Homework 14: Functions

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Question 1: Let $f : X \to Y$ and $g : Y \to X$ both be functions. Determine whether each of the following statement is true or false. If a statement is true, prove it. Otherwise, provide a counter example.

(a) if $g \circ f$ is injective, then f is injective.

True.
Proof Suppose $g \circ f$ is injective.
So then by def. for any $a, a' \in X$, $g \circ f(a) \neq g \circ f(a')$ when $a \neq a'$. It follows that $g(f(a)) \neq g(f(a'))$ and $f(a) \neq f(a')$ (otherwise $g \circ f$ would not be injective).
So then for any $a, a' \in X$, when $a \neq a'$ it follows that $f(a) \neq f(a')$.
Therefore f is injective.

(b) if $g \circ f$ is injective, then g is injective.

False, g can be surjective.
Counter example…
Let Y and X be the sets $Y = \{a, b\}$ and $X = \{1, 2, 3\}$.
Let g and f be the functions $g = \{(1, a), (2, b), (3, b)\}$ and $f = \{(a, 1), (b, 2)\}$, so then $g \circ f = \{(a, a), (b, b)\}$.
Here $g \circ f$ is injective, but g is surjective.

Question 3: Define $g : R \to R$ by $g(z) = 3z^2 - 4$. Find each of the following sets.

```
# writing function of g(z) = 3z^2 - 4
g = function(z){
  3*z^2 - 4
}

# Inverse function. Given the range will return the domain
gi = function(z){
  sqrt((z - 4)/3)
}
```

(c) $g([-2, 4])$

Answer: I'm inputting integers, but the range is the same. So it's [8, 44].

```
g(-2:4)
```

```
## [1]  8 -1 -4 -1  8 23 44
```

(d) $g^{-1}((-10, 1))$

Answer: The function is undefined at this interval. A negative in the square root symbol will return an imaginary number. So I'll go out on a limb and say $g^{-1}((-10, 1)) = \emptyset$.

```r
gi(-10:1)
```

```
##  [1] NaN NaN NaN NaN NaN NaN NaN NaN NaN NaN NaN NaN
```

(e) $g(\emptyset)$

Answer: Hmm... $\emptyset$?

(f) $g^{-1}(\emptyset)$

Answer: Hmm... $\emptyset$?

I originaly thought this... $(-\inf, 4]$ but am going with the empty set .

Question 6: Let $f : X \to Y$ be a function, and let $A, B \subset X$ and $C, D \subset Y$. Determine if each statement is true or false. If it's true prove it, if it's false give a counterexample.

*Note: It is not stated that f is bijective, so while f^{-1} is a relation, f^{-1} may not be a function.

(k) $A \subset f^{-1}(f(A))$

True.
Informal proof f^{-1}'s range is a subset of the domain of f. Here the domain of f is A, so then the range of f^{-1} is a subset of A.

(l) $f^{-1}(f(A)) \subset A$

False. *Informal counter example* Let f be surgective but not injective, so that f^{-1} is not a function. This means the $f^{-1}(A)$ can map back to $A \cup B$ as long as $A \cap B$ are subsets of the domain of f, in this case $A \cap B \subset X$.

(m) $C \subset f(f^{-1}(C))$

True.
Proof $c \in C$ and $f(c) = d \in f^{-1}(C)$ then $c \in f(f^{-1}(C))$. So any element c that is in $f(f^{-1}(C))$ is also in C. Therefore $C \subset f(f^{-1}(C))$.
reference: (theorem 12.4 pg. 243 in textbook, and an online source at www.math.unl.edu)

(m) $f(f^{-1}(C)) \subset C$

True.
Proof Let $c \in f(f^{-1}(C))$, then $c = f(d)$ for some $d \in f^{-1}(C)$, and
$f(d) \in C$. So any element c that is in $f(f^{-1}(C))$ is also in C. Therefore $c = f(d) \in C$.
reference: (theorem 12.4 pg. 243 in textbook, and an online source at www.math.unl.edu)