Homework 8

1. Let J be a matrix which is in Jordan normal form. Suppose that J has the following blocks on its diagonal: $J_{5,\sqrt{2}}$ (a 5×5 block with $\sqrt{2}$ on the diagonal), 3 copies of $J_{4,7}$, 2 copies of $J_{2,7}$, 4 copies of $J_{1,7}$, two copies of $J_{3.\pi}$, and one copy of $J_{2,\pi}$.

(a) What is the characteristic polynomial of J?
(b) What is the minimal polynomial of J?
(c) For each eigenvalue λ of J, what is the dimension of the generalized eigenspace corresponding to λ?
(d) For each eigenvalue λ of J, find the dimension of $\mathrm{Ker}(J-\lambda I)^k$ for every k.

2. Let W be the space of all continuous functions $\mathbb{R} \to \mathbb{R}$, and let $V \subset W$ be the subspace spanned by the functions x, x^2, x^3, e^x, e^{-x}. Let $T : V \to V$ be linear transformation that sends a function to its derivative (so $T(f) = f'$). Find the Jordan normal form of T.

3. Let V be the space of all continuous functions $f : [0,1] \to \mathbb{R}$. Define the following inner product on V: $\langle f, g\rangle = \int\limits_0^1 f(t)g(t)dt$. Let $f(x) = 0$ be the constant function that is equal to 0, let $g(x) = x^2$, $h(x) = x^3 - 1$. Find the lengths of the sides and the angles of the triangle formed by f, g, h.

4. Let W be the space of all continuous functions $\mathbb{R} \to \mathbb{R}$, and let $V \subset W$ be the subspace spanned by the functions x, x^2, x^3, e^x, e^{-x}. Let $U \subset W$ be the subspace spanned by x, x^2, x^3. Find the orthogonal projection of e^x to U.

5. Let $V = M_3(\mathbb{R})$ be the space of 3×3 matrices. Define an inner product on V by setting $\langle A, B\rangle = \mathrm{trace}(A^T B)$. Let $U \subset V$ be the subspace of antisymmetric matrices (a matrix A is antisymmetric if $A^T = -A$. Find:

(a) The closest point in U to the identity matrix.
(b) The distance between the identity matrix and the space U.