Homeswork 8

This is as far as I got by the end of teh day friday, but I plan to do more over the weekend. Maybe I'll do the thing where I email you (the professor) what I've done more to show I've put in the effort.

1. Let J be a matrix which is in Jordan normal form. Suppose that J has the following blocks on its diagonal: $J_{5,\sqrt{2}}$ (a 5 x 5 block with 2 on the diagonal), 3 copies of $J_{4,7}$, 2 copies of $J_{2,7}$, 4 copies of $J_{1,7}$, two copies of $J_{3,\pi}$, and one copy of $J_{2,\pi}$.
(a) What is the characteristic polynomial of J?
(b) What is the minimal polynomial of J?
(c) For each eigenvalue λ of J, what is the dimension of the generalized eigenspace corresponding to λ?
(d) For each eigenvalue λ of J, find the dimension of $Ker(J-\lambda I)^k$ for every k.

$$P_J(x) = (x-\sqrt{2})^5(x-7)^{20}(x-\pi)^8$$
$$m_J(x) = (x-\sqrt{2})^5(x-7)^4(x-\pi)^3$$

dimension of general eigenspace of $\lambda = \sqrt{2}$ is 5.
dimension of general eigenspace of $\lambda = 7$ is 20.
dimension of general eigenspace of $\lambda = \pi$ is 8.

$Ker(J-\sqrt{2}I)^5 = 5$
$Ker(J-\sqrt{2}I)^4 = 4$
$Ker(J-\sqrt{2}I)^3 = 3$
$Ker(J-\sqrt{2}I)^2 = 2$
$Ker(J-\sqrt{2}I)^1 = 1$

$Ker(J-7I)^{20} = 20$
$Ker(J-7I)^4 = 20$
$Ker(J-7I)^3 = 17$
$Ker(J-7I)^2 = 14$
$Ker(J-7I)^1 = 9$

$Ker(J-\pi I)^8 = 8$
$Ker(J-\pi I)^3 = 8$
$Ker(J-\pi I)^2 = 6$
$Ker(J-\pi I)^1 = 3$

1. Let W be the space of all continuous functions $R \to R$, and let $V \subset W$ be the subspace spanned by the functions x, x^2, x^3, e^x, e^{-x}. Let $T : V \to V$ be linear transformation that sends a function to its derivative (so $T(f) = f'$). Find the Jordan normal form of T .

Well, I really wish I did this question before the test today. Goofed that one up. Turning to inspirtion from the internet.

So

$$T(x) = 1$$
$$T(x^2) = 2x$$
$$T(x^3) = 3$$
$$T(e^x) = e^x$$
$$T(e^{-x}) = -e^{-x}$$

This is what our coordinat maps will look like.

$$\begin{pmatrix} \frac{d}{dx}x \\ \frac{d}{dx}x^2 \\ \frac{d}{dx}x^3 \\ \frac{d}{dx}e^x \\ \frac{d}{dx}e^{-x} \end{pmatrix}$$

$$T(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, T(x^2) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}, T(x^3) = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}, T(e^x) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, T(e^{-x}) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix},$$

Put those bad boys together... This is close I think but maybe not quit there. I don't fully understand it.

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{pmatrix},$$

1. Let V be the space of all continuous functions $f : [0, 1] \to R$. Define the following inner product on $V :< f, g >= \int_0^1 f(t)g(t)dt$. Let $f(x) = 0$ be the constant function that is equal to 0, let $g(x) = x^2$, $h(x) = x^3 - 1$. Find the lengths of the sides and the angles of the triangle formed by f, g, h.

1. Let W be the space of all continuous functions $R \to V \subset W$ be the subspace spanned by the functions x, x^2, x^3, e^x, e^{-x}. Let $U \subset W$ be the subspace spanned by x, x^2, x^3. Find the orthogonal projection of e^x to U.

1. Let $V = M_3(R)$ be the space of 3×3 matrices. Define an inner product on V by setting $hA, Bi = trace(ATB)$. Let $U \subset V$ be the subspace of antisymmetric matrices (a matrix A is antisymmetric if $A^T = -A$. Find:
   (a) The closest point in U to the identity matrix.
   (b) The distance between the identity matrix and the space U.