HW5 MTH 321

Jamie Ash

2022-09-15

Question 1: If a and b are both odd, then ab is odd.
Hypothesis: a and b are both odd
Conclusion: ab is odd
Truth value: True
Proof: Suppose a and b are both odd.
Then $a = 2k + 1$ and $b = 2l + 1$ where $k, l\ inZ$.
Likewise $ab = (2k + 1)(2l + 1) = 4kl + 2k + 2l + 1$.
Let $(4kl + 2L + 2k) = 2(2kl + L + k) = 2h$ where $h \in Z$.
So $ab = 2h + 1$.
Therefore ab is odd.

Question 2: If a is even or b is even then ab is even.
Hypothesis: a is even or b is even
Conclusion: ab is even
Truth value: True
Proof:
Suppose a is even or b is even. We approach this proof by addressing the two relevant cases.
Case 1: Without loss of generality, suppose a is even and b is even.
It follows that $a = 2k$ and $b = 2l$ where $k, l \in Z$.
Then $ab = 2k \times 2l = 2(2kl) = 2h$ where $h \in Z$.
Lastly $ab = 2h$ where $h \in Z$.
Therefore ab is even.

Case 2: Without loss of generality, suppose a is even and b is odd.
It follows that $a = 2k$ and $b = 2l + 1$ where $k, l \in Z$.
Then $ab = 2k(2l + 1) = 2(2kl + k) = 2h$ where $h \in Z$.
Simply $ab = 2h$ where $h \in Z$.
Therefore ab is even.

Therefore in all cases of a is even or b is even, then ab is even.

Question 3: If $x \in Z$ is odd, then $x^2 - 1$ is divided by 4.
Hypothesis: $x \in Z$ is odd
Conclusion: $x^2 - 1$ is divided by 4
Truth value: True
Counter example: I was not sure if this statement is true, so I tested it for a few values of $x \in Z$.

```
# creating integers
z = seq(from = -1000000000, to = 1000000000, by = 1)
# if x is odd then...
x = 2*z+1
# creating x^2 - 1
c = x^2 - 1
# c %% 4 returns a vector of the remainders of c / 4
```

```r
# unique(c %% 4) returns a vector of all the unique values in c %% 4
unique(c %% 4)
```

```
## [1] 0
```

```r
# The only unique remainder is 0, so all values in c are divisible by 4

# cleaning the environment
rm(z, x, c)
```

All of the remainders of $\frac{x^2-1}{4}$ are 0 when $[x \in Z : -1000000000 \leq x \leq 1000000000]$, therefore it's likely that **Question 3** is a true statement.

Question 4: If $a|b$ then $a^2|b^2$.
Hypothesis: $a|b$
Conclusion: $a^2|b^2$
Truth value: True
Suppose $a|b$
Then $a/b = c$ where $c \in Z$.
Furthermore $b^2/a^2 = a/b * a/b = c * c = c^2$.
If $c \in Z$, then $c^2 \in Z$.
So $b^2/a^2 = k$ where $k \in Z$.
Therefore $a^2|b^2$.

Question 5: If $a|c$ and $b|c$ then $ab|c$.
Hypothesis: $a|c$ and $b|c$
Conclusion: $ab|c$
Truth value: False
counter:
Ff c is even and $|c| > 2$, and $b = c$ while $a = c/2$, then $ab > c$.
Therefore, in general ab does not divide c.

example:

```r
c = 4
b = 4
a = 2

# a/b
b/a
```

```
## [1] 2
```

```r
# b/c
c/b
```

```
## [1] 1
```

```r
# ab/c
c / (a*b)
```

```
## [1] 0.5
```

Question 6: If $d|a$ and $d|b$ then $d|(a+b)$.
Hypothesis: $b|a$ and $d|b$
Conclusion: $d|(a+b)$
Truth value: True *Proof:* Suppose $d|a$ and $d|b$.
So (a/d) = k where $k \in Z$.
Similarly $(a/d) = n$ where $n \in Z$.
Using $(a+b)/b = (a/d) + (b/d)$.
Therefore $d|(a+b)$.

Question 7: If $a|r$ and $c|q$ then $ac|cq$.
Hypothesis: $a|r$ and $c|q$
Conclusion: $ac|cq$
Truth value: True
Proof: Suppose $a|r$ and $c|q$.
Then $r/a = k$ and $c/q = l$ where $k, l \in A$.
So $\frac{r}{a} \times \frac{q}{c} = \frac{rq}{ac} = k \times l$ where $k, l \in Z$.
Furthermore $k \times l = h$ where $h \in Z$.
Lastly $\frac{rq}{ac} = h \in Z$.
Therefore $ac|bd$

Question 8: If $n \in Z$ then $3n^2 + n + 114$ is even.
Hypothesis: $n \in Z$
Conclusion: $3n^2 + n + 114$ is even
Truth value: False
Counter:
Suppose $n \in Z$.
Case 1: n is odd, then $3n^2 + n + 114$ is odd.
Case 2: n is even, then $3n^2 + n + 114$ is even.
Therefore $3n^2 + n + 114$ is not generaly even when $n \in Z$.

Question 9: If $x, y \in R$ satisfy $x < y$, then $x^2 < y^2$.
Hypothesis: $x, y \in R$ satisfy $x < y$
Conclusion: $x^2 < y^2$
Truth value: False
Counter:

When x is positive and y is negative, $x^2 < y^2$ is false.
Here are a few cases where x^2 and y^2 are equal…

```
x  = seq(from = -1, to = -1000, by = -1)
y  = x * -1
x2 = x^2
y2 = y^2
# all are FALSE
unique(x2 < y2)
```

```
## [1] FALSE
```

```
unique(x2 == y2)
```

```
## [1] TRUE
```

Question 10: If $x \in (0, 4)$, then $\frac{4}{x(4-x)} \geq 1$.
Hypothesis: $x \in (0, 4)$

Conclusion: $\frac{4}{x(4-x)} \geq 1$
Truth value: False?
Counter:
Does this mean x can be either 0 or 4? If x is 0, then the conclusion has a 0 in the denominator.