Untitled

2022-08-29

Question 2:

This statement is a disjunction that can be broken down into two propositions: 1) the number of heads in a pile is equal to the number of tails in the other pile (Q) 2) the number of tails in a pile is equal to the number of heads in the other pile (R)

So the statement can be represented as...

$$Q \vee R$$

It is assumed that each coin is two sided with one side that is heads and the other side is tails. Therefore the number of heads up coins plus the number of tails up coins equals the size of the pile.

$$H_{og} + T_{og} = P_{og}$$

Similarly when the coins are separated into two separate piles (P_1 and P_2) the size of those piles are equal to the sum of heads and tails coins.

$$H_1 + T_1 = P_1$$
$$H_2 + T_2 = P_2$$

By extension the number of heads in pile one (H_1) plus the number of heads in pile two (H_2) is equal the he number of heads in the original pile (P_{og}). The same is true for the number of tails.

$$H_1 + H_2 = H_{og}$$
$$T_2 + T_2 = T_{og}$$

Using this notation, the propositions Q and R becomes...

$$Q : H_1 = T_2$$
$$R : T_1 = H_2$$

If we set the size of P_1 and P_2 to be equal to the number of heads and tails in P_{og}, and solve for either in H_1, H_2, T_1, and T_2 in the proposition $Q \vee R$, we find the truth value of Q to always be TRUE and R to always be false.

$$P_1 = T_{og}$$
$$P_2 = H_{og}$$

Showing Q is always true when $P_2 = H_{og}$:

$$\begin{aligned} T_2 &= H_1 \\ P_2 - H_2 &= \\ H_{og} - H_2 &= \\ H_1 + H_2 - H_2 &= \\ H_1 &= H_1 \end{aligned}$$

Showing R is only true when $T_0 = H_0$ when $P_1 = T_{og}$ in way too many steps:

$$\begin{aligned}
T_1 = H_2 \\
T_{og} - H_2 = \\
T_{og} - (P_2 - H_2) \\
(P_{og} - H_0) - (P_2 - H_2) = \\
(P_{og} - H_0) - (H_0 - H_2) = \\
P_{og} - H_0 - H_0 + H_2 = \\
P_{og} - 2H_0 = 0 \\
P_{og} = 2H_0 \\
T_0 + H_0 = 2H_0 \\
T_0 = H_0
\end{aligned}$$

If we swapped $P_2 = H_{og}$ and $P_1 = T_{og}$ so that $P_2 = T_{og}$ and $P_1 = H_{og}$, then then the truth value of Q and R would be swapped as well.

When $H_0 \neq T_0$, and we set $P_2 = H_{og}$ and $P_1 = T_{og}$, then Q is always true, and R is always false. So the proposition $Q \vee R$ is always true.

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