Zernike Conventions

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Todo list

1. Wavefront Model

The basic model for light propagation is Fresnel Diffraction:

$$\Psi(x,y,z,t) = \exp\left(\frac{\imath 2\pi}{\lambda}(z-t)\right) u_z(x,y) \ , \tag{1}$$

where we take the initial wavefront $u_{z=0}$ to be

$$u_0(x,y) \propto \exp(P(x,y)) \exp(\imath\Phi(x,y))) \ , \tag{2}$$

where r is the two-dimensional radial coordinate, P is the pupil function, and Φ are the aberrations. u_0 is convolved with an atmospheric seeing kernel (Kolmogorov), then propagated to the focal plane. We measure $|u_z(x,y)|^2$. Aberrations in the wavefront ($\Phi \neq 0$) can introduce spurious shapes.

1.1. alt:

The basic model for light propagation is Fresnel Diffraction:

$$\Psi(x,y,z,t) = \exp\left(\frac{\imath 2\pi}{\lambda}(z-t)\right) u_z(x,y) \ , \tag{3}$$

where we take the initial wavefront $u_{z=0}$ to be

$$u_0(x,y) \propto \exp(P(x,y)) \exp(\imath\Phi(x,y))) \ , \tag{4}$$

where r is the two-dimensional radial coordinate, P is the pupil function, and Φ are the aberrations.

u_0 is convolved with an atmospheric seeing kernel (Kolmogorov), then propagated to the focal plane:

$$u_z = \frac{\exp\left(\imath\pi r^2/\lambda z\right)}{\imath\lambda z} \mathrm{FT}\left[\exp\left(\frac{\imath\pi r^2}{\lambda z}\right) u_0(x,y)\right] \ . \tag{5}$$

What we measure is the irradiance:

$$I_z(x,y) = |u_z(x,y)|^2. \tag{6}$$

Aberrations in the wavefront ($\Phi \neq 0$) can introduce spurious shapes.

These aberrations vary across the focal plane; $d, a\ldots$ are really $d(x,y)$, functions of focal plane location. Call $d_0, a_0\ldots$ the ‘normal’ optics contribution to wavefront aberrations. Defocus and misalignment of the focal plane can be characterized by linear shifts and tilts of the normal wavefront, so that for some exposure i the defocus d is:

$$d_i(x,y) = (1+\Delta_d+\theta_{y,d}x+\theta_{x,d}y)d_0(x,y) \; . \tag{7}$$

Measurements of these variations using the focus chips is how the active optics system works.

We would like to obtain this same information for the focal plane, not just the alignment chips. But because the focal plane is near focus, directly modeling Φ is difficult. Instead, parameterize the shape into a sum of moments:

$$M_{pq,z} = \frac{\int \mathrm{d}A \; w(x,y)I_z(x,y)(x-\bar{x})^p(y-\bar{y})^q}{\int \mathrm{d}A \; w(x,y)I_z(x,y)} \; , \tag{8}$$

where $\bar{x}$ and $\bar{y}$ are the centroids of the image and w is a weighting function.

We measure the moments here using the *hsm* weighting scheme, an iterative weighting optimized for measuring second moments used in galsim.

2. Zernikes – Noll, Complex Number

Want table like one from cpd2014MayCollaboration but also including azimuthal etc degree and form as a complex number z and $x+\imath y$

also astigmatism -x and y as well as trefoil need to be renamed

Type	Noll	$(n,\ m)$	Variable	Polar Polynomial
Piston	1	(0, 0)	0	1
Tilt-0	2	(0, 1)	0	$2r\cos\theta$
Tilt-90	3	(0, 1)	0	$2r\sin\theta$
Defocus	4	(2, 0)	d	$\sqrt{3}(2r^2-1)$
Astigmatism-45	5	(2, 2)	$\Im[a]$	$\sqrt{6}r^2\sin 2\theta$
Astigmatism-0	6	(2, 2)	$\Re[a]$	$\sqrt{6}r^2\cos 2\theta$
Coma-90	7	(3, 1)	$\Im[c]$	$\sqrt{8}(3r^3-2r)\sin\theta$
Coma-0	8	(3, 1)	$\Re[c]$	$\sqrt{8}(3r^3-2r)\cos\theta$
Trefoil-30	9	(3, 3)	$\Im[t]$	$\sqrt{8}r^3\sin 3\theta$
Trefoil-0	10	(3, 3)	$\Re[t]$	$\sqrt{8}r^3\cos 3\theta$
Spherical Defocus	11	(4, 0)	s	$\sqrt{5}(6r^4-6r^2+1)$

3. Moments and Adaptive Moments

both the conventional definition and talk about how I use Hirata and Seljak adaptive moments.

4. Whisker Conventions

Lift from whiskerconvention for w?

5. Higher Order Moments

In analogy with gravitational shear and flexion, these moments can be decomposed into linear combinations with convenient rotational symmetries:

$$e_0 = M_{20} + M_{02} \tag{9}$$
$$e_1 = M_{20} - M_{02} \tag{10}$$
$$e_2 = 2M_{11} \tag{11}$$
$$\zeta_1 = M_{30} + M_{12} \tag{12}$$
$$\zeta_2 = M_{03} + M_{21} \tag{13}$$
$$\delta_1 = M_{30} - 3M_{12} \tag{14}$$
$$\delta_2 = -M_{03} + 3M_{21} \tag{15}$$
$$\tag{16}$$

$\epsilon = (e_1 + \imath e_2)/e_0$ is one of the common definitions for ellipticity, while ζ and δ correspond to unnormalized F-1 and F-3 flexion, respectively.

Is it ϵ or χ?

Wavefront aberrations lead to measurable moments. Plugging (5) into (8) and carrying out the math for the Zernike aberrations defocus, astigmatism, coma, trefoil, and spherical defocus

(d, a, c, t, s) we find analytically:

$$
\begin{aligned}
e_0 = &\quad 24d^2 + 16\sqrt{15}\,ds + 120s^2 + 12|a|^2 + 56|c|^2 + 24|t|^2 \\
\mathbf{e} = e_1 + \imath e_2 = &\quad 8\sqrt{2}(3d + \sqrt{15}\,s)a + 32c^2 + 45\bar{c}t \\
\boldsymbol{\zeta} = \zeta_1 + \imath\zeta_2 = &\quad 64(3d + 2\sqrt{15}\,s)(2a\bar{c} + \bar{a}t) + \\
&\quad 16\sqrt{2}(18d^2 + 22\sqrt{15}\,ds + 120s^2 + 6|a|^2 + 25|c|^2 + 15|t|^2)c \\
&\quad + 48\sqrt{2}\,a^2\bar{t} + 240\sqrt{2}\,\bar{c}^2 t \\
\boldsymbol{\delta} = \delta_1 + \imath\delta_2 = &\quad 192(3d + 2\sqrt{15}\,s)ac \\
&\quad + 144\sqrt{2}(2d^2 + 2\sqrt{15}\,ds + 12s^2 + 5|c|^2)t \\
&\quad + 144\sqrt{2}\,a^2\bar{c} + 160\sqrt{2}\,c^3
\end{aligned}
$$

6. Zernike v Seidel

maybe work out the field aberration stuff with seidel first and then with zernike (the maths certainly will be at least a /little/ simpler...)