Math 321: Intro to Advanced Mathematics
November 6, 2022

Cardinality and Countability

Question 1. Prove each of the following. In each case, you should create a bijection between the two sets. Briefly justify that your functions are in fact bijections.

(a) $|\{\heartsuit, \clubsuit, \spadesuit\}| = |\{\circ, \square, \triangle\}|$

(b) $|\mathbb{N}| = |\{\text{odd natural numbers}\}|$.

(c) $|A \times \{1\}| = |A|$, where A is any set.

(d) **(Challenge)** $\big|[a,b]\big| = \big|[c,d]\big|$, where $a, b, c, d \in \mathbb{R}$, $a < b$ and $c < d$. Remember that these are closed intervals.

Question 2. Let $\mathcal{F}$ denote the set of all functions from $\mathbb{N}$ to $\{0,1\}$.

(a) Describe at least three functions in the set $\mathcal{F}$.

(b) Prove that $|\mathcal{F}| = |\mathcal{P}(\mathbb{N})|$.

Question 3. Let X be a set. Prove that "has the same cardinality as" is an equivalence relation on $\mathcal{P}(X)$.

Question 4. Prove or disprove: The set $\{a_1, a_2, a_3, \cdots : a_i \in \mathbb{Z}\}$ of infinite sequences of integers is countably infinite.

Question 5. Prove that if A and B are finite sets with $|A| = |B|$, then any injection $f : A \to B$ is also a surjection. Show this is not necessarily true if A and B are not finite.