Untitled

Jamie Ash

2022-11-04

Question 4.

Rewriting question. for any $n \in \mathbb{N}$ we can write write it as $n = 2^{x_1} + 2^{x_2} + ...2^{x_i}$ where x_0 to x_i are elements of the set $x \in P(\mathbb{N})$.

(Base case) Showing 1 to 7 can be writen as $2^{x_1} + 2^{x_2} + ...2^{x_i}$.

```
raise = function(x){
  sum(2^x)
}

# first few natural numbers
n = 0:2
# produce the power set of n
sets = powerSet(n, rev=TRUE)
# raise by 2 and sum each subset of n (in the powerset)
u = lapply(sets, raise)
# Just changing the data class to vector (from a list), and sriting them
u = unlist(u)
sort(u)
```

Let X be the set $\{x \in P(\mathbb{N}); 2^{x_1} + 2^{x_2} + ...2^{x_i}\}$. Suppose for the sake of argument that there is some $n \in \mathbb{N}$ where $n \notin X$.

Let k be this number, and t be the greatest factor of two that is less than k, and ℓ be the smallest factor of two greater than k. Such that $t < k < \ell$ and $2t = \ell$.

Then $k - t = m$ and $k = m + t$ where $m \in \mathbb{N}$, and $m < t < k$.
Since $m < t < k$ and k is the smallest number not in X, it follows that $t, m \in X$.

Similarly, because $t > m$, then $t + m \in X$. Note, if it where the case where $m > t$ then ℓ would be the greatest factor of two that is less than k, not t.
So $t + m = k$ and $t + m \in X$ but $k \notin X$. This is a contradiction.
Therefore there all $n \in \mathbb{N}$ are in X.