



# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

Masaki Waga

Kyoto University  
12 May 2021, YR-OWLS  
Based on the paper at FORMATS'19



Monitoring

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# Safety Critical CPSs

## Self-driving car crash in Arizona: Red light runner hits Waymo van



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Technology

### Tesla Model 3: Autopilot engaged during fatal crash

17 May 2019

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The Tesla Model 3 after the crash

<https://www.abc15.com/news/region-southeast-valley/chandler/waymo-car-involved-in-chandler-arizona-crash>

<https://www.bbc.com/news/technology-48308852>

# Monitoring

## Specification:

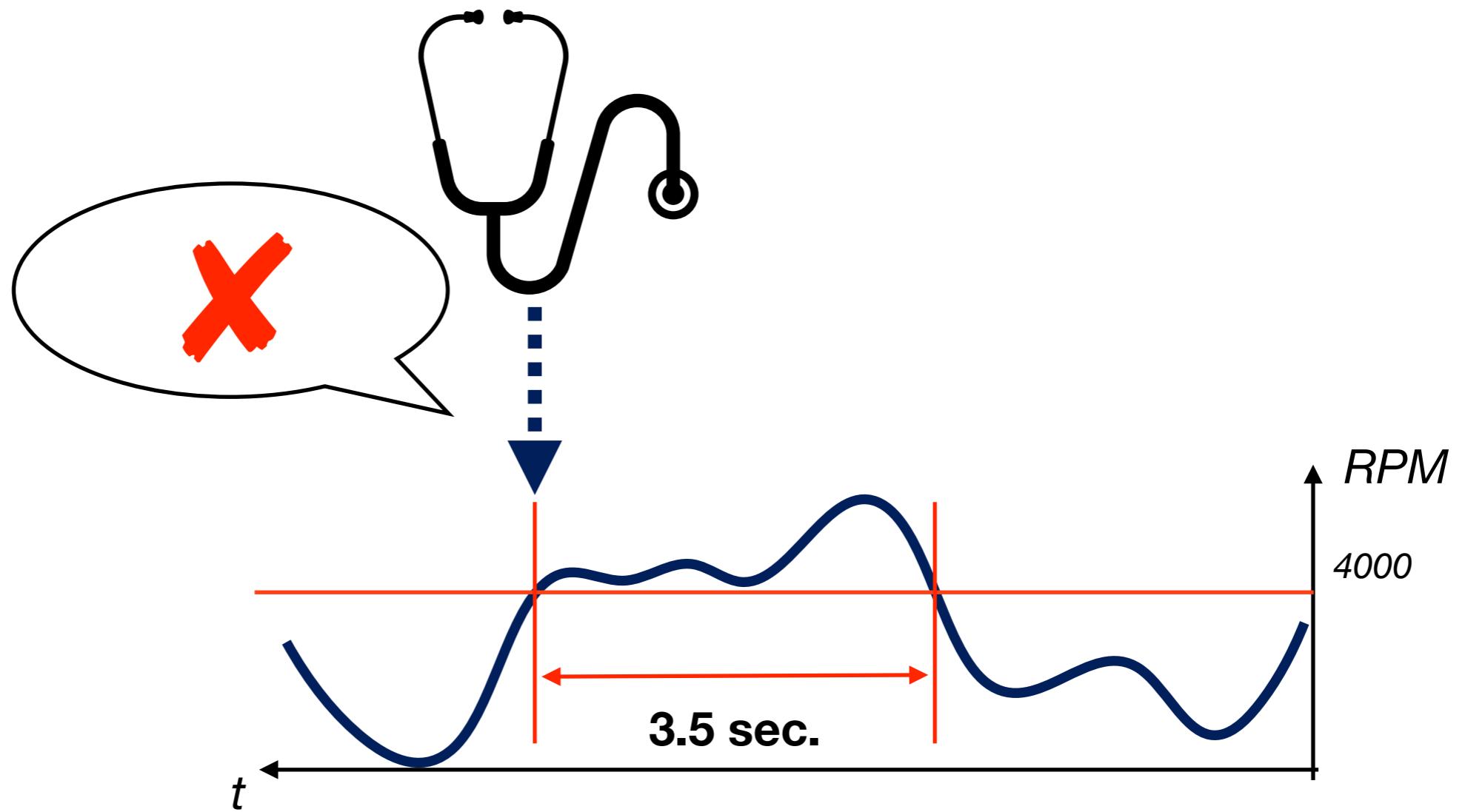
No (RPM > 4000) for > 1 sec.



# Monitoring

## Specification:

No ( $RPM > 4000$ ) for  $> 1$  sec.



# Timed Pattern Matching

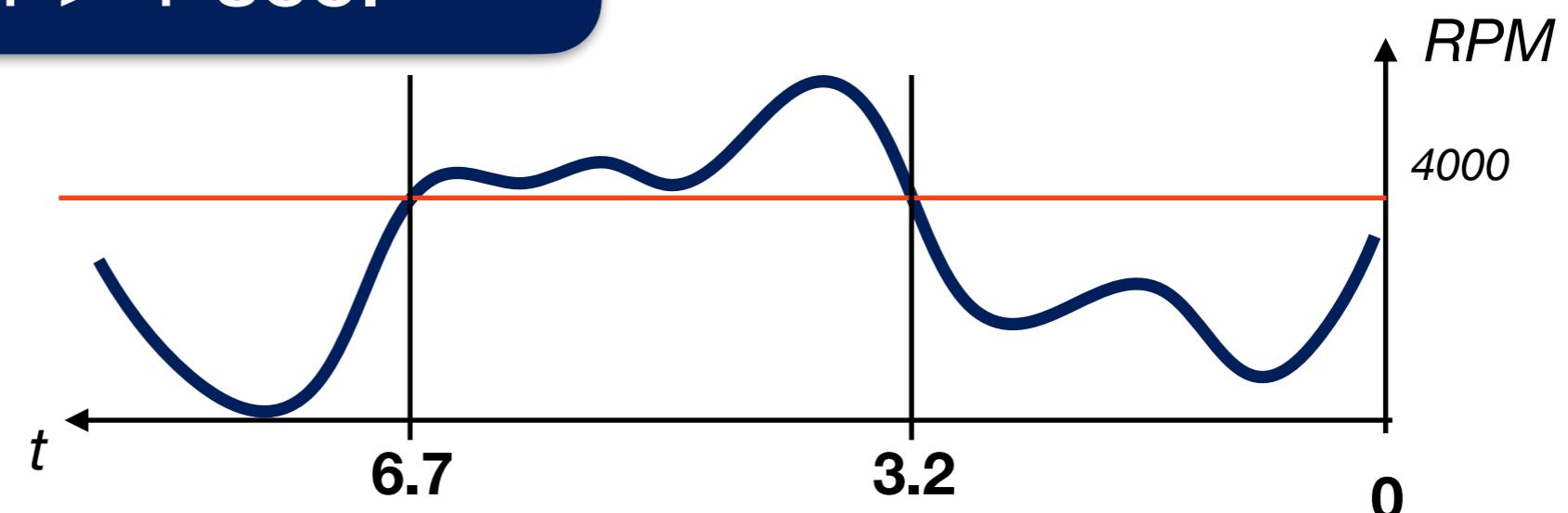
[Ulus+, FORMATS'14]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

## Pattern Specification:

(RPM > 4000) for > 1 sec.



# Timed Pattern Matching

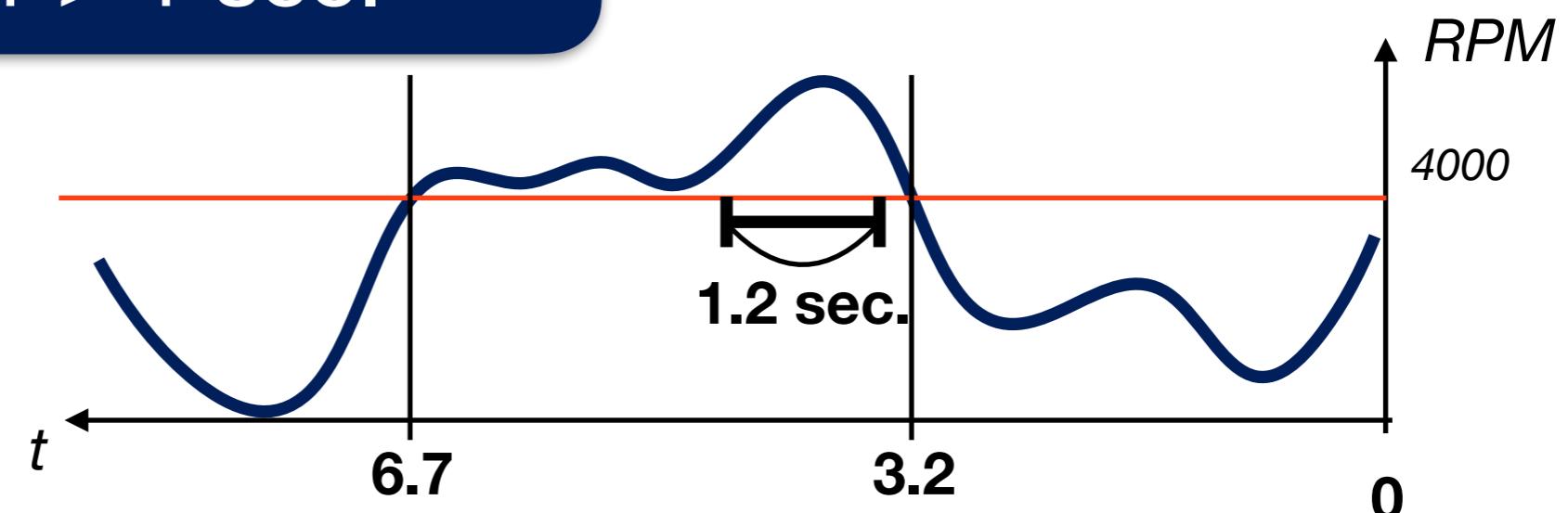
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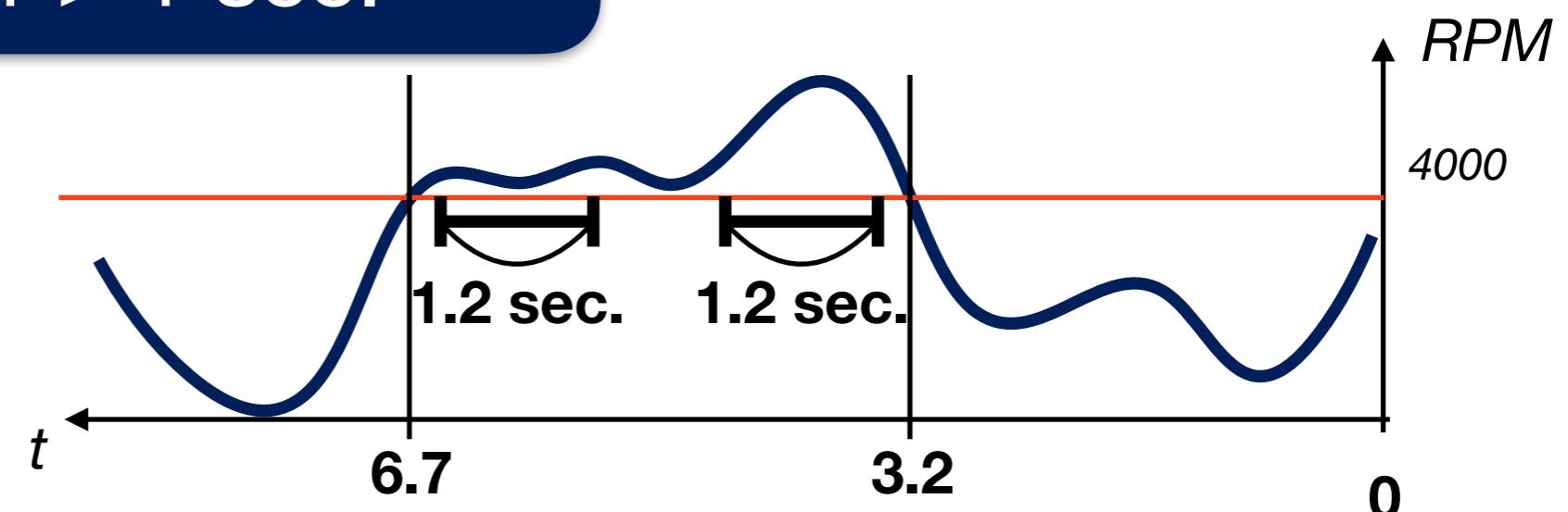
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# Timed Pattern Matching

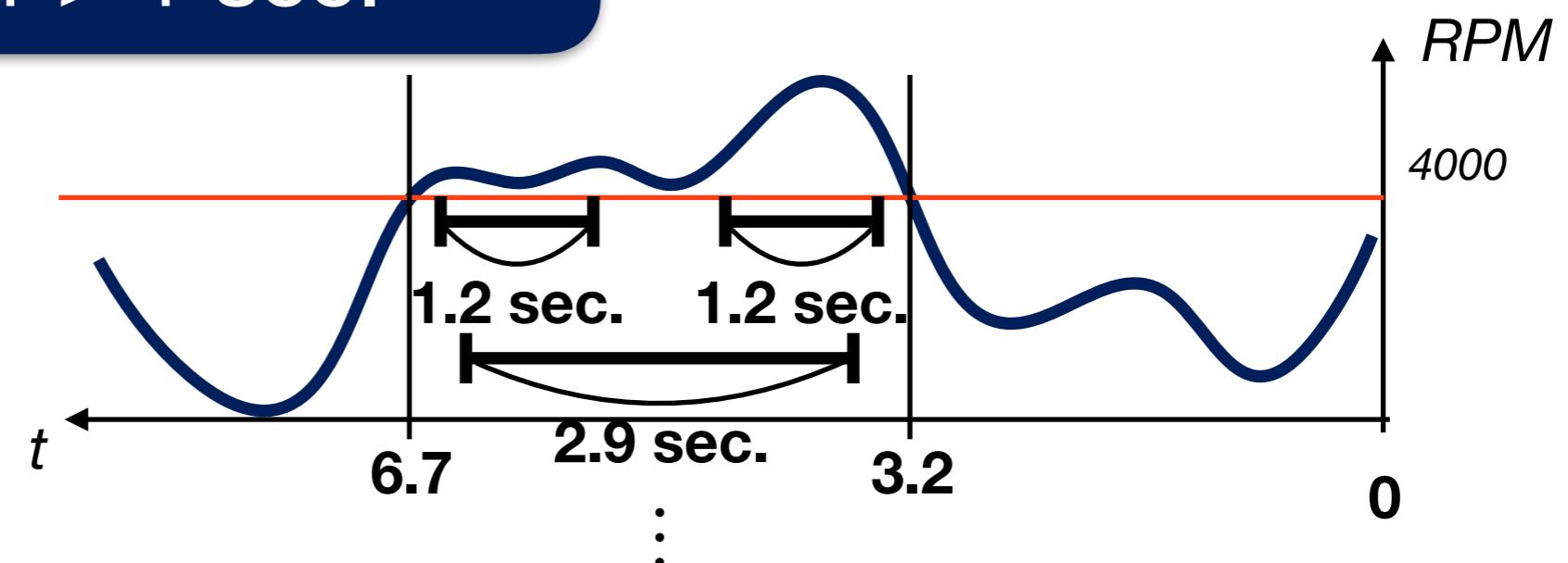
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# Timed Pattern Matching

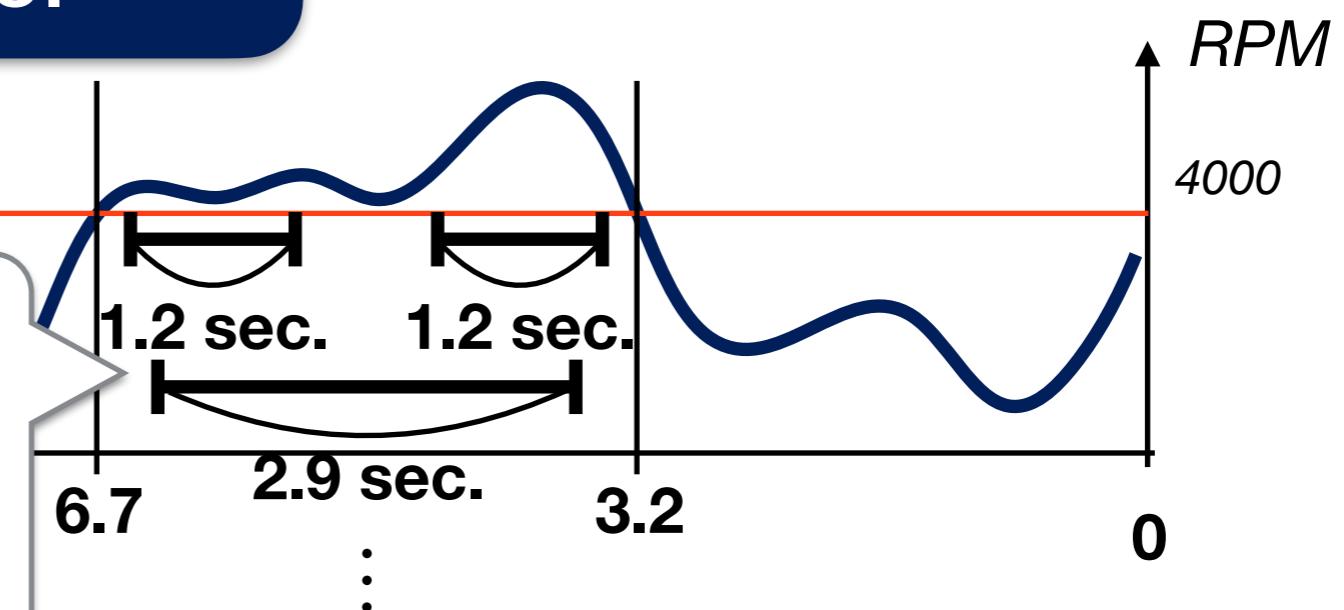
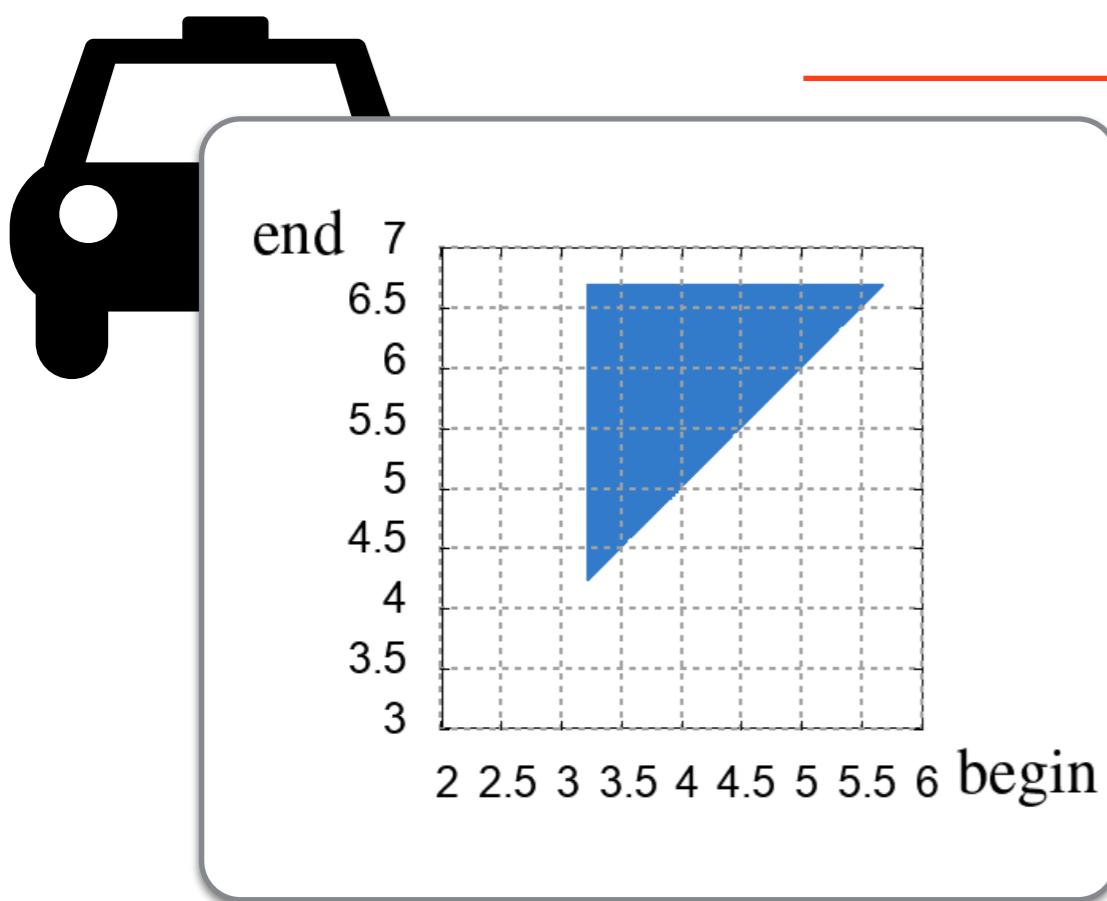
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Given: Signal and Pattern Spec.

Goal: Find all the matching intervals

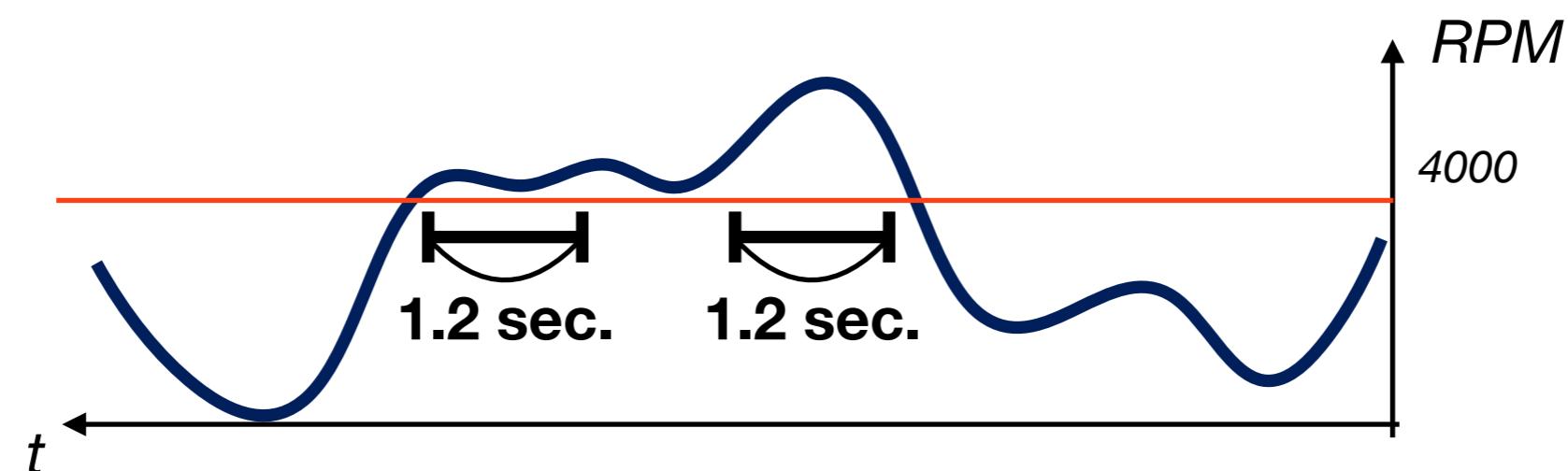
## Pattern Specification:

(RPM > 4000) for > 1 sec.



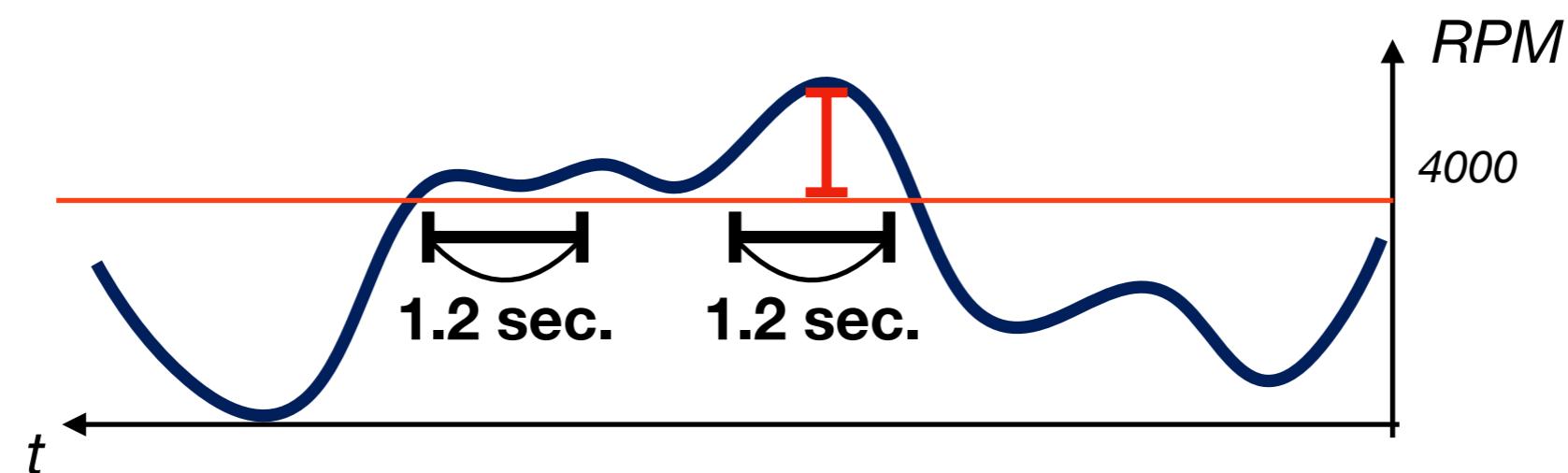
# Qualitative → Quantitative

Pattern Specification:  
 $(RPM > 4000) \text{ for } > 1 \text{ sec.}$



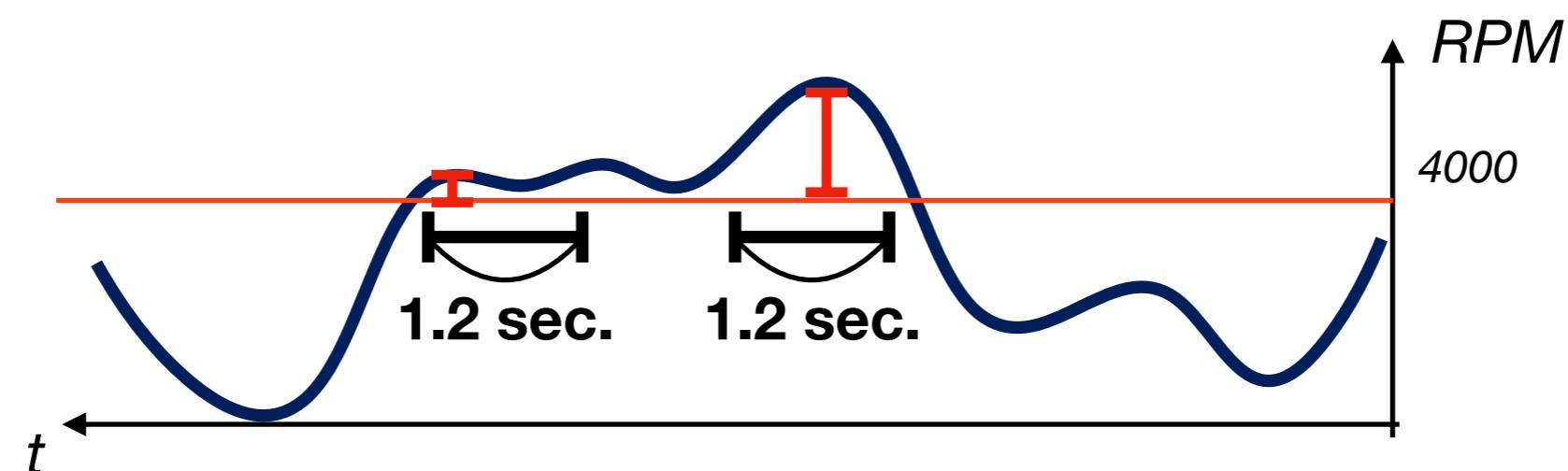
# Qualitative → Quantitative

Pattern Specification:  
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Pattern Specification:  
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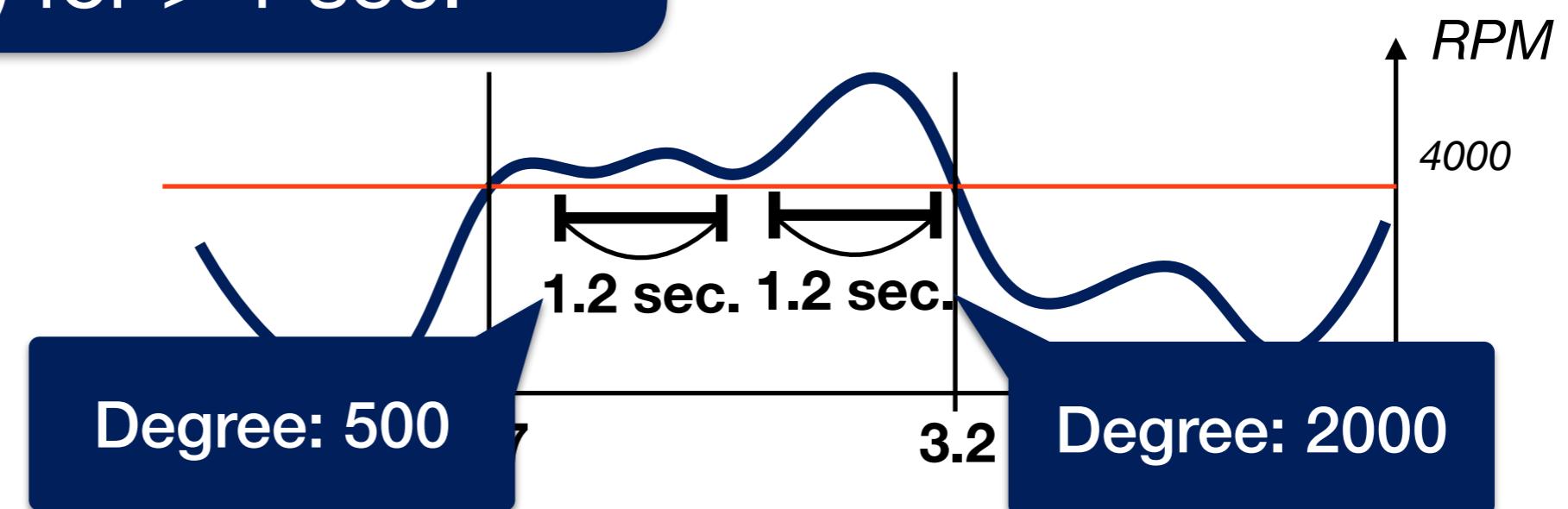
# Quantitative Timed Pattern Matching

[Bakhirkin+, FORMATS'17]

Given: Signal and Pattern Spec.

Goal: Find all the matching intervals + **satisfaction degree**

Pattern Specification:  
 $(RPM > 4000) \text{ for } > 1 \text{ sec.}$



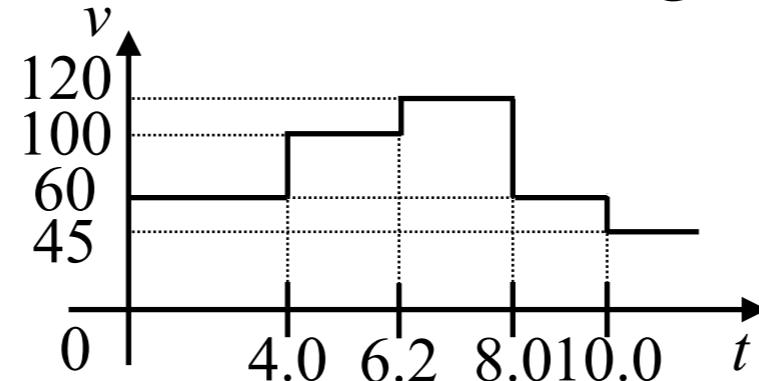
# Quantitative Timed Pattern Matching

## Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal  $\sigma$**

- e.g.,



- **Pattern Specification  $\mathcal{W}$**

- **Spec.** to be monitored

- e.g., The velocity should not keep  $> 80$  for  $> 1$  sec.

## Output

- **Function** assigning the satisfaction degree to each **subsignal**  $\sigma([t,t'])$ 
  - . e.g.,  $\mathcal{M}(\sigma, \mathcal{W})(2.0, 4.0) = -20$ ,  $\mathcal{M}(\sigma, \mathcal{W})(6.5, 7.8) = 40$ , ...

satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0))$

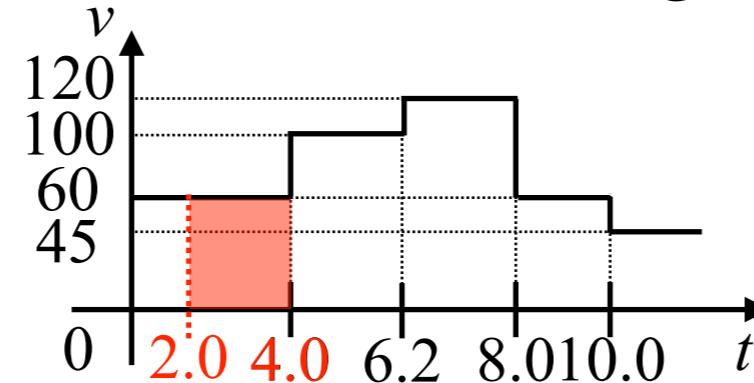
# Quantitative Timed Pattern Matching

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[Bakhirkin+, FORMATS'17]

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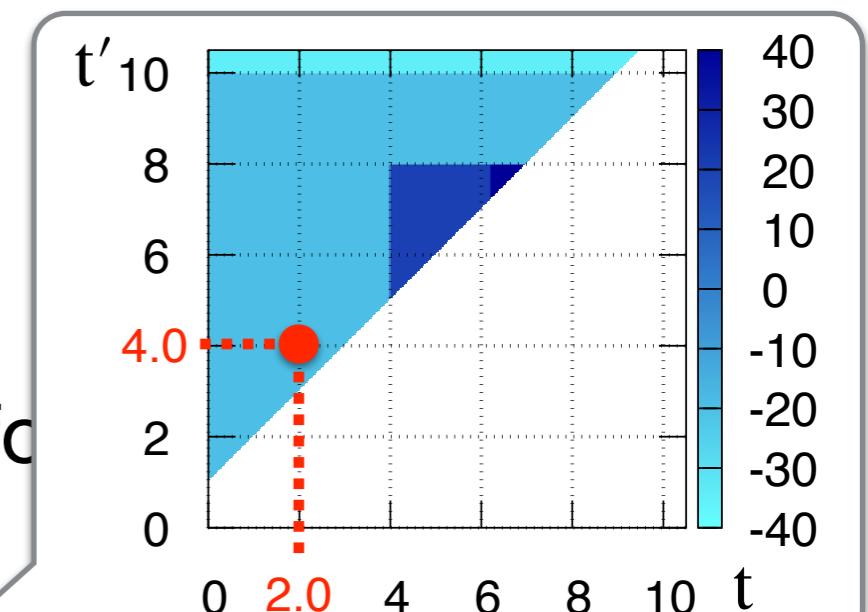
- e.g.,



- **Pattern Specification  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep  $> 80$  for  $t \in [2.0, 4.0]$



## Output

- **Function** assigning the satisfaction degree to each **subsignal**  $\sigma([t, t'])$ 
  - e.g.,  $M(\sigma, \mathcal{W})(2.0, 4.0) = -20, M(\sigma, \mathcal{W})(6.5, 7.8) = 40, \dots$

satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0])$

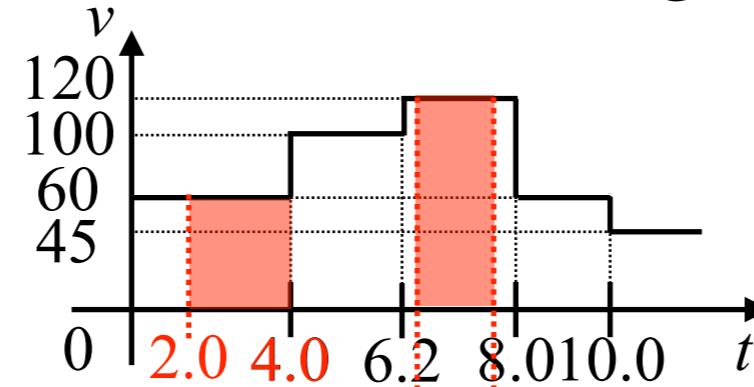
# Quantitative Timed Pattern Matching

## Input

[Bakhirkin+, FORMATS'17]

- **Real-valued piecewise-constant signal  $\sigma$**

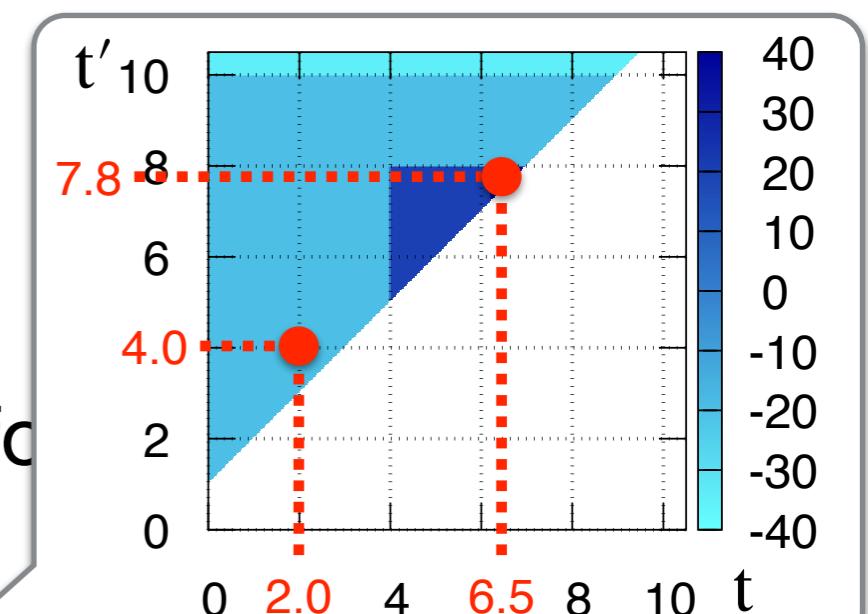
- e.g.,



- **Pattern Specification  $\mathcal{W}$**

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- e.g., The velocity should not keep  $> 80$  for  $t \in [2.0, 4.0]$



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satisfaction degree of  $\mathcal{W}$  for  $\sigma([2.0, 4.0])$

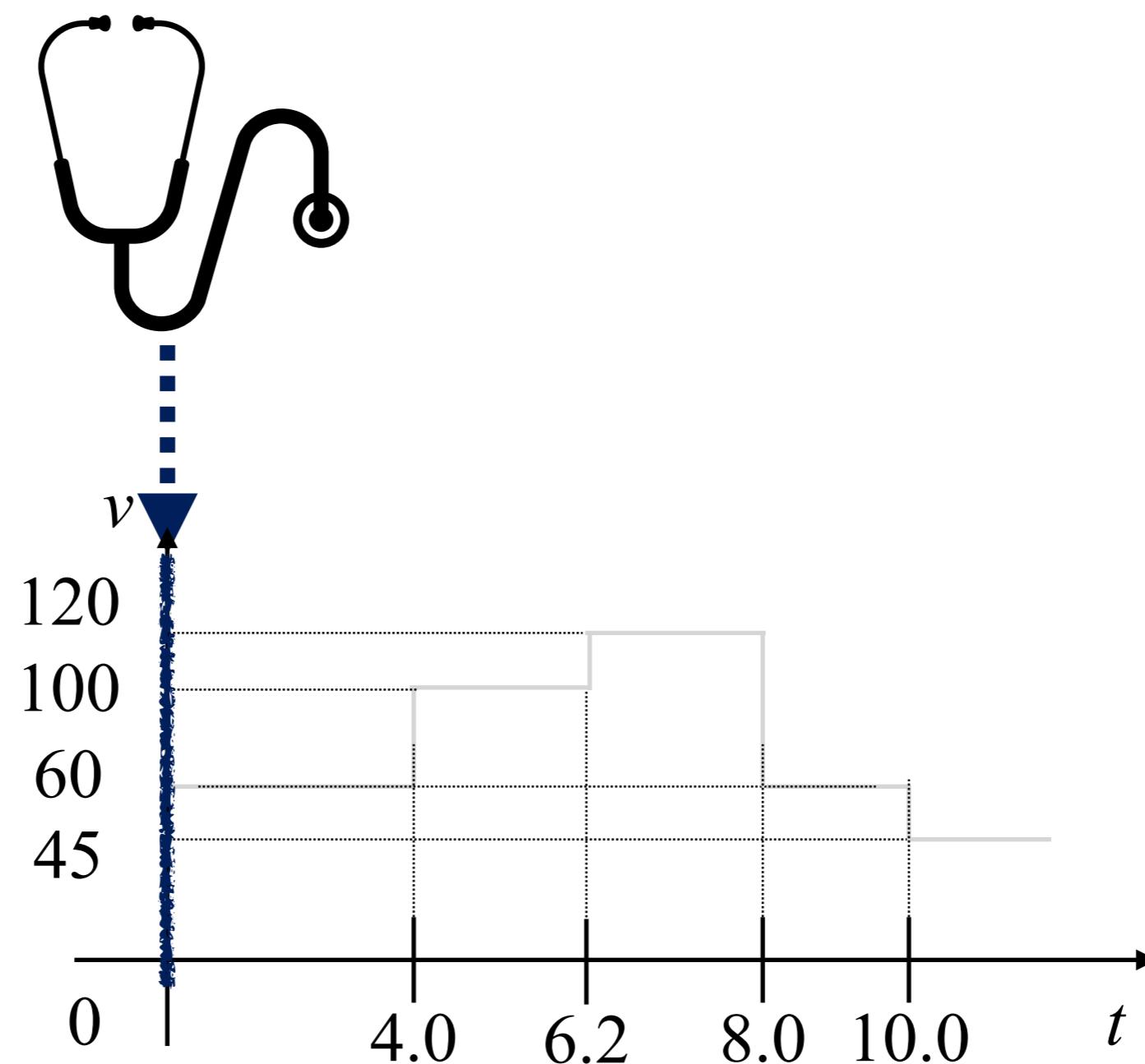


# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

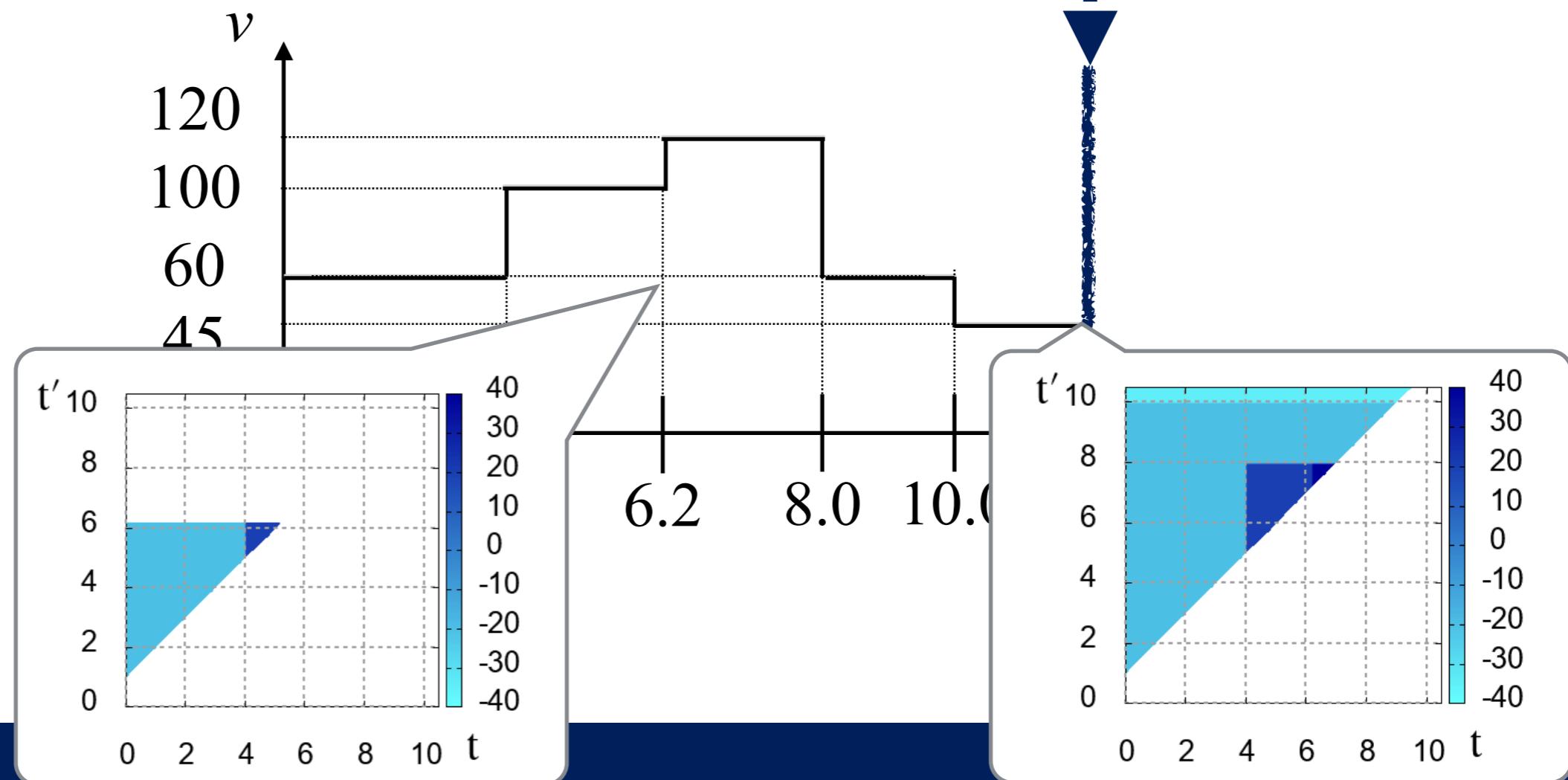
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# Online Pattern Matching



# Online Pattern Matching





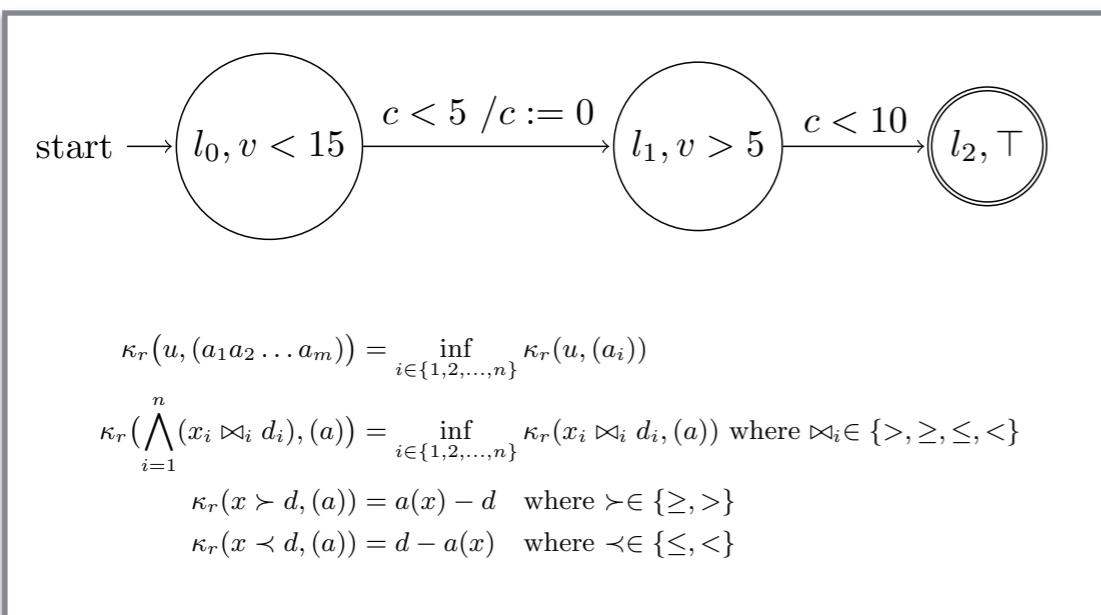
# Online Quantitative Timed Pattern Matching with Semiring- Valued Weighted Automata

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12 May 2021, YR-OWLS  
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# Timed symbolic weighted automata (TSWA)

- New formalism for spec.
  - Automata structure is good for online monitoring
- Generality of semiring (same as the usual WFA)

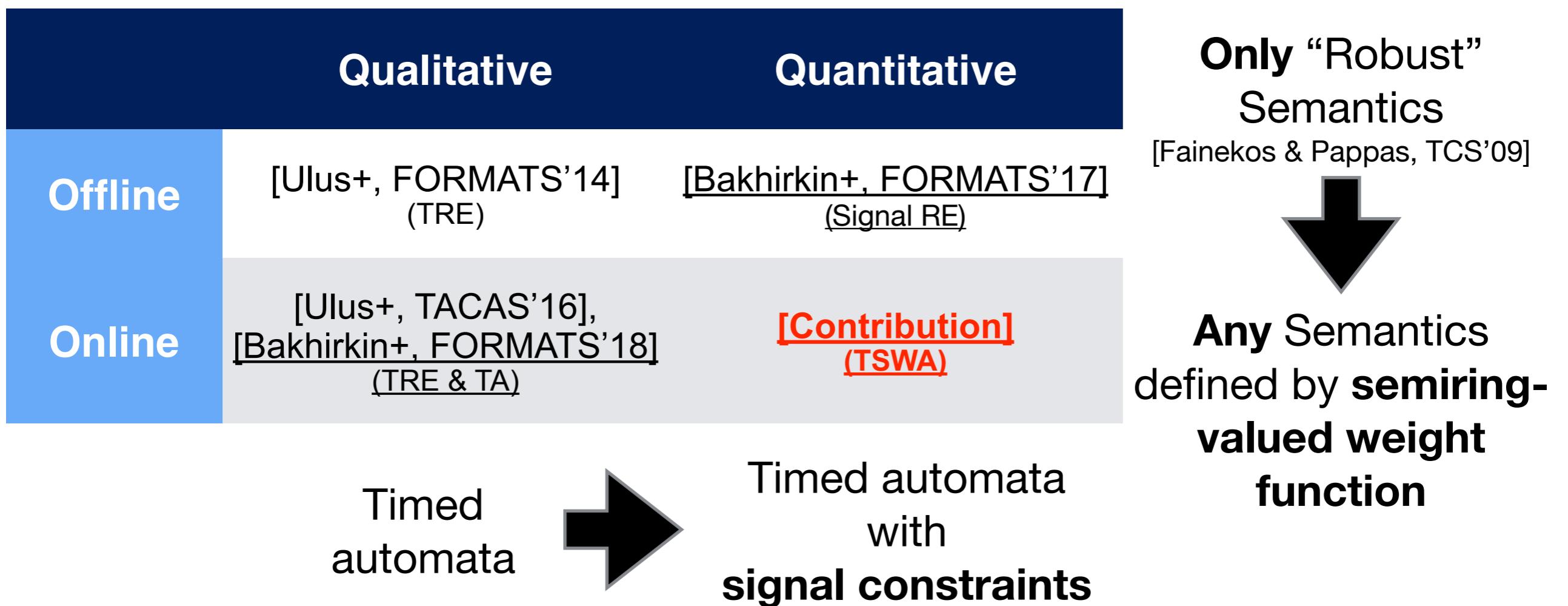


	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

# Contribution

- Introduced **timed symbolic weighted automata (TSWA)**
- **TSWA:** timed automata with signal constraints (TSA)
  - + semiring-valued weight function
  - Quantitative semantics**
- Gave **online** algorithm for quantitative timed pattern matching
- Implementation + experiments → **Scalable!!**

# Related Works

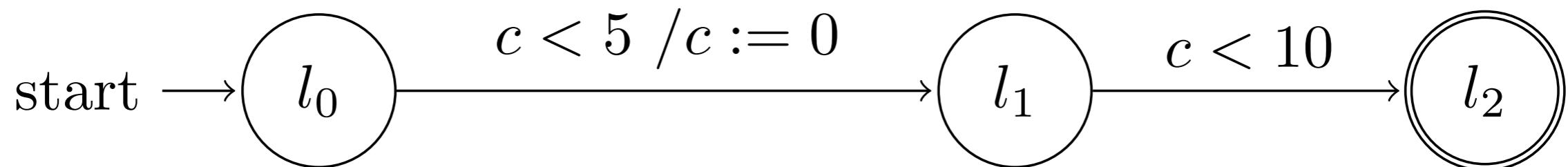


# Outline

- Motivation + Introduction
- Technical Part
  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
  - Quantitative monitoring/timed pattern matching algorithm
    - Idea: zone construction with weight
- Experiments

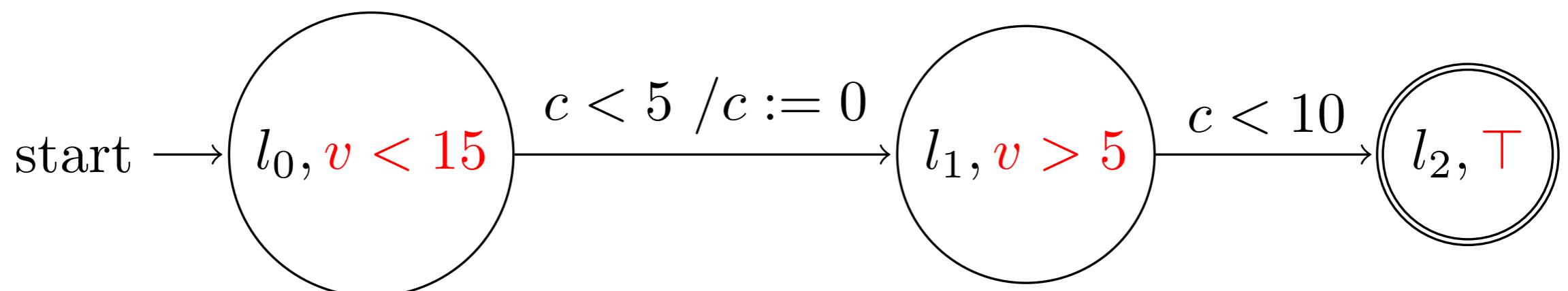
# TSWA: TA with signal constraints + weight function

## Timed Automaton (TA)



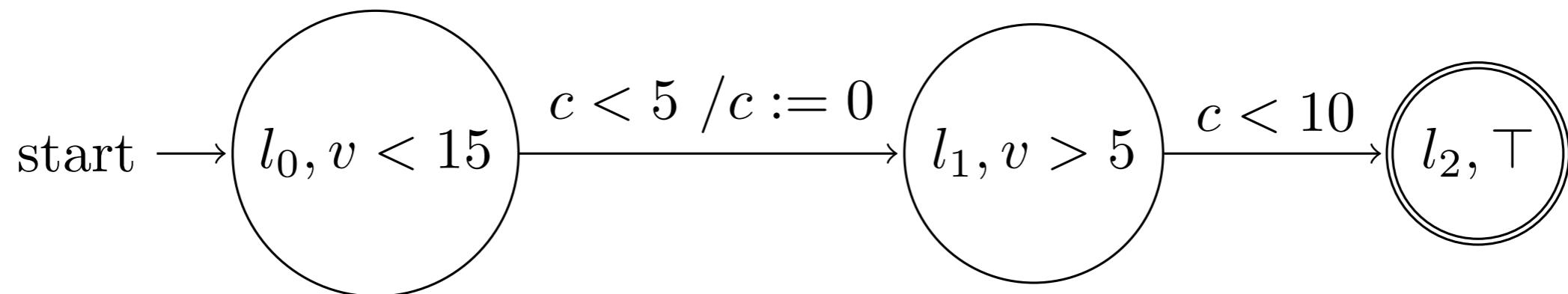
# TSWA: TA with signal constraints + weight function

## Timed Symbolic Automaton (TSA)



# TSWA: TA with signal constraints + weight function

## Timed Symbolic Weighted Automaton (TSWA)



+

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

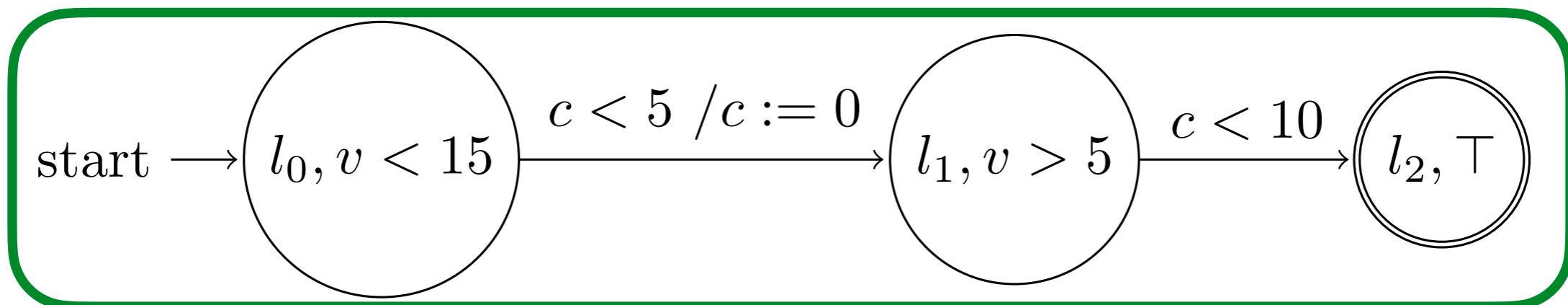
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

# TSWA: TA with signal constraints + weight function

## Timed Symbolic Weighted Automaton (TSWA)



Automata structure

+

Quantitative  
semantics

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

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$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

# Weight function

$$\kappa: \Phi(X, \mathbb{D}) \times (\mathbb{D}^X)^* \rightarrow S$$

Constraints on signal values at the location

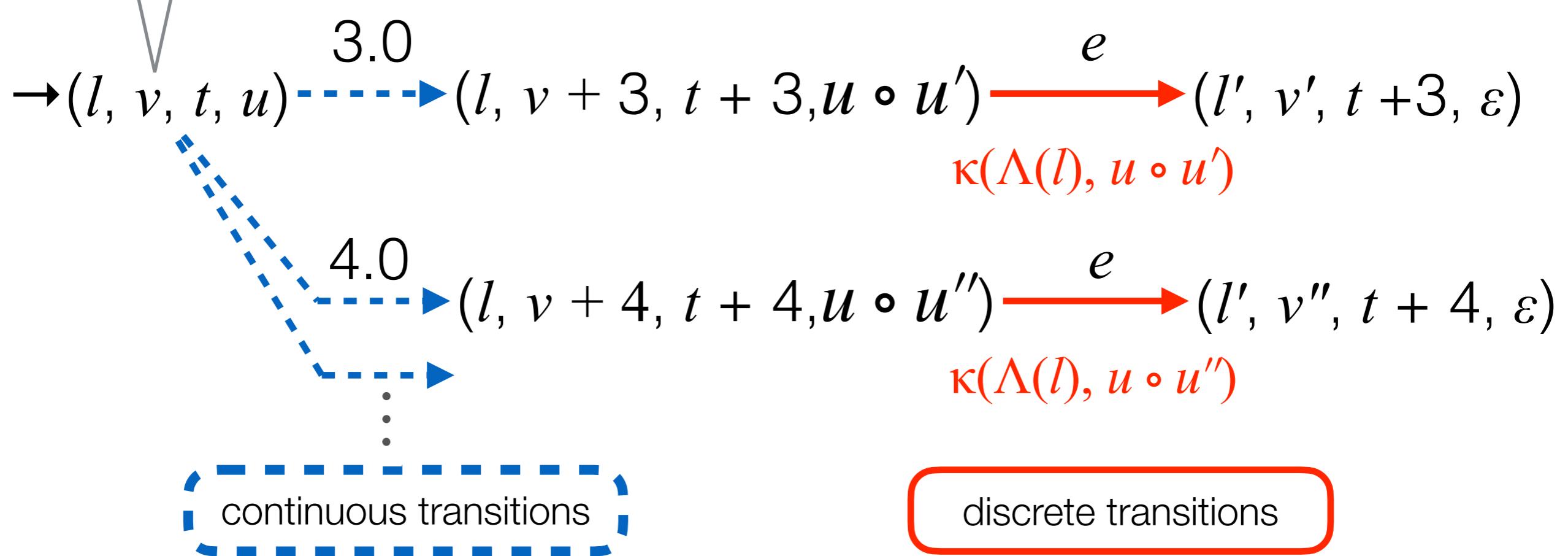
Sequence of signal values at the location

Semiring value

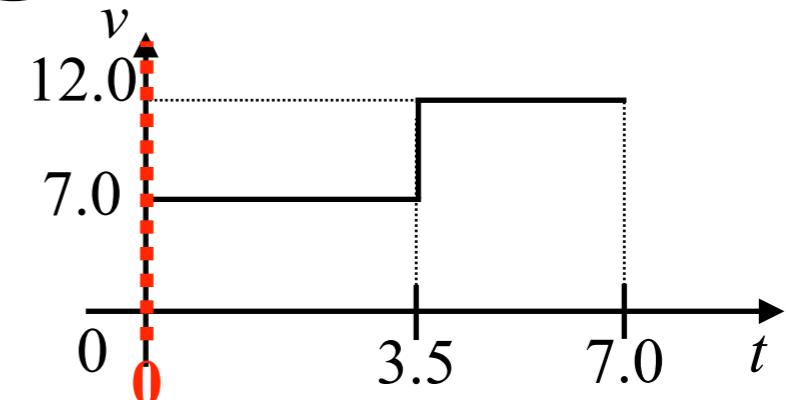
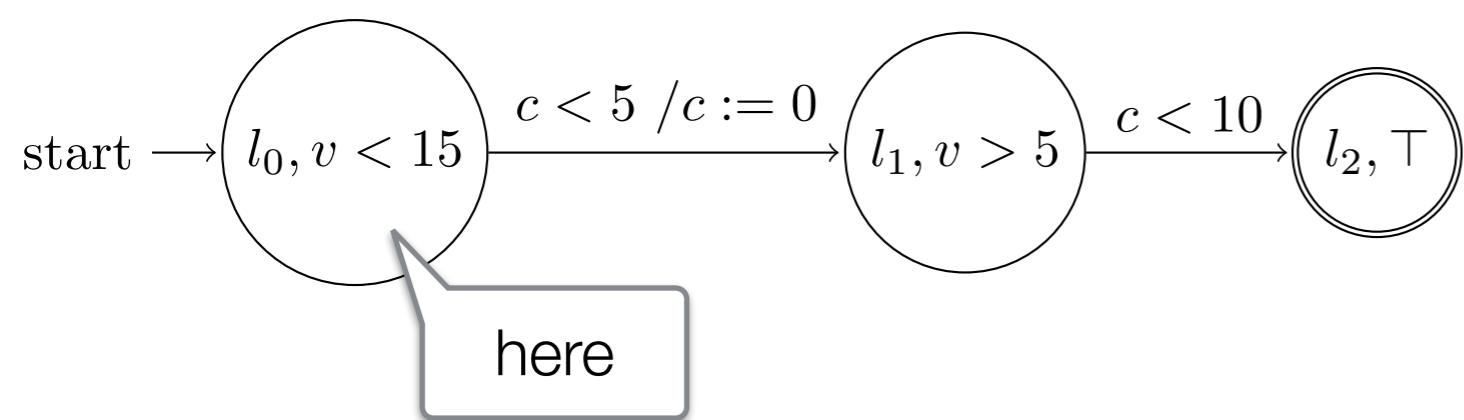
- $\kappa(\Lambda(l), u)$ : weight for the stay at  $l$  with signal values  $u$
- Semiring: set  $S$  with accumulating operators  $\oplus$  and  $\otimes$
- We can use any complete and idempotent semiring

# Semantics: Weighted TTS

- $l$ : location
- $v$ : clock valuation
- $t$ : absolute time
- $u$ : sequence of signal values after the latest discrete transition

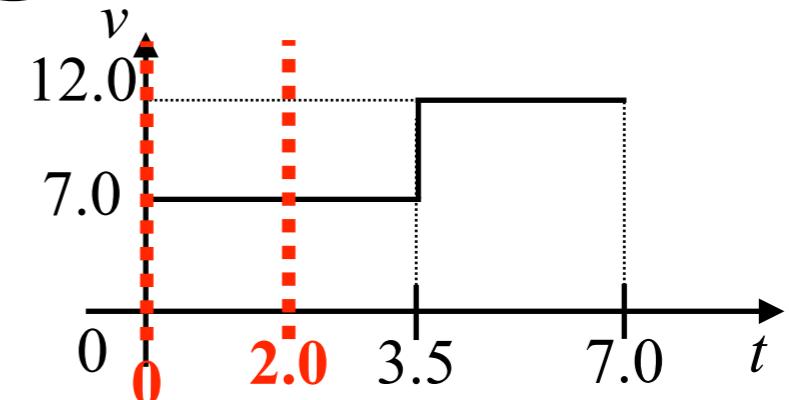
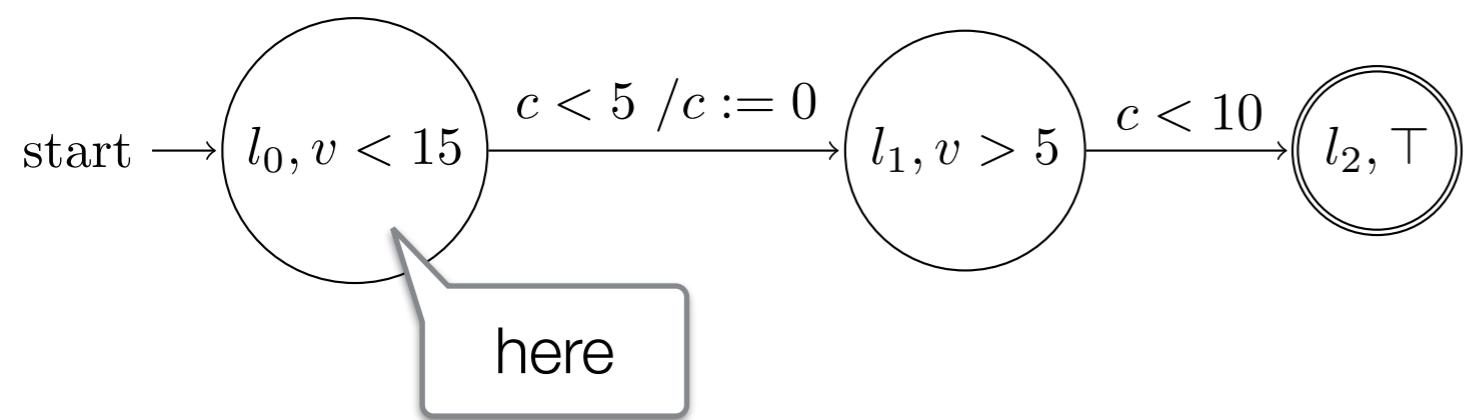


# One path in Weighted TTS



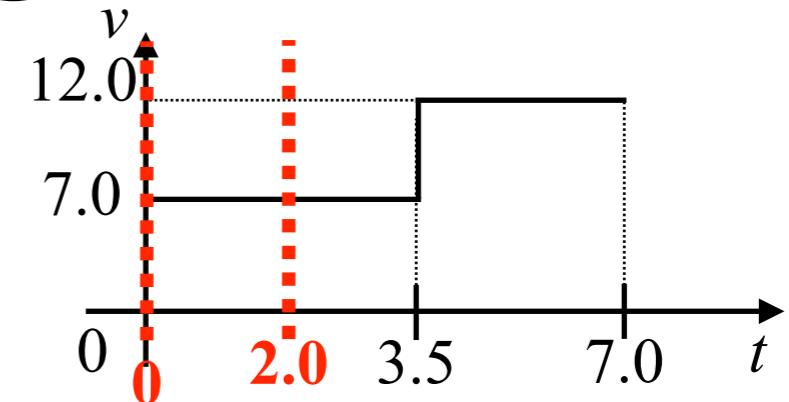
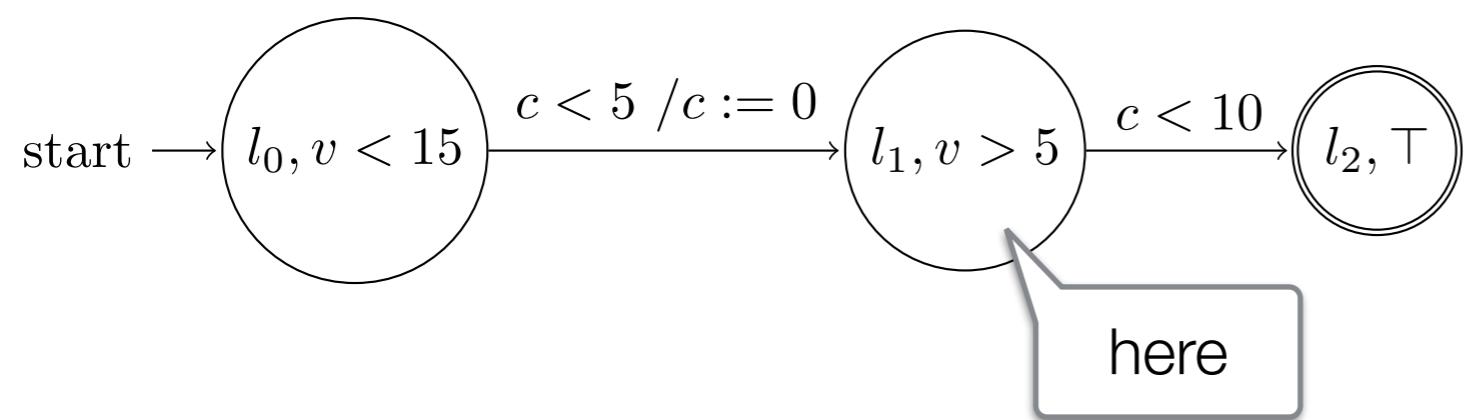
$\rightarrow(l_0, c=0, 0, \varepsilon)$

# One path in Weighted TTS



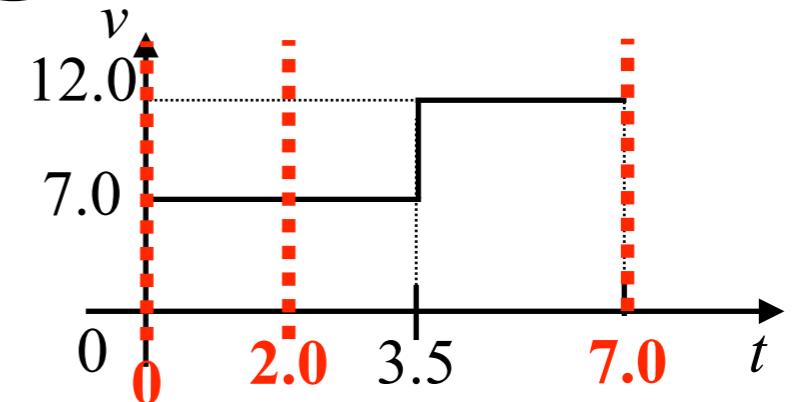
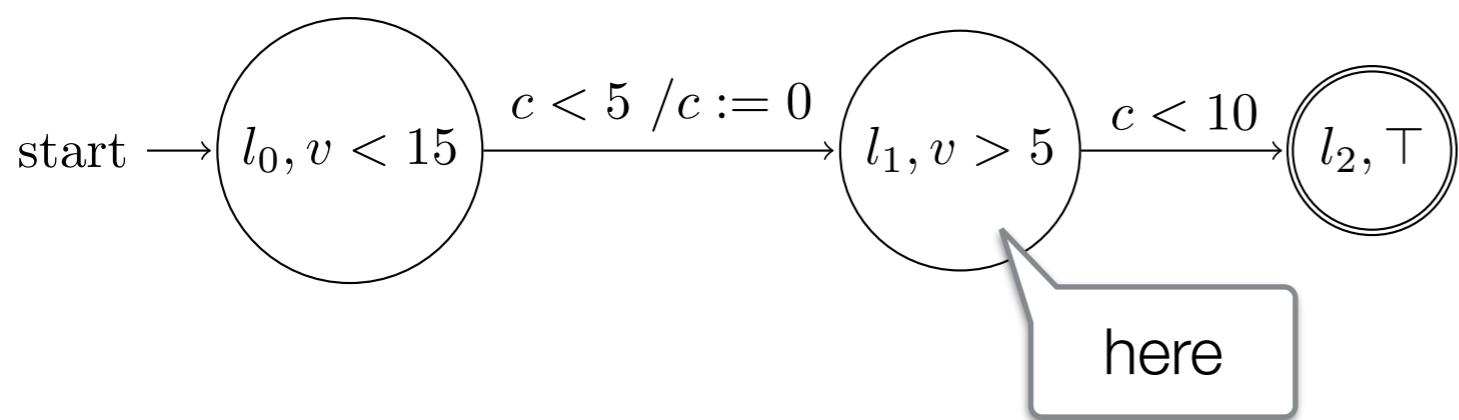
$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\})$

# One path in Weighted TTS



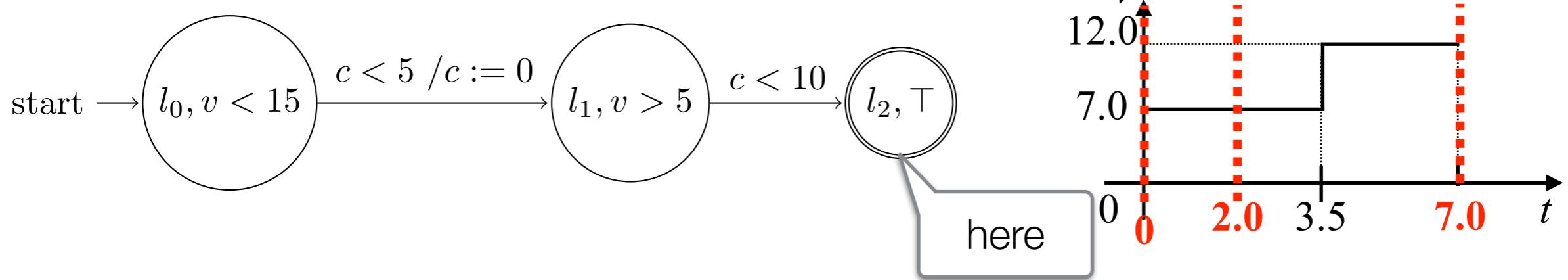
$$\xrightarrow{2.0} (l_0, c=0, 0, \varepsilon) \xrightarrow{\text{---}} (l_0, c=2, 2, \{v = 7\}) \xrightarrow[e]{\kappa(v < 15, \{v=7\})} (l_1, c=0, 2, \varepsilon)$$

# One path in Weighted TTS

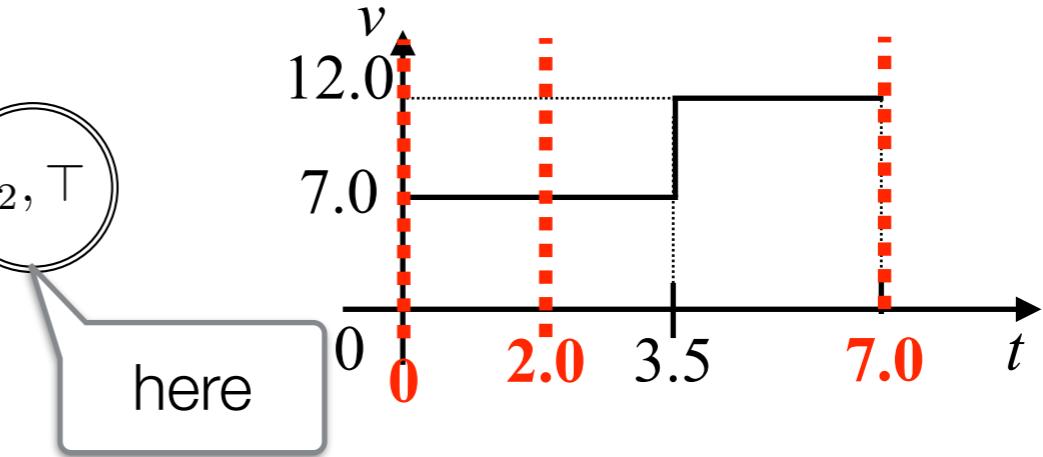
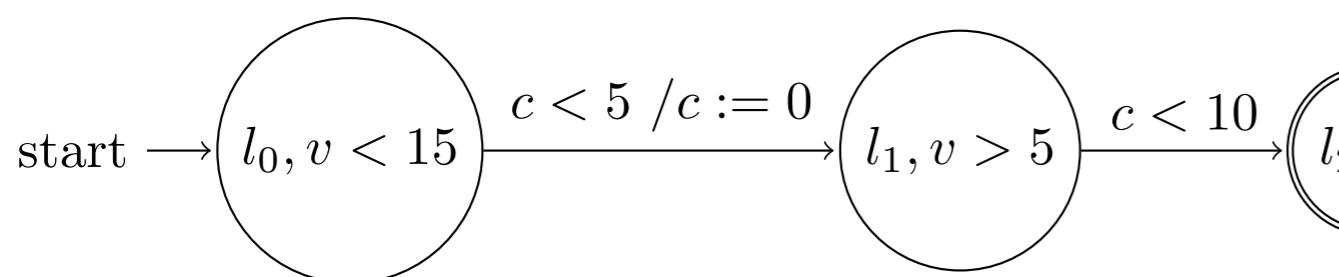


$$\begin{aligned}
 &\xrightarrow{2.0} (l_0, c=0, 0, \varepsilon) \xrightarrow{\text{---}} (l_0, c=2, 2, \{v = 7\}) \xrightarrow[e]{\kappa(v < 15, \{v=7\})} (l_1, c=0, 2, \varepsilon) \\
 &\xrightarrow[5.0]{\text{---}} (l_1, c=5, 7, \{v = 7\}\{v = 12\})
 \end{aligned}$$

# One path in Weighted TTS


 $\xrightarrow{2.0} (l_0, c=0, 0, \varepsilon) \xrightarrow{\kappa(v < 15, \{v=7\})} (l_0, c=2, 2, \{v = 7\}) \xrightarrow{e} (l_1, c=0, 2, \varepsilon)$ 
 $\xrightarrow{5.0} (l_1, c=5, 7, \{v = 7\} \{v = 12\}) \xrightarrow{\kappa(v > 5, \{v=7\} \{v=12\})} (l_2, c=5, 7, \varepsilon)$

# One path in Weighted TTS



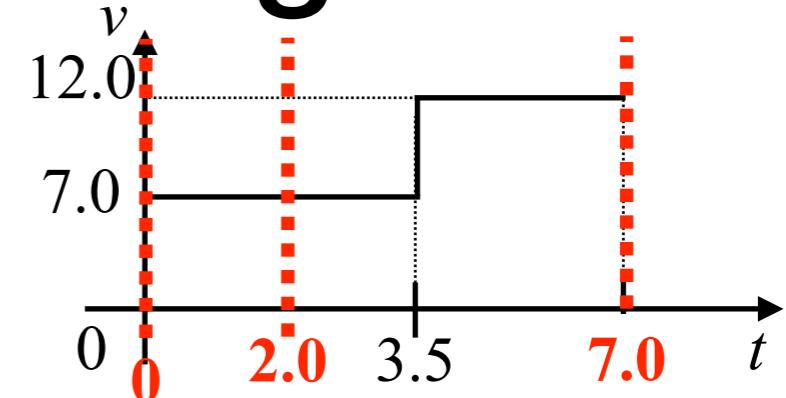
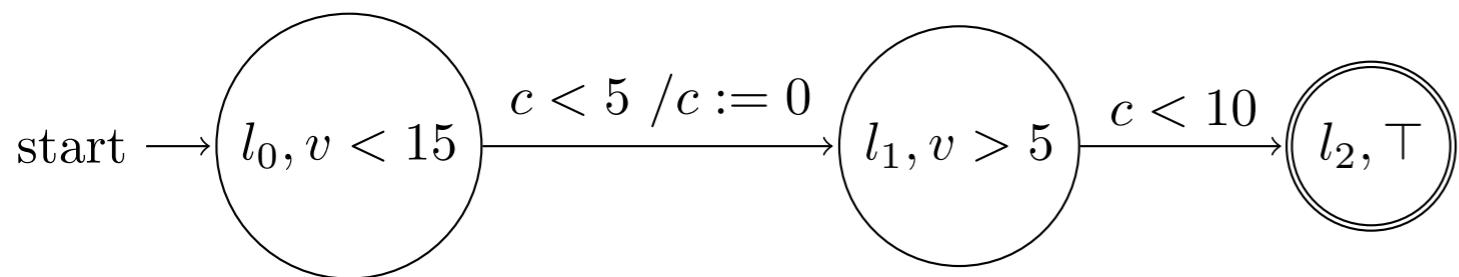
$$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\}) \xrightarrow[e]{\kappa(v < 15, \{v=7\})} (l_1, c=0, 2, \varepsilon)$$

$$\xrightarrow[5.0]{\quad\quad\quad} (l_1, c=5, 7, \{v = 7\} \{v = 12\}) \xrightarrow[\kappa(v > 5, \{v=7\} \{v=12\})]{\otimes \quad e} (l_2, c=5, 7, \varepsilon)$$

Boolean    sup-inf    tropical

$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	sup	inf
$\otimes$	$\wedge$	inf	+

# Accumulating paths in Weighted TTS



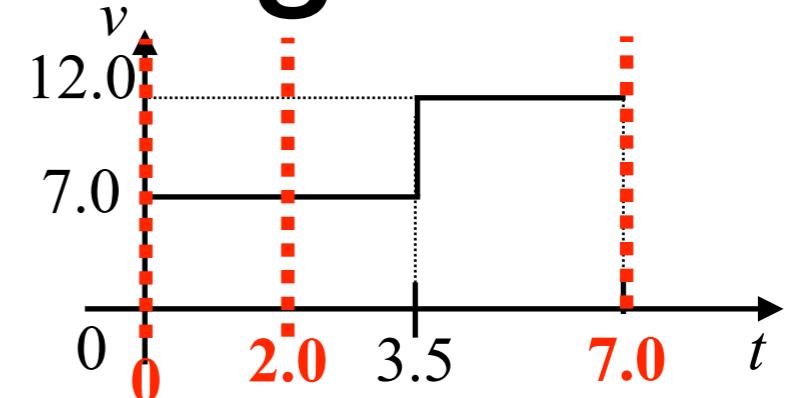
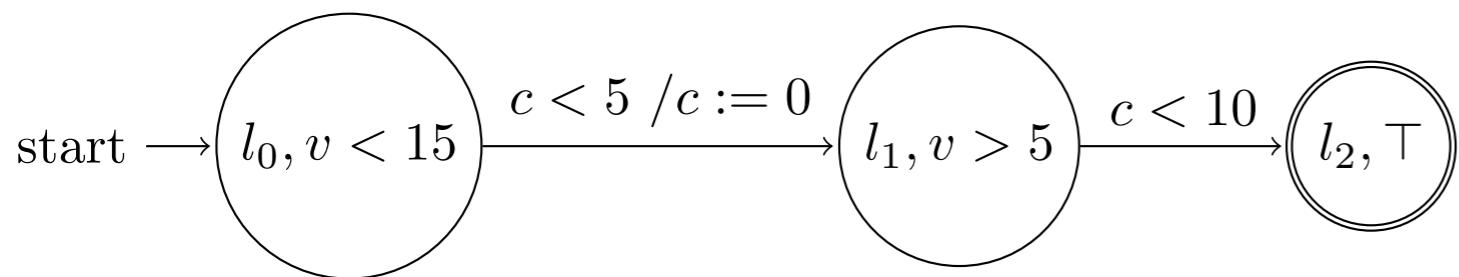
$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v = 7\}) \xrightarrow{e} (l_1, c=0, 3, \varepsilon) \xrightarrow{5.0} (l_1, c=5, 7, \{v = 7\}\{v = 12\}) \xrightarrow{e} (l_2, c=5, 7, \varepsilon)$   
 $\kappa(v < 15, \{v=7\}) \otimes \kappa(v > 5, \{v=7\}\{v=12\})$

$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{4.0} (l_0, c=4, 4, \{v = 7\}\{v = 12\}) \xrightarrow{e} (l_1, c=0, 4, \varepsilon) \xrightarrow{3.0} (l_1, c=3, 7, \{v = 12\}) \xrightarrow{e} (l_2, c=3, 7, \varepsilon)$   
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⋮

	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
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# Accumulating paths in Weighted TTS



$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{2.0} (l_0, c=2, 2, \{v=7\}) \xrightarrow{e} (l_1, c=0, 3, \varepsilon) \xrightarrow{5.0} (l_1, c=5, 7, \{v=7\}\{v=12\}) \xrightarrow{e} (l_2, c=5, 7, \varepsilon)$

$\kappa(v < 15, \{v=7\}) \otimes \kappa(v > 5, \{v=7\}\{v=12\})$



$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{4.0} (l_0, c=4, 4, \{v=7\}\{v=12\}) \xrightarrow{e} (l_1, c=0, 4, \varepsilon) \xrightarrow{3.0} (l_1, c=3, 7, \{v=12\}) \xrightarrow{e} (l_2, c=3, 7, \varepsilon)$

$\kappa(v < 15, \{v=7\}\{v=12\}) \otimes \kappa(v > 5, \{v=12\})$

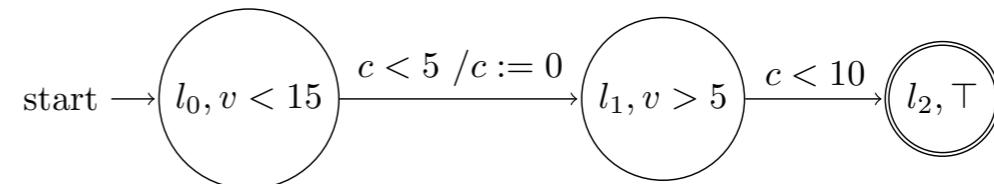


$\vdots$

	Boolean	sup-inf	tropical
$\oplus$	$S$	$\{\text{True/False}\}$	$\mathbb{R} \cup \{\pm\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

# Timed symbolic weighted automata (TSWA)

TSA: the automata structure



Weight function ( $\kappa$ ): the one-step semantics  
(weight on each transition)

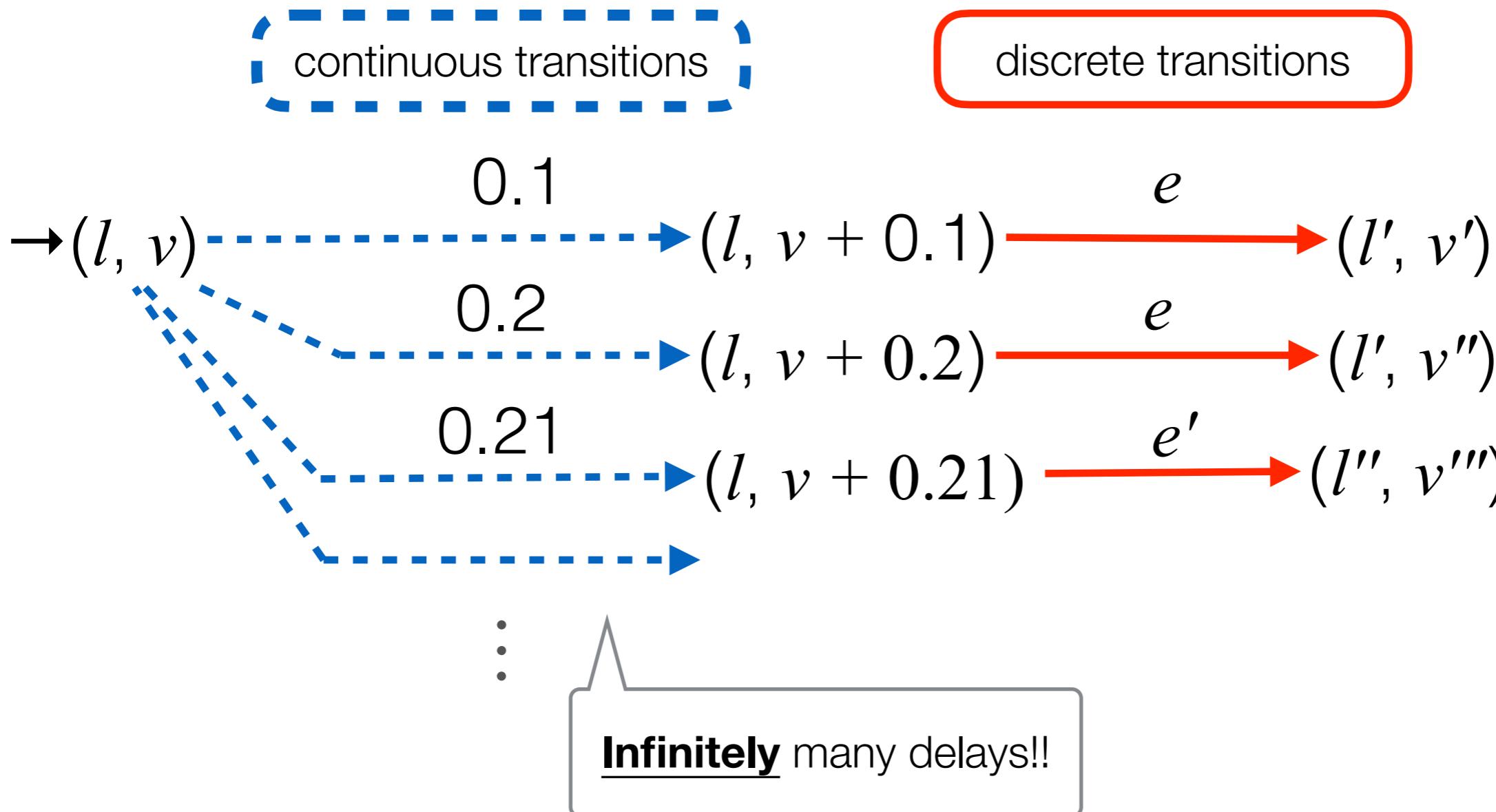
Semiring operations ( $\otimes, +$ ): how to accumulate weights

One-step semantics  $\rightarrow$  semantics for a path/TSWA

# Outline

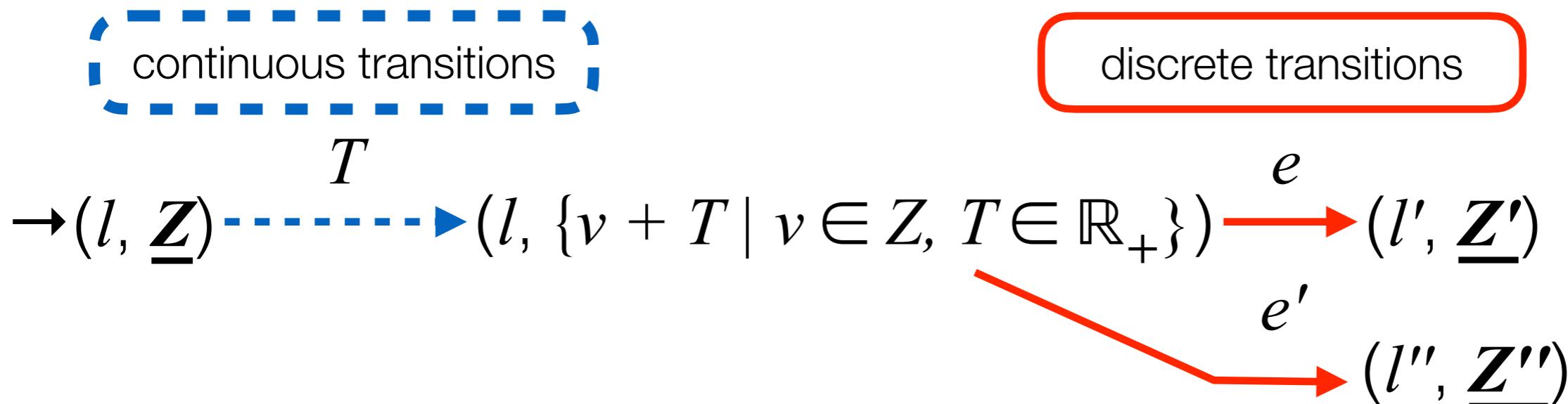
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  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
    - Quantitative monitoring/timed pattern matching algorithm
      - Idea: zone construction with weight
  - Experiments

# Review: Reachability by zones



Ininitely many reachable states!!  $\rightarrow$  symbolic analysis by zones

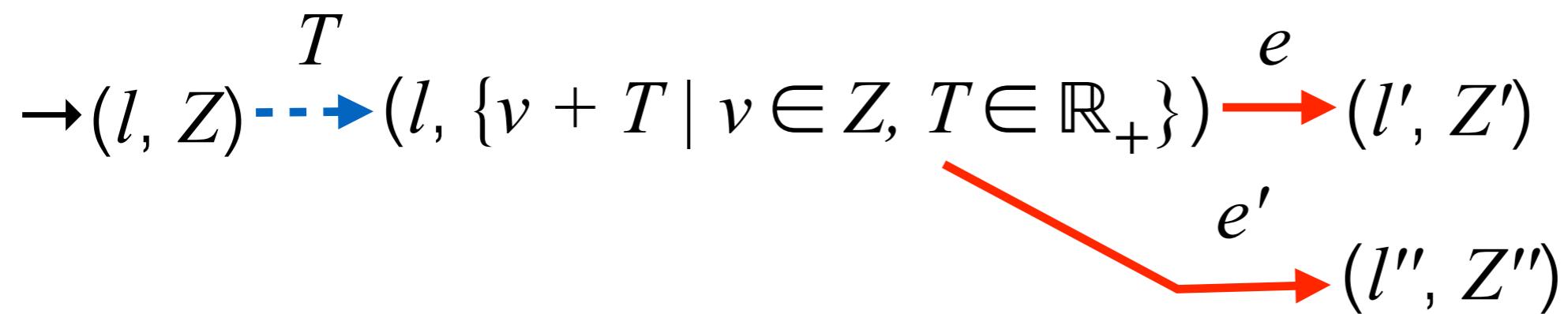
# Review: Reachability by zones



**Ininitely** many reachable states!!  $\rightarrow$  symbolic analysis by **zones**

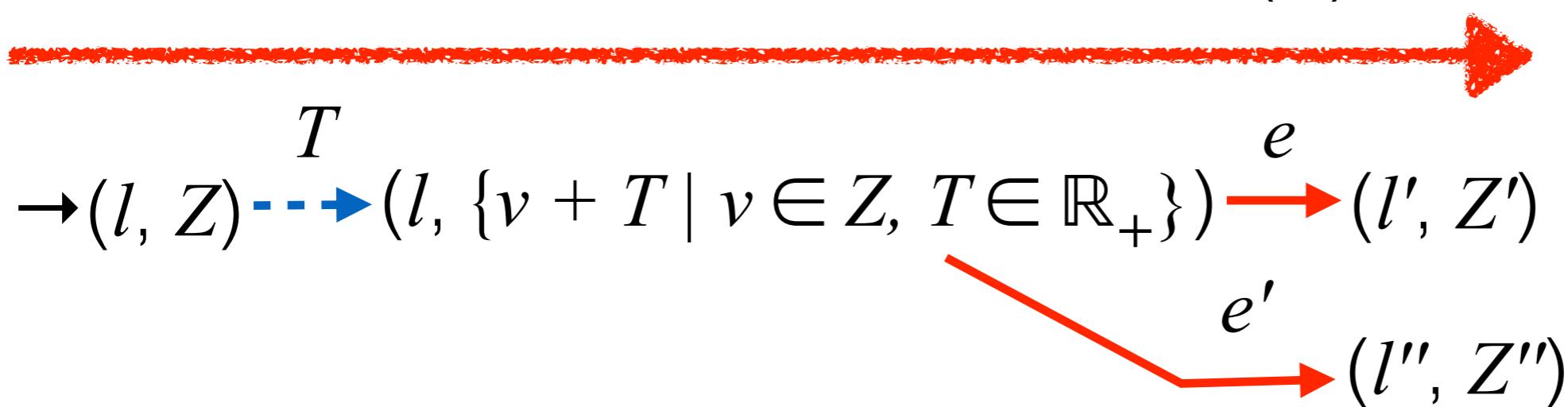
**Observation:** reachability  
is shortest distance over  
Boolean semiring!

**Observation:** reachability is shortest distance over Boolean semiring!



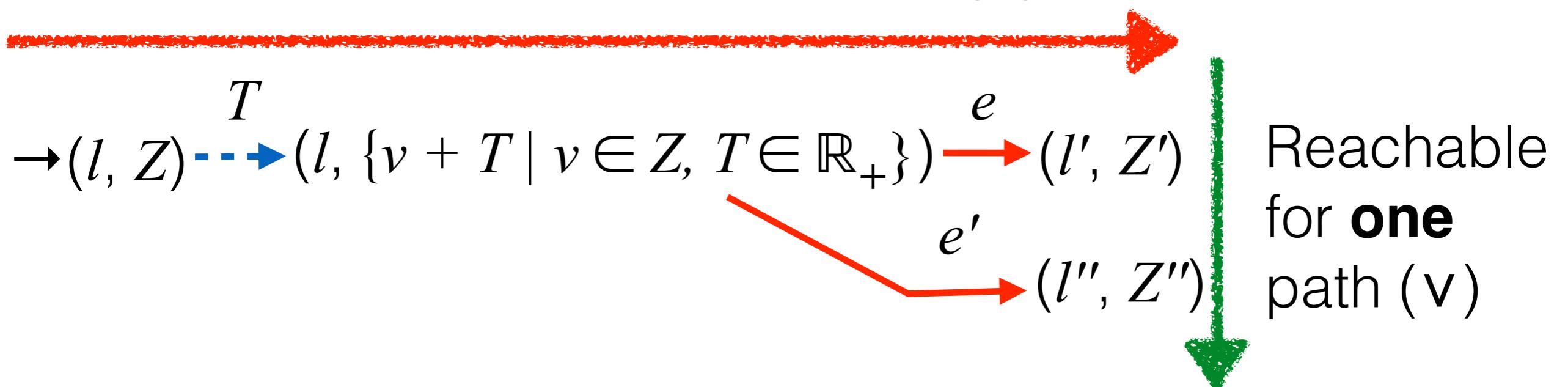
# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )

$$\xrightarrow{T} \rightarrow(l, Z)$$

$\cdots$

$$\xrightarrow{\{v + T \mid v \in Z, T \in \mathbb{R}_+\}} (l, \{v + T \mid v \in Z, T \in \mathbb{R}_+\})$$

$$\xrightarrow{e} (l', Z') \quad \xrightarrow{e'} (l'', Z'')$$

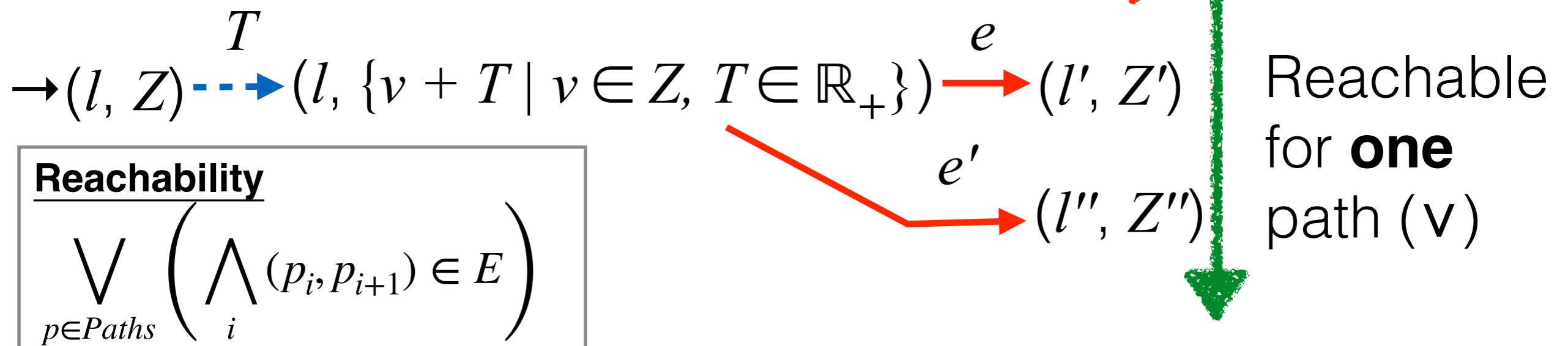
Reachable  
for **one**  
path ( $v$ )

Reachability

$$\bigvee_{p \in Paths} \left( \bigwedge_i (p_i, p_{i+1}) \in E \right)$$

# Observation: reachability is shortest distance over Boolean semiring!

Reachable at **all** the transitions ( $\wedge$ )



## Shortest Distance (for semiring)

$$\bigoplus_{p \in \text{Paths}} \left( \bigotimes_i w(p_i, p_{i+1}) \right)$$

	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

M. Waga (NII)

# Observation: reachability is shortest distance over Boolean semiring!

## Qualitative

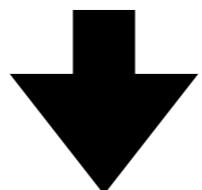
[Bakhirkin+, FORMATS'18]

TA  $\xrightarrow{\text{zone constr.}}$

zone graph  
(with Boolean weight)

reachability checking  
(Boolean shortest distance)

reachability  
(qualitative semantics)



## Quantitative [Contribution]

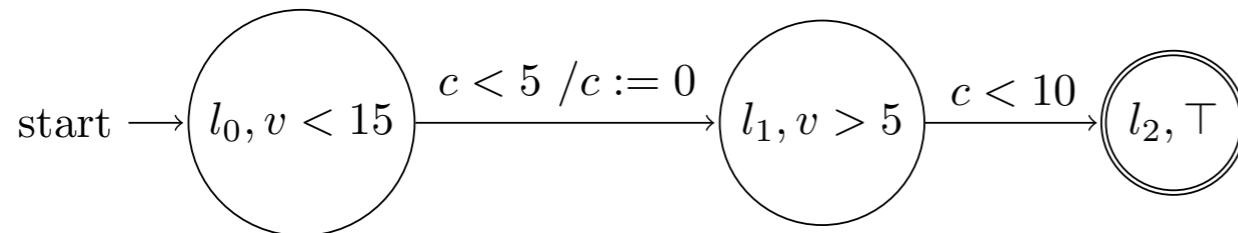
TSWA  $\xrightarrow{\text{zone constr.}}$

zone graph  
with semiring weight

semiring  
shortest distance

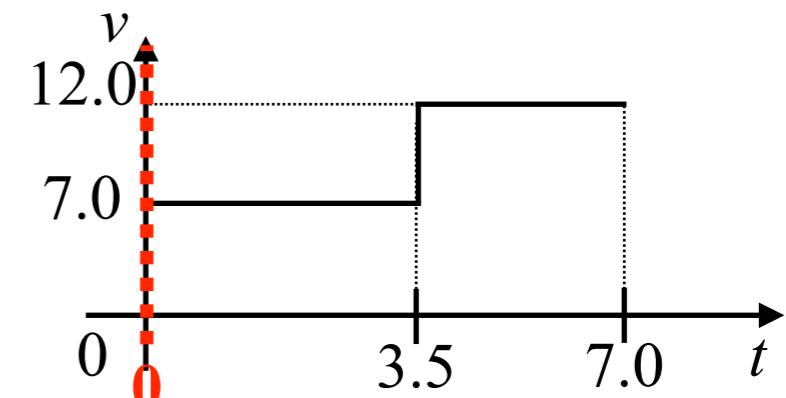
quantitative  
semantics

# Zone construction with weight



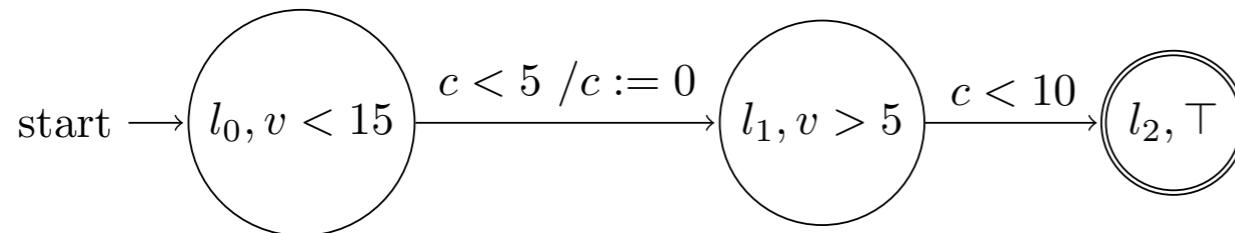
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



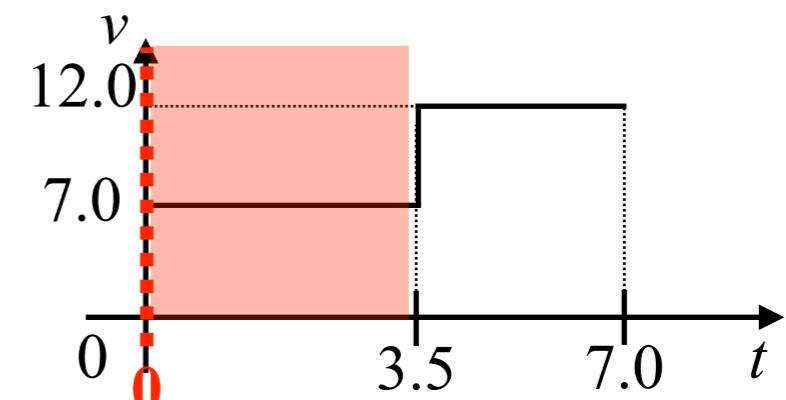
$\rightarrow (l_0, c = T = 0, \varepsilon)$

# Zone construction with weight



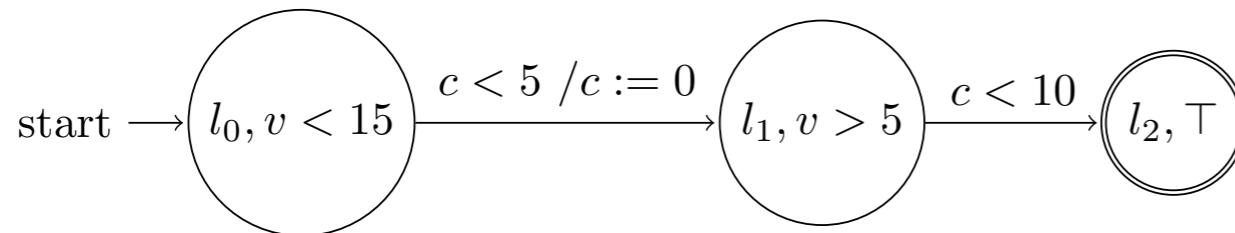
- $T$ : absolute time
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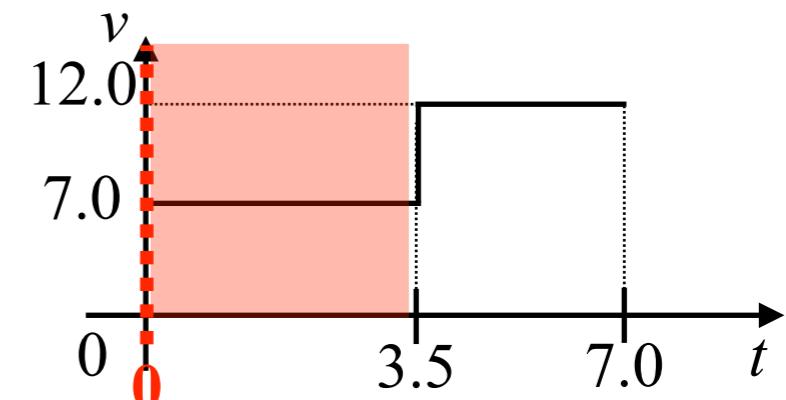
$\rightarrow (l_0, c = T = 0, \varepsilon) \xrightarrow{\text{-----}} (l_0, 0 < c = T < 3.5, \{v = 7\})$

# Zone construction with weight



- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

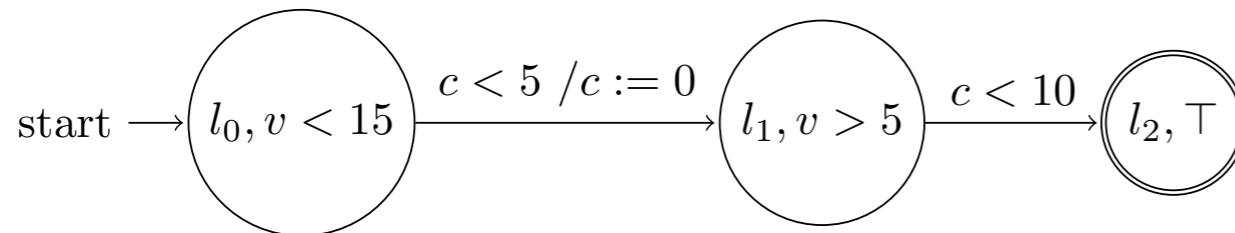


$$\rightarrow (l_0, c = T = 0, \varepsilon) \xrightarrow{\text{-----}} (l_0, 0 < c = T < 3.5, \{v = 7\})$$

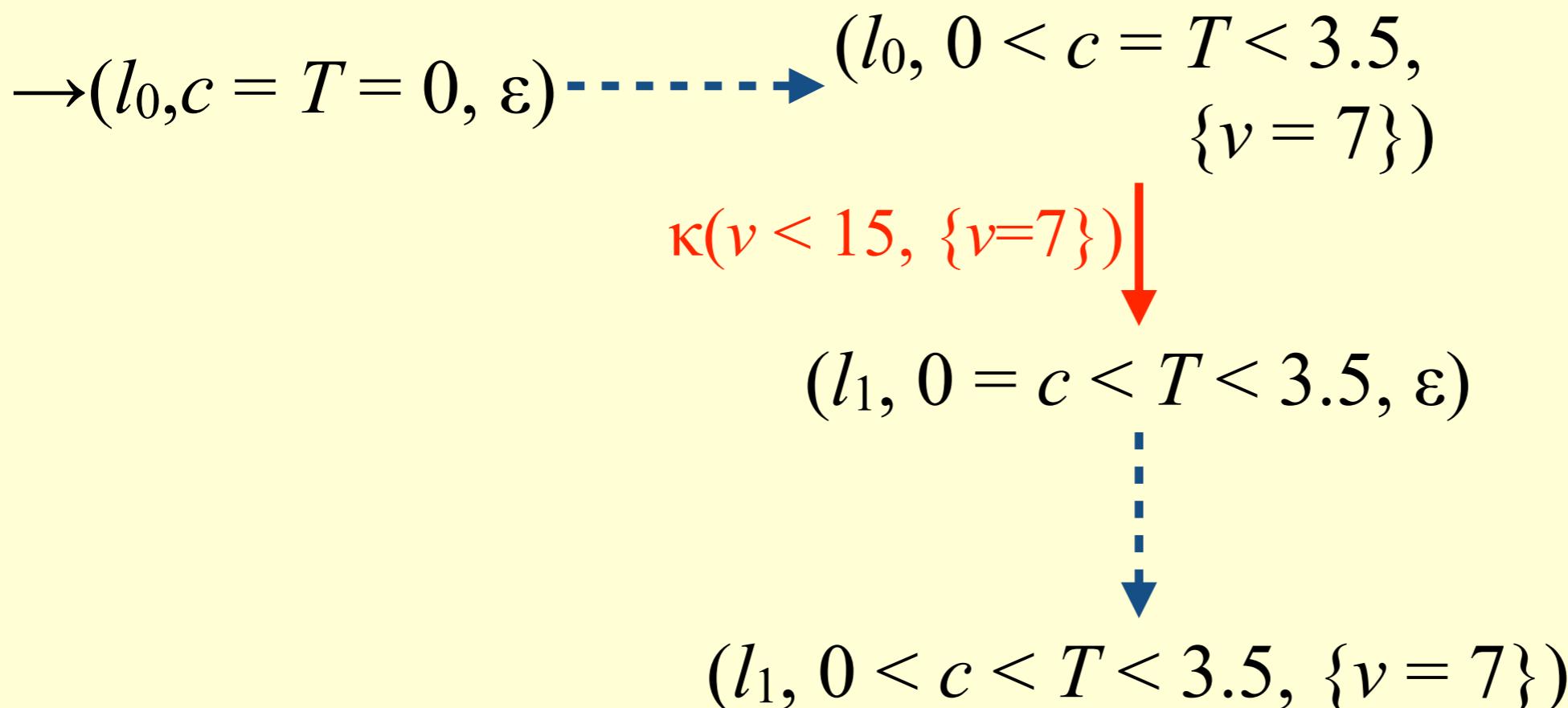
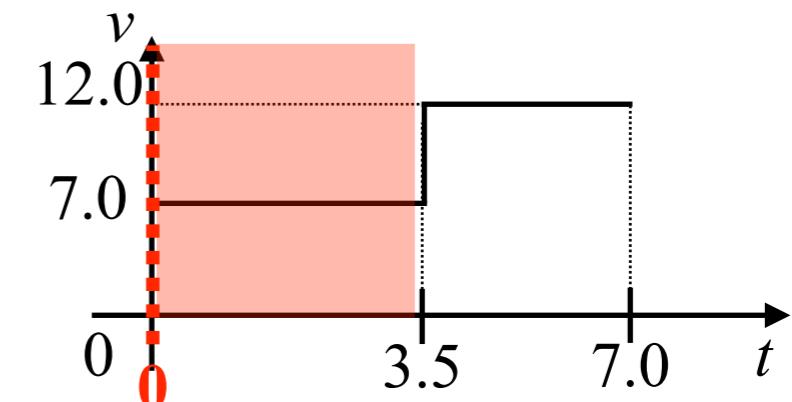
$\kappa(v < 15, \{v=7\})$

$$\downarrow$$
$$(l_1, 0 = c < T < 3.5, \varepsilon)$$

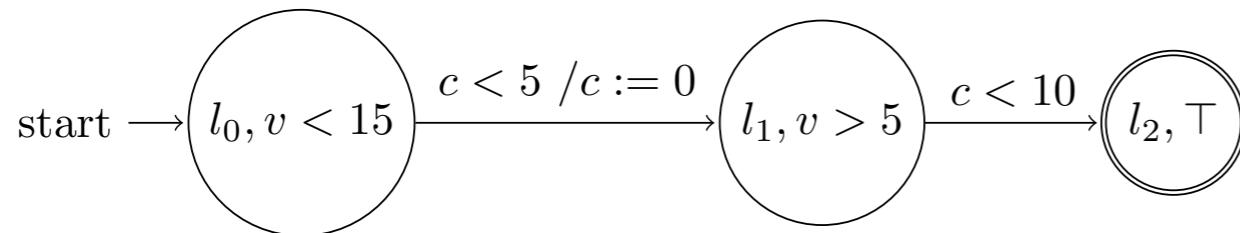
# Zone construction with weight



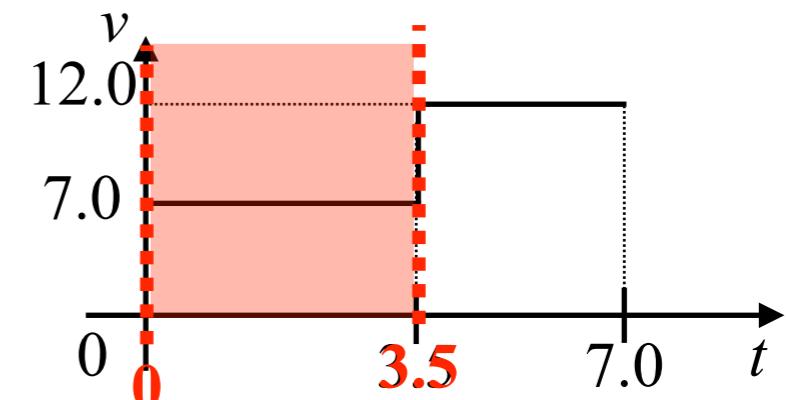
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



# Zone construction with weight



- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



$\rightarrow (l_0, c = T = 0, \varepsilon) \xrightarrow{\quad} (l_0, 0 < c = T < 3.5,$   
 $\quad \quad \quad \{v = 7\})$

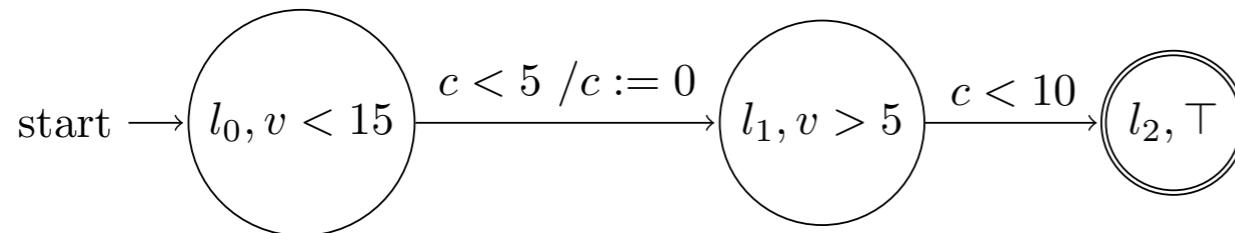
$\kappa(v < 15, \{v=7\})$

$(l_1, 0 = c < T < 3.5, \varepsilon)$



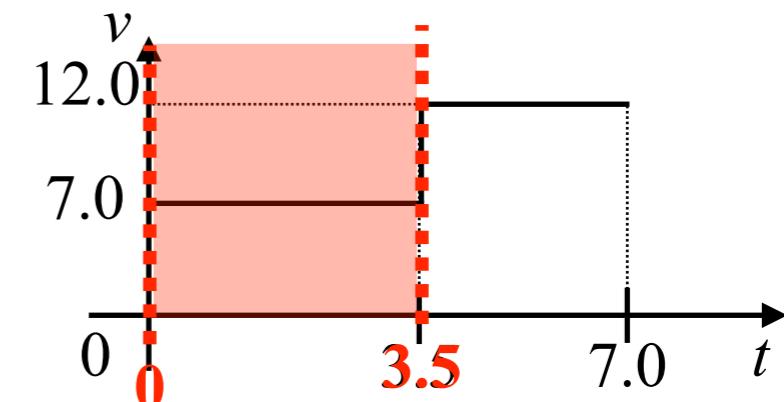
$(l_1, 0 < c < T < 3.5, \{v = 7\})$

# Zone construction with weight



- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring



$$\rightarrow (l_0, c = T = 0, \varepsilon) \xrightarrow{\quad} (l_0, 0 < c = T < 3.5, \{v = 7\}) \xrightarrow{\quad} (l_0, c = T = 3.5, \{v = 7\})$$

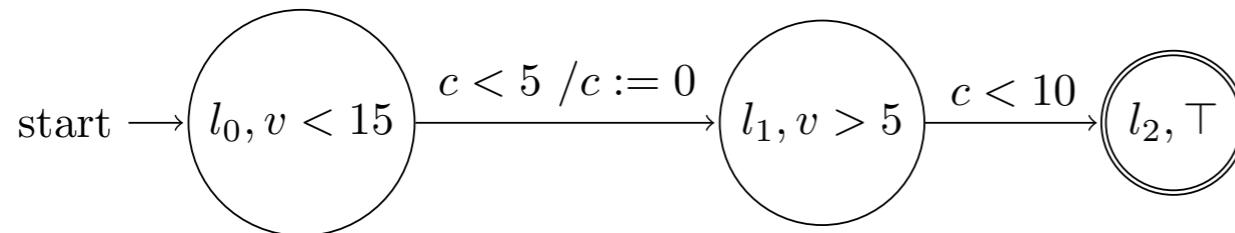
$\kappa(v < 15, \{v=7\})$

$(l_1, 0 = c < T < 3.5, \varepsilon)$



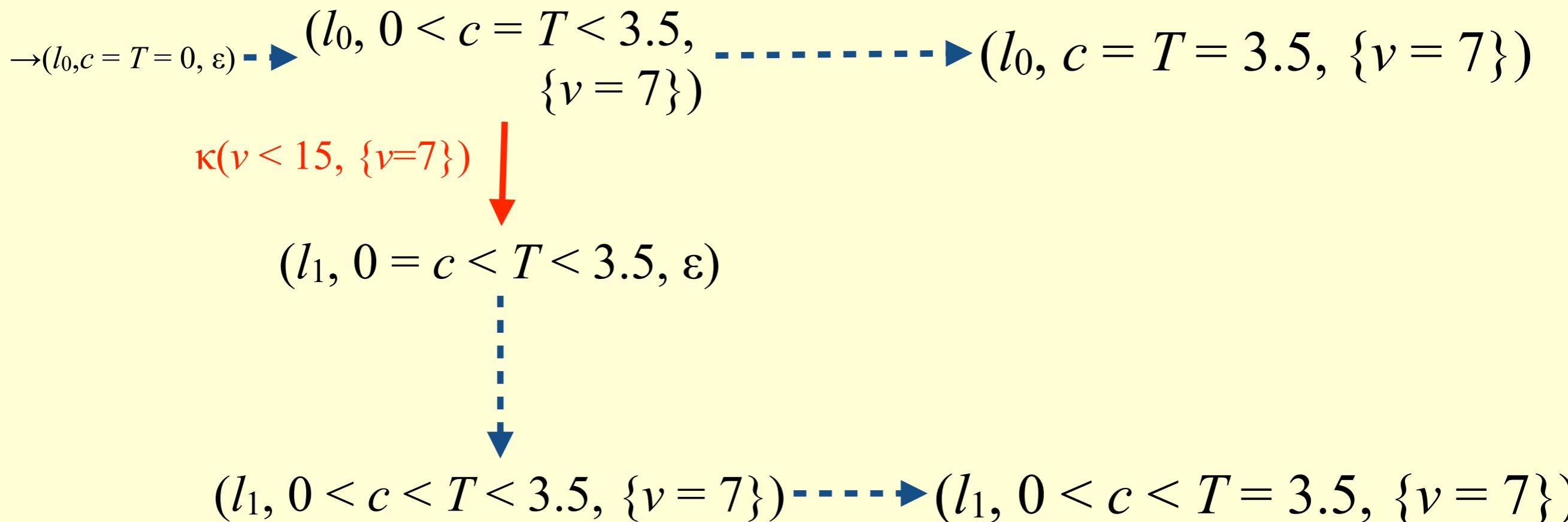
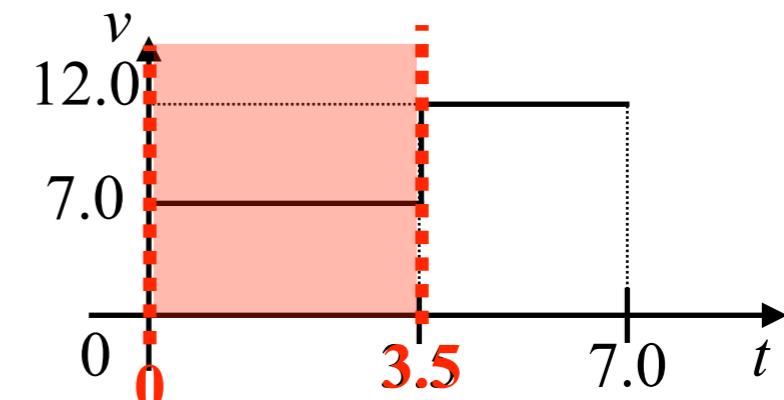
$(l_1, 0 < c < T < 3.5, \{v = 7\})$

# Zone construction with weight

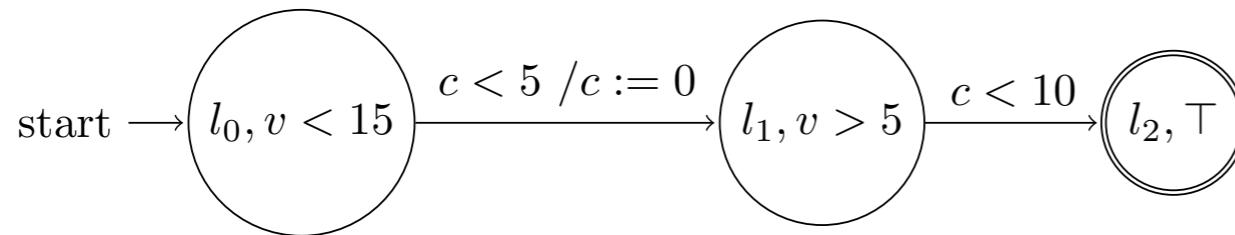


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

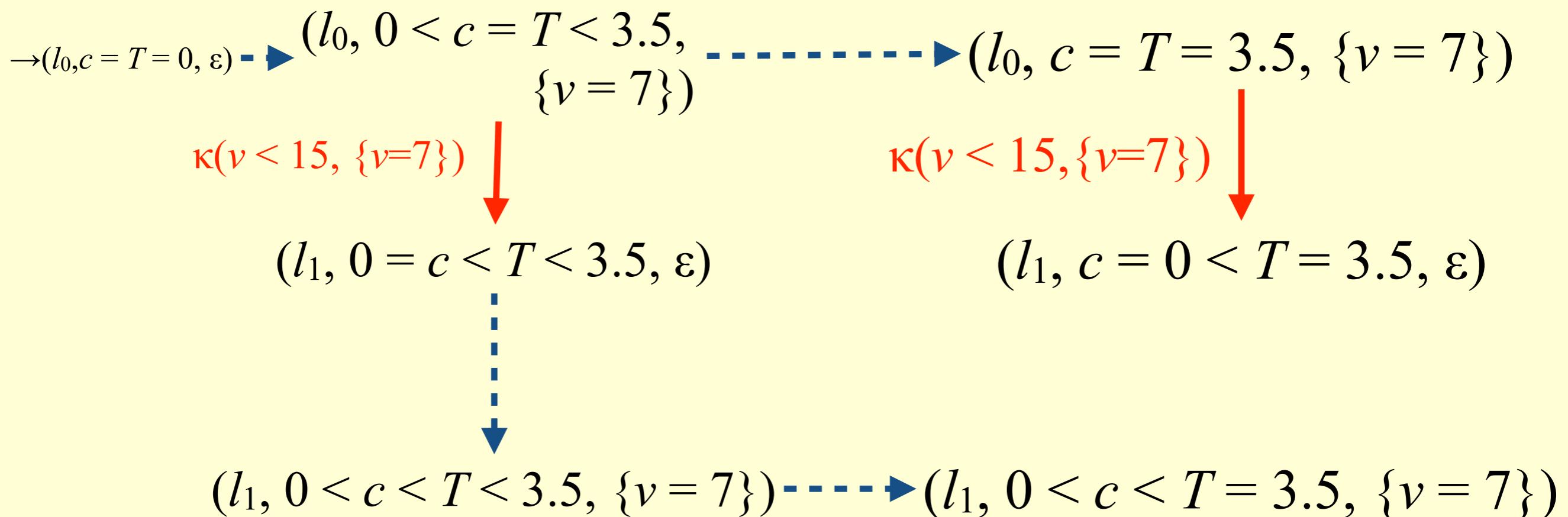
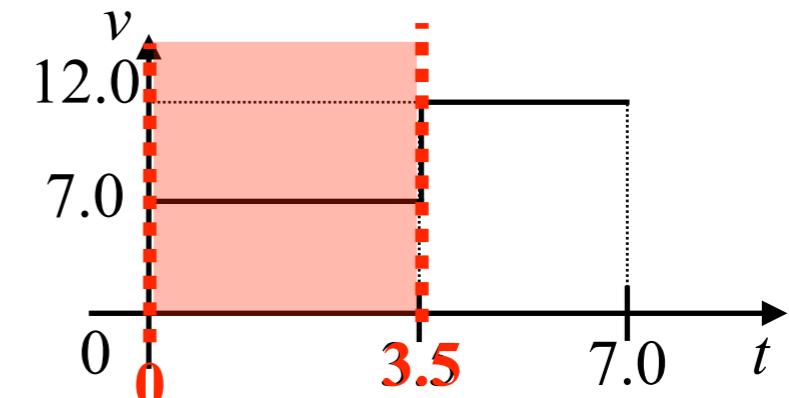


# Zone construction with weight

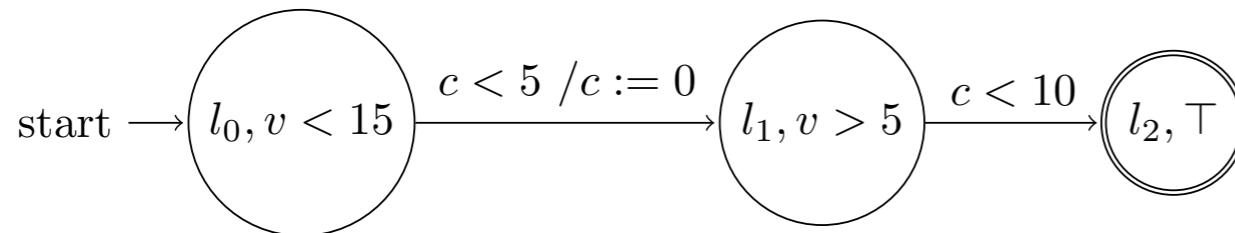


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

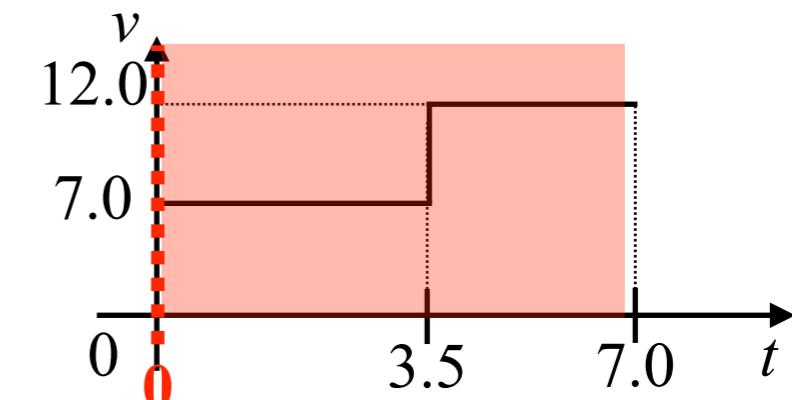
This is OK for monitoring



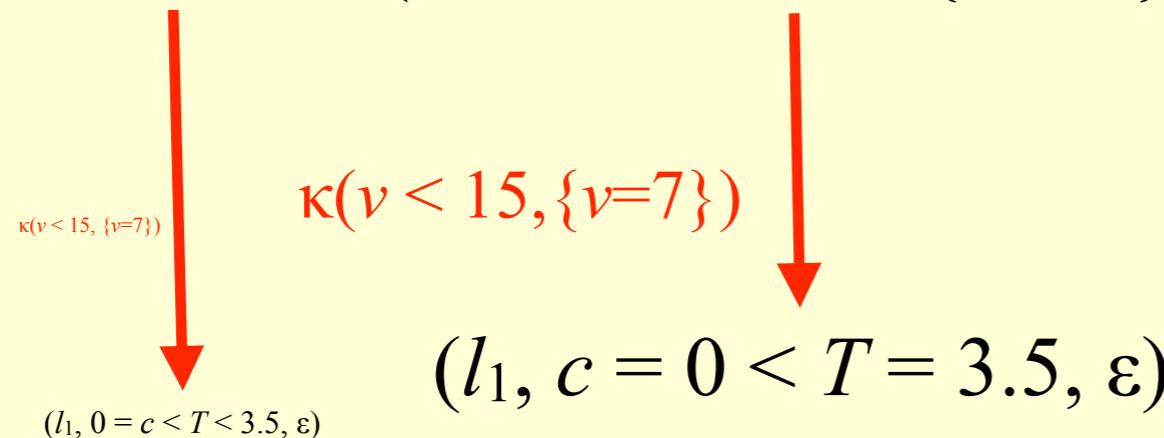
# Zone construction with weight



- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

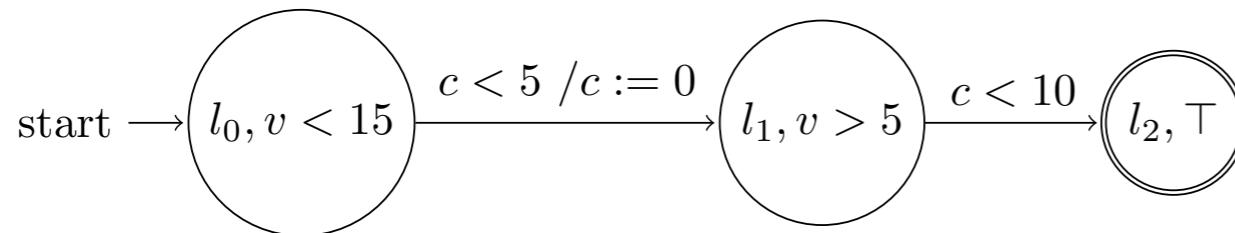


$\rightarrow (l_0, c = T = 0, \varepsilon) \xrightarrow{(l_0, 0 < c = T < 3.5, \{v = 7\})} (l_0, c = T = 3.5, \{v = 7\})$

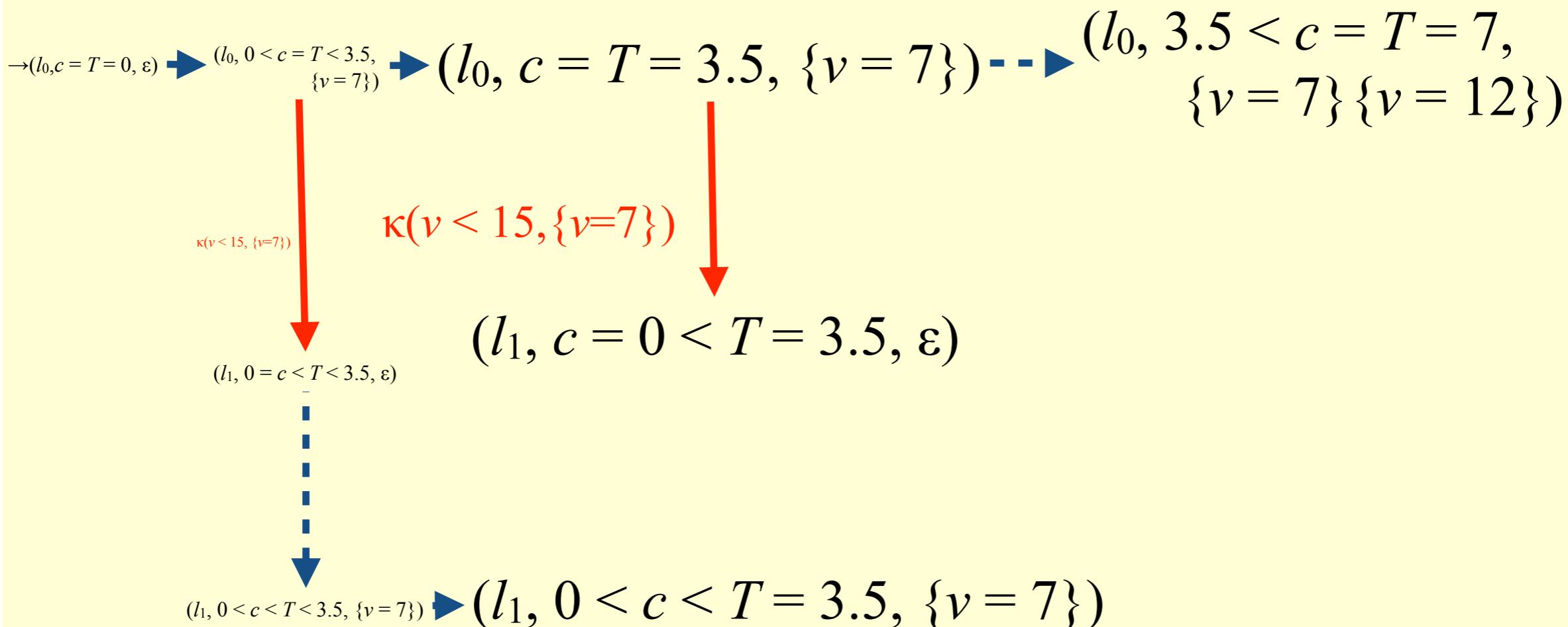
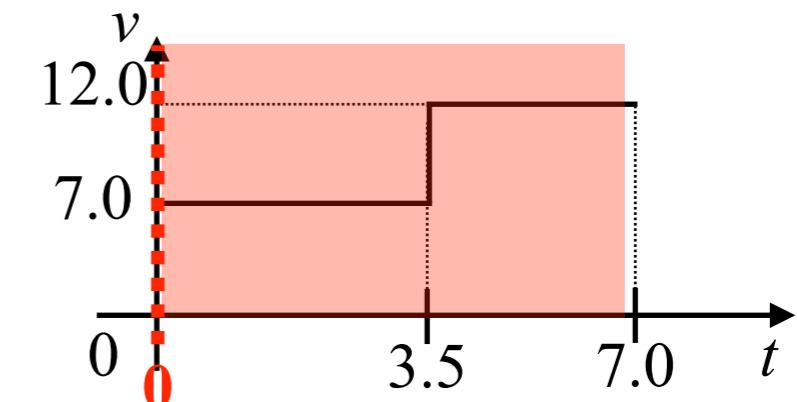


$\rightarrow (l_1, 0 < c < T < 3.5, \{v = 7\}) \xrightarrow{} (l_1, 0 < c < T = 3.5, \{v = 7\})$

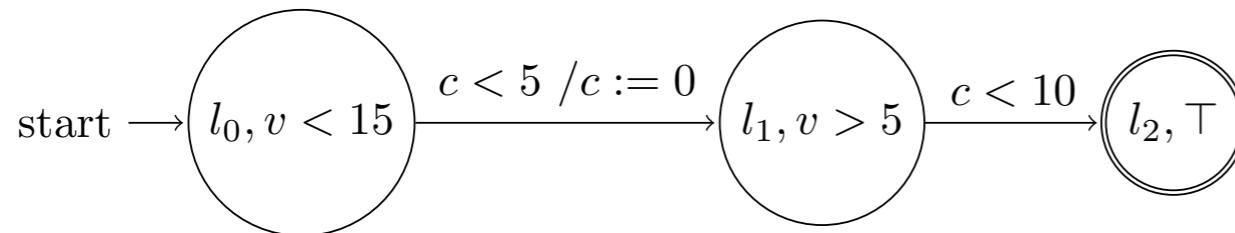
# Zone construction with weight



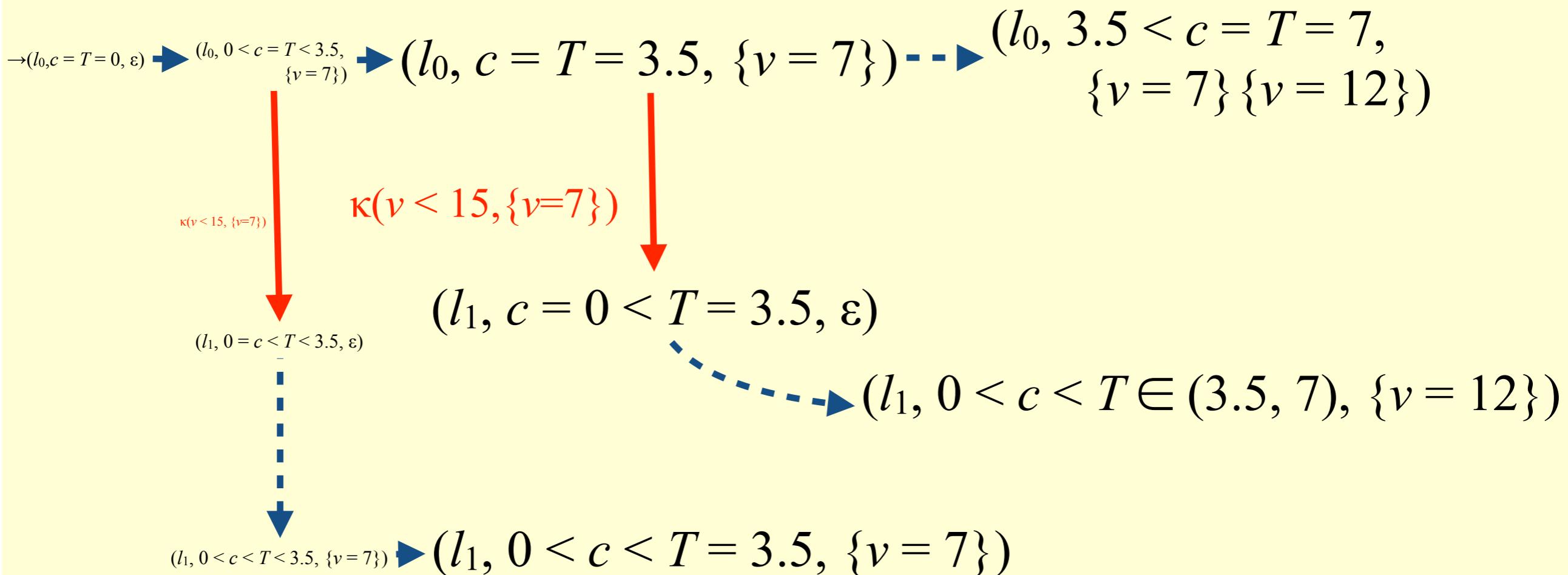
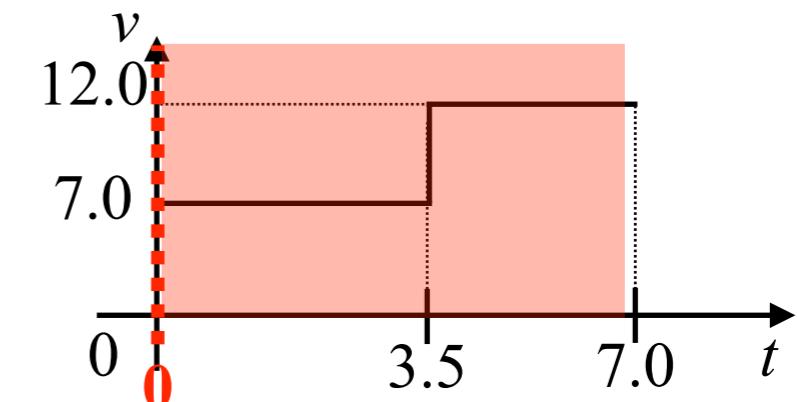
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



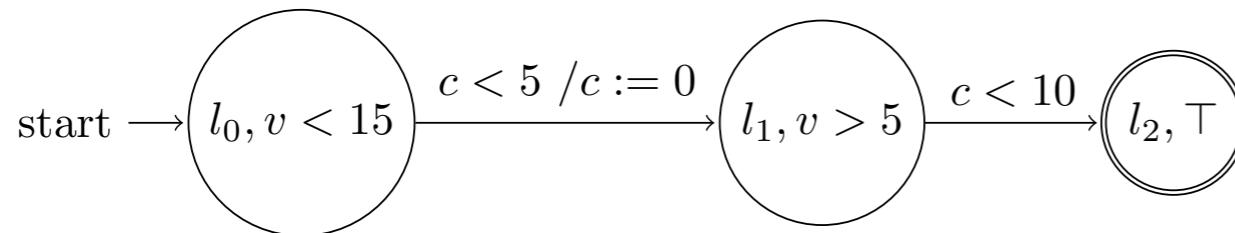
# Zone construction with weight



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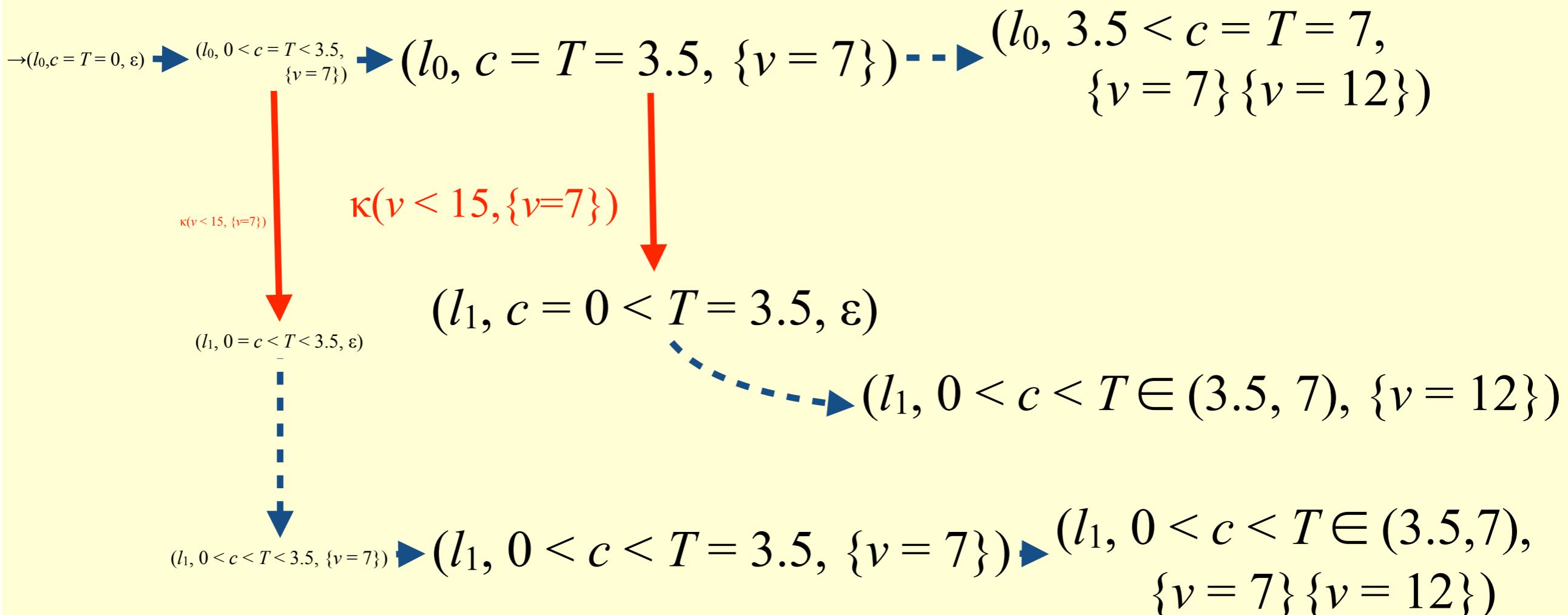
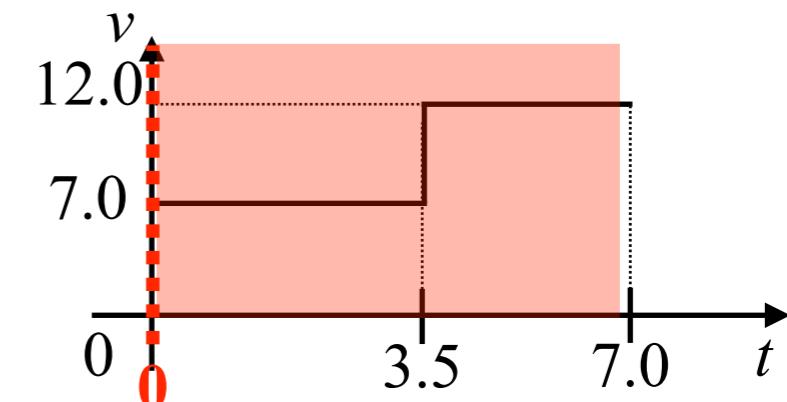


# Zone construction with weight

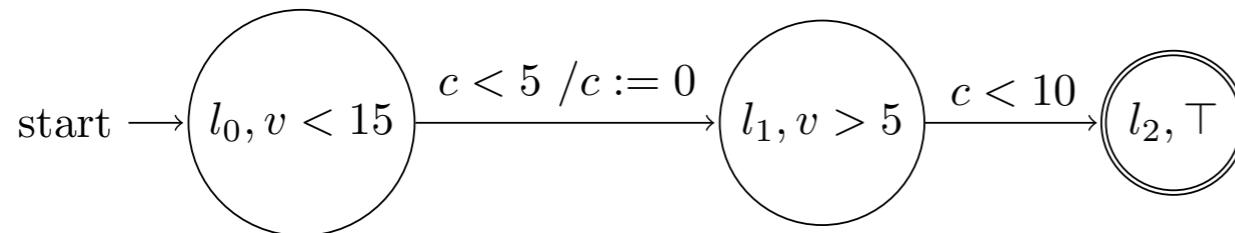


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

This is OK for monitoring

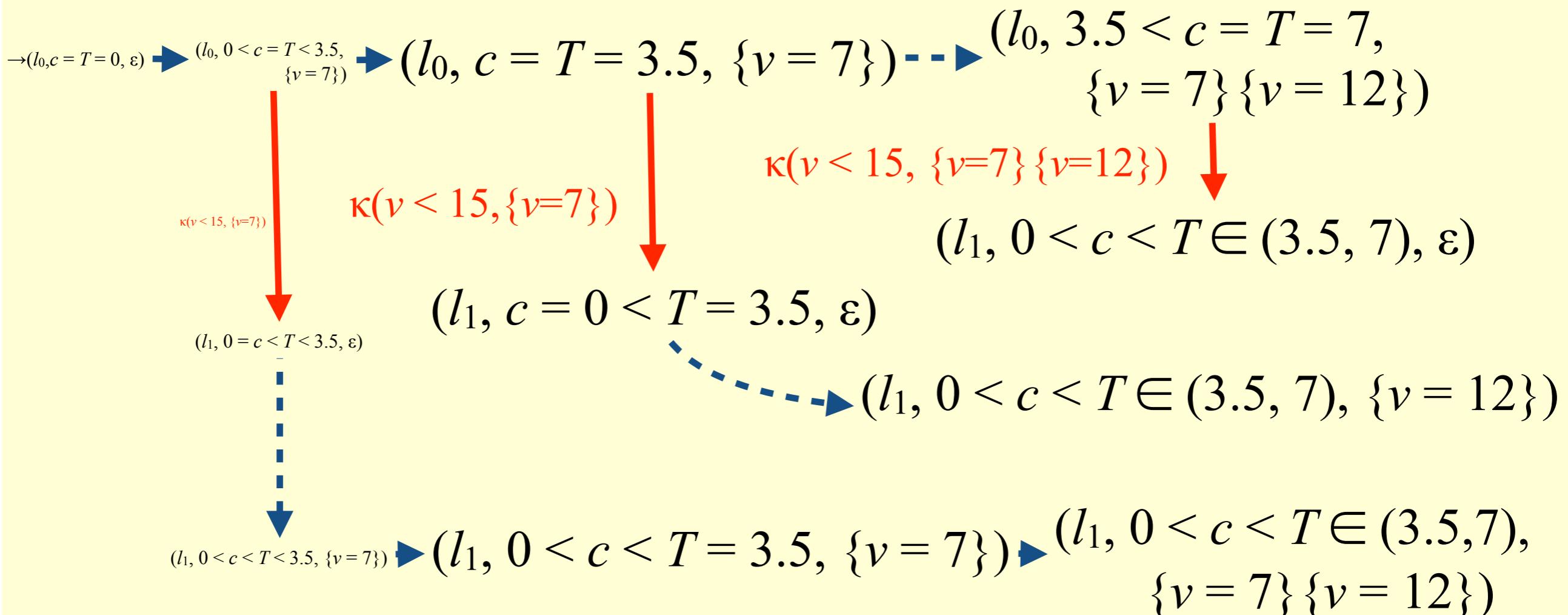
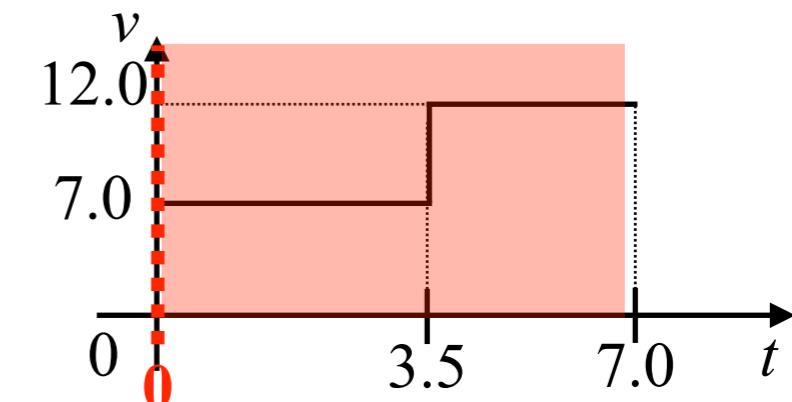


# Zone construction with weight

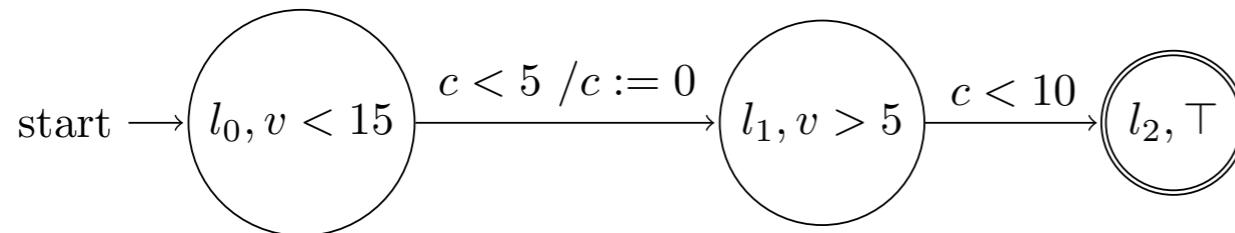


- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$

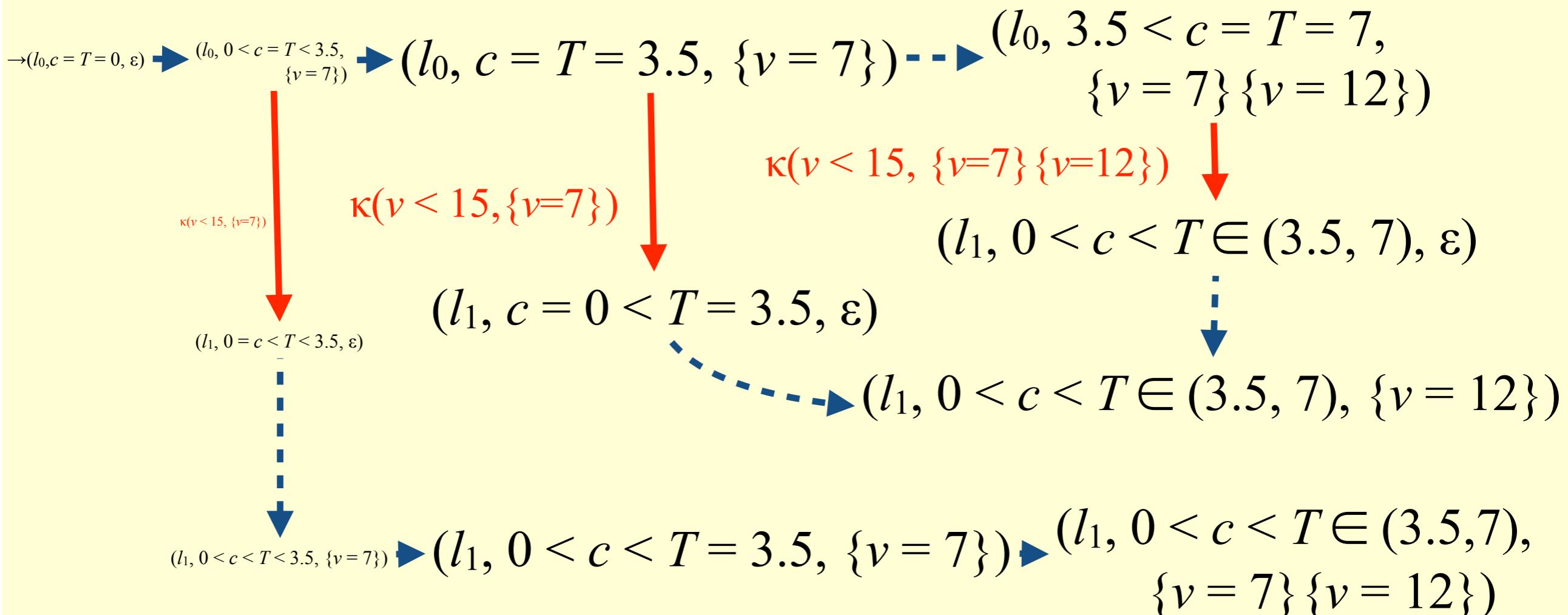
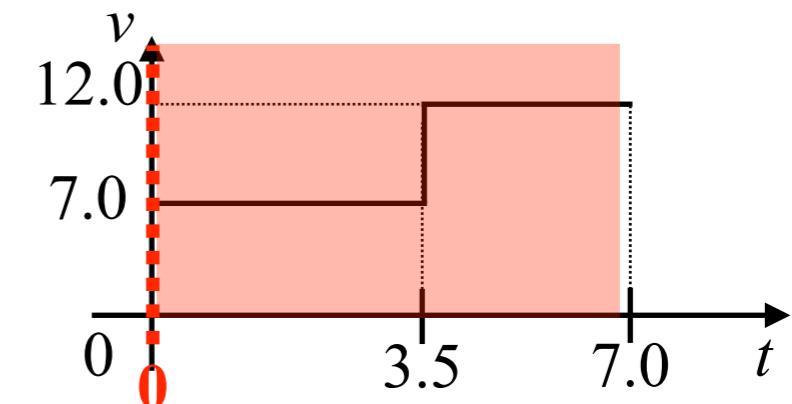
This is OK for monitoring



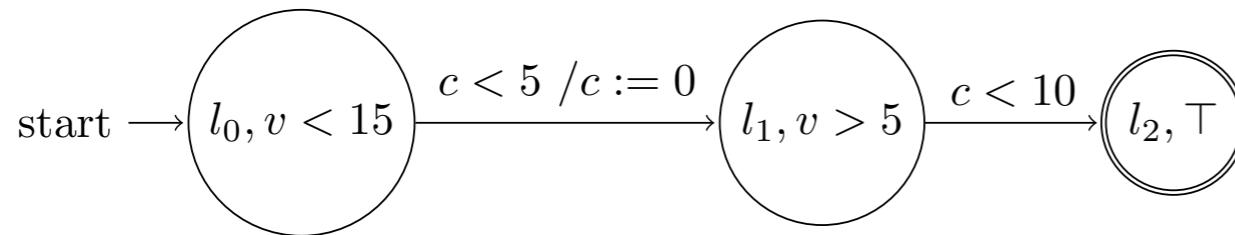
# Zone construction with weight



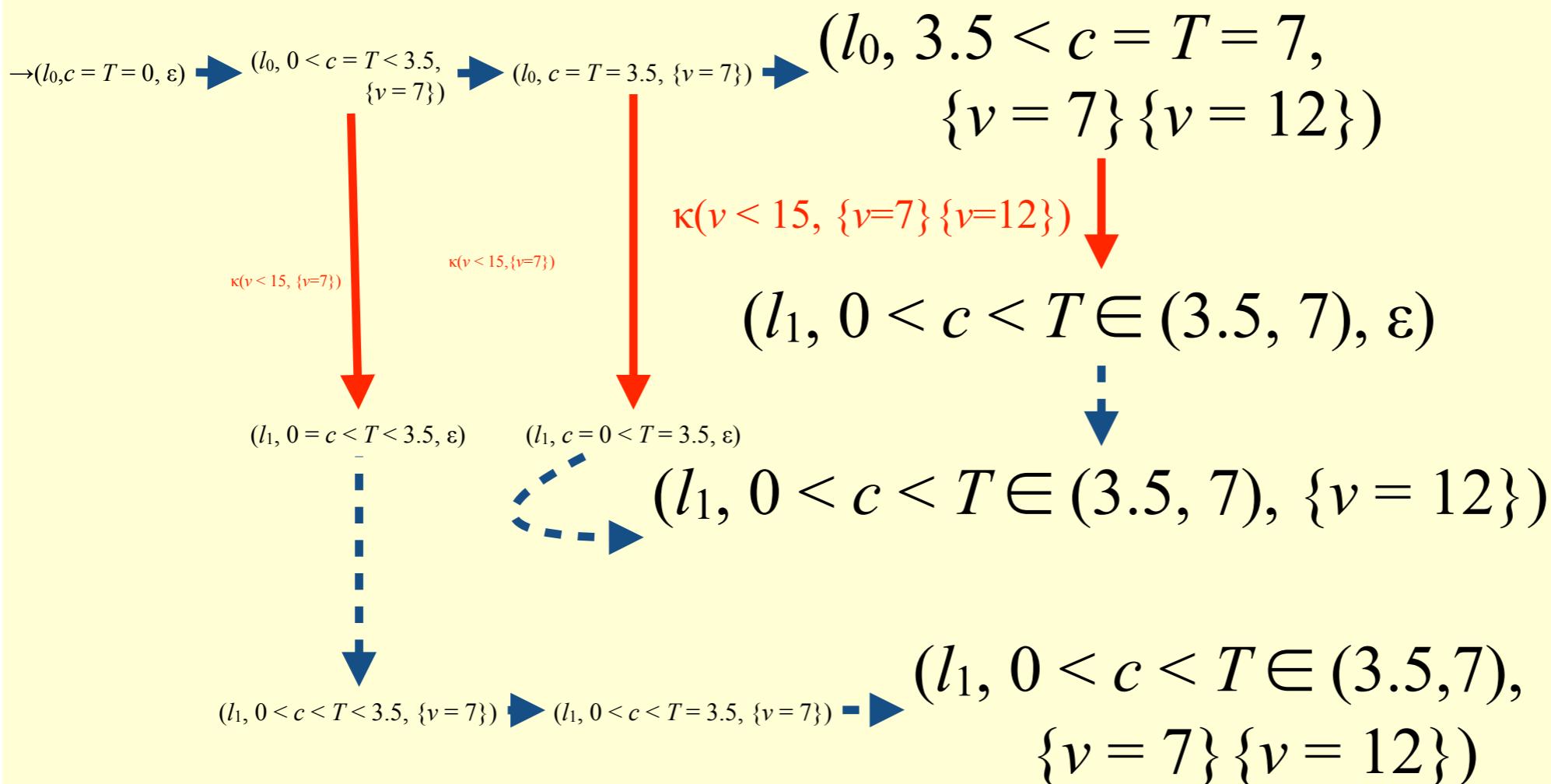
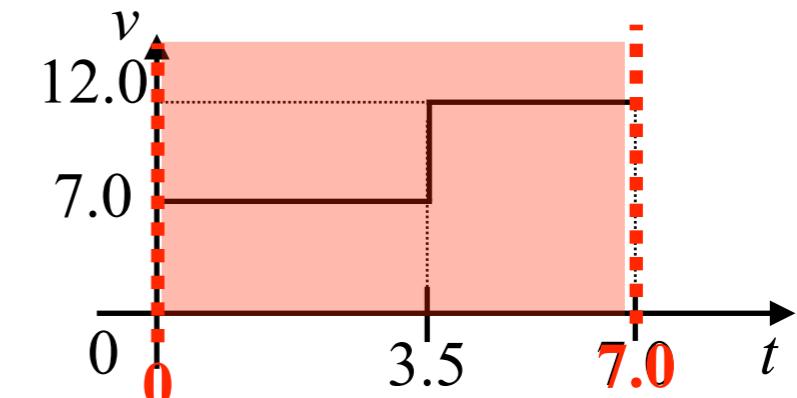
- $T$ : absolute time This is OK for monitoring
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



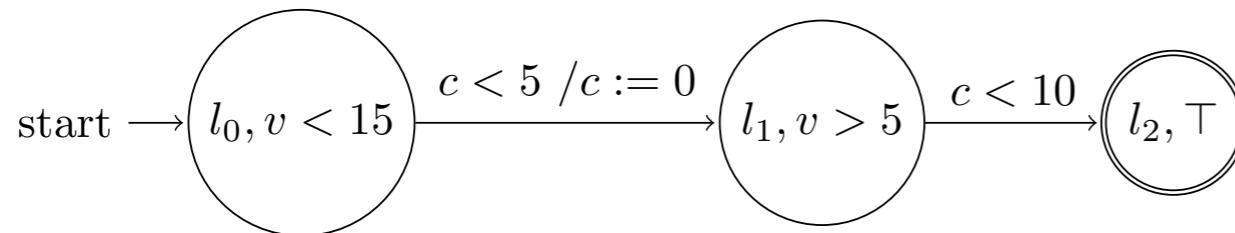
# Zone construction with weight



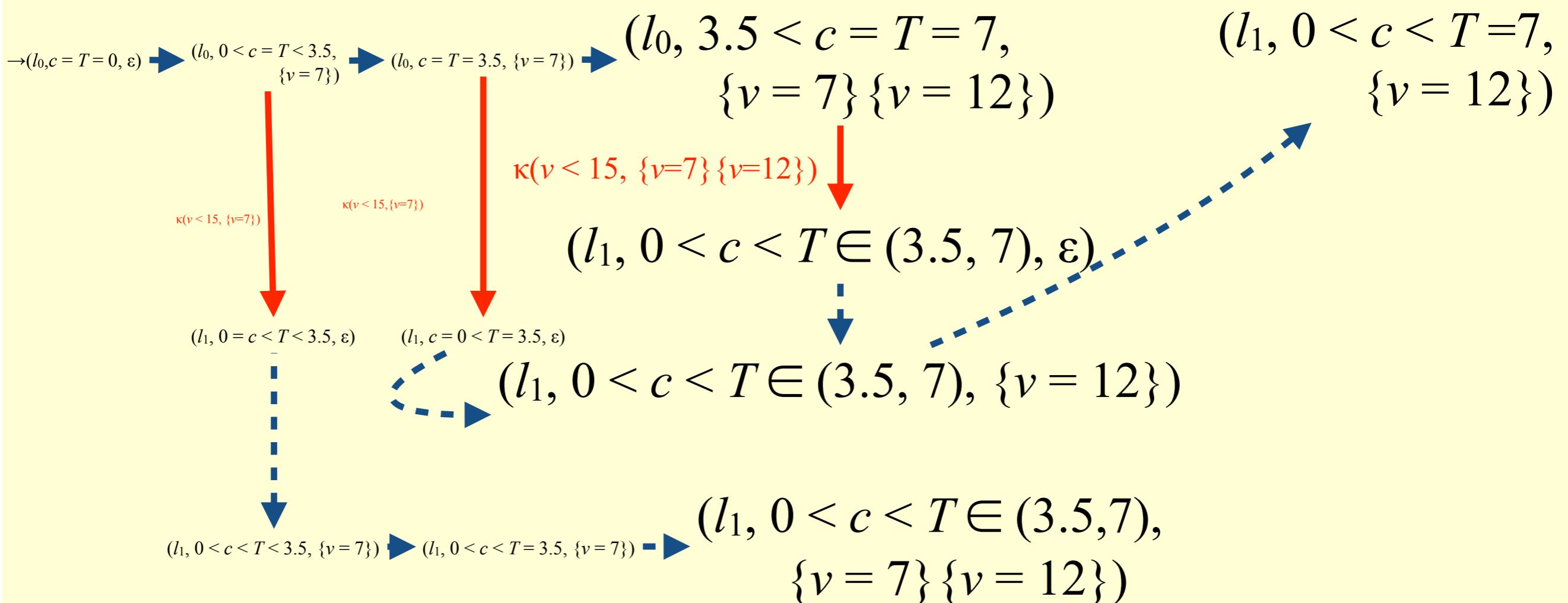
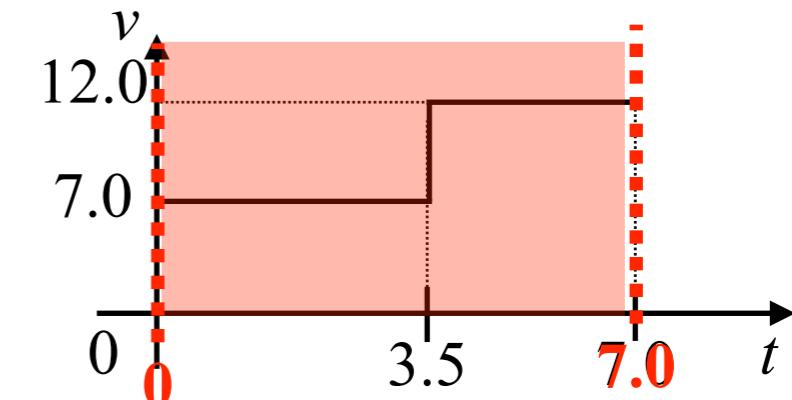
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



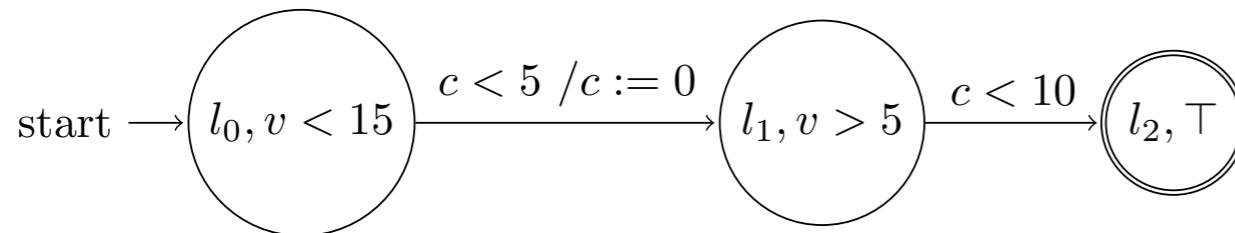
# Zone construction with weight



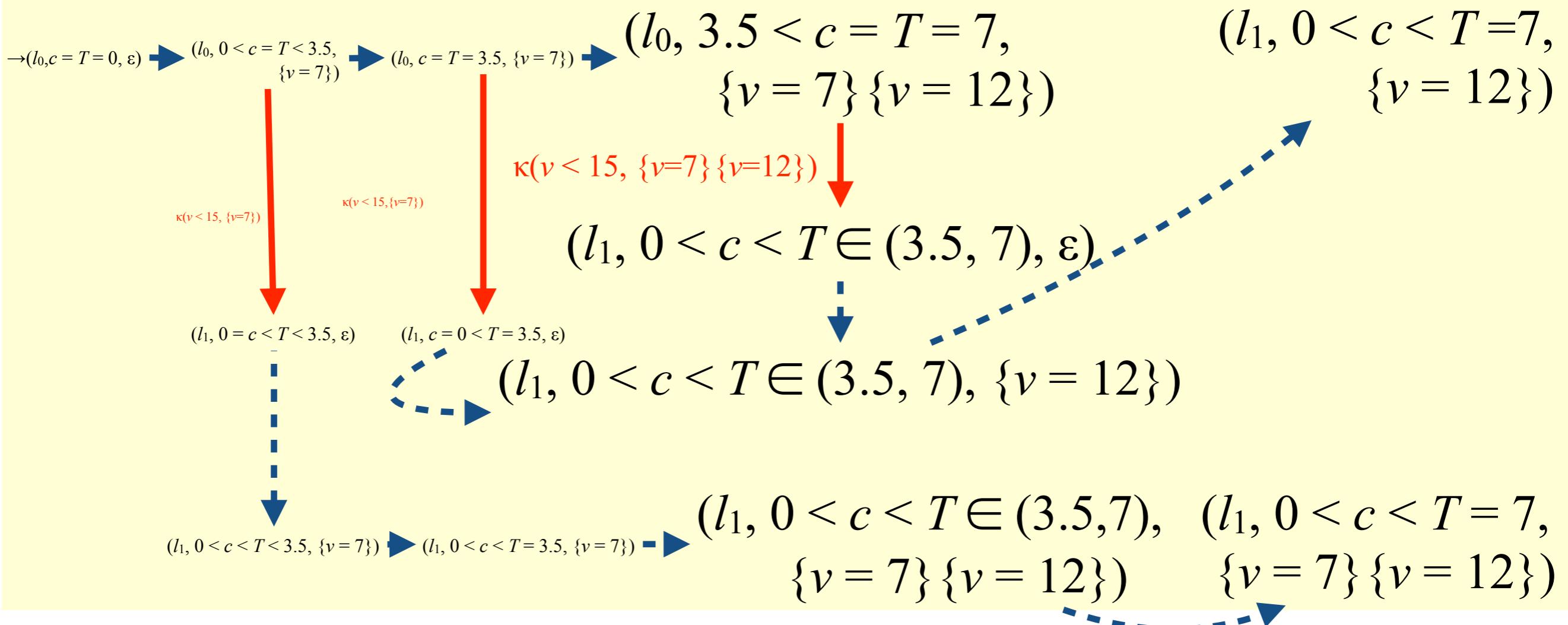
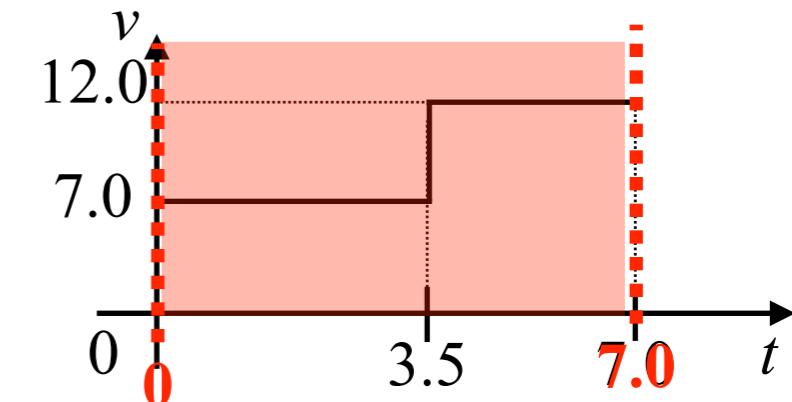
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



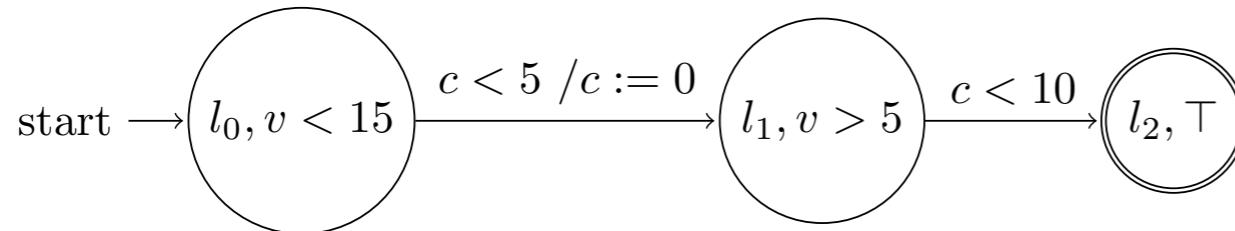
# Zone construction with weight



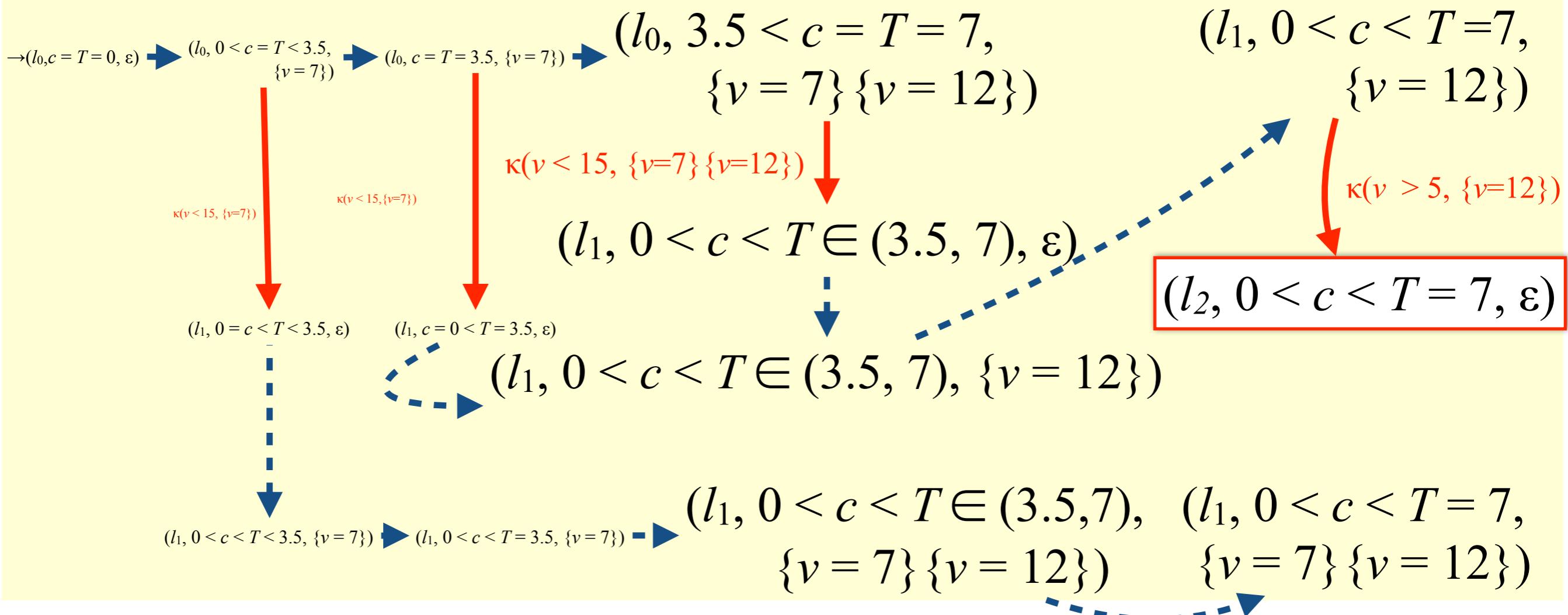
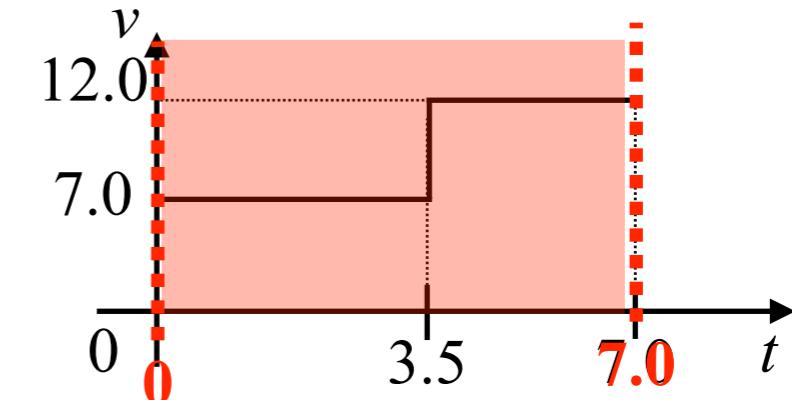
- $T$ : absolute time
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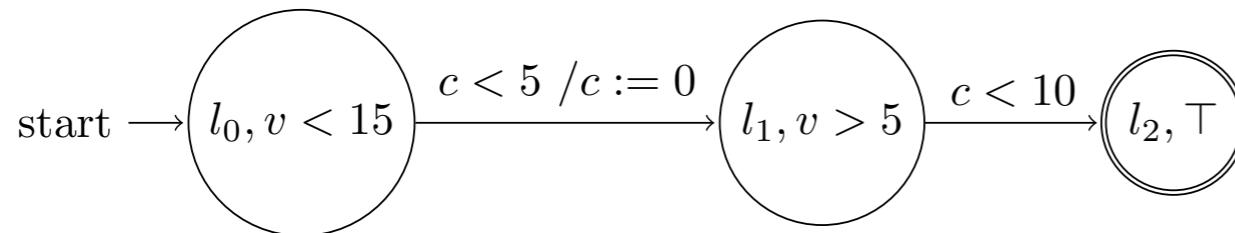
# Zone construction with weight



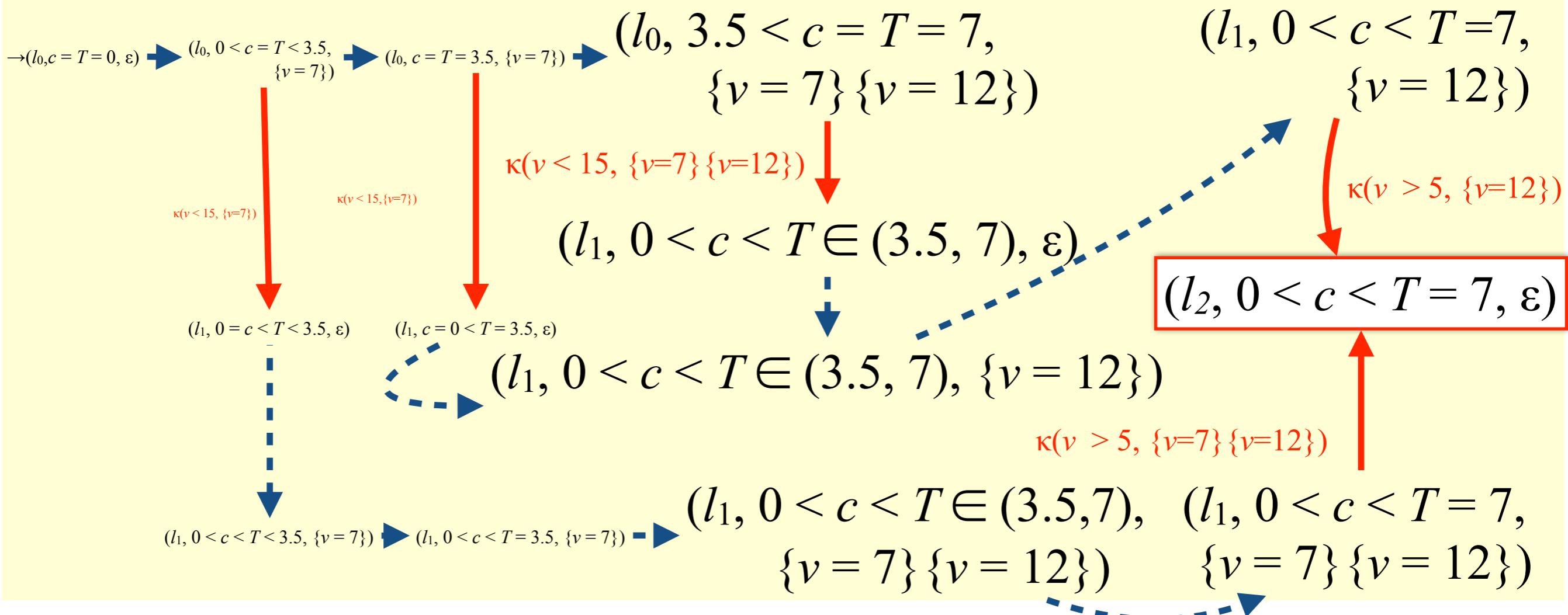
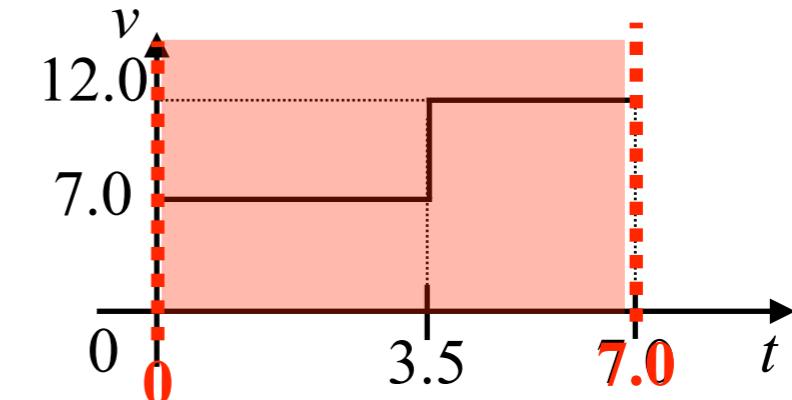
- $T$ : absolute time
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



# Zone construction with weight



- $T$ : absolute time This is OK for monitoring
- Accepted  $\Leftrightarrow$  transit to acc. loc. at  $T = |\sigma| (= 7.0)$



# Main Theorem: Correctness

## Thm.

The shortest distance in the zone graph with weight is same as the shortest distance in the weighted TTS for any complete and idempotent semiring.

All of them work!!

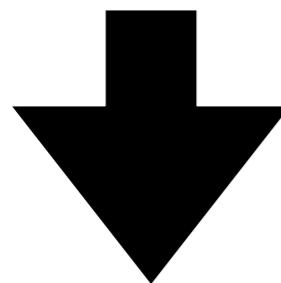
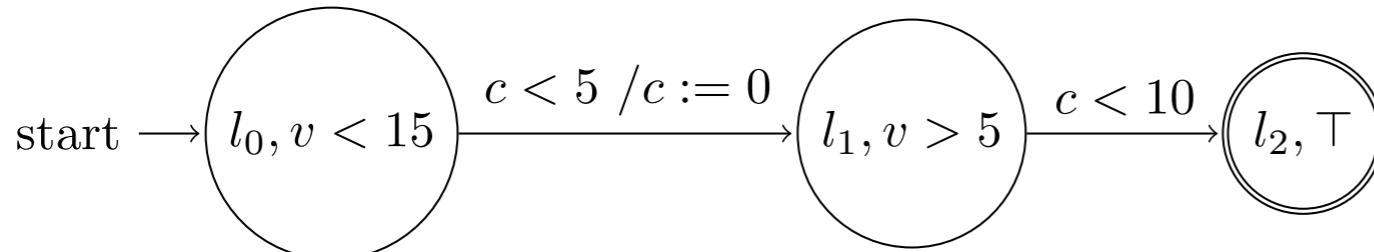
	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

# Local Conclusion: Zone Construction with Weight

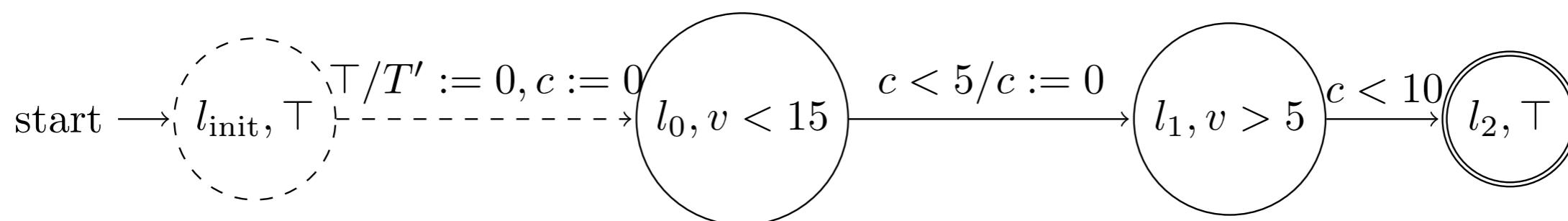
- The construction is basically same as the usual zone construction
- Weights are same as weighted TTS
- The state space is **finite** thanks to **zones** and **finite horizon** of the input signal

# Matching Automata for Pattern Matching

[Bakhirkin+, FORMATS'18]



- Add  $l_{\text{init}}$  to wait for the beginning of the matching
- Add clock variable  $T'$  for the beginning of the matching



# Outline

- Motivation + Introduction
- Technical Part
  - Timed symbolic weighted automata (TSWA)
    - TSWA: TA with signal constraints + weight function
  - Quantitative monitoring/timed pattern matching algorithm
    - Idea: zone construction with weight
- Experiments

# Environment of Experiments

- **Semirings:** sup-inf ( $\mathbb{R} \cup \{\pm \infty\}$ , sup, inf) and tropical ( $\mathbb{R} \cup \{+\infty\}$ , inf, +)
- Used 3 original benchmarks (automotive):
  - Inspired by ST-Lib [Kapinski+, SAE Technical Paper'16]

- **Overshoot:**  $|\nu_{\text{ref}} - \nu|$  gets large after  $\nu_{\text{ref}}$  changed
  - Only matches the sub-signals of **length < 150 time units**

- **Ringing:**  $\nu(t) - \nu(t-10)$  gets positive and negative repeatedly
  - Only matches the sub-signals of **length < 80 time units**

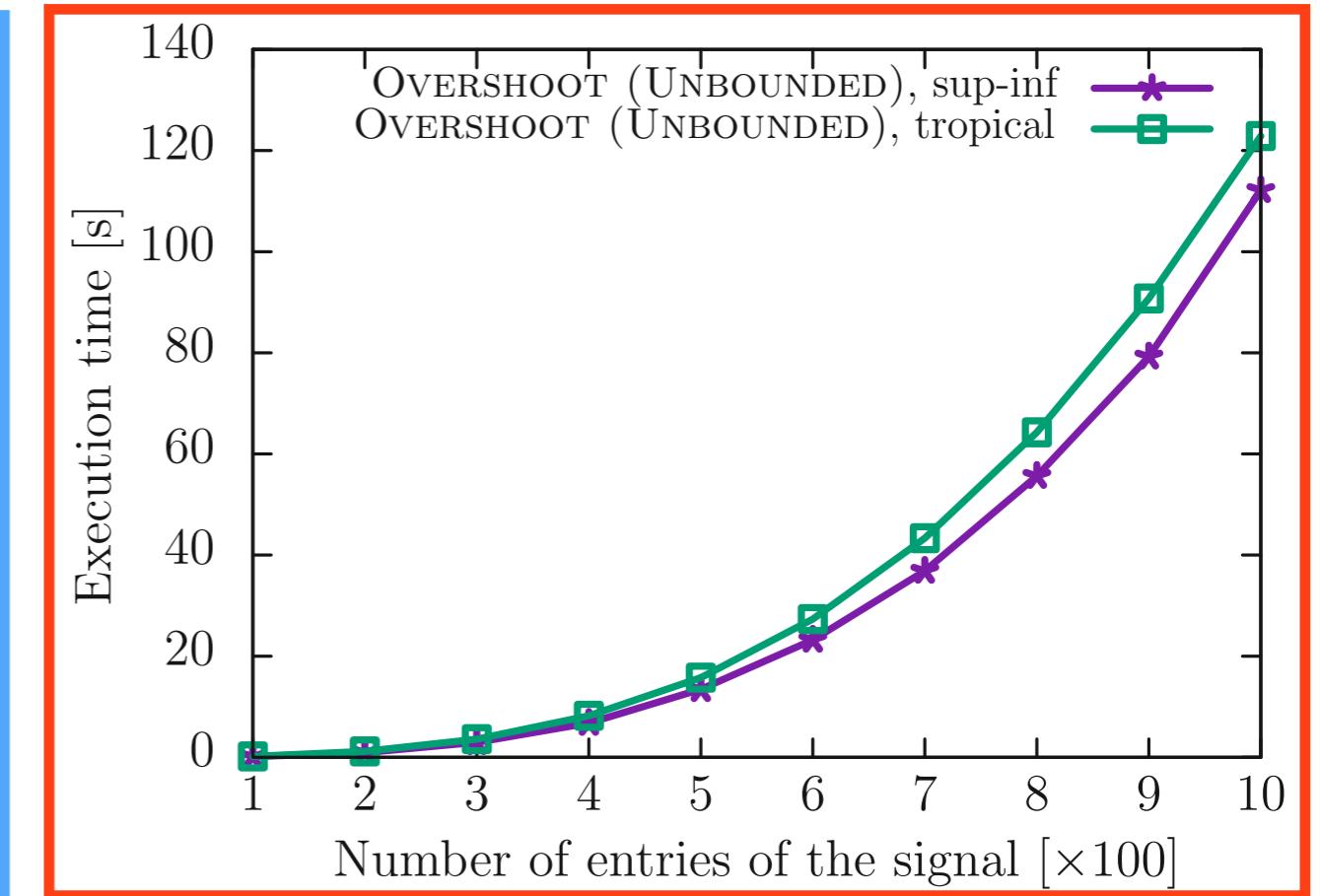
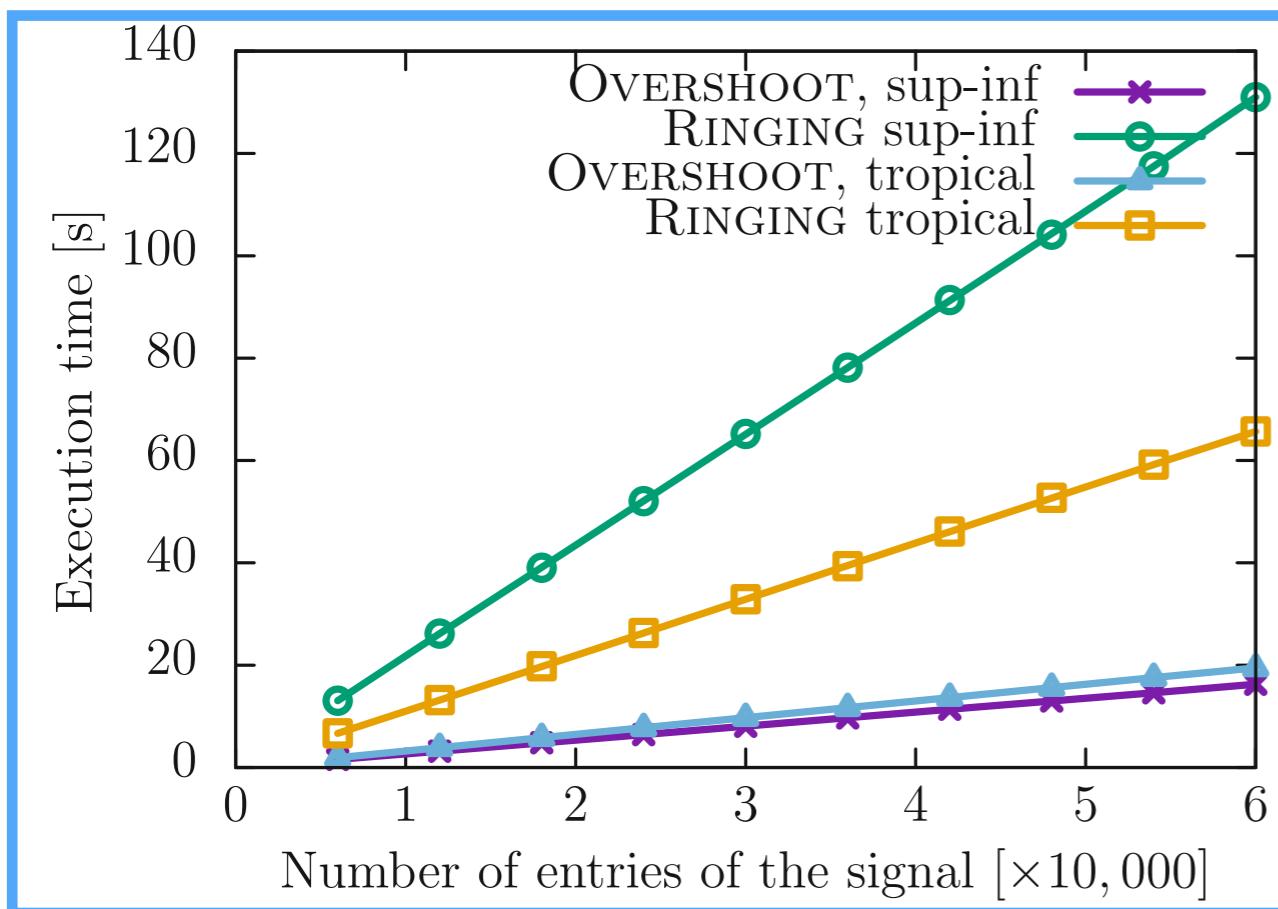
- **Overshoot (unbounded):**  $|\nu_{\text{ref}} - \nu|$  gets large after  $\nu_{\text{ref}}$  changed
  - No such *bounded*

- Amazon EC2 c4.large instance / Ubuntu 18.04 LTS (64 bit)
  - 2.9 GHz Intel Xeon E5-2666 v3, 2 vCPUs, 3.75 GiB RAM

# Execution Time

## Bounded

## Unbounded



- Execution time is **linear** for the bounded spec.
  - 1,000 entries / 1 or 2 sec.
- Execution time explodes for the unbounded spec.

# Conclusion

- Introduced timed symbolic weighted automata (**TSWA**)
  - **TSWA**: TA with signal constraints + weight function
- Gave quantitative monitoring/timed pattern matching algorithm
  - **Idea**: zone construction with weight
- Implementation + experiments
  - scalable for bounded specifications

# Appendix

# Example: “Robust” Semantics

**Weight Function:** minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

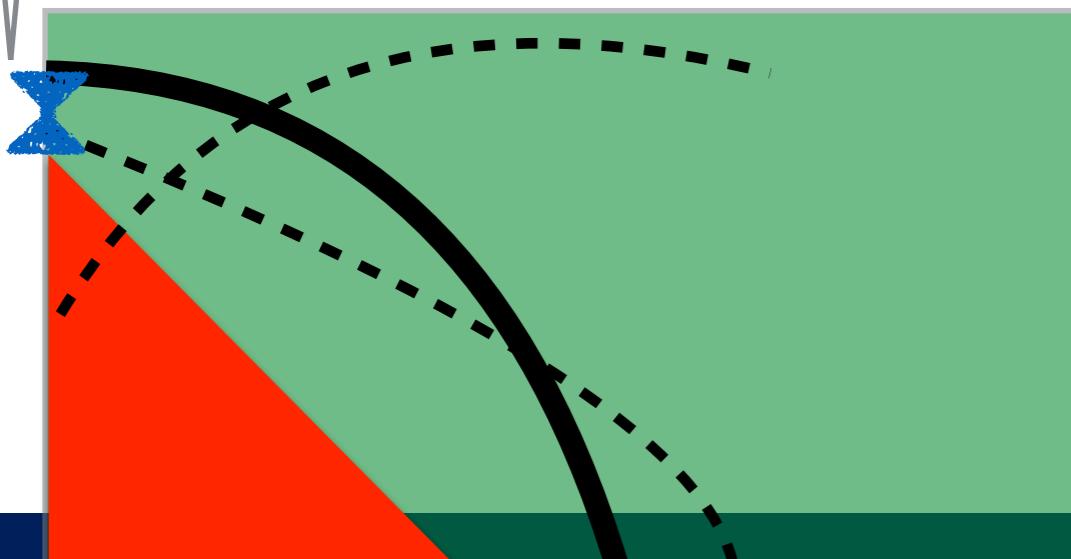
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

Robustness

**Semiring:** sup-inf semiring



# Example: “Robust” Semantics

**Weight Function:** minimum distance from the threshold

$$\kappa_r(u, (a_1 a_2 \dots a_m)) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(u, (a_i))$$

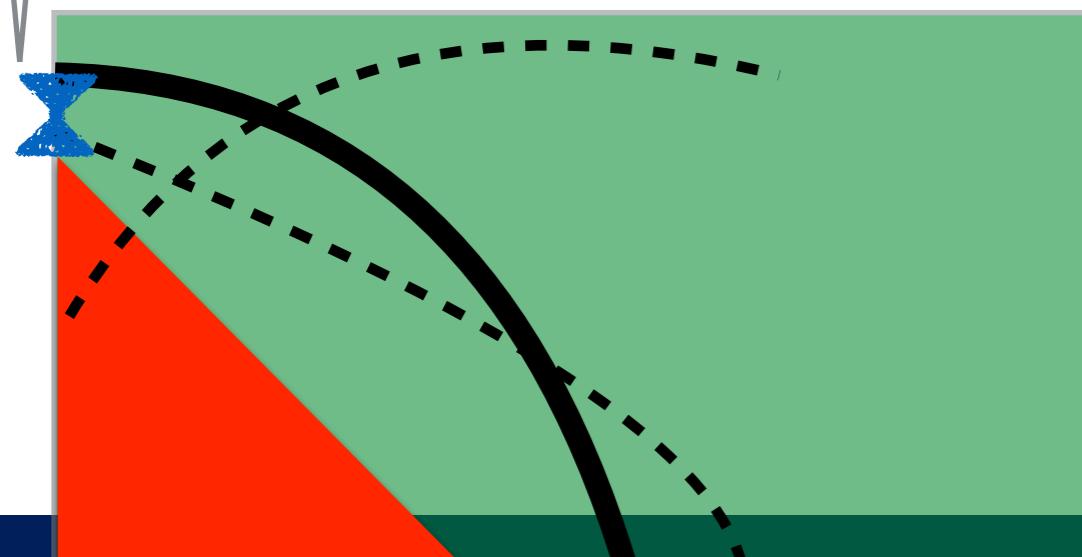
$$\kappa_r\left(\bigwedge_{i=1}^n (x_i \bowtie_i d_i), (a)\right) = \inf_{i \in \{1, 2, \dots, n\}} \kappa_r(x_i \bowtie_i d_i, (a)) \text{ where } \bowtie_i \in \{>, \geq, \leq, <\}$$

$$\kappa_r(x \succ d, (a)) = a(x) - d \quad \text{where } \succ \in \{\geq, >\}$$

$$\kappa_r(x \prec d, (a)) = d - a(x) \quad \text{where } \prec \in \{\leq, <\}$$

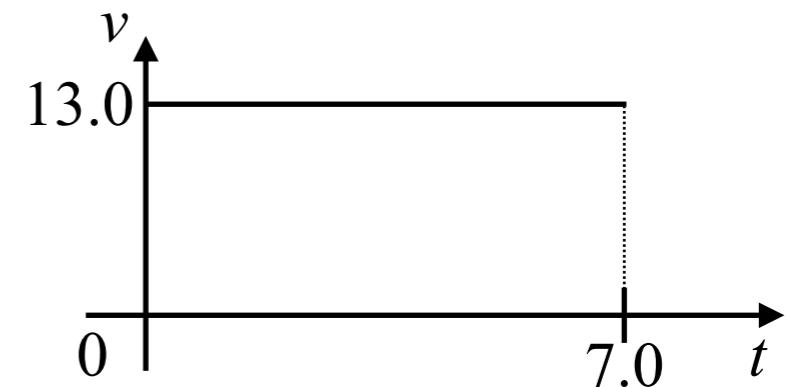
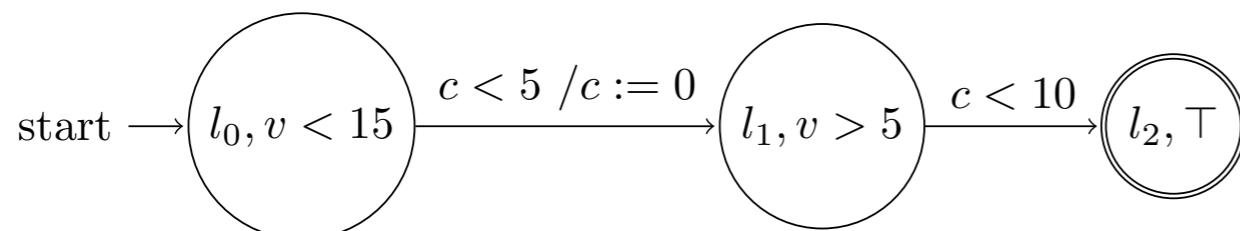
Robustness

**Semiring:** sup-inf semiring



	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

# Comparison of the semiring



$\rightarrow (l_0, c=0, 0, \varepsilon) \xrightarrow{\quad} (l_0, c=2, 2, \{v = 7\}) \xrightarrow{\quad} (l_1, c=0, 3, \varepsilon) \xrightarrow{\quad} (l_1, c=5, 7, \{v = 7\} \{v = 12\}) \xrightarrow{\quad} (l_2, c=5, 7, \varepsilon)$

$$\kappa(v < 15, \{v=13\}) = 2 \quad \otimes \quad \kappa(v > 5, \{v=13\}) = 8$$

## Sup-inf semiring

$$2 \otimes 8 = \inf(2, 8) = 2$$

## Tropical semiring

$$2 \otimes 8 = 2 + 8 = 10$$

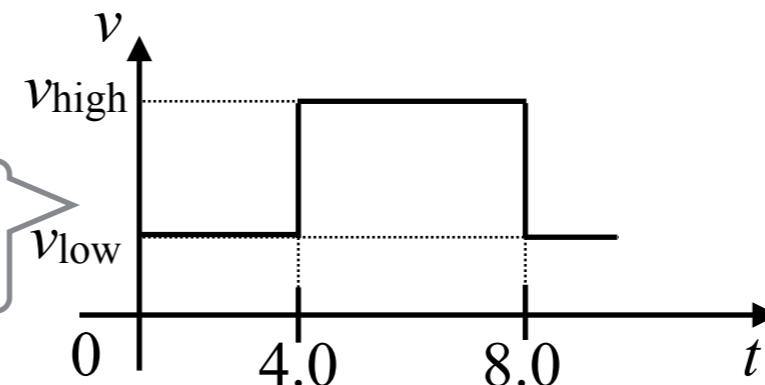
	Boolean	sup-inf	tropical
$S$	{True/False}	$\mathbb{R} \cup \{\pm\infty\}$	$\mathbb{R} \cup \{+\infty\}$
$\oplus$	$\vee$	$\sup$	$\inf$
$\otimes$	$\wedge$	$\inf$	$+$

# (Qualitative) timed pattern matching

## Input

[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$** 
  - System ***log*** 
    - e.g.,
- **Real-time spec.  $\mathcal{W}$** 
  - **Spec.** to be monitored
    - e.g., The velocity should not keep high for > 1 sec.



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the ***log*** satisfies the **spec.**
  - e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

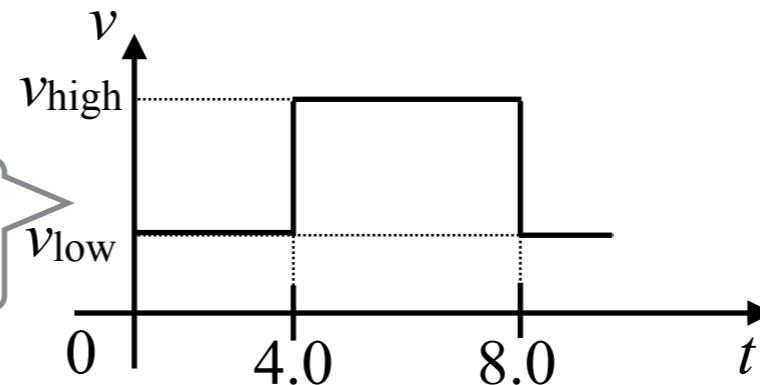
## Input

[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System  **$\log$**

discretized!!

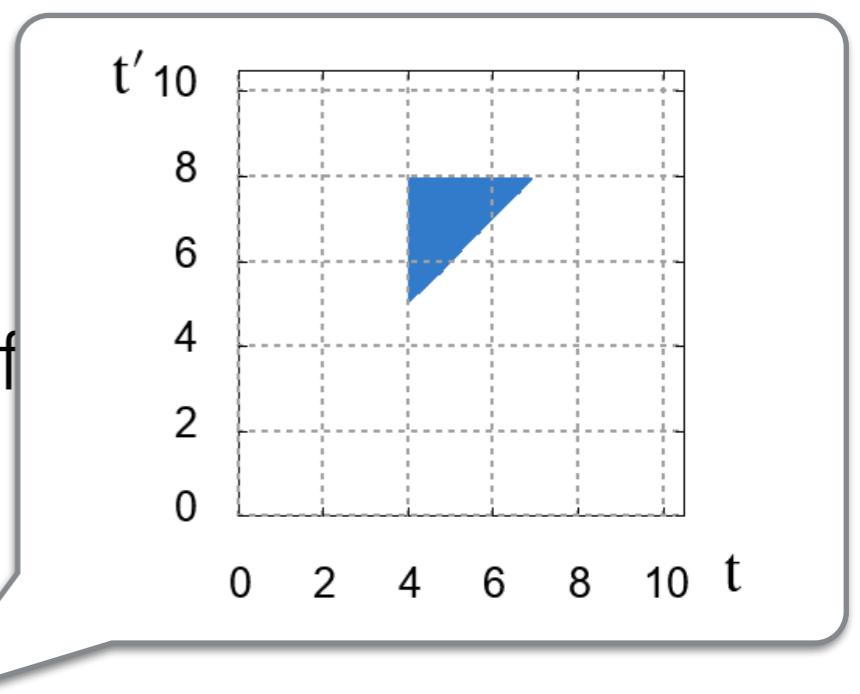


- e.g.,

- **Real-time spec.  $\mathcal{W}$**

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the  **$\log$**  satisfies the **spec.**
- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

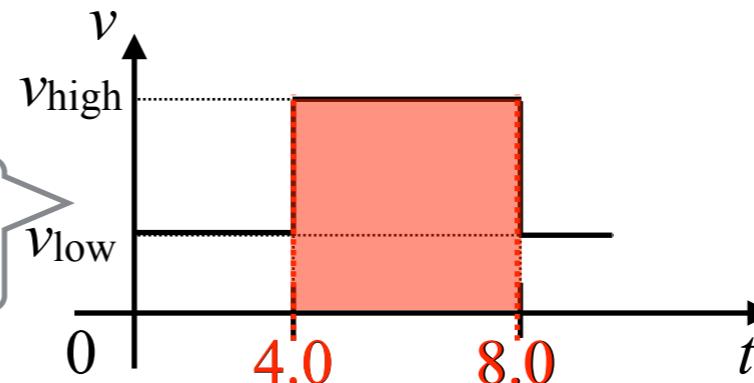
[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System log

discretized!!

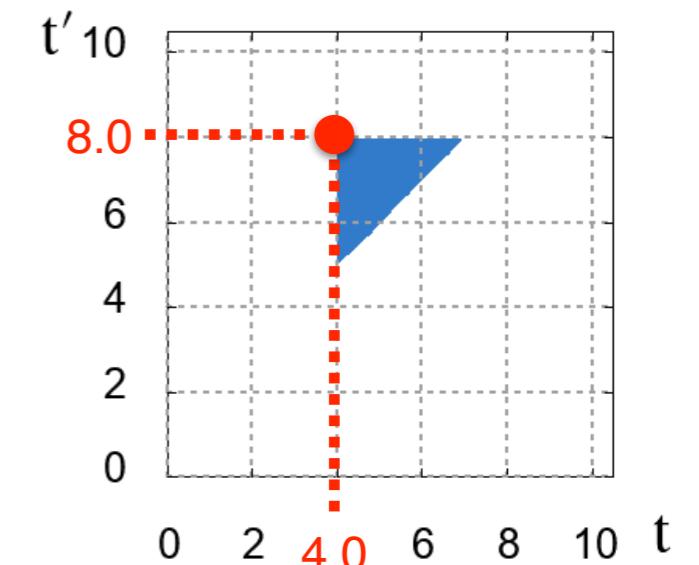
- e.g.,



- **Real-time spec.  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep high f



## Output

- All the subsignals  $\sigma([t,t'])$  of the log satisfies the spec.
- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

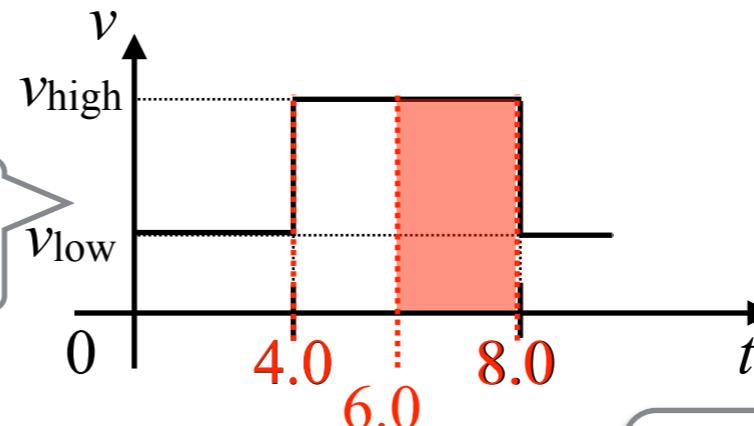
[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System  **$\log$**

discretized!!

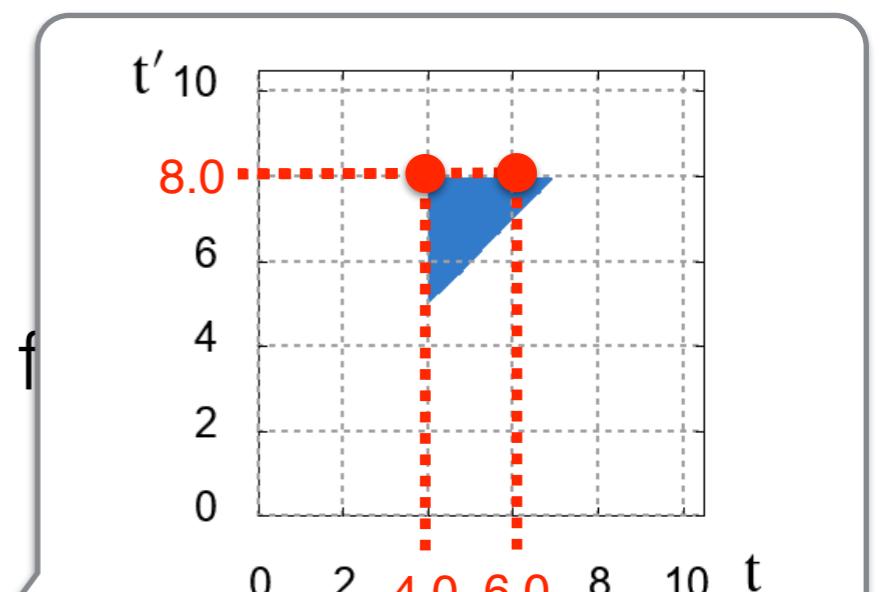
- e.g.,



- **Real-time spec.  $\mathcal{W}$**

- **Spec.** to be monitored

- e.g., The velocity should not keep high f



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the  **$\log$**  satisfies the **spec.**
  - e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

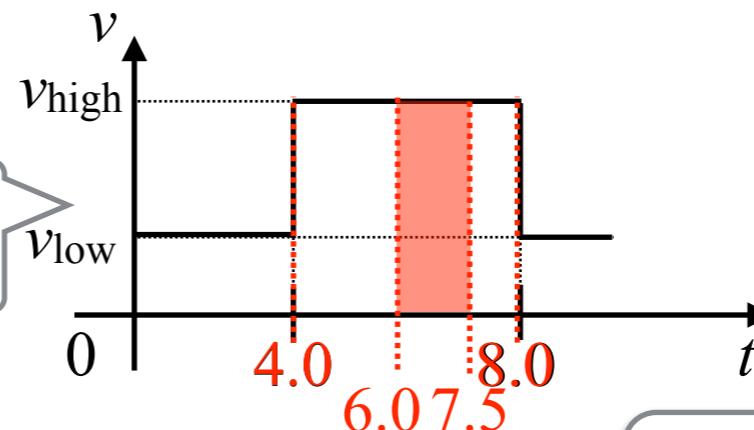
[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System  **$\log$**

discretized!!

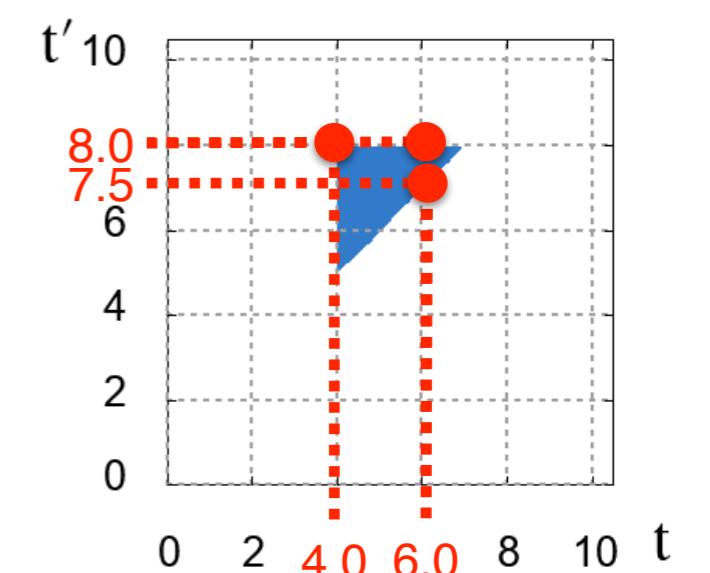
- e.g.,



- **Real-time spec.  $\mathcal{W}$**

- **Spec.** to be monitored

- e.g., The velocity should not keep high for long



## Output

- **All** the subsignals  $\sigma([t,t'])$  of the  **$\log$**  satisfies the **spec.**
  - e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

# (Qualitative) timed pattern matching

## Input

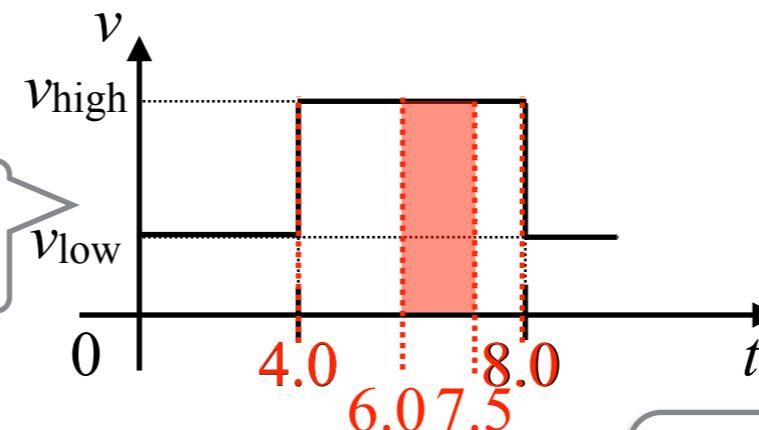
[Ulus+, FORMATS'14]

- **Finite-valued signal  $\sigma$**

- System log

discretized!!

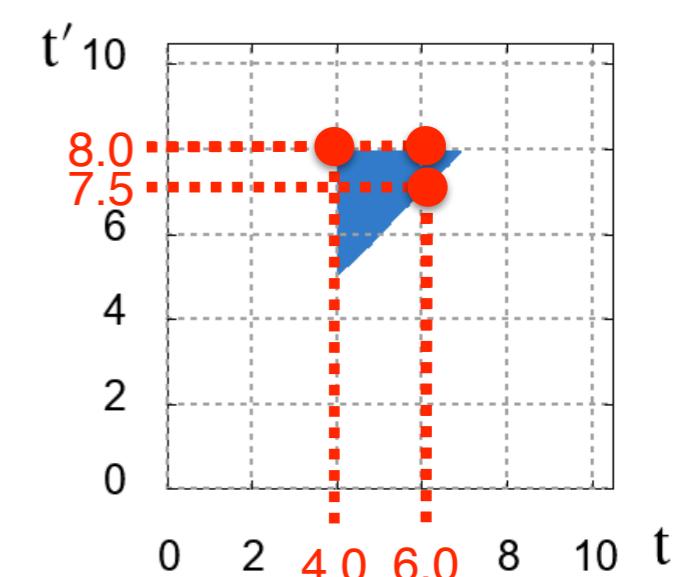
- e.g.,



- **Real-time spec.  $\mathcal{W}$**

- Spec. to be monitored

- e.g., The velocity should not keep high for too long



## Output

- All the subsignals  $\sigma([t,t'])$  of the log satisfies the spec.
- e.g.,  $\sigma([4.0,8.0])$ ,  $\sigma([6.0,8.0])$ ,  $\sigma([6.0,7.5])$ , ...

We want to know how robustly the spec. is satisfied!!