Homework 8: James Ash

Part 1: Toulmin Analysis

In your own words, what is meant in the paper by "data"? By "claim"? By "warrant"? By "backing"?

Data is a accepted truths, axioms, or statements that have been previously proven to be true, like lemons. Similarly previews statemnts of a proof that have been proven true and relevant fit this bill.

Warrants are miniature propositions, and are used to show a logical step from the data to the claim. Backings are additional warrants or data that is provided if the a current warrant is not initially accepted as true by the demographic. Backings can be provided preemptively.

A **claim** is a statement or proposition that is being presented as true. Claims can become data for the preseding warrant.

Pick two of the "Sample Proofs" on the worksheet attached, one that you think is good and one that is bad. Do a Toulmin analysis on these two proofs, giving the claim, data, and warrant for each step.

Proposition 2, proof 2b, good proof:

Step 1: data: "Assume $\sqrt{15} \leq \sqrt{2} + \sqrt{6}$"
warrant: "Then... square both sides" claim: $\sqrt{15} \leq (\sqrt{2} + \sqrt{6})^2 = 8 + 2\sqrt{12}$

Step 2: data: $\sqrt{15} \leq (\sqrt{2} + \sqrt{6})^2 = 8 + 2\sqrt{12}$
warrant: Subtract 8 from both sides
claim: $7 \leq 2\sqrt{12}$

Step 3: data: $7 \leq 2\sqrt{12}$
warrant: Square both sides claim: $49 < 48$

Step 3: data: $49 < 48$
warrant: This is a contradiction... claim: ...so we must have $\sqrt{2} + \sqrt{6} < \sqrt{15}$

I liked this proof because the data for each warrant was the claim from the previous statement, so not much digging was needed. After doing the Toulmin analysis I liked the proof a little less, only because the first set of data - warrant - claim didn't have the same structure as the preseding data - warrant - claim. That is, it had an equal sign in the statement rather than putting the result in the next line below.

Question 1, proof 1A, bad proof:

Step 1: data: no data
warent: $n = (2k) = (2k)^2 = 4k$
claim: so it's divisible by 4

The proof does not provide sufficient data for the warrant $(2k)^2 = 4k$. A definition of even numbers would be useful, and the assertion that $k \in Z$ is needed. The major gap is that k is undefined. Furthermore the claim "so it's divisible by 4" is ambiguous. It is difficult to discern what "it's" represents.

Part 2: Combinatorial Proofs:

Disclaimer: I drew heavily from example 3.28 in the textbook to answer this question. I really like this section but also find it very difficult, and would like to get better at doing combinatorial proofs.

Question 4:

Show $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

$\binom{2n}{n}$ describes the number of unique permutations that can be created by drawing n elements from a set of length $2n$. Lets call this set with length $2n$ set D.

To count the total number of unique permutations in set D is we can divide D into two equal parts A and B, both of length n so that $|A| = n$ and $|B| = n$. In this way $D = A \cup B$ and $A \cap B = \varnothing$.

For any k with $0 \leq k \leq n$, because $\sum_{k=0}^{n}$, n things from D can be taken by pulling k things from A and $n-k$ things from B for a total of $k + (n-k) = n$ things drawn. So that's $\binom{n}{k}$ from A and $\binom{n}{n-k}$ things from B. Using the multiplication principle we find that's $\binom{n}{n-k}\binom{n}{k}$ drawn in total from set D.

k can be any number from $0 to n$ so to total number of ways to select n things from D is the sum of all k values as follows...

$$\binom{n}{0}\binom{n}{n-0} + \binom{n}{1}\binom{n}{n-1} + \binom{n}{2}\binom{n}{n-2} + ... + \binom{n}{n}\binom{n}{0}$$

Since $\binom{n}{n-k} = \binom{n}{k}$ thgis expression becomes...

$$\binom{n}{0}\binom{n}{0} + \binom{n}{1}\binom{n}{1} + \binom{n}{2}\binom{n}{2} + ... + \binom{n}{n}\binom{n}{n}$$

which can be expressed as...

$\sum_{k=0}^{n} \binom{n}{k}^2$

So I've shown two ways to count the unique permutations of set D by drawing n elements two different ways. First as $\sum_{k=0}^{n} \binom{n}{k}^2$ and then as $\binom{2n}{n}$.

Therefore $\sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$.

Part 3: Reflection

Finally caught up with all the class work and readings after taking the trip to Vietnam.I find this section difficult, but would really like to improve. I like statistics and combinitorics seems pretty relevant to that especially with the binomial theorem involved.