Carnival

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Equivalence classes

Question 15. Let $A = \{cat, dog, monkey\}$. Find all equivalence relations on A. Justify that you have them all.

Let Nickelodeon be the set $A = \{cat, dog, monkey\}$.

I've listed all equivalence classes for the set Nickelodeon E_1 through E_5 below.

Reflexive. Each relation is symmetric because every element in set Nickelodeon is related to itself.

Symmetric. Whenever one element from set Nickelodeon is related to another element, that element is related back to the original element i.e. $\{(cat, dog), (dog, cat)\}$ or $catRdog \implies dogRcat$. Symmetry is not broken if no elements are related to another (i.e. E_1).

Transitive. Each relation is transitive because there is a relation aRb and bRc there is the relation aRc where a, b, c are elements in Nickelodeon.

For each relation I made the elements related to themselves i.e. reflexive. Then I thought there would be the cases where there where 0, 1, 2, or 3 symmetric relations.

case 1: 0 symmetric relations. One set. E_1 has 0 symmetric or transitive relations.

case 2: 1 symmetric relations. Three relations. Sets E_2 through E_4.

case 3: 2 symmetric relations. No relations. Any set with two symmetric relations will also need to be transitive. A third symmetric relation would need to exist so that the transitivity and symmetry is upheld.

case 4: 3 symmetric relations. One set E_5. This includes every eelement and every relation.

$$E_1 = \{(cat, cat), (dog, dog), (monkey, monkey)\}$$

$$E_2 = \{(cat, dog), (cat, cat), (dog, cat), (dog, dog), (monkey, monkey)\}$$

$$E_3 = \{(cat, cat), (dog, monkey), (dog, dog), (monkey, dog), (monkey, monkey)\}$$

$$E_4 = \{(cat, monkey), (cat, cat), (dog, dog), (monkey, cat), (monkey, monkey)\}$$

$$\begin{aligned} E_5 = \{&(cat, dog), (cat, monkey), (cat, cat), (dog, monkey), (dog, cat), (dog, dog), \\ &(monkey, dog), (monkey, cat), (monkey, monkey)\} \end{aligned}$$

■

Modular arithmatic

Question 17.

Prove for every $n \in \mathbb{N}$.

$$9|(4^{3n} - 1)$$

Rewording proof...

If $n \in \mathbb{N}$ then $9|(4^{3n} - 1)$.

Proof Suppose $n \in \mathbb{N}$. We proceed by induction.

(base case) $n = 1$

$$\begin{aligned} 4^{3(1)} - 1 \\ 64 - 1 \\ 63 \\ \frac{63}{9} = 7 \end{aligned}$$

(inductive step) Here we will show that k implies $k + 1$, for $k \geq 1$.

Suppose $9|(4^{3k} - 1)$. By definition...

$$9c = (4^{3k} - 1)$$

This simplifies to...

$$9c = (64^k - 1)$$

Multiply both sides by 64...

$$\begin{aligned} 9c \times 64 &= (64^k - 1) \times 64 \\ 9c \times 64 &= 64^{k+1} - 64 \\ 9c \times 64 + 64 &= 64^{k+1} \end{aligned}$$

Add 1 to both sides...

$$\begin{aligned} 9c \times 64 + 64 - 1 &= 64^{k+1} - 1 \\ 9c \times 64 + 63 &= 64^{k+1} - 1 \end{aligned}$$

Observe the left side of the equation $9c \times 64 + 63$. It is divisible by 9 without remainder because we can factor out a 9 so that $9(c \times 64 + 7) = 0(mod(9))$. By substituting the left hand side of this equation with $0(mod(9))$ we get...

$$0(mod(9)) = 64^{k+1} - 1$$

Returning equation to it's OG form

$$0(mod(9)) = 4^{3k+1} - 1$$

and thus...

$$9|(4^{3k+1} - 1)$$

Therefore $9|(4^{3k} - 1) \implies 9|(4^{3k+1} - 1)$ and this proof holds for all natural numbers $n \in \mathbb{N}$. ■

Quantifier proof disproof.

Question 12. Prove or disprove: There exists a positive real number x for which $x^2 > \sqrt{x}$

Proof Let x be the positive real number 0.5 such that

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x = 0.5
x^2 < sqrt(x)
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## [1] TRUE
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Therefore there exists a positive real number x for which which $x^2 > \sqrt{x}$. ■