1 Lecture Outline

1. Tie up any loose ends from Thursday.
2. Review of power laws.
   - Definition
   - "Paradoxes" – your friends have more friends than you do; most people live in larger-than-average cities; etc. etc. Get show of hands on last one.
   - Mathematical view: heavy tails, variance is not finite, no WLLN / concentration around the mean.
3. Where have power laws been claimed?
   - Original data sets from Barabási and Albert (1999): actor collaboration, WWW, power grid.
   - Others (collected in Albert and Barabási (2002)): citations, protein-protein interaction, academic coauthorships.
   - **Note:** The Barabasi-Albert collaboration (1999-2002) has accumulated over 70,000 citations – roughly 10 per day for 20 years.
4. Why is this so interesting?
   - **A theory of everything for networks?** "Yet, probably the most surprising discovery of modern network theory is the *universality of the network topology*: Many real networks, from the cell to the Internet, independent of their age, function, and scope, converge to similar architectures. *It is this universality that allowed researchers from different disciplines to embrace network theory as a common paradigm.*" Barabási (2009)
   - **Interesting theoretical properties** (some of this foreshadows later lectures)
     a) No epidemic threshold: A conspiracy theory has a nonzero probability to spread to a large portion of a scale-free network, no matter how silly!
     b) VERY small world: scale free networks have even smaller diameter scaling than classical small-world models. Cohen and Havlin (2003)
     c) Robustness to random failures.
5. But wait – how do we know all these networks are scale-free? In other words, how do we connect **models** to **data**?
   a) Many papers plot the degree-histogram on log-log axes and observe a linear fit. But... (show Fig. 4.1 of Clauset et al. (2009), make students guess which one is the power law.).
   b) Introduce two fundamental tasks here: *inference* and *model selection.*
   c) Overview of methods from Clauset et al. (2009) for inference and model selection (with math, but relatively light).

d) Findings from Clauset et al. (2009): in many claimed power laws (not just networks), other degree distributions are at least as plausible.

6. Contemporary discussion (subject to squeezing under time pressure)

   a) Review of methods and findings from Broido and Clauset (2017): only about 10% of a large, 1,000 network data set favored the scale-free hypothesis over some simple alternative hypotheses. *"Taken together, these results indicate that genuinely scale-free networks are remarkably rare, and scale-free structure is not a universal pattern."*

   b) Lively twitter discussion; coverage in The Atlantic, Quanta.

References

Albert, R. and Barabási, A.-L. (2002). Statistical mechanics of complex networks. *Reviews of Modern Physics*, 74(January):47.

Barabási, A. L. (2009). Scale-free networks: A decade and beyond. *Science*, 325(5939):412–413.

Barabási, A.-L. and Albert, R. (1999). Emergence of scaling in random networks. 286(5439):11.

Broido, A. and Clauset, A. (2017). Scale-free networks are rare. (June):26–28.

Clauset, A., Shalizi, C. R., and Newman, M. E. J. (2009). Power-law distributions in empirical data. *SIAM review*, 51(4):661–703.

Cohen, R. and Havlin, S. (2003). Scale-Free Networks Are Ultrasmall. *Physical Review Letters*, 90(5):4.