Untitled

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Question 1. Prove each of the following. I each case, you should create a bijection between the two sets.Briefly justify that your function are in fact bijections.

(a) $|\{\heartsuit, \clubsuit, \spadesuit\}| = |\{\circ, \square, \triangle\}|$

(b) $|\mathbb{N}| = |\{oddnaturalnumbers\}|$

(c) $|A \times \{1\}| = |A|$, where A is any set.

Question 2. Let $\digamma$ denote the set of all functions from $\mathbb{N}$ to $\{0, 1\}$.

(a) Describe at least three functions in the set $\digamma$.

$f : \mathbb{N} \to \{0, 1\}$ where $f(x) = x * 0$, or $f(x) = x^0$, or $f(x) = \{$if x is even assign a 1, if x is odd assign a 0$\}$.

(b) Prove that $|\digamma| = |P(\mathbb{N})|$.

Proof We proceed by proof of equivalent cases (I am making this term up I think). First I will show that $|\digamma| = 2^{|\mathbb{N}|}$. Then I will show that $|P(\mathbb{N})| = 2^{|\mathbb{N}|}$

case 1. By the definition of equal functions, for each element $n \in \mathbb{N}$ there are only two possible functions that can send $n \to \{0, 1\}$. So then given the multiplcatory rule of combinitorics (pg. 69 of textbook), there are $2^{|\mathbb{N}|}$ functions that can send $\mathbb{N} \to \{0, 1\}$. Thus $|\digamma| = 2^{|\mathbb{N}|}$.

case 2. In class we showed that $|P(\mathbb{N})| = 2^{|\mathbb{N}|}$.

Therefore $|P(\mathbb{N})| = 2^{|\mathbb{N}|} = |\digamma|$, meaning $|P(\mathbb{N})| = |\digamma|$.

Question 3. Let X be a set. Prove that "it has the same cardinality as" is an equivalence relation on $P(X)$.

Question 4. Prove or disprove: The set $\{a_1, a_2, a_3 ... a_i : a \in \mathbb{Z}\}$ of infinite sequence of integers is countably infinite.

Question 5: Prove that A and B are finite sets with $|A| = |B|$, then any injection $f : A \to B$ is also a surjection. Show this is not necessarily true if A and B are not finite.

Direct proof!!! (p.s. I do not have all my direct proof check marks)...
Suppose that A and B are finite sets with $|A| = |B|$.
Then the number of inductive functions that can be made from A to B $f : A \to B$, is described using the binomial function $\binom{|A|}{|B|}$. Similarly, the number of bijective functions from $A \to B$ is given by $\binom{|B|}{|A|}$.
Since A and B are of equal cardinality, then $\binom{|B|}{|A|} = \binom{|A|}{|B|}$.
Thus the number of injective relationships equals the number of bijective relationships.
This means all injective functions between A and B are surjective.
Therefore any injection, $f : A \to B$, is also a surjection.

*Note: When describing the number of bijective functions as $\binom{|A|}{|B|}$, if $|A| < |B|$ then $\binom{|A|}{|B|} = 0$, which agrees with the pigeon hole principle that if $|A| < |B|$ then there is no surjective function from A to B, and thus no bijective function from A to B.