

## Homework 1

1. Consider the game we described in class that involved clicking on points on a grid. There are three colors (red, green, blue), and clicking on a square in the grid cycles the colors of that square and every other square in its row and column. Suppose the game was played on a  $3 \times 3$  grid instead of a  $2 \times 2$ . Can you start with an all red configuration and end with a configuration where the first column is blue, the second column is red, and the third column is green?
2. A game is played on a  $7 \times 7$  board. The board wraps around - if a piece is situated on the rightmost column and moves one to the right, it ends up on the leftmost column. A piece on the top row that moves one up ends up on the bottom row. A piece is placed in the  $(1, 1)$  square. The piece uses the knight move set from Chess (a knight can move in L shapes - two to the right and one down, two to the right and one up, two up and one left, etc). Can the piece reach the  $(2, 2)$  square? If so, describe the moves it needs to do so. Hint: this is a linear algebra problem that requires solving a system of equations over the field  $F_7$ .
3. A circle passes through the points  $(2, 6)$ ,  $(-1, 7)$ ,  $(-4, -2)$ . Find an equation for the circle (Hint: every circle can be written in the form  $x^2 + y^2 + ax + by + c = 0$ . Your variables should be  $a, b, c$ .)
4. A marketplace trades four different commodities: abacuses, bananas, crystals, and drums. You can trade 3 abacuses, 8 bananas, and four crystals for five drums. You can trade one crystal for three abacuses and four drums. You can trade one abacus and one drum for two bananas and a crystal. You can trade one abacus for two bananas and three crystals. A person comes to the market with one abacus. They left with five drums. What were the trades that they made?