Homework 4: Ven Diagrams

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Question 1:

Is it always, sometimes, or never true that union distributes over intersection:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Answer: It is *always* true that union distributes over intersection

Here we show that $(A \cup B) \cap A \cup C \subseteq (A \cup B) \cap (A \cup C)$

If $x \in A \cup (B \cap C)$ then $x \in (A \cup B) \cap (A \cup C)$

Suppose $x \in A \cup (B \cap C)$

case 1: So $x \in A$ It follows that $x \in A \cup B$ and $x \in A \cup C$ therefore $x \in (A \cup B) \cap (A \cup C)$

case 2: Let $x \in (B \cup C)$ So $x \in A$ and $x \in C$ then $x \in (C \cup A)$ and $x \in (B \cup A)$ for that matter $x \in (C \cup X)$ and $x \in (B \cup X)$ where X is any set. therefore $x \in (A \cup B) \cap (A \cup C)$

Now we show that $(A \cup B) \cap (A \cup C) \subseteq (A \cup B) \cap A \cup C$

If $x \in (A \cup B) \cap (A \cup C)$ then $x \in A \cup (B \cap C)$

Suppose $x \in (A \cup B) \cap (A \cup C)$ Then $x \in A$ or $x \in B$ and $x \in A$ or $x \in C)$ implies that $x \in A$ or $x \in B$ and $x \in C$ therefore $x \in A \cup (B \cap C)$

Plain Language: If a random element x is in set A then $(A \cup B) \cap (A \cup C)$ is true, and the only other case where this is true is when $x \in B$ and $x \in C$. Therefore $x \in A \cup (B \cap C)$.

Question 2:

Is it always, sometimes, or never true that intersection is associative:

$$A \cap (B \cap C) = (A \cap B) \cap C$$

Answer: It is always true that intersection is associative.

Proof: To prove that that intersection is always associative and $A \cap (B \cap C) = (A \cap B) \cap C$ I will show first that $A \cap (B \cap C) \subseteq (A \cap B) \cap C$ and then that $(A \cap B) \cap C \subseteq A \cap (B \cap C)$.

If $x \in A \cap (B \cap C)$ then $x \in (A \cap B) \cap C$ Suppose $x \in A \cap (B \cap C)$. So $x \in B$ and $x \in C$, and $x \in A$. Which implies that $x \in A$ and $x \in B$, and $x \in C$ Therefore $(A \cap B) \cap C$

If $x \in (A \cap B) \cap C$ then $x \in A \cap (B \cap C)$ Let $x \in (A \cap B) \cap C$ So $x \in A$ and $x \in B$ and $x \in C$ implies that $x \in B$ and $x \in C$ and $x \in A$ therefore $x \in A \cap (B \cap C)$

Question 3:

Is it always, sometimes, or never true that set difference distributes over intersection:

$$A - (B \cap C) = (A - B) \cap (A - C)$$

Answer: Is it sometimes true that set difference distributes over intersection.

For the case of $A - (B \cap C) = (A - B) \cap (A - C)$ the set difference distributes over intersections when $A = \langle a, c, d, f\rangle$ and $B = \langle g, h, j, k\rangle$, and $C = \langle l, m, n, o\rangle$.

for the left side of the equation $A - (B \cap C)$...

$$A - (B \cap C)\langle a, c, d, f\rangle - (\langle g, h, j, k\rangle \cap \langle l, m, n, o\rangle)\langle a, c, d, f\rangle - \varnothing\langle a, c, d, f\rangle$$

For the right side of the equation $(A - B) \cap (A - C)$...

$$(A - B) \cap (A - C)(\langle a, c, d, f\rangle - \langle g, h, j, k\rangle) \cap (\langle a, c, d, f\rangle - \langle l, m, n, o\rangle)(\langle a, c, d, f\rangle) \cap \langle a, c, d, f\rangle\langle a, c, d, f\rangle$$

But for the case of $A - (B \cap C) = (A - B) \cap (A - C)$ the set difference does not distributes over intersections when $A = \langle a, c, d, f\rangle$ and $B = \langle g, h, j, k\rangle$, and $C = \langle a, c, n, o\rangle$.

for the left side of the equation $A - (B \cap C)$...

$$A - (B \cap C)\langle a, c, d, f\rangle - (\langle g, h, j, k\rangle \cap \langle a, c, n, o\rangle)\langle a, c, d, f\rangle - \varnothing\langle a, c, d, f\rangle$$

for the right side of the equation $(A - B) \cap (A - C)$...

$$(A - B) \cap (A - C)(\langle a, c, d, f\rangle - \langle g, h, j, k\rangle) \cap (\langle a, c, d, f\rangle - \langle a, c, n, o\rangle)\langle a, c, d, f\rangle \cap \langle d, f\rangle\langle d, f\rangle$$

Therefore $(A - B) - C$ is sometimes equal to A - (B - C).

Question 4:

Is it always, sometimes, or never true that set difference is associative:

$$A - (B - C) = (A - B) - C$$

Answer: Is it sometimes true that set difference is associative:

For the case of $A - (B - C) = (A - B) - C$ the set difference is associative when $A = \langle a, c, d, f\rangle$ and $B = \langle g, h, j, k\rangle$, and $C = \langle l, m, n, o\rangle$.

First we find the left side of $A - (B - C) = (A - B) - C$

$$A - (B - C)\langle a, c, d, f\rangle - (\langle g, h, j, k\rangle - \langle l, m, n, o\rangle)\langle a, c, d, f\rangle - \langle g, h, j, k\rangle\langle a, c, d, f\rangle A - (B - C) = \langle a, c, d, f\rangle$$

First we find the right side of $A - (B - C) = (A - B) - C$

$$(A - B) - C(\langle a, c, d, f\rangle - \langle g, h, j, k\rangle) - \langle l, m, n, o\rangle\langle a, c, d, f\rangle - \langle l, m, n, o\rangle\langle a, c, d, f\rangle(A - B) - C = \langle a, c, d, f\rangle$$

For this same case the set difference is not associative when $A = \langle a, c, d, f\rangle$ and $B = \langle g, h, j, k\rangle$, and $C = \langle a, c, n, o\rangle$.

For the left side of the equation $A - (B - C)$...

$$A - (B - C)\langle a, c, d, f\rangle - (\langle g, h, j, k\rangle - \langle a, c, n, o\rangle)\langle a, c, d, f\rangle - \langle g, h, j, k\rangle\langle a, c, d, f\rangle$$

For the right side of the equation $(A - B) - C$...

$$(A - B) - C(\langle a, c, d, f\rangle - \langle g, h, j, k\rangle) - \langle a, c, n, o\rangle\langle a, c, d, f\rangle - \langle a, c, n, o\rangle\langle d, f\rangle$$

Therefore $(A - B) - C$ is sometimes equal to $A - (B - C)$.