**Homework 6**

**1.** Divide the following polynomials with remainder.

(a) Divide $x^7 - x^3 + 1$ by $x^4 + x + 2$.
(b) Divide $X^8 - 1$ by $x + 1$.
(c) Divide $x^3 + 2x^2 - 3x + 1$ by $x^2 + x - 1$.

**2.** Find the minimal polynomials of the following matrices.

(a) $\begin{pmatrix} 5 & -1 \\ 1 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

**3.** For every pair of non-constant polynomials $p, q$ where $q$ divides $p$, show how to find a matrix $A$ whose characteristic polynomial is $p$ and whose minimal polynomial is $q$.

**4.** What's wrong with the following "proof" of the Cayley Hamilton theorem: Let $A$ be an $n \times n$ matrix. Then $p_A(A) = \det(A - A) = \det(0) = 0$. Where does the "proof" fail? Read the direct algebraic proof of the Cayley Hamilton theorem that appears in the wikipedia article about the theorem.

**5.** We say that $B$ is a square root of $A$ if $B^2 = A$. Over the field $\mathbb{C}$, show that the identity matrix has infinitely many square roots. Explain why each such root is diagonalizable.