

HW 7: Jamie Ash

2022-10-09

Part 2: I hate to Contradict you...

Question 1: $\log_2(3)$ is irrational.

Proof:

Suppose for the sake of contradiction that $\log_2(3)$ is not irrational.

By definition, irrational numbers are $\mathbb{R} - \mathbb{Q}$.

So if $\log_2(3)$ is not irrational, then it is rational.

Let $\log_2(3) = x$ where $x \in \mathbb{Q}$.

With a little algebra, we get $3 = 2^x$.

Because $x \in \mathbb{Q}$, x can be represented as k/l where $k \in \mathbb{Z}$ and $l \in \mathbb{N}$.

So $3 = 2^x$ becomes $3 = 2^{\frac{k}{l}}$, where $k \neq 0$ because $3 \neq 1$.

We can rearrange $3 = 2^{\frac{k}{l}}$ as $3^l = 2^k$.

But $3^l = 2^k$ is a contradiction, because 3^l is odd for all $l \in \mathbb{N}$, and 2^k is even for all $k \in \mathbb{Z}, k \neq 0$.

Therefore $\log_2(3)$ is irrational.

Question 6: For integers a , b and c , if $a^2 + b^2 = c^2$ then a is even or b is even.

Proof:

Suppose for the sake of contradiction that $a^2 + b^2 = c^2$ and a is odd and b is odd.

By the proof given in class, the product of any two odd numbers is odd and the product of any two even numbers is even.

Let $a = 2k + 1$ and $b = 2l + 1$ where $k, l \in \mathbb{Z}$.

It follows that $a^2 = 2h + 1$ and $b^2 = 2m + 1$ where $h, m \in \mathbb{Z}$.

With substitution $a^2 + b^2 = c^2$ becomes $(2h + 1) + (2m + 1) = c^2$.

$(2h + 1) + (2m + 1) = c^2$ simplifies to $2h + 2m + 2 = c^2$.

We proceed by addressing one of the two possible cases: c^2 is odd (case 1) or c^2 is even (case 2).

case 1:

Suppose c^2 is odd. So $c^2 = 2p + 1$, where $p \in \mathbb{Z}$.

Using substitution $2h + 2m + 2 = 2p + 1$.

With some algebra...

$$2h + 2m + 1 = 2p$$

$$2(h + m) + 1 = 2p$$

$$2t + 1 = 2p \text{ where } t \in \mathbb{Z}$$

But $2t + 1 = 2p$ is a contradiction because $2t + 1$ is odd and $2p$ is even.

Therefore if $a^2 + b^2 = c^2$, then a is even or b is even.

Question 8: There exists real numbers x and y so that $(y + x)^2 = x^2 + y^2$.

Proof:

Suppose for the sake of contradiction that for all real numbers x and y , $(y + x)^2 \neq x^2 + y^2$.

Let x and y be the real number 0, so that $x = 0$ and $y = 0$.

Through substitution $(y + x)^2 \neq x^2 + y^2$ becomes $(0 + 0)^2 \neq 0^2 + 0^2$.

Which simplifies to $0 \neq 0$, but this is a contradiction because $0 = 0$.

Therefore, there exists real numbers x and y so that $(y + x)^2 = x^2 + y^2$.

Part 1: Converse

Statement with a true converse: If I have an odd number of cats, then I don't have an even number of cats.

Statement with a false converse: If I have three cats, then I have an odd number of cats.

Part 3: Weekly reflection

Everything is good. I'm a little behind because I attended a conference in Vietnam for a week. I'm a little less than a week behind on the homework as of the writing of this reflection. I was able to keep up with the reading while I was away.

The conference was called Ocean Optics XII. Most of the scientists there work with ocean color satellites. Ocean color satellites are a classification of satellites that measure the reflectance of sunlight from the ocean's surface. From ocean reflectance, a lot of the properties of the ocean water can be inferred, most notably the phyto-plankton* content of ocean water. I work with one of the satellite products, CHL, and until recently had very little knowledge of how the CHL satellite products is produced (I still mostly don't understand). Basically the satellite measures the oceans reflectance with a sensor similar to a common camera, and from that back calculates what the light leaving the ocean would be after accounting for atmospheric effects. There's a good bit of math and hand waiving involved.

I'm mentioning all this because my master's program is coming to a close, and I'm thinking about pursuing a Phd. I wanted to do something more mathy, which is why I'm taking this course, and another math class. I feel like getting into ocean optics could be a good path because it combines mathematics with oceanography. A lot of the other students at their conference were in math, physics, or engineering programs.

*phyto-plankton are plankton that photosynthesis. Plankton are any organism in the ocean who's location/movement is subject to or dominated by the ocean currents. Phyto-plankton make up the base of the ocean's food web, and are responsible for about half the oxygen in the atmosphere.