Homework 2 - Classwork 2

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Question 1. Complete the following definitions:

Definition 1. An integer n is even if it can be divided by two and the result is an integer.

$$\frac{n}{2} = i \tag{1}$$

Where n is some number, and i is an integer.

Definition 2. An integer m is odd if we can add one then divide by two and the result is an integer.

$$\frac{(n+1)}{2} = i \tag{2}$$

Where n is some number, and i is an integer.

Question 2. Use your definitions to prove the following statement.

Proposition 3. If n is an even integer then n^2 is an even integer.

If n is an even integer then $n = 2i$, where i is some integer. Then n^2 is an even integer if $n^2/2 = i$.

When we substitute $2i$ for n in $n^2/2 = i$ we get the following.

$$\frac{(2i)^2}{2} = i \tag{3}$$

We rearrange the equation to be...

$$\frac{1}{2}(2i)(2i) \tag{4}$$

Then if we factor out the 2 we get...

$$(i)(2i) \tag{5}$$

When two integers are added or multiplied the result is an integer. So the result of $(i)(2i)$ is always an integer, and n^2 is always an even number.

This can be extended to n^x where x is some natural number. So step 4 would become:

$$i = \frac{1}{2}(2i)(2i)...i = (i)(2i)... \tag{6}$$

Similarly $(i)(2i)...$ is entirely composed of integers being multiplied by one another, so the result is an integer, and n^x is even.