

# Homework 10: Jamie Ash

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## Part 1: Paradox

I did not come up with these, but also didn't just google it. The first one is an oldy, the second one I saw on reddit.

**Paradox 1** An omnipotent god creates a stone that they can't lift. What happens if they try to lift the stone?

**Paraadox 2** What is the probability of guessing the correct answer to this question? a) 25% b) 50% c) 25% d) 60%

## Part 2: Set Proofs

**Sets and power sets:** I did not do this part

*Question 2:* Remember that for a set  $A$ , the power set of  $A$  is the set of all subsets of  $A$ . We write  $P(A)$ .

**Computation:**

Question 4: Find all values of  $n$  that make the congruence true.

- (a)  $5 \equiv 25 \pmod{n}$  answer:  $n$  is any of  $\{2, 4, 5, 10\}$
- (b)  $20 \equiv 0 \pmod{n}$  answer:  $n$  is any of  $\{2, 4, 5, 10\}$
- (c)  $37 \equiv 1 \pmod{n}$  answer:  $n$  is any of  $\{2, 4, 6, 12, 18\}$

**Proofs:****Question 5:** If  $n$  is odd, then  $8|(n^2 - 1)$ .*Proof* We proceed by inference and cases.

By the definition of divisibility,  $8|(n^2 - 1)$  implies  $8l = n^2 - 1$  where  $l \in \mathbb{Z}$ . By the definition of odd numbers, we have  $n$  is  $2k + 1 \in \mathbb{Z}$ . We proceed with two cases, first where  $n$  is any positive odd number, and then where  $n$  is any negative odd number.

(Base step) Consider the series of positive odd numbers,  $H$ ,  $(k \in \mathbb{N}, 0 : 2k + 1)$  that is ordered from smallest to largest (not sure how to use series/set notation to denote order). Now consider  $m \in H$ , where first possible value of  $m$  is 1, corresponding to the first possible value of  $k$  being 0. Through substitution we get...

$$8l = m^2 - 1$$

$$8l = 1^2 - 1$$

$$8l = 0$$

$$l = 0$$

*or*

$$8l = (2k + 1)^2 - 1$$

$$8l = (2(0) + 1)^2 - 1$$

$$8l = (1)^2 - 1$$

$$8l = 0$$

$$l = 0$$

(intuitive step) Suppose  $8|(n^2 - 1)$  where  $n$  is any positive odd number.

By the definition of divisibility and odd numbers we have...

definition of divisibility add  $(8k+8)$  to both sides factor right side pull out the 8 addition of integers  $l + k + 1 = p$  where  $p \in \mathbb{Z}$

$$8l = (2k + 1)^2 - 1$$

$$8l + (8k + 8) = (2k + 1)^2 - 1 + (8k + 8)$$

$$8l + 8k + 8 = (2k + 3)^2 - 1$$

$$8(l + k + 1) = (2k + 3)^2 - 1$$

$$8p = (2k + 3)^2 - 1$$

therefore  $8|(2l + 3)^2 - 1$  or  $8|(n + 2)^2 - 1$  i.e.  $S_{k+1}$ .

For the case where  $n$  is all negative odd numbers, we use the same proof because  $n^2$  where  $n$  is any negative odd number results in an equivalent set as  $n^2$  for any positive odd number.

So  $S_k \implies S_{k+1}$ . Through inference if  $n$  is odd, then  $8|(n^2 - 1)$ .

## Part 3: Reflection

Two nights ago I had a dream where I was walking on a sidewalk along a suburban street. It was night time and there were a few restaurants on the other side of the way. The row of restaurants ended at an intersection, beyond which was obscured by a canopy of large overhanging trees. From the adjacent corner of the intersection a cat ran across the street towards me, and as the cat approached the sidewalk that I was on, it formed into a human figure. I felt threatened by the once cat now human shape.

At this point I became aware of someone walking alongside me. I had the vague sense that they were a friend, a good friend, but the kind of good friend whose friendship forms over a year or two. I took a step to place myself between the figure and this good friend. The figure came closer, and I intently stared at it's form as it moved. Either I could not see it's form clearly, or the memory of the figure's appearance is lost with the rest of my forgotten dreams. I'm not sure. I do know that the figure was ragged and grey, possibly without a face.

My fear slowly pulled me into the waking world, and I gradually became aware of the surroundings in my room. The figure was now silhouetted against the curtain at the foot of my bed. I couldn't move and struggled to speak. This was the same felling you get in a dream when you attempt to run but can't go faster than a walk. Eventually I let out a yell... "Your sandwiches suck!!" which jolted me into full consciousness and mobility. One of the restaurants across the street, where the cat originated, was a sandwich shop.

Although I yelled something dumb, and it's kind of funny, I can't stop wondering about the timing of this dream. That same night I had gone to have dinner with a few people at my friend Petra's house. My other friend Bailey, who used to live in that same house, and happens to be very "in tune" with spirits, claims there is both a demon in the garage, and a ghost cat that wanders the premise. Maybe the cat and demon are one in the same?

At the dinner party Petra and I poked fun of the garage demon (at least the idea of a garage demon), and mildly insinuated that Bailey is off her rocker. Maybe this upset the cat demon, and it followed me home. In which case this poltergeist is contagious, and all of the Math 321 class has now been exposed. Happy Halloween.