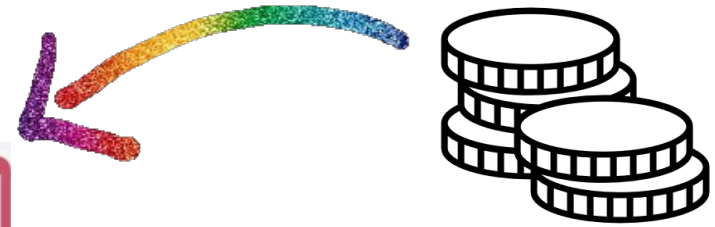
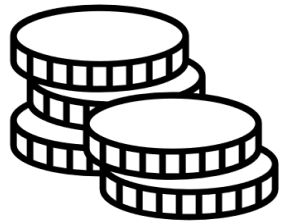
A Mixed Linear and Graded Logic

Victoria Vollmer
University of Kent

Joint work with some really cool people

Intuitionistic linear logic





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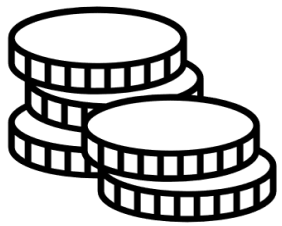

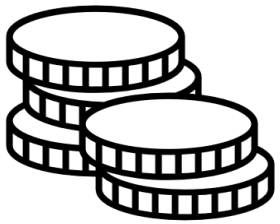

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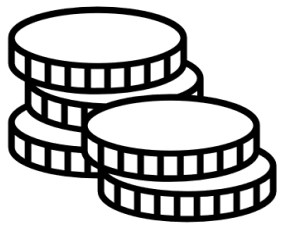

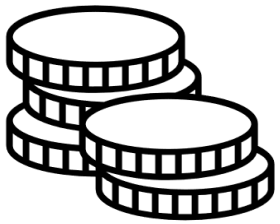

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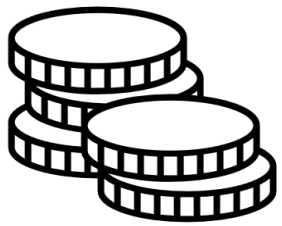

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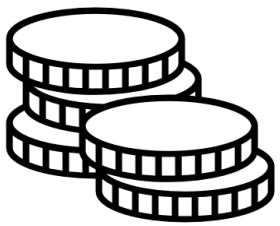

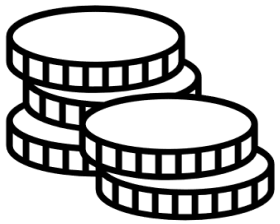

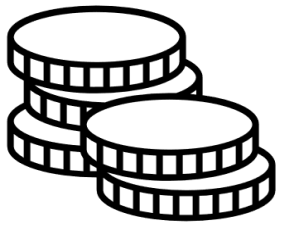

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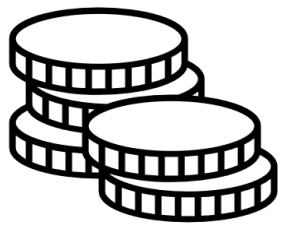

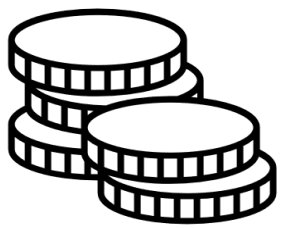

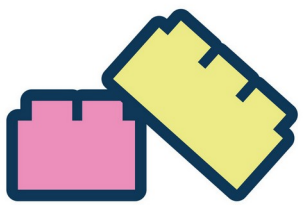

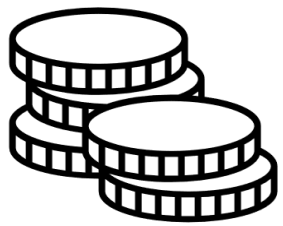

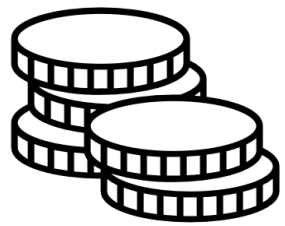

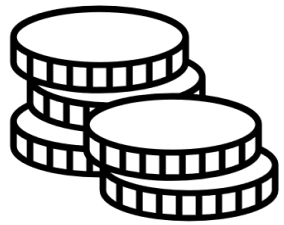

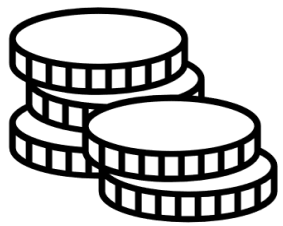

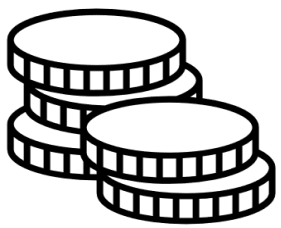

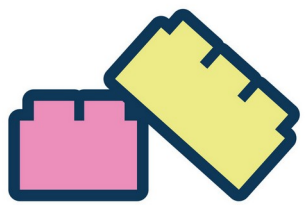

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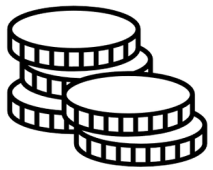

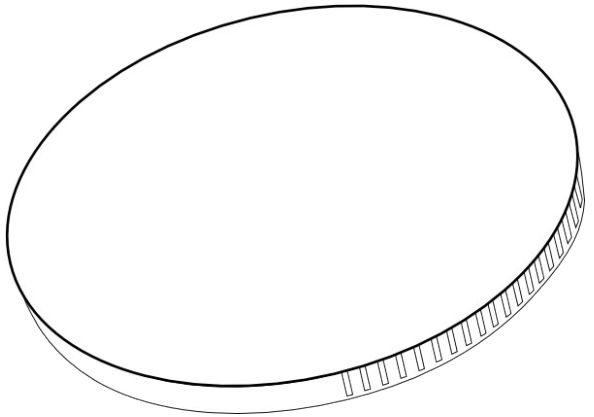

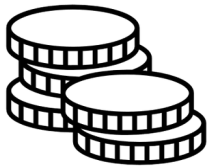

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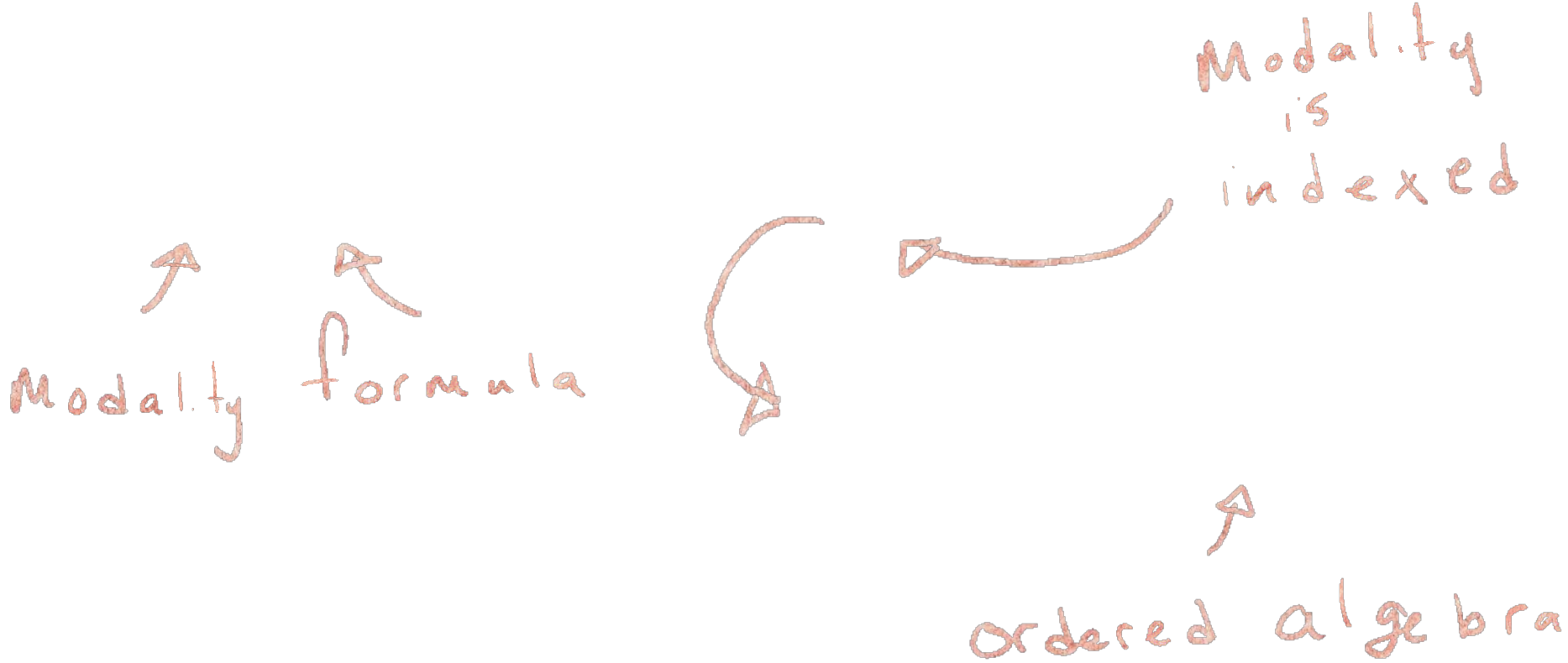
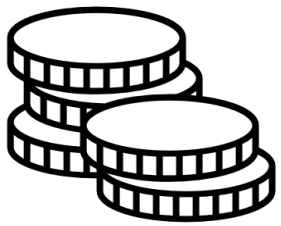
Benton's LNL

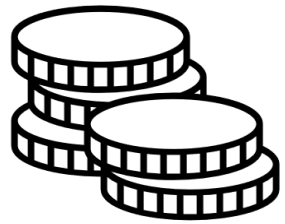
Graded modal logic

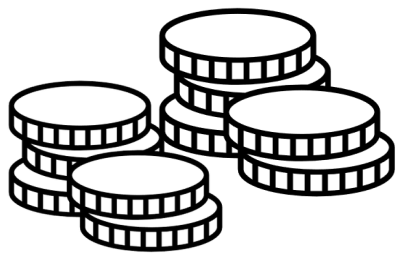
Graded modal logic



Graded modal logic







Mixed Graded Linear Logic (mGL)

mGL

mGL

mGL

mGL

mGL

Fujii, Soichiro, Shin-ya Katsumata, and Paul-André Mellies. “Towards a formal theory of graded monads.” *FOSSACS 2016*

Goal

Goal

Goal

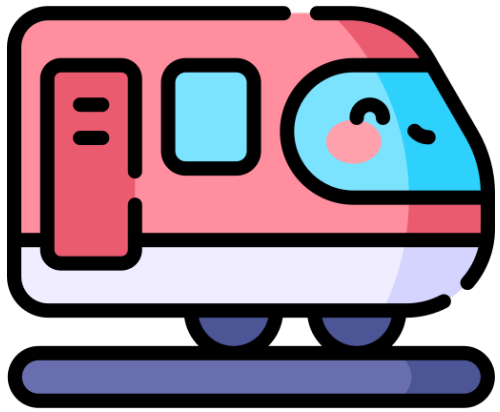
Goal

Goal

Fun Fact

Many Buses

F



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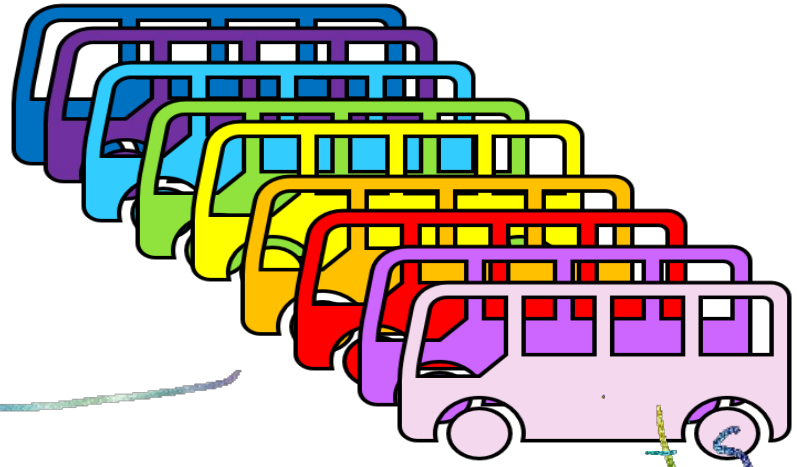

GF

is a family

Graded multi-category

Graded multi-category

Structural natural transformations

$$\frac{\langle \Phi, \Psi \rangle \xrightarrow{f} Z}{\langle \Phi, 0 * \Phi', \Psi \rangle \xrightarrow{\mathsf{weak}_{\Phi,\Phi',\Psi,Z}(f)} Z}$$

$$\frac{\langle \Phi, (r, X), (s, X), \Psi \rangle \xrightarrow{f} Z}{\langle \Phi, (r + s, X), \Psi \rangle \xrightarrow{\mathsf{contr}_{\Phi,r,s,X,\Psi,Z}(f)} Z}$$

$$\frac{\langle \Phi, \Phi_1, \Phi_2, \Psi \rangle \xrightarrow{f} Z}{\langle \Phi, \Phi_2, \Phi_1, \Psi \rangle \xrightarrow{\mathsf{ex}_{\Phi,\Phi_1,\Phi_2,\Psi,Z}(f)} Z}$$

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