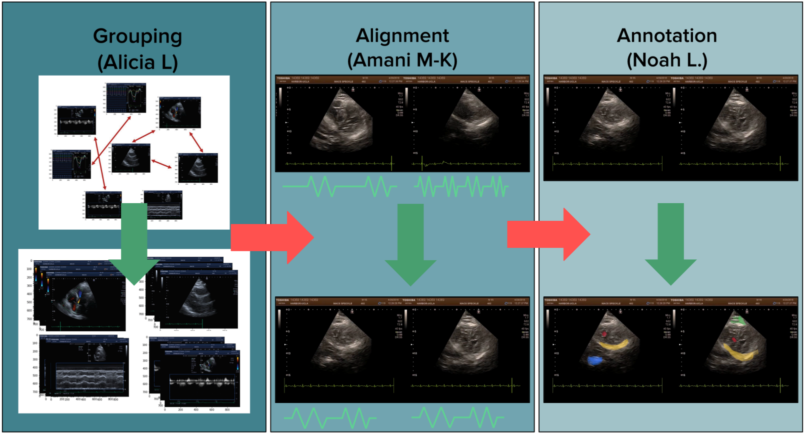
- Develop tensor-based topic models that respect the natural multi-modal structure of our data and allow for the incorporation of flexible expert supervision information.
- Design efficient training methods for these tensor-based topic models and produce open-source implementations.
- Illustrate the promise of these models and methods by applying them to ECG analysis.

Goal

- Goal: Carry out a semi-supervised tensor decomposition of echo-cardiograms
- Input: several varieties of echocardiograms from different patients

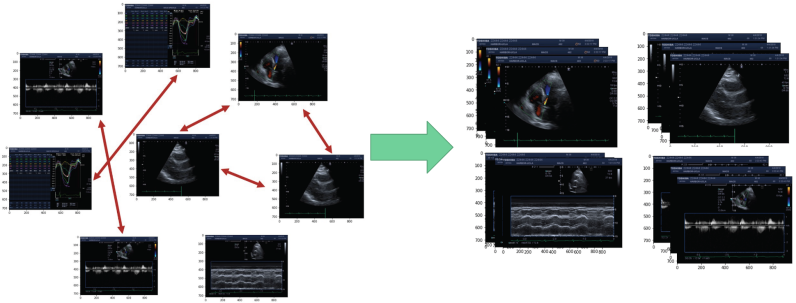
How can we make this data suitable for a tensor decomposition?

The Data Pipeline

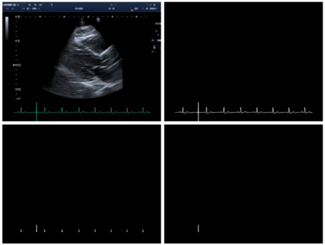


Grouping

Alicia Lu developed an algorithm to create clusters of similar data types.



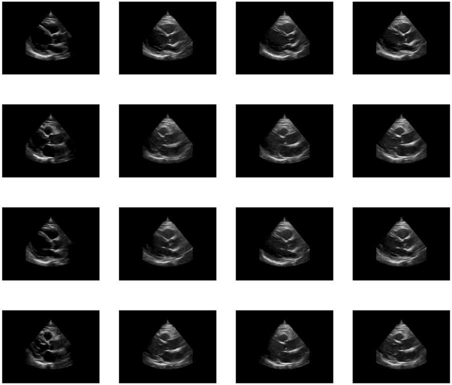
Video Alignment



A. Maina-Kilaas "Automatic Alignment of ECG Sequences

1. **Isolate Frame Line**
   - Mask Green Pixels
   - Eliminate Lower Pixels
   - Largest bounding box
2. **Find Frames with Heart Beats**
   - Find frame line
   - Analyze preceding pixels

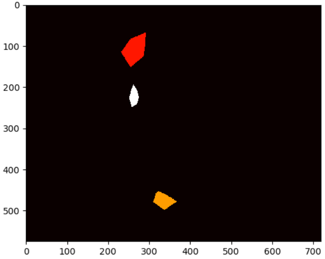
Video Alignment



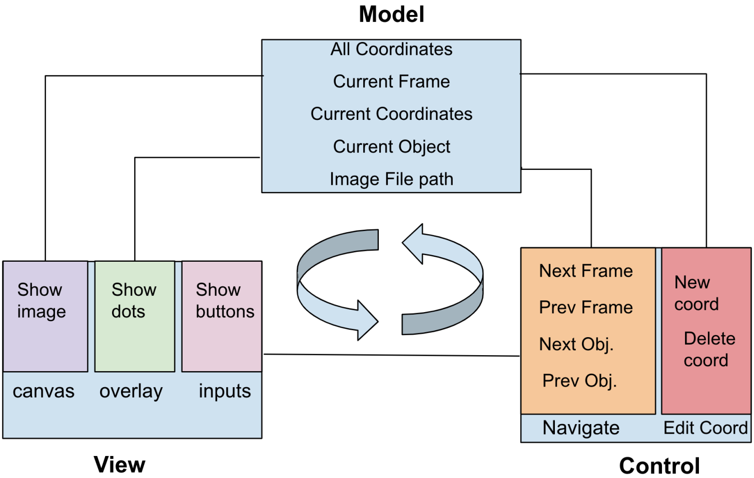
1. Separate Heartbeats
2. Timescale Heartbeats
3. Recombine

Annotation


mainwindow
choose dir
new object
clear object
finished Annotation
add prev frame objects
clear the following object:
0
confirm clear object
prev image
next image


0
100
200
300
400
500
0
100
200
300
400
500
600
700

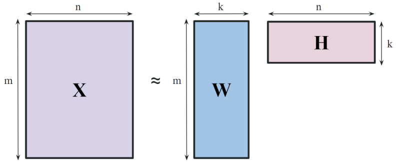
Model View Control



Nonnegative Matrix Factorization (NMF)

Given $\mathbf{X} \in \mathbb{R}_{\geq 0}^{m \times n}$, find $\mathbf{W} \in \mathbb{R}_{\geq 0}^{m \times k}$ and $\mathbf{S} \in \mathbb{R}_{\geq \mathbf{0}}^{\mathbf{k} \times \mathbf{n}}$ such that

$$\mathbf{X} \approx \mathbf{WH}.$$



Often, this is formulated as an optimization problem of the model error using (most commonly) the Frobenius norm or the Information Divergence.

Entrywise-l1 Norm

$$||\mathbf{X} - \mathbf{WH}||_1 = \sum_{ij} |x_{ij} - (\mathbf{WH})_{ij}|$$

- More robust to outliers.
- Goal: Derive multiplicative updates to train NMF on this error objective function.

Gradient Descent

To optimize, we can employ a coordinate-wise iterative gradient descent method.

$$x_{n+1} = x_n - \alpha_n \nabla F(x_n).$$

In our case, the function F is the entrywise-l1 norm. We will use an alternating multiplicative update method, in which we hold W constant to update H, or vice versa.

Because of the nonconvexity of our problem, we are not guaranteed (or likely) to converge at a global min. However, we will converge at a local min.

Derivation

$$\frac{\partial ||\mathbf{X} - \mathbf{W}\mathbf{H}||_1}{\partial w_{st}} = \sum_i^m \sum_j^n \frac{\partial}{\partial w_{st}} |x_{ij} - (\mathbf{W}\mathbf{H})_{ij}|$$

$$= \sum_i^m \sum_j^n \frac{x_{ij} - (\mathbf{W}\mathbf{H})_{ij}}{|x_{ij} - (\mathbf{W}\mathbf{H})_{ij}|} \frac{\partial}{\partial w_{st}} (x_{ij} - (\mathbf{W}\mathbf{H})_{ij})$$

$$= \sum_i^m \sum_j^n \frac{x_{ij} - (\mathbf{W}\mathbf{H})_{ij}}{|x_{ij} - (\mathbf{W}\mathbf{H})_{ij}|} \frac{\partial}{\partial w_{st}} (x_{ij} - \sum_q^k w_{iq} h_{qj})$$

$$= -\sum_j^n \frac{x_{sj} - (\mathbf{W}\mathbf{H})_{sj}}{|x_{sj} - (\mathbf{W}\mathbf{H})_{sj}|} h_{tj}$$

From Additive to Multiplicative Updates

We want to guarantee nonnegativity.

$$w_{st} \leftarrow w_{st} + \alpha_{st} \sum_{j}^{n} \frac{x_{sj} - (\mathbf{WH})_{sj}}{|x_{sj} - (\mathbf{WH})_{sj}|} h_{tj}$$

$$w_{st} \leftarrow w_{st} + \alpha_{st} \sum_{j}^{n} \frac{x_{sj}}{|x_{sj} - (\mathbf{WH})_{sj}|} h_{tj} - \alpha_{st} \sum_{j}^{n} \frac{(\mathbf{WH})_{sj}}{|x_{sj} - (\mathbf{WH})_{sj}|} h_{tj}$$

Let

$$\alpha_{st} = \frac{w_{st}}{\sum_j^n \frac{(\mathbf{WH})_{sj}}{|x_{sj} - (\mathbf{WH})_{sj}|} h_{tj}}.$$

Entrywise Updates

$$w_{st} \leftarrow \Big(\frac{w_{st}}{\sum_j^n \frac{(\mathbf{WH})_{sj} h_{tj}}{|x_{sj} - (\mathbf{WH})_{sj}|}}\Big)\Big(\sum_j^n \frac{x_{sj} h_{tj}}{|x_{sj} - (\mathbf{WH})_{sj}|}\Big)$$

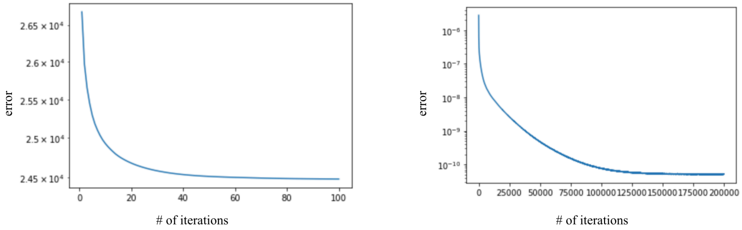
$$h_{tz} \leftarrow \Big(\frac{h_{tz}}{\sum_i^m \frac{(\mathbf{WH})_{iz} w_{iz}}{|w_{it} - (\mathbf{WH})_{iz}|}}\Big)\Big(\sum_i^m \frac{x_{iz} w_{it}}{|x_{iz} - (\mathbf{WH})_{iz}|}\Big)$$

Matrix Updates

$$\mathbf{W} \leftarrow \frac{\mathbf{W}}{\frac{\mathbf{WH}}{|\mathbf{X\text{-}WH}|}\mathbf{H}^T} \odot \frac{\mathbf{X}}{|\mathbf{X\text{-}WH}|}\mathbf{H}^T$$

$$\mathbf{H} \leftarrow \frac{\mathbf{H}}{\mathbf{W}^T\frac{\mathbf{WH}}{|\mathbf{X\text{-}WH}|}} \odot \mathbf{W}^T\frac{\mathbf{X}}{|\mathbf{X\text{-}WH}|}$$

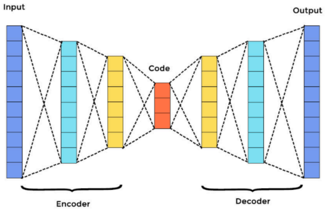
Experimental Results



Left: Randomly initialized X, W, and H.
Right: Randomly initialized W and H, and initialized X as WH. Then, added noise to W and H.

- We are continuing to develop a supervised tensor-based topic modeling method using expert annotations to aid in interpreting ECG data.
- We have made significant progress in file organization/clustering, video alignment, and GUI development.
- We have derived multiplicative updates for entrywise-l1 NMF.

Autoencoder



Autoencoders educba.com

1. **Structure of an Autoencoder**
   - encoder + decoder
   - bottle neck
2. **Goals of an Autoencoder**
   - Reconstruct noisy images
   - Understand important features
   - Hierarchical representation

Application to Our Project

Tensor Models for ECGs

- Look into the possible applications of autoencoders for this project.
- Finish developing the GUI for image annotation.
- Apply a SSNCPD model to groups of videos (identified using the clustering algorithm) formulated into a tensor using supervision data from medical experts.

Entrywise-l1 NMF

- Prove that this objective function is nonincreasing over the updates we derived.
- Run more experiments to show strengths and weaknesses of these updates.
- Apply these updates to a tensor case (NCPD).