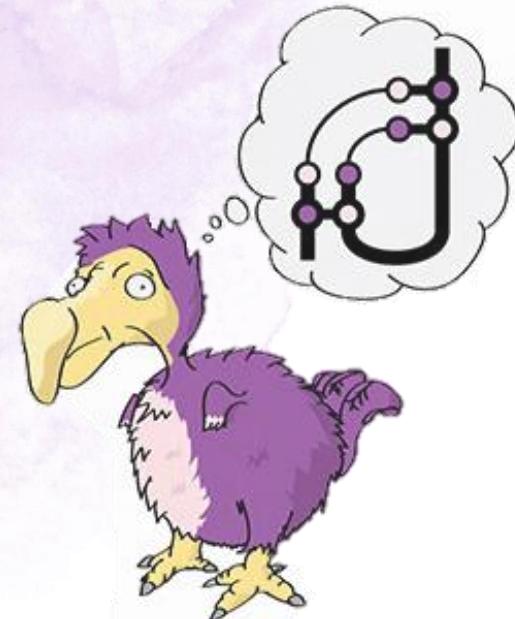


Focked-up ZX Calculus

Picturing continuous-variable
quantum computation

Razin A. Shaikh, Lia Yeh and Stefano Gogioso
University of Oxford



DEPARTMENT OF
**COMPUTER
SCIENCE**

Plan

1. Background and motivation
2. Focked-up ZX calculus
 - Generators, rules and common gates
 - Gaussian completeness
3. Applications
 - Quantum Error Correction with the GKP code
 - Gaussian Boson sampling
4. Conclusion and future work

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Qubit quantum computing

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Symmetric monoidal category of finite dimensional Hilbert spaces

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States	Vectors in 2^n dimensional complex Hilbert space
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Operations Linear maps

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Sequential composition Matrix multiplication

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Sequential composition Matrix multiplication

Parallel composition Tensor product

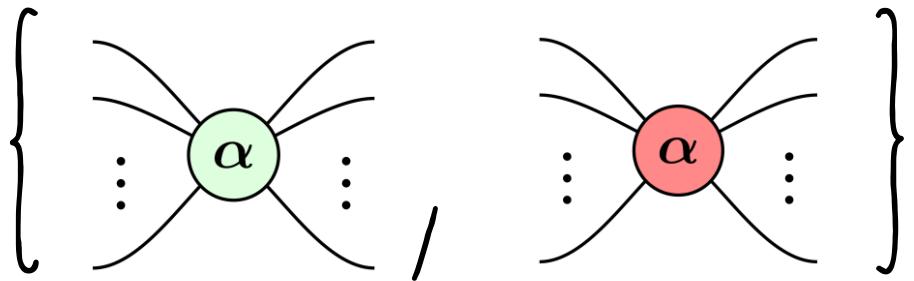
Qubit ZX calculus

$$\begin{array}{ccc} m : & \text{---} & \alpha & \text{---} & n : \\ & | & & | & \\ & \vdots & \text{---} & \vdots & \vdots \\ & | & & | & \\ & \vdots & & \vdots & \vdots \\ & | & & | & \\ & \vdots & & \vdots & \vdots \end{array} \quad \xrightarrow{\llbracket \cdot \rrbracket} \quad |0\rangle^{\otimes n} \langle 0|^{\otimes m} + e^{i\alpha} |1\rangle^{\otimes n} \langle 1|^{\otimes m}$$

$$\begin{array}{ccc} m : & \text{---} & \alpha & \text{---} & n : \\ & | & & | & \\ & \vdots & \text{---} & \vdots & \vdots \\ & | & & | & \\ & \vdots & & \vdots & \vdots \\ & | & & | & \\ & \vdots & & \vdots & \vdots \end{array} \quad \xrightarrow{\llbracket \cdot \rrbracket} \quad |+\rangle^{\otimes n} \langle +|^{\otimes m} + e^{i\alpha} |-\rangle^{\otimes n} \langle -|^{\otimes m}$$

where $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad |\pm\rangle = |0\rangle \pm |1\rangle$

Qubit ZX calculus

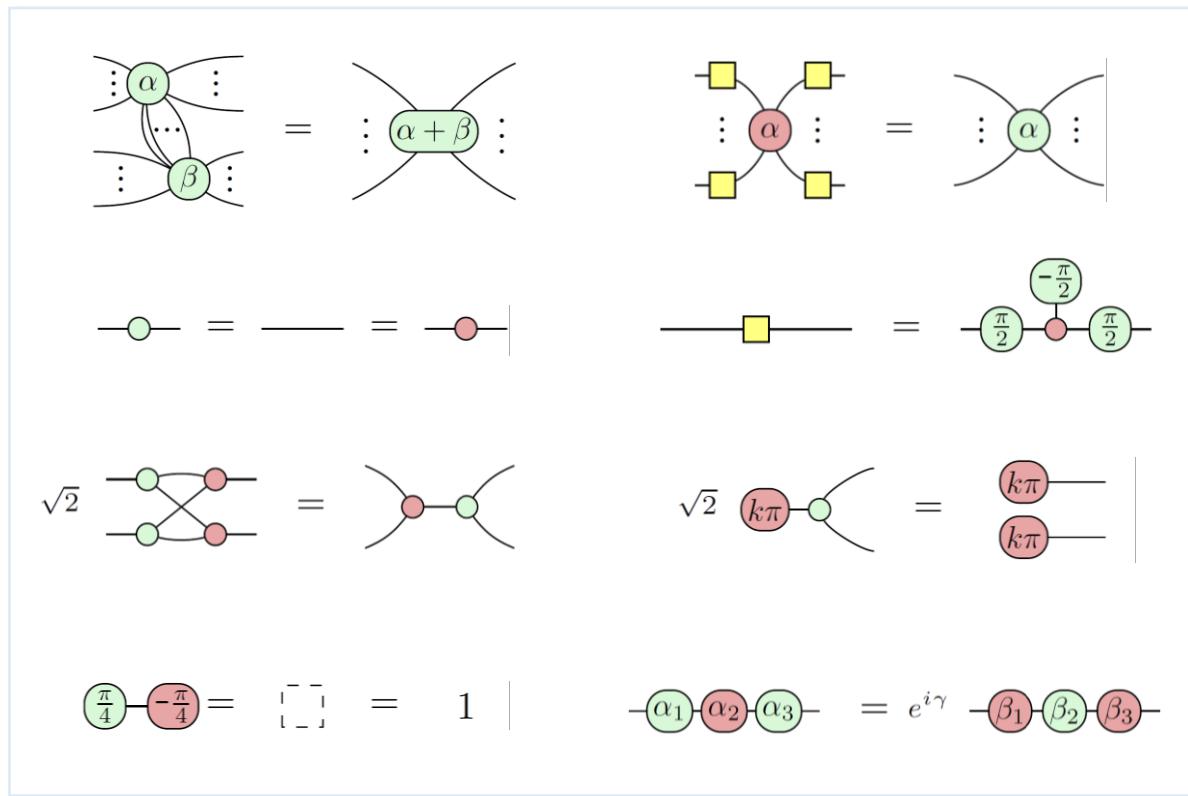


You can draw any quantum circuit using just these two types of spiders.

Universality:

These suffice to represent any linear map on any number of qubits.

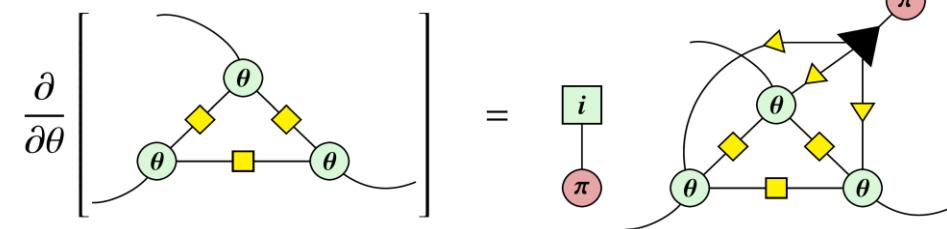
Qubit ZX calculus



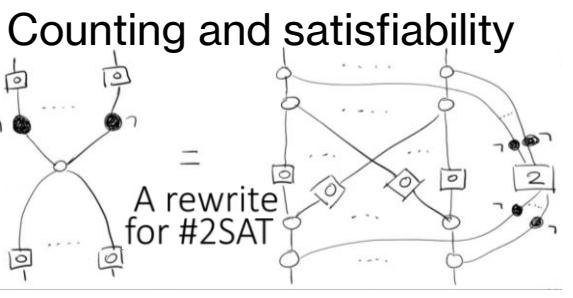
Completeness:

You can prove any equalities of qubit linear maps using just these 8 rules.

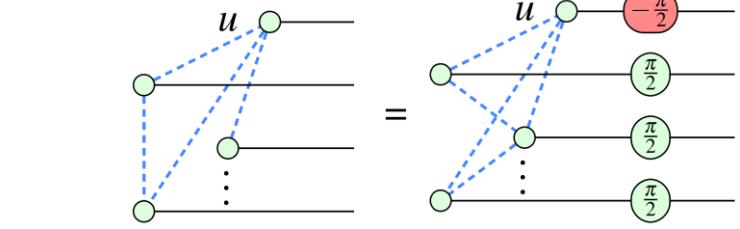
Differentiation for Quantum Machine Learning



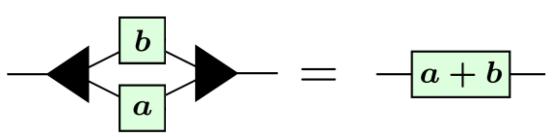
Counting and satisfiability



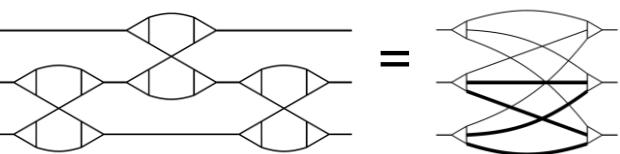
Measurement-Based Quantum Computing



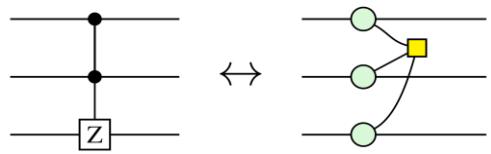
◀ is good for sums



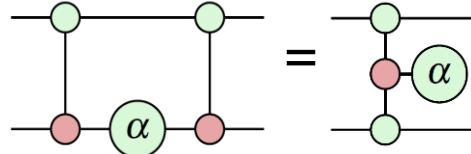
▶ is good for linear optics



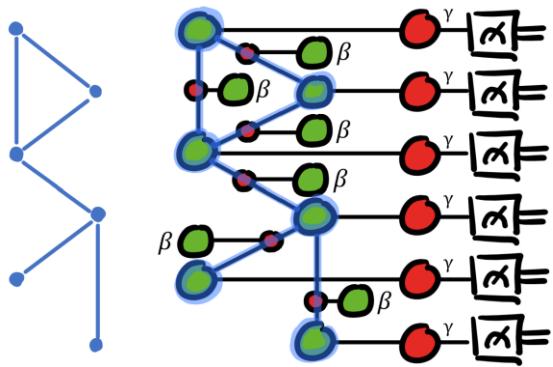
Toffoli+Hadamard



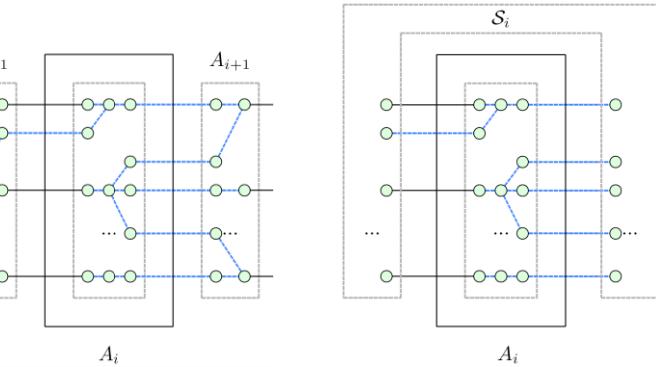
Quantum Circuit Optimization



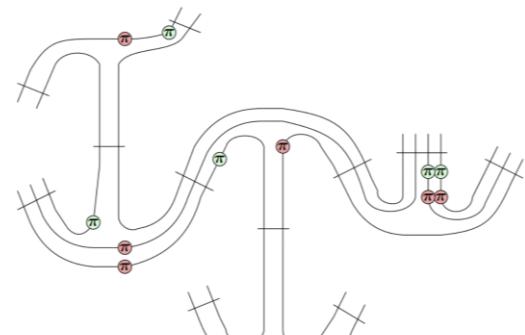
QAOA



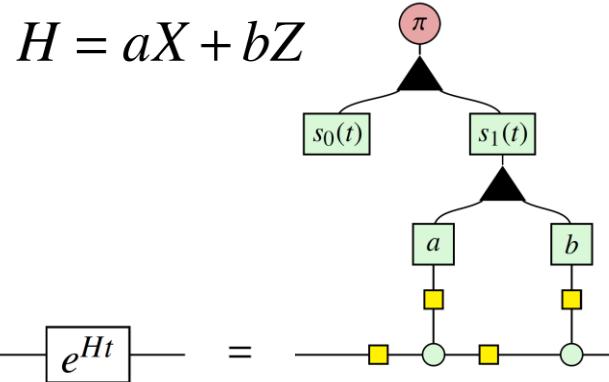
Blind Quantum Computation



Spin-network diagrams



Hamiltonian Exponentiation



2007



Formulation



Development



2017 - 18



Completeness



Present



Applications



ZX-calculus is Complete for Finite-Dimensional Hilbert Spaces

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The ZX-calculus is a graphical language for reasoning about quantum computing and quantum information theory. As a complete graphical language, it incorporates a set of axioms rich enough to derive any equation of the underlying formalism. While completeness of the ZX-calculus has been established for qubits and the Clifford fragment of prime-dimensional qudits, universal completeness beyond two-level systems has remained unproven until now. In this paper, we present a proof establishing the completeness of finite-dimensional ZX-calculus, incorporating only the mixed-dimensional Z-spider and the qudit X-spider as generators. Our approach builds on the completeness of another graphical language, the finite-dimensional ZW-calculus, with direct translations between these two calculi. By proving its completeness, we lay a solid foundation for the ZX-calculus as a versatile tool not only for quantum computation but also for various fields within finite-dimensional quantum theory.

Why continuous-variable quantum computation?

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- Native simulation of Bosonic systems / quantum field theories

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Why continuous-variable quantum computation?

- Native simulation of Bosonic systems / quantum field theories
- Quantum error correction with Bosonic codes
- Scalable hardware implementation
 - On photonics, superconducting and trapped-ions platforms

CVQC – what is a qumode?

	Qubit	Qudit	Qumode
Hilbert space	\mathbb{C}^2		
State	$\alpha 0\rangle + \beta 1\rangle$		

CVQC – what is a qumode?

	Qubit	Qudit	Qumode
Hilbert space	\mathbb{C}^2	\mathbb{C}^d	
State	$\alpha 0\rangle + \beta 1\rangle$	$\sum_{n=0}^{d-1} a_n n\rangle$	

CVQC – what is a qumode?

	Qubit	Qudit	Qumode
Hilbert space	\mathbb{C}^2	\mathbb{C}^d	$L^2(\mathbb{R})$
State	$\alpha 0\rangle + \beta 1\rangle$	$\sum_{n=0}^{d-1} a_n n\rangle$	$\int_{\mathbb{R}} \psi(x) x\rangle dx$ or $\sum_{n=0}^{\infty} a_n n\rangle$

CVQC – Orthogonal bases

Position basis

$$|x\rangle_{x \in \mathbb{R}}$$

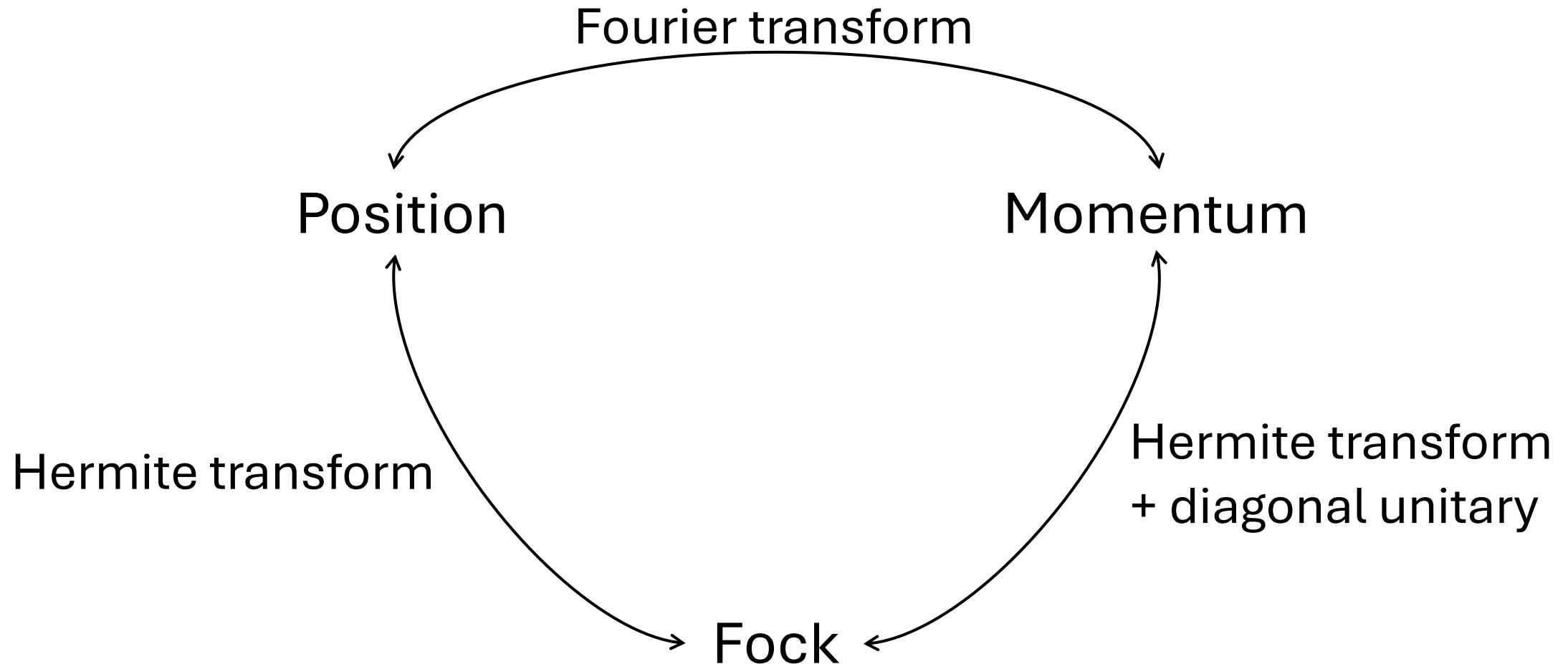
Momentum basis

$$|p\rangle_{p \in \mathbb{R}}$$

Fock basis

$$|n\rangle_{n \in \mathbb{N}}$$

CVQC – Orthogonal bases



Example of Hermite transform

$$|n\rangle = \int \psi_n(x) |x\rangle dx$$

where $\psi_n(x)$ is the n -th Hermite function.

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$$|n\rangle = \int \psi_n(x) |x\rangle dx$$

where $\psi_n(x)$ is the n -th Hermite function.

$$\psi_0(x) = \pi^{-\frac{1}{4}} e^{-\frac{1}{2}x^2},$$

$$\psi_1(x) = \sqrt{2} \pi^{-\frac{1}{4}} x e^{-\frac{1}{2}x^2},$$

$$\psi_2(x) = \left(\sqrt{2} \pi^{\frac{1}{4}} \right)^{-1} (2x^2 - 1) e^{-\frac{1}{2}x^2},$$

$$\psi_3(x) = \left(\sqrt{3} \pi^{\frac{1}{4}} \right)^{-1} (2x^3 - 3x) e^{-\frac{1}{2}x^2},$$

$$\psi_4(x) = \left(2\sqrt{6} \pi^{\frac{1}{4}} \right)^{-1} (4x^4 - 12x^2 + 3) e^{-\frac{1}{2}x^2},$$

$$\psi_5(x) = \left(2\sqrt{15} \pi^{\frac{1}{4}} \right)^{-1} (4x^5 - 20x^3 + 15x) e^{-\frac{1}{2}x^2}.$$

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ZX calculus - Generators

ZX calculus - Generators

$$\begin{array}{ccc} \text{Diagram: } & \xrightarrow{\llbracket \cdot \rrbracket} & \int f(x) |x\rangle^{\otimes b} \langle x|^{\otimes a} dx \\ \text{A green circle labeled } f \text{ with two input ports labeled } a : \text{ and two output ports labeled } : b. & & \text{(Z-SPIDER)} \end{array}$$

ZX calculus - Generators

$$\begin{array}{ccc} \text{Diagram: } & \xrightarrow{\llbracket \cdot \rrbracket} & \int f(x) |x\rangle^{\otimes b} \langle x|^{\otimes a} dx \\ \text{A green circle labeled } f \text{ with two input wires labeled } a \text{ and two output wires labeled } b. & & \text{(Z-SPIDER)} \end{array}$$

$$\begin{array}{ccc} \text{Diagram: } & \xrightarrow{\llbracket \cdot \rrbracket} & \sum_{n=0}^{\infty} g(n) |n\rangle^{\otimes b} \langle n|^{\otimes a} \\ \text{An orange circle labeled } g \text{ with two input wires labeled } a \text{ and two output wires labeled } b. & & \text{(FOCK-SPIDER)} \end{array}$$

ZX calculus - Generators

$$\begin{array}{ccc} \text{Diagram: } & \xrightarrow{\llbracket \cdot \rrbracket} & \int f(x) |x\rangle^{\otimes b} \langle x|^{\otimes a} dx \\ \text{A green circle labeled } f \text{ with two input wires labeled } a \text{ and two output wires labeled } b. & & \text{(Z-SPIDER)} \end{array}$$

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$$\begin{array}{ccc} k & \xrightarrow{\llbracket \cdot \rrbracket} & k \\ & & \text{(GLOBAL-SCALAR)} \end{array}$$

ZX calculus – Notations

$$\begin{array}{c} \text{---} \\ | \quad \quad \quad | \\ a \quad \vdots \quad \vdots \quad b \end{array} \quad := \quad \begin{array}{c} \text{---} \quad i^{\hat{n}} \quad \text{---} \\ a \quad \vdots \quad \text{---} \quad -i^{\hat{n}} \quad \vdots \quad b \\ \text{---} \quad i^{\hat{n}} \quad \text{---} \quad -i^{\hat{n}} \quad \text{---} \end{array} \quad \xrightarrow{[\![\cdot]\!]} \quad \int f(p) \ |p\rangle^{\otimes b} \langle p|^{\otimes a} \ dp \quad (\text{X-SPIDER})$$

where $\text{---} \quad (-i)^{\hat{n}} \quad \text{---}$ is the Fourier transform.

ZX calculus – Notations

$$\begin{array}{c} \text{Diagram: } \text{A red circle labeled } f \text{ with two input wires labeled } a \text{ and } b \text{ meeting at the center.} \\ := \\ \text{Diagram: } \text{A green circle labeled } f \text{ with four wires: top-left } i^{\hat{n}}, \text{ top-right } -i^{\hat{n}}, \text{ bottom-left } i^{\hat{n}}, \text{ and bottom-right } -i^{\hat{n}}. \\ \xrightarrow{[\cdot]} \int f(p) |p\rangle^{\otimes b} \langle p|^{\otimes a} dp \quad (\text{X-SPIDER}) \end{array}$$

where $\text{--- } (-i)^{\hat{n}} \text{ ---}$ is the Fourier transform.

$$\begin{array}{ccc} \text{Diagram: } \text{A green circle with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. & := & \text{Diagram: } \text{A green circle labeled } 1 \text{ with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. \\ \text{Diagram: } \text{A red circle with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. & := & \text{Diagram: } \text{A red circle labeled } 1 \text{ with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. \\ \text{Diagram: } \text{An orange circle with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. & := & \text{Diagram: } \text{An orange circle labeled } 1 \text{ with two input wires labeled } \vdots \text{ and two output wires labeled } \vdots. \end{array}$$

ZX calculus – Notations

$$\begin{array}{ccc} \textcolor{red}{\circlearrowleft} \chi_x & \xrightarrow{\llbracket \cdot \rrbracket} & |x\rangle \\ \textcolor{green}{\circlearrowleft} \bar{\chi}_p & \xrightarrow{\llbracket \cdot \rrbracket} & |p\rangle \\ \textcolor{orange}{\circlearrowleft} \delta_n & \xrightarrow{\llbracket \cdot \rrbracket} & |n\rangle \end{array}$$

where $\chi_x(p) = e^{-i2\pi px}$, $\bar{\chi}_p(x) = e^{i2\pi px}$, and δ_n is the Kronecker delta at n .

ZX calculus – Notations

$$\chi_x \longrightarrow \xrightarrow{[\![\cdot]\!]} |x\rangle$$

$$\bar{\chi}_p \longrightarrow \xrightarrow{[\![\cdot]\!]} |p\rangle$$

$$\delta_n \longrightarrow \xrightarrow{[\![\cdot]\!]} |n\rangle$$

where $\chi_x(p) = e^{-i2\pi px}$, $\bar{\chi}_p(x) = e^{i2\pi px}$, and δ_n is the Kronecker delta at n .

$$\overbrace{\hspace{1cm}}^{\textstyle \Rightarrow m} := \overbrace{\hspace{1cm}}_{m \in \mathbb{R}} \xrightarrow{[\![\cdot]\!]} \int |mx\rangle_X \langle x|_X \, dx \quad (\text{MULTIPLIER})$$

The diagram shows a horizontal line with a brace underneath it labeled $m \in \mathbb{R}$. This is followed by a colon and an equals sign. Then there is another horizontal line with four components: a red oval labeled $i\hat{n}$, a green oval labeled $e^{i\pi \frac{\hat{x}^2}{m}}$, a red oval labeled $e^{i\pi m\hat{x}^2}$, and a green oval labeled $e^{i\pi \frac{\hat{x}^2}{m}}$. An arrow labeled $\xrightarrow{[\![\cdot]\!]}$ points from this sequence to an integral expression.

ZX calculus – Notations

W-node

$$\begin{array}{c} \text{Input node} := \text{Output node} \\ \text{Input node} := \text{Output node} \\ \text{Input node} := \pi^{-\frac{1}{4}} \text{ (Diagram)} \end{array}$$

The diagram for $\pi^{-\frac{1}{4}}$ shows a sequence of nodes: two green circles labeled $e^{\frac{-x^2}{2}}$, a red circle, a blue rectangle labeled $\sqrt{2}$, another green circle labeled $e^{\frac{x^2}{2}}$, and an orange circle labeled $2^{\frac{n}{2}}$. The first two green nodes are connected by a curved arrow pointing to the red node. The red node is connected to the blue rectangle. The blue rectangle is connected to the green node, which is then connected to the orange node.

$$\text{Control node} := \dots$$

The diagram for the control node shows a vertical ellipsis above a triangular node, which is then connected to a horizontal line. This is followed by a sequence of nodes: a blue triangle, a red triangle, a blue triangle, a red triangle, and so on, with ellipses indicating repetition.

ZX calculus – Notations

W-node

$$\begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array} := \quad \begin{array}{c} \delta_0 \text{---} \\ \text{---} \end{array}$$

$$\begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array} := \quad \begin{array}{c} \pi^{-\frac{1}{4}} \quad \begin{array}{c} e^{\frac{-\hat{x}^2}{2}} \\ \text{---} \\ e^{\frac{-\hat{x}^2}{2}} \end{array} \\ \text{---} \quad \text{---} \quad \text{---} \quad \text{---} \end{array}$$

$$\begin{array}{c} \vdots \quad \text{---} \\ \vdots \quad \text{---} \end{array} := \quad \begin{array}{c} \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \\ \text{---} \quad \text{---} \end{array}$$

$$k \vdots \quad \xrightarrow{[\![\cdot]\!]} \quad \sum_{n_1, \dots, n_k \geq 0} \sqrt{\frac{(\sum_i n_i)!}{\prod_i n_i!}} \left| \sum_i n_i \right\rangle \langle n_1, \dots, n_k \right|$$

ZX calculus – Rules 1

$$\begin{array}{c} \text{Diagram showing } f_1 \text{ and } f_2 \text{ as separate nodes with multiple inputs and outputs.} \\ \text{Diagram showing } f_1 f_2 \text{ as a single node with the same connections, indicating composition.} \end{array} =$$

$$\begin{array}{c} \text{Diagram showing two nodes } g_1 \text{ and } g_2 \text{ connected by multiple edges, each with a self-loop, followed by an equals sign and a single node } g_1g_2 \text{ with all edges merged into one.} \\ = \\ \end{array}$$

$$\begin{array}{c} \text{Diagram showing two configurations of strands merging at a point, separated by an equals sign.} \\ = \\ \text{Diagram showing the strands merged into a single bundle, followed by the label (FUSION).} \\ (\text{FUSION}) \end{array}$$

$$\chi_x \circ f = f(x)$$

$$\begin{array}{ccc} \text{Diagram A} & = & \text{Diagram B} \end{array}$$

$$\begin{array}{ccc} \delta_n & f & \vdots \\ \text{---} & \text{---} & \text{---} \end{array} = f(n) \begin{array}{c} \delta_n \\ \vdots \\ \delta_n \end{array} \quad (\text{COPY})$$

$$\begin{array}{c} \text{Diagram A} \\ \text{Diagram B} \end{array} = \begin{array}{c} \text{Diagram C} \\ \text{Diagram D} \end{array}$$

$$\begin{array}{c} \text{Diagram showing a node } \gamma_k \text{ with two incoming edges labeled } a \text{ and two outgoing edges labeled } \gamma_k. \\ = \\ \text{Diagram showing a node } \gamma_k \text{ with two incoming edges labeled } a \text{ and two outgoing edges labeled } \gamma_k, with internal connections between the incoming and outgoing edges. \end{array}$$

$$\begin{array}{c} \text{Diagram A} \\ \text{Diagram B} \end{array} = \begin{array}{c} \text{Diagram C} \\ \text{Diagram D} \end{array}$$

$$\text{---} \circ \text{---} = \text{---} \bullet \text{---} = \text{---} \square \text{---} = \text{---}$$

(IDENTITY)

ZX calculus – Rules 2

$$\text{---} \circ f = \text{---} \circ \mathcal{F}(f) \quad (\text{FOURIER})$$

$$\text{---} \circ f = \text{---} \circ \left(\int f(x) \psi_{\hat{n}}(x) dx \right) \quad (\text{HERMITE})$$

$$\begin{array}{c} \text{---} \circ r \circ f(\hat{x}) \circ r \circ r \\ \vdots \\ \text{---} \circ r \circ f(\hat{x}) \circ r \end{array} = \begin{array}{c} \vdots \\ \text{---} \circ f(r\hat{x}) \circ \vdots \end{array} \quad (\text{MULT})$$

$$\text{---} \circ r \circ s = \text{---} \circ r \cdot s \quad (\text{TIMES})$$

$$1 = \boxed{} \quad (\text{ONE})$$

ZX calculus – Rules 3

$$\text{---} \circ e^{i\theta \hat{n}} \text{---} = \text{---} \circ e^{i\pi \hat{x}^2 \tan \frac{\theta}{2}} \text{---} \circ e^{i\pi \hat{p}^2 \sin \theta} \text{---} \circ e^{i\pi \hat{x}^2 \tan \frac{\theta}{2}} \text{---} \quad (\text{EULER})$$

The diagram illustrates the equivalence between two circuit representations. On the left, four beam splitter gates are shown in series, each with parameters $\cos \hat{n}\theta$, $\sin \hat{n}\theta$, $-\sin \hat{n}\theta$, and $\cos \hat{n}\theta$. These correspond to the sequence of rotation gates on the right: $e^{i\pi \hat{x}^2 \tan \frac{\theta}{2}}$, $e^{i\pi \hat{p}^2 \sin \theta}$, and $e^{i\pi \hat{x}^2 \tan \frac{\theta}{2}}$.

The diagram shows the equivalence between a triforce gate (represented by three stacked triangles) and a single rotation gate $e^{i\pi \hat{x}^2 / 2}$. The triforce gate is labeled with $\pi^{-\frac{1}{4}}$.

Operators

Creation and annihilation operators $a^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$ and $a|n\rangle = \sqrt{n}|n-1\rangle$

$$\text{Diagram: } \begin{array}{c} \text{A green circle labeled } \delta_1 \text{ is connected to a vertex of a triangle. The other two vertices of the triangle are connected by a horizontal line.} \\ \xrightarrow{[\cdot]} \quad a^\dagger \end{array} \quad \text{and} \quad \begin{array}{c} \text{A red circle labeled } \delta_1 \text{ is connected to a vertex of a triangle. The other two vertices of the triangle are connected by a horizontal line.} \\ \xrightarrow{[\cdot]} \quad a \end{array}$$

Quadrature operators $\hat{x}|x\rangle = x|x\rangle$ and $\hat{p}|p\rangle = p|p\rangle$

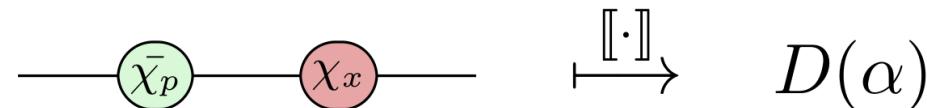
$$\text{Diagram: } \begin{array}{c} \text{A green circle labeled } \hat{x} \text{ is connected to a horizontal line.} \\ \xrightarrow{[\cdot]} \quad \hat{x} \end{array} \quad \text{and} \quad \begin{array}{c} \text{A red circle labeled } \hat{p} \text{ is connected to a horizontal line.} \\ \xrightarrow{[\cdot]} \quad \hat{p} \end{array}$$

Number operator $\hat{n}|n\rangle = n|n\rangle$

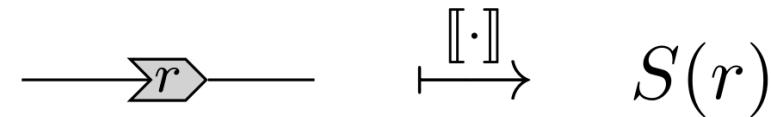
$$\text{Diagram: } \begin{array}{c} \text{A blue circle labeled } n \text{ is connected to a horizontal line.} \\ \xrightarrow{[\cdot]} \quad \hat{n} \end{array}$$

Gaussian gates – single mode

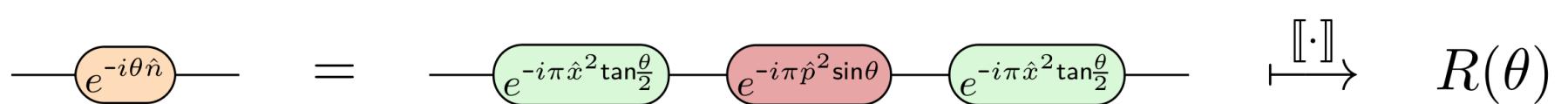
Displacement



Squeezing



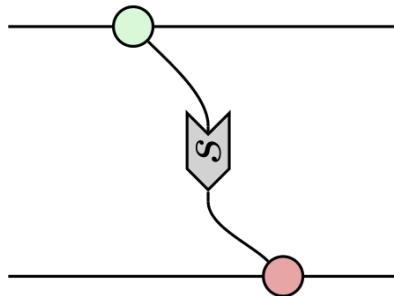
Rotation



Gaussian gates – two modes

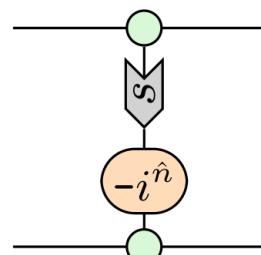
Controlled-X

$$CX(s) |x\rangle |y\rangle = |x\rangle |y + sx\rangle$$



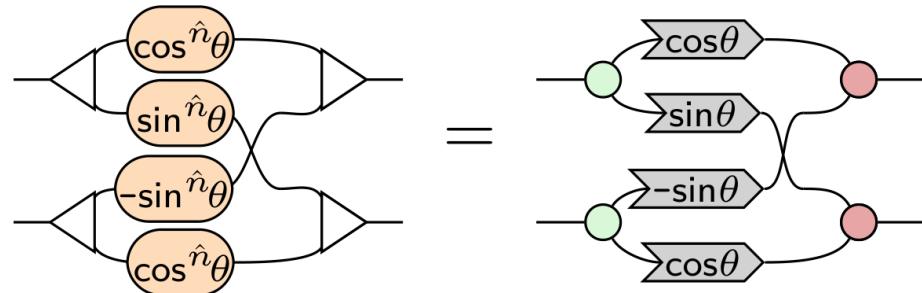
Controlled-phase

$$CZ(s) |x\rangle_X |y\rangle_X = e^{i2\pi sxy} |x\rangle_X |y\rangle_X$$



Beam splitter

$$B(\theta, \phi) = \exp\left(\theta(e^{i\phi}a_1a_2^\dagger - e^{-i\phi}a_1^\dagger a_2)\right)$$



Non-Gaussian gates

Cubic phase

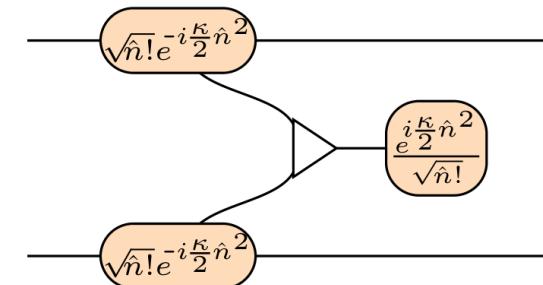
$$V(\gamma) = e^{i\gamma \hat{x}^3} = \text{---} \left(e^{i\gamma \hat{x}^3} \right) \text{---}$$

Kerr

$$K(\kappa) = e^{i\kappa \hat{n}^2} = \text{---} \left(e^{i\kappa \hat{n}^2} \right) \text{---}$$

Cross-Kerr

$$CK(\kappa) = e^{i\kappa \hat{n}_1 \hat{n}_2} =$$



Plan

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 - Generators, rules and common gates
 - Gaussian completeness
3. Applications
 - Quantum Error Correction with the GKP code
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4. Conclusion and future work

Completeness for the Gaussian fragment

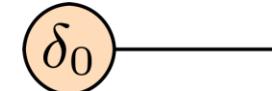
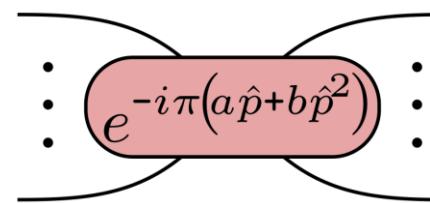
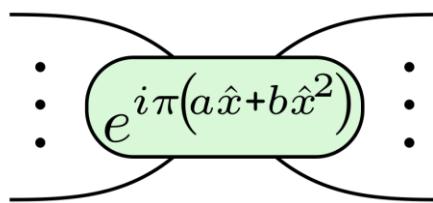
Completeness for the Gaussian fragment

Universal CV gate set: Gaussian gates + 1 non-Gaussian gate

Completeness for the Gaussian fragment

Universal CV gate set: Gaussian gates + 1 non-Gaussian gate

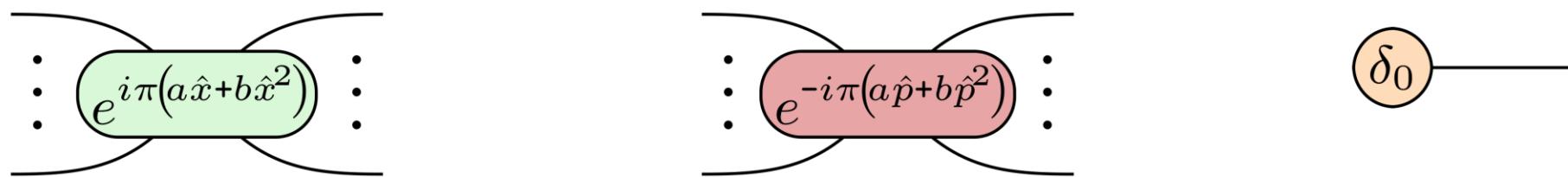
ZX Gaussian fragment:



Completeness for the Gaussian fragment

Universal CV gate set: Gaussian gates + 1 non-Gaussian gate

ZX Gaussian fragment:



Theorem 4.6. ZX_G is complete for the Gaussian fragment of CVQC: For any two diagrams D_1 and D_2 in ZX_G , if $\llbracket D_1 \rrbracket = \llbracket D_2 \rrbracket$, then $\text{ZX}_G \vdash D_1 = D_2$.

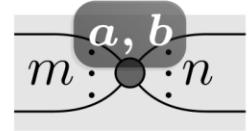
Completeness via translation

Graphical symplectic algebra – (Booth, Carette, Comfort 2024)

Completeness via translation

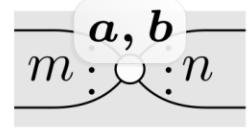
Graphical symplectic algebra – (Booth, Carette, Comfort 2024)

GSA generators:



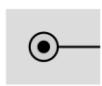
$\boxed{\cdot}$

$$\left\{ \left(\begin{bmatrix} \underline{z} \\ x \\ \vdots \\ m \\ x \end{bmatrix}, \begin{bmatrix} \underline{z}' \\ x \\ \vdots \\ n \\ x \end{bmatrix} \right) \middle| \begin{array}{l} \underline{z} \in \mathbb{R}^m, \underline{z}' \in \mathbb{R}^n, x \in \mathbb{R} \text{ such that} \\ \sum_{j=0}^{m-1} z_j - \sum_{k=0}^{n-1} z'_k + bx = a \end{array} \right\}$$



$\boxed{\cdot}$

$$\left\{ \left(\begin{bmatrix} z \\ \vdots \\ m \\ z \\ \underline{x} \end{bmatrix}, \begin{bmatrix} -z \\ \vdots \\ n \\ -z \\ \underline{x}' \end{bmatrix} \right) \middle| \begin{array}{l} \underline{x} \in \mathbb{R}^m, \underline{x}' \in \mathbb{R}^n, z \in \mathbb{R} \text{ such that} \\ \sum_{j=0}^{m-1} x_j + \sum_{k=0}^{n-1} x'_k - bz = a \end{array} \right\}$$



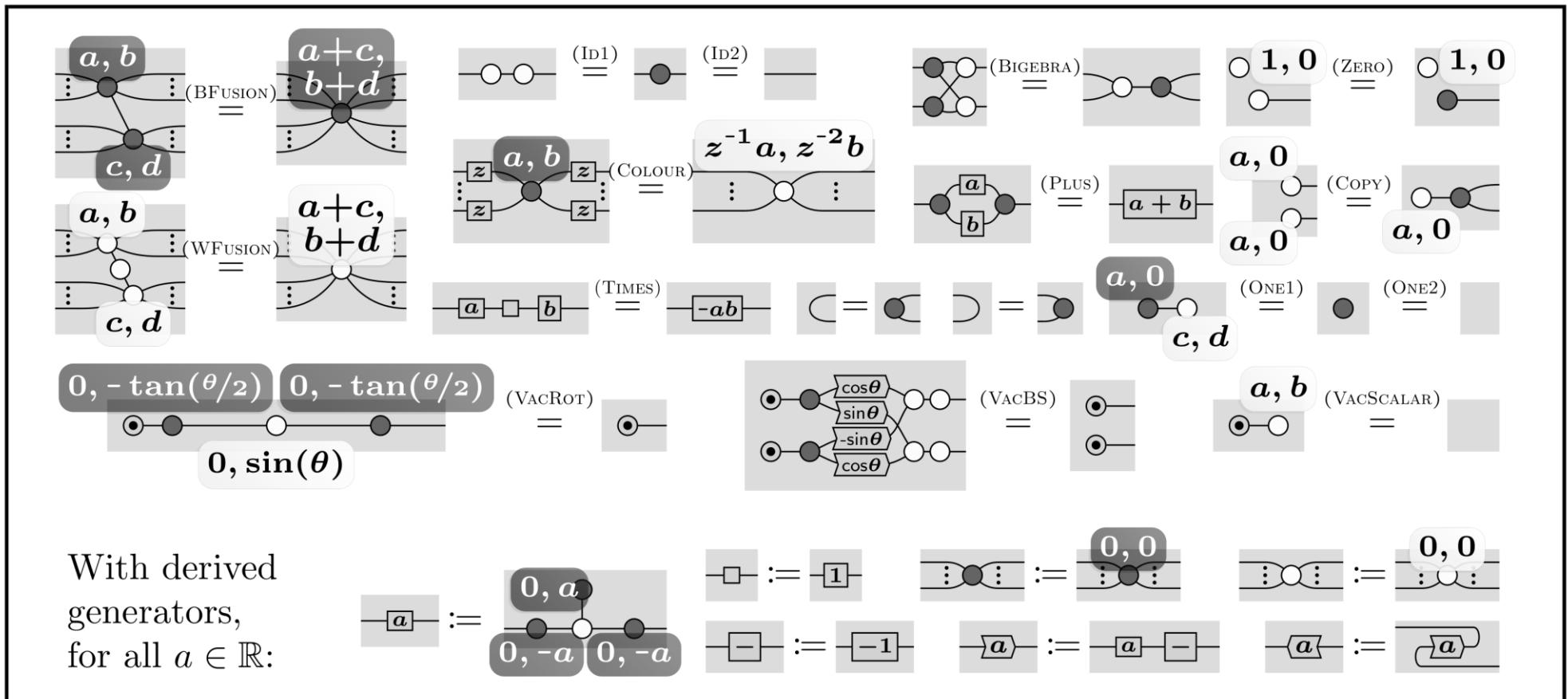
$\boxed{\cdot}$

$$\left\{ \left(\bullet, \begin{bmatrix} ix \\ x \end{bmatrix} \right) \middle| x \in \mathbb{R} \right\}$$

Completeness via translation

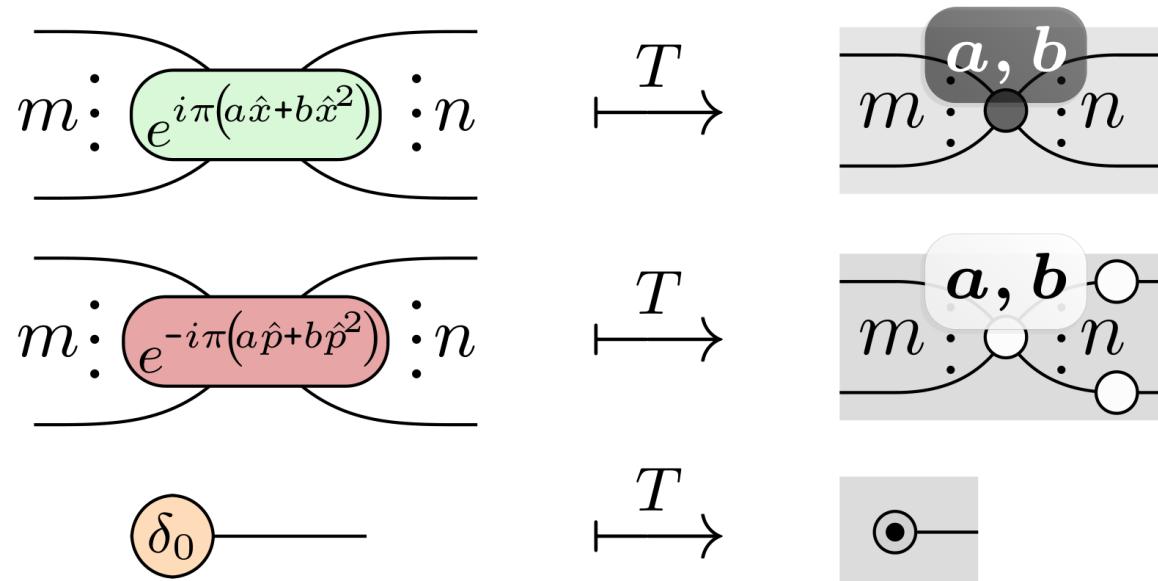
Graphical symplectic algebra – (Booth, Carette, Comfort 2024)

GSA rules:



Completeness via translation

Invertible translation functor $T : \text{ZX}_G \rightarrow \text{GSA}$



Completeness via translation

All GSA rules are derivable

Proposition 4.5. *For diagrams D_1 and D_2 in GSA, if $\text{GSA} \vdash D_1 = D_2$, then $\text{ZX}_G \vdash T^{-1}(D_1) = T^{-1}(D_2)$.*

Proof. By the functoriality of T^{-1} , it is sufficient to show that all the axioms of GSA (Figure 3) are derivable in ZX_G . The table below summarizes the proofs for each rule.

GSA rule	Follows from
(BFusion)	(FUSION)
(WFusion)	(FUSION)
(Id)	(IDENTITY)
(Bialgebra)	(BIALGEBRA)
(Zero)	Lemma C.1 & 2.2
(Colour)	(X-SPIDER) & (MULT)
(Plus)	(PLUS)
(Copy)	(COPY)
(Times)	Lemma C.2
(One)	Lemma C.3 & C.4
(VacRot)	Lemma C.5
(VacBS)	Lemma C.6
(VacScalar)	Lemma C.7

□

Completeness via translation

Theorem 4.6. ZX_G is complete for the Gaussian fragment of $CVQC$: For any two diagrams D_1 and D_2 in ZX_G , if $\llbracket D_1 \rrbracket = \llbracket D_2 \rrbracket$, then $\text{ZX}_G \vdash D_1 = D_2$.

Plan

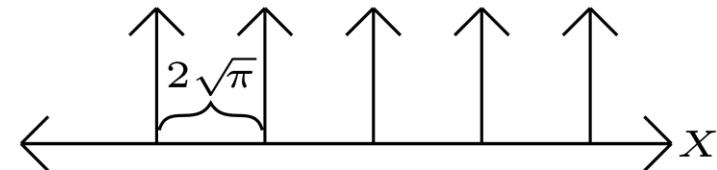
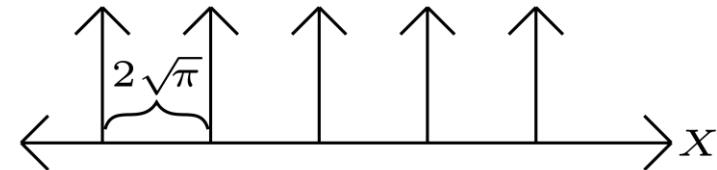
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Quantum Error Correction with the GKP code

Encoding a qubit in an oscillator:

$$|0_L\rangle := \sum_{k \in \mathbb{Z}} |2k\sqrt{\pi}\rangle_X$$

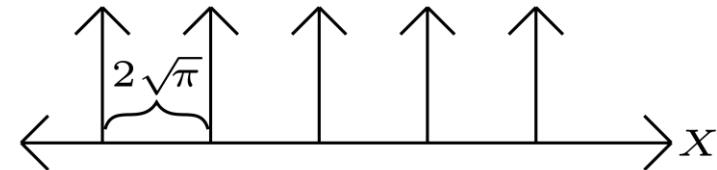
$$|1_L\rangle := \sum_{k \in \mathbb{Z}} |(2k + 1)\sqrt{\pi}\rangle_X$$



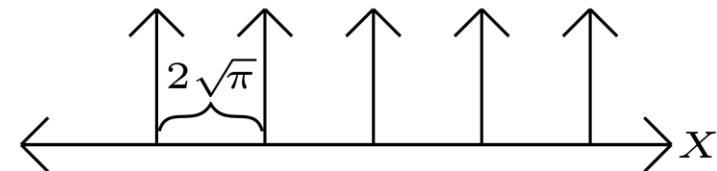
Quantum Error Correction with the GKP code

Encoding a qubit in an oscillator:

$$|0_L\rangle := \sum_{k \in \mathbb{Z}} |2k\sqrt{\pi}\rangle_X$$



$$|1_L\rangle := \sum_{k \in \mathbb{Z}} |(2k + 1)\sqrt{\pi}\rangle_X$$



$$|0_L\rangle = \textcircled{0}_L \text{---}$$

$$|1_L\rangle = \textcircled{1}_L \text{---}$$

where $0_L(x) = \sum_{k \in \mathbb{Z}} \delta(x - 2k\sqrt{\pi})$ and $1_L(x) = \sum_{k \in \mathbb{Z}} \delta(x - (2k + 1)\sqrt{\pi})$.

Quantum Error Correction with the GKP code

Encoding a qubit in an oscillator – finite squeezing

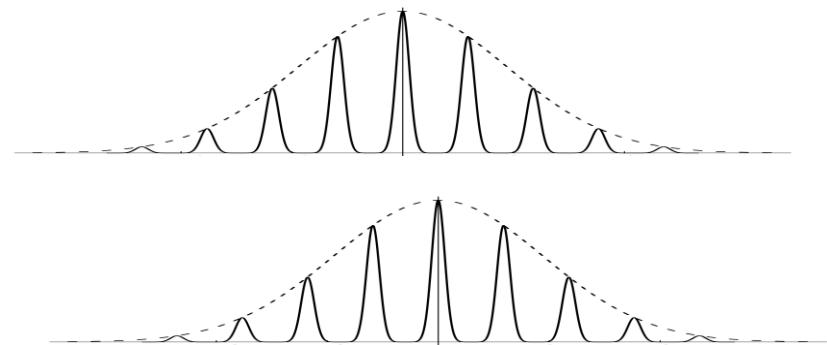
Quantum Error Correction with the GKP code

Encoding a qubit in an oscillator – finite squeezing

$$|\tilde{0}_L\rangle = \int \sum_{k \in \mathbb{Z}} e^{-2\pi\Delta^2 k^2} e^{-\frac{x^2}{2\Delta^2}} |x + 2k\sqrt{\pi}\rangle dx$$

$$|\tilde{1}_L\rangle = \int \sum_{k \in \mathbb{Z}} e^{-2\pi\Delta^2 k^2} e^{-\frac{x^2}{2\Delta^2}} |x + (2k+1)\sqrt{\pi}\rangle dx$$

where Δ is the width of the Gaussian



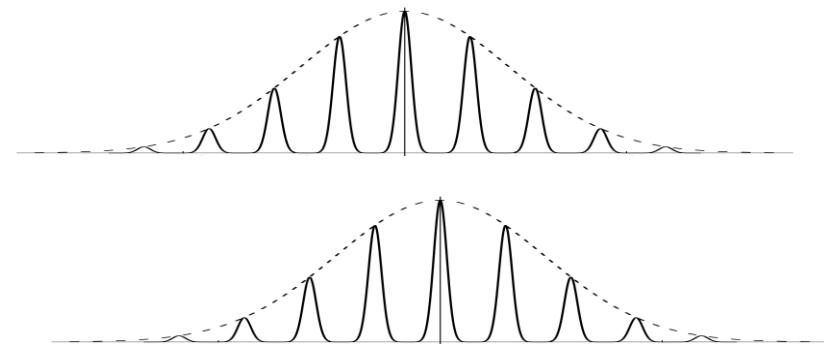
Quantum Error Correction with the GKP code

Encoding a qubit in an oscillator – finite squeezing

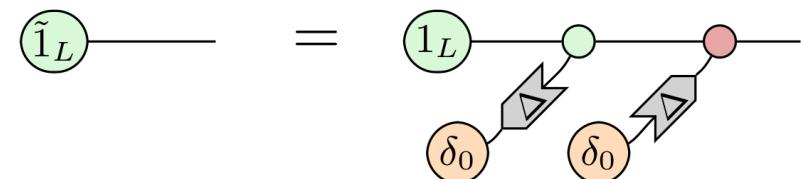
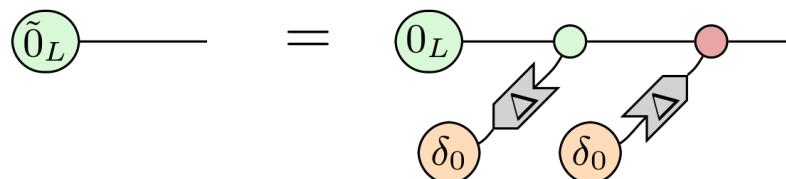
$$|\tilde{0}_L\rangle = \int \sum_{k \in \mathbb{Z}} e^{-2\pi\Delta^2 k^2} e^{-\frac{x^2}{2\Delta^2}} |x + 2k\sqrt{\pi}\rangle dx$$

$$|\tilde{1}_L\rangle = \int \sum_{k \in \mathbb{Z}} e^{-2\pi\Delta^2 k^2} e^{-\frac{x^2}{2\Delta^2}} |x + (2k + 1)\sqrt{\pi}\rangle dx$$

where Δ is the width of the Gaussian



Proposition 5.1.

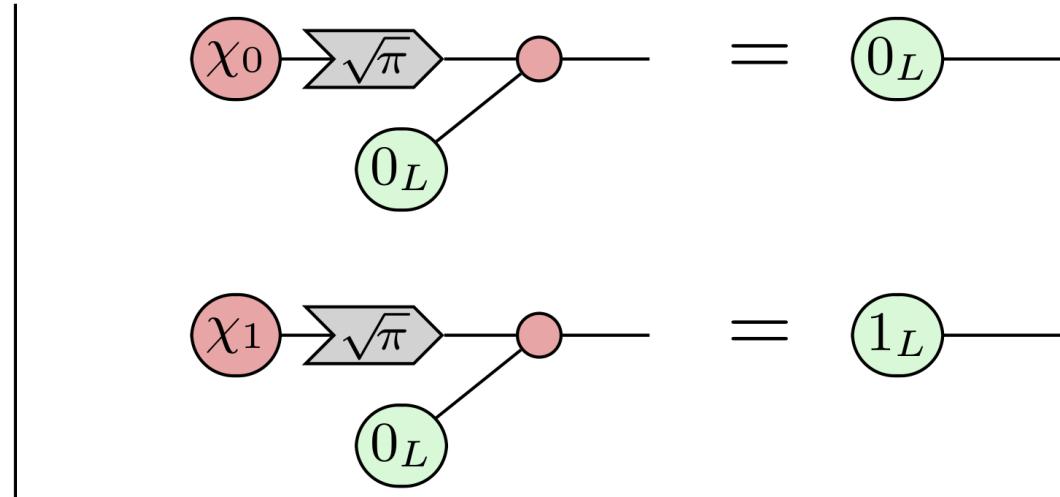
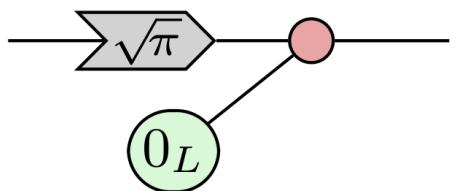


GKP Encoder

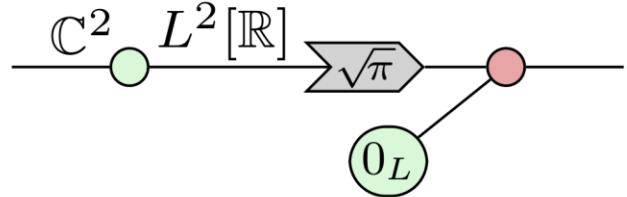
$$0_L - \chi_{\sqrt{\pi}} - = 1_L -$$

GKP Encoder

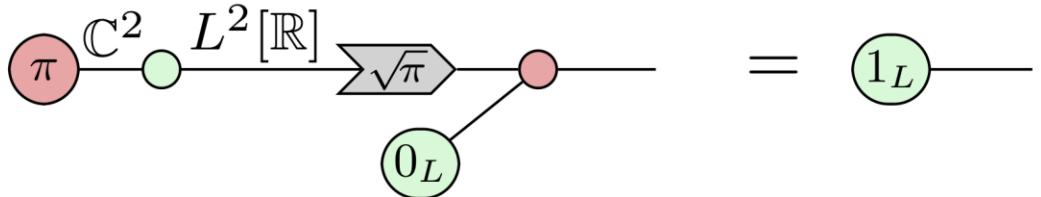
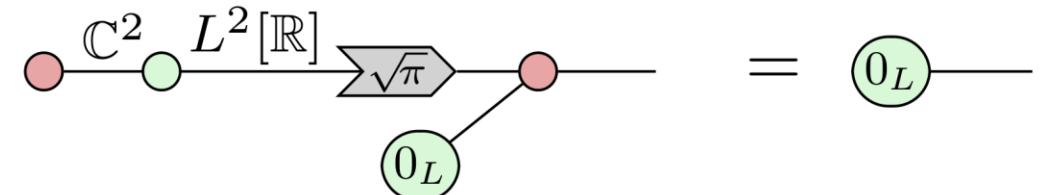
$$0_L \xrightarrow{\chi_{\sqrt{\pi}}} = 1_L$$



GKP Encoder



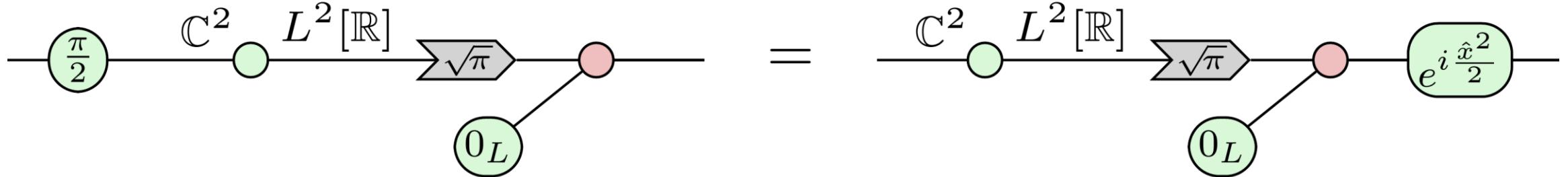
|



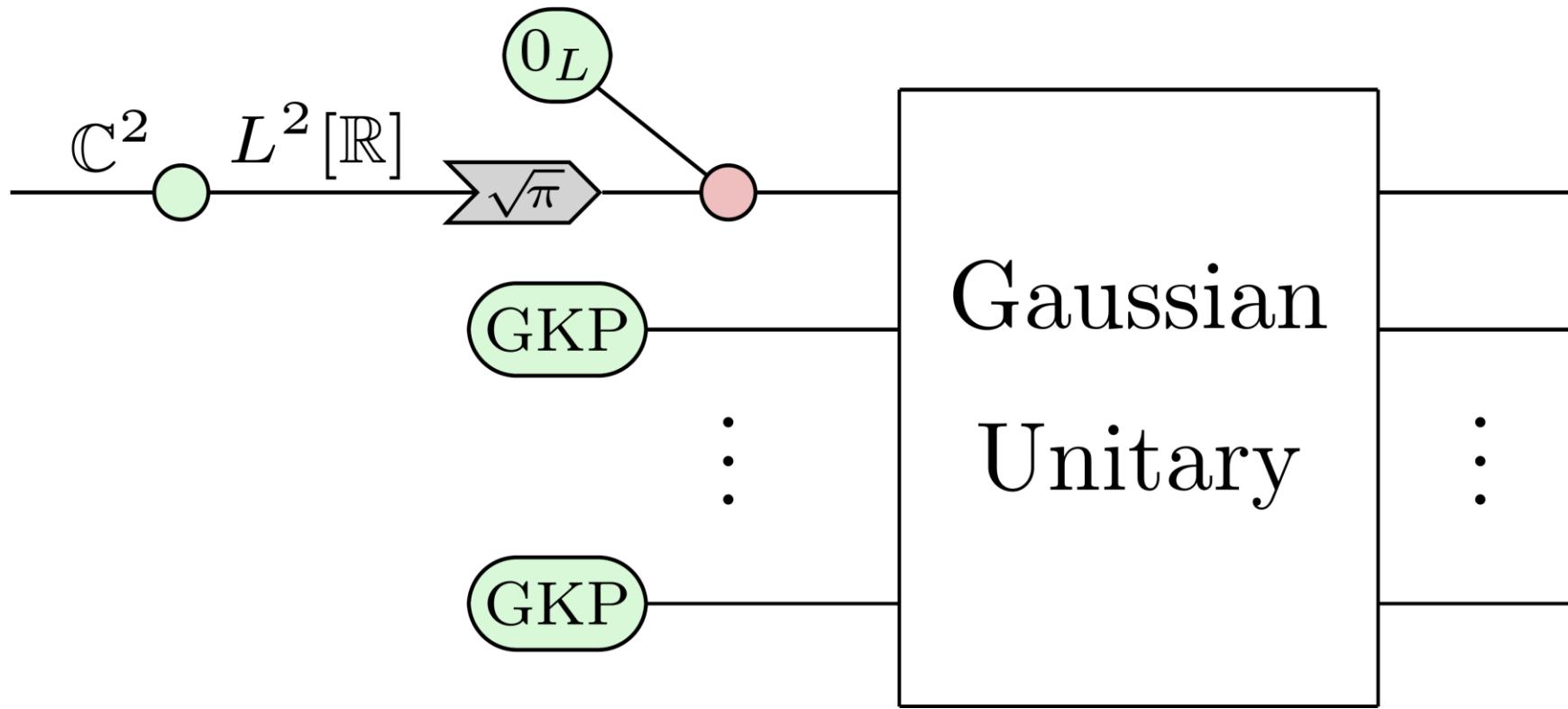
Mixed-dimensional Z spider sends $|0\rangle$ to $|0\rangle_X$ and $|1\rangle$ to $|1\rangle_X$

Logical operators by pushing-through-the-encoder

Example: $\frac{\pi}{2}$ Z rotation (or S gate)

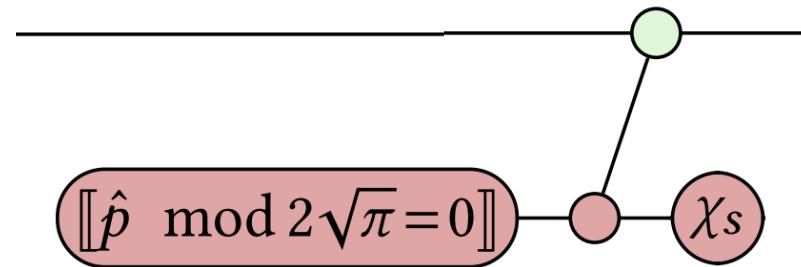


General GKP encoder

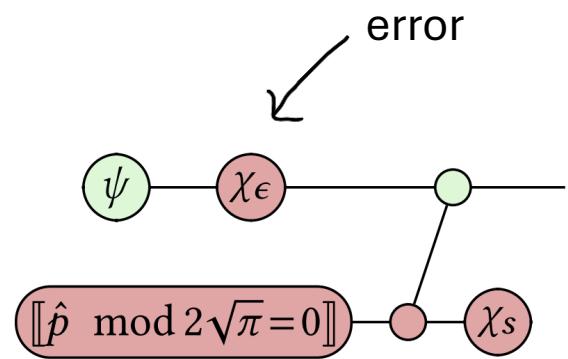


Error detection and correction

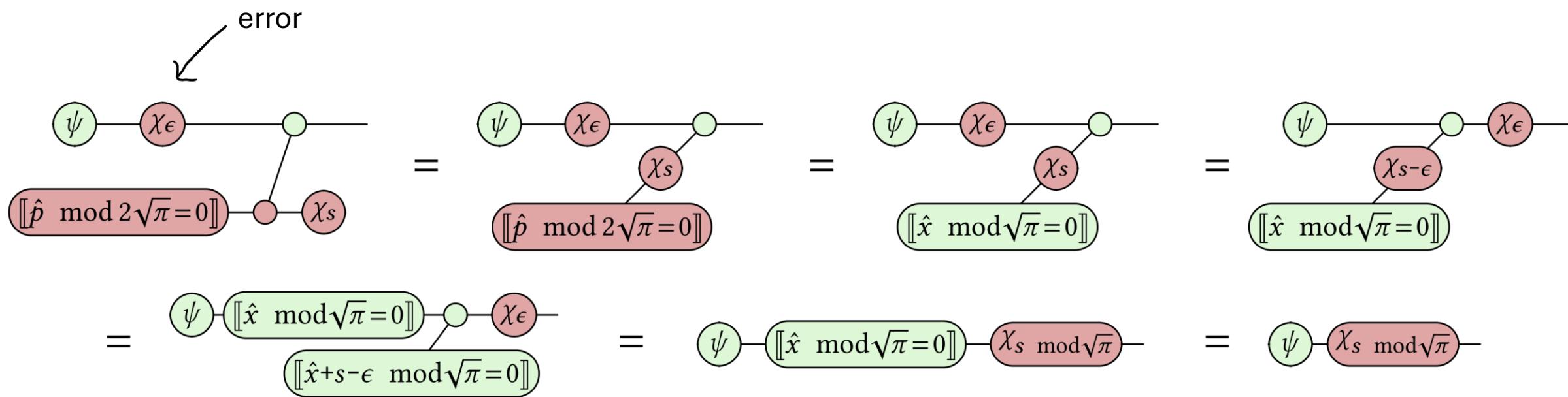
Syndrome measurement circuit:



Error detection and correction



Error detection and correction



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Gaussian boson sampling

- Non-universal model of quantum computation
- Samples from #P-hard problem
- Experimental demonstrations of quantum advantage

We prove hardness of gaussian boson sampling diagrammatically

Gaussian boson sampling

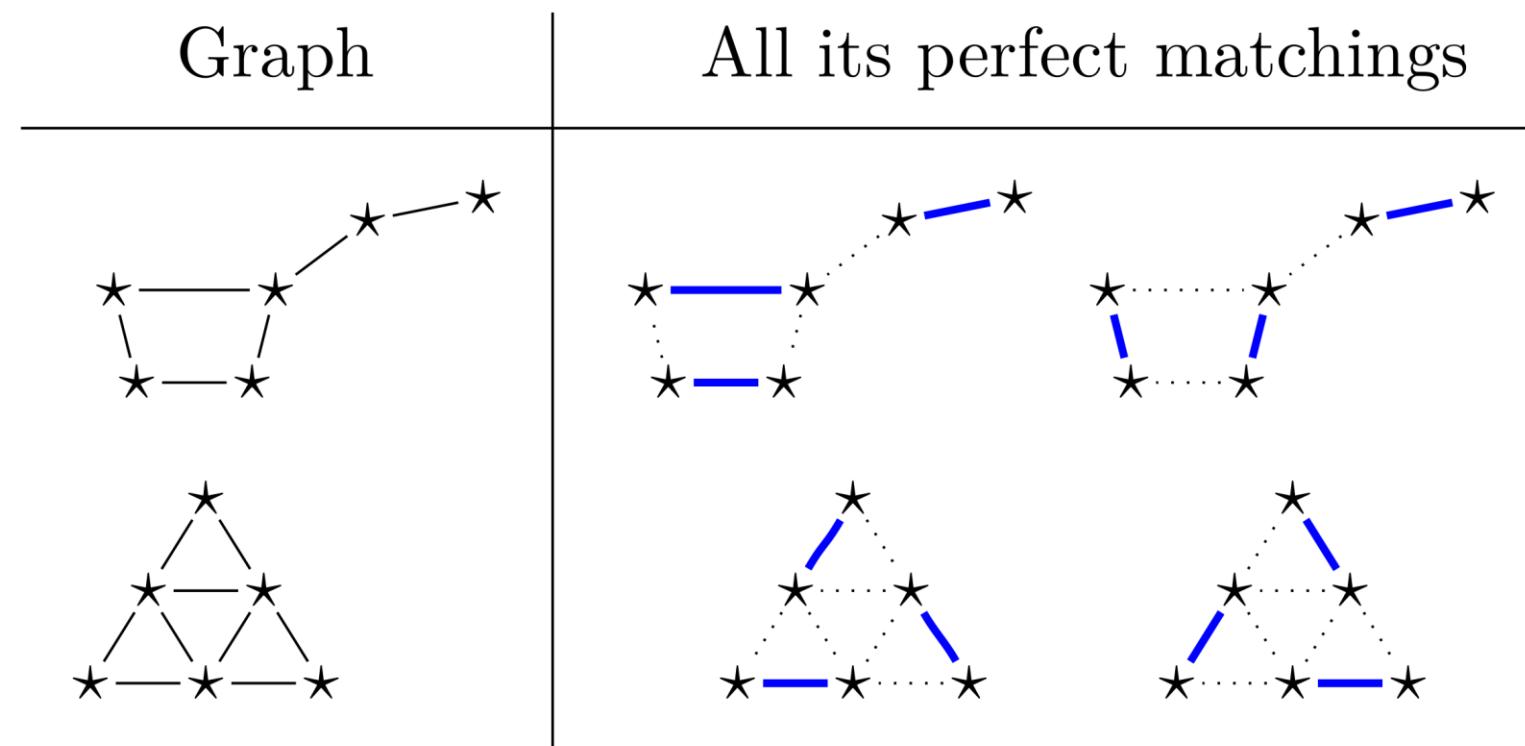
$$\langle n_1, \dots, n_s | \psi \rangle \propto \text{Haf} \left(U \bigoplus_{i=1}^s \tanh(r_i) U^T \right)_{\text{sub}}$$

where Haf - hafnian function

U - interferometer matrix

r_i - squeezing parameters

Perfect matchings of graphs

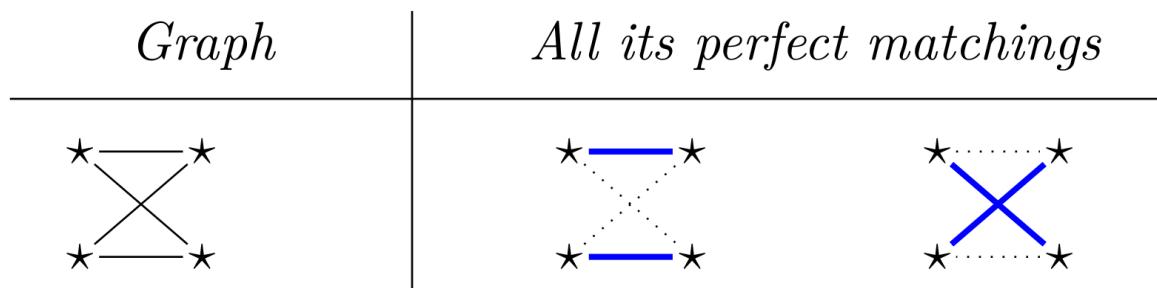


Hafnian is the (weighted) sum of perfect matchings of a graph

Perfect matchings using W-node

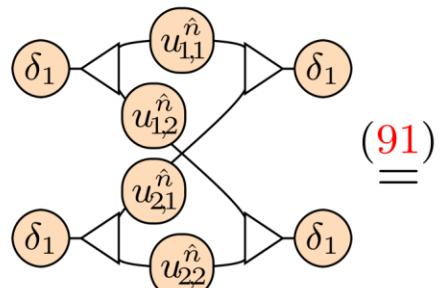
$$\text{W-node} = \begin{array}{c} \delta_0 \\ \delta_1 \\ \hline \end{array} + \begin{array}{c} \delta_1 \\ \delta_0 \\ \hline \end{array}$$

$$\begin{array}{c} \delta_1 \\ \delta_1 \\ \hline \end{array} + \begin{array}{c} \delta_1 \\ \delta_1 \\ \hline \end{array} = \begin{array}{c} \delta_1 & \delta_1 \\ \delta_0 & \delta_0 \\ \hline \end{array} + \begin{array}{c} \delta_0 & \delta_0 \\ \delta_1 & \delta_1 \\ \hline \end{array} = 1 + 1 = 2$$



Perfect matchings using W-node

$$\delta_1 \text{---} \begin{cases} \delta_0 \\ \delta_1 \end{cases} = \begin{cases} \delta_0 \\ \delta_1 \end{cases} + \begin{cases} \delta_1 \\ \delta_0 \end{cases}$$



$\stackrel{(91)}{=}$

$$\sum_{a,b,c,d=0}^1 \begin{cases} \delta_a & \delta_c \\ \delta_{\neg a} & \delta_{\neg c} \\ \delta_b & \delta_d \\ \delta_{\neg b} & \delta_{\neg d} \end{cases} = \begin{cases} \delta_1 & \delta_1 \\ \delta_0 & \delta_0 \\ \delta_1 & \delta_1 \\ \delta_0 & \delta_0 \end{cases} + \begin{cases} \delta_0 & \delta_0 \\ \delta_1 & \delta_1 \\ \delta_1 & \delta_1 \\ \delta_0 & \delta_0 \end{cases} = u_{1,1}u_{2,2} + u_{1,2}u_{2,1}$$

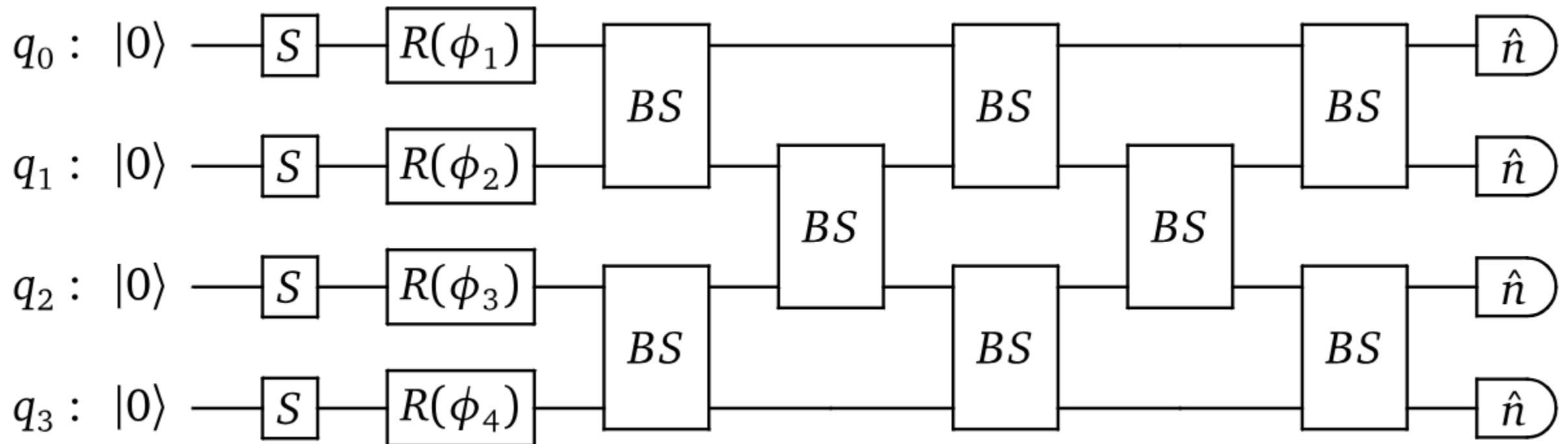
Perfect matchings using W-node

Proposition 6.2. *For a weighted adjacency matrix A of a graph with s vertices,*

$$\begin{array}{ccc} \text{Diagram showing a weighted adjacency matrix } A \text{ with } s \text{ rows and columns, and two orange nodes labeled } \delta_1 \text{ connected by a path.} & = & \text{haf}(A) \end{array}$$

Proof is by induction

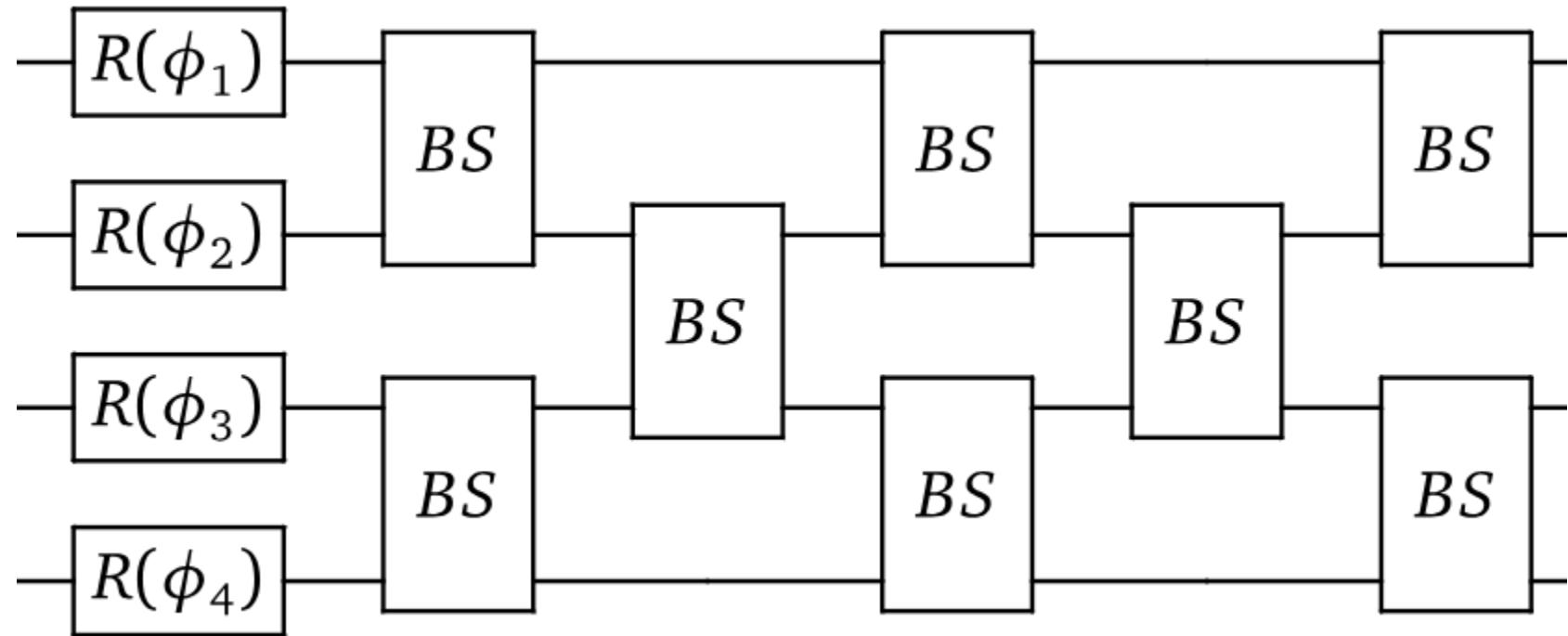
Gaussian boson sampling



An example 4-mode Gaussian boson sampling circuit

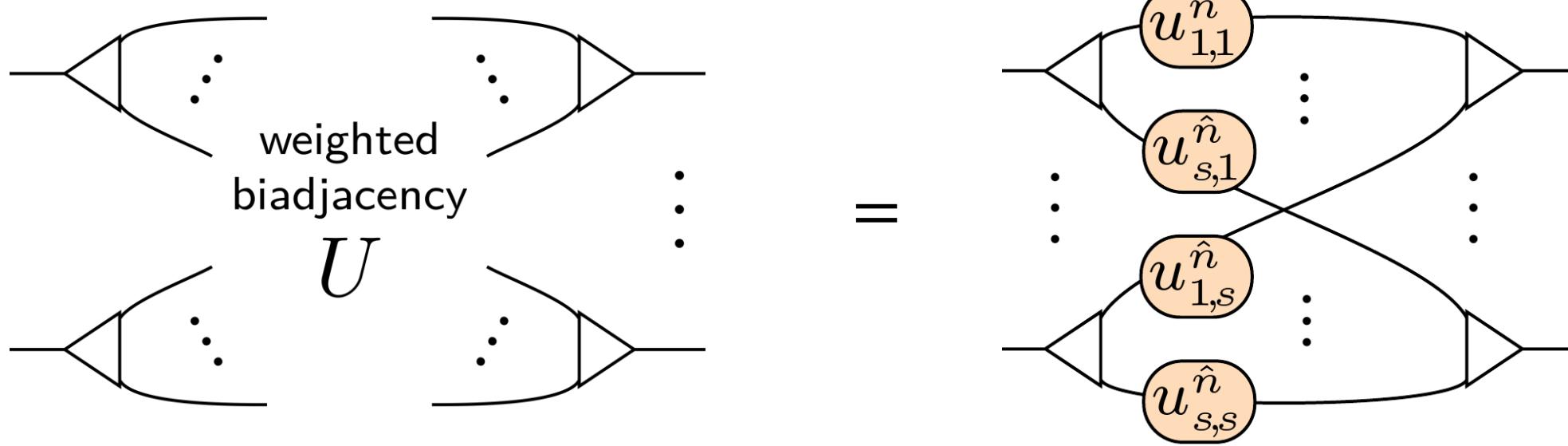
Figure taken from: https://strawberryfields.ai/photonics/demos/run_gaussian_boson_sampling.html

Gaussian boson sampling



An example 4-mode interferometer

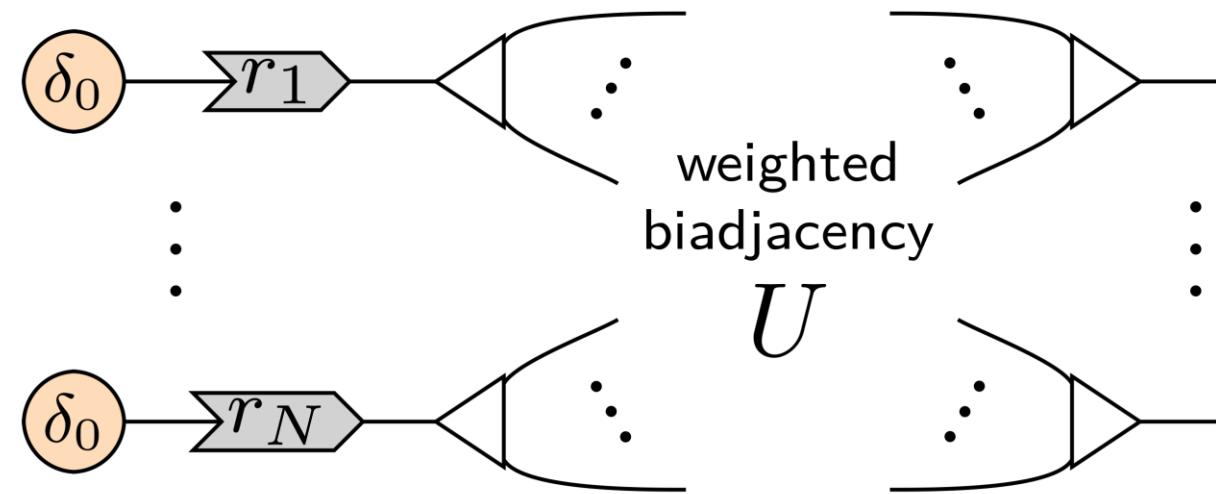
Gaussian boson sampling



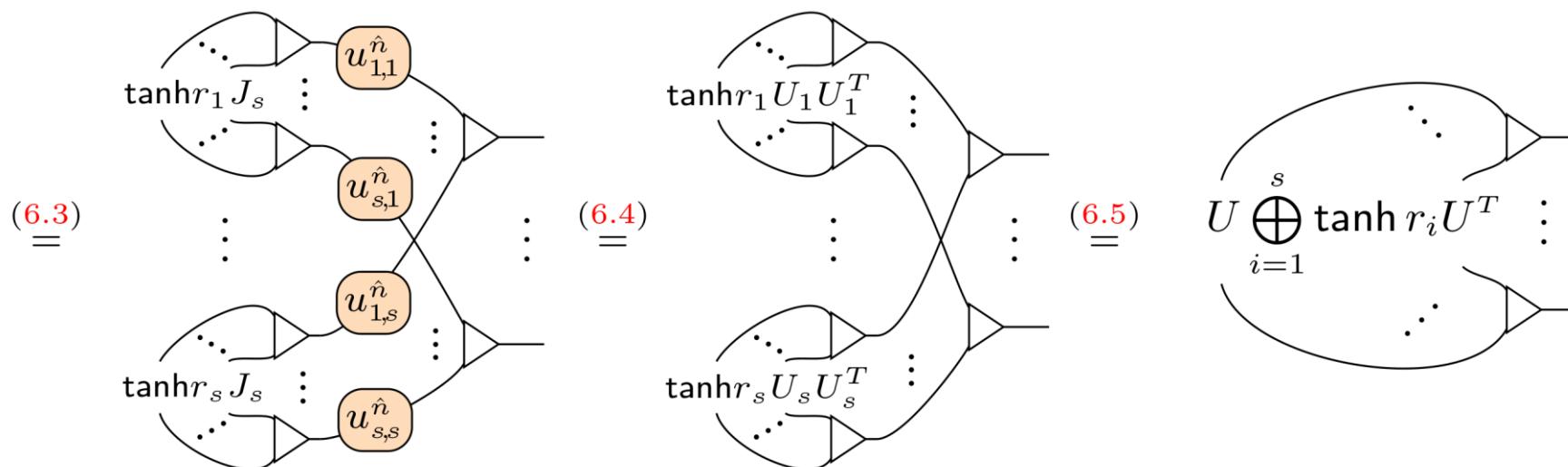
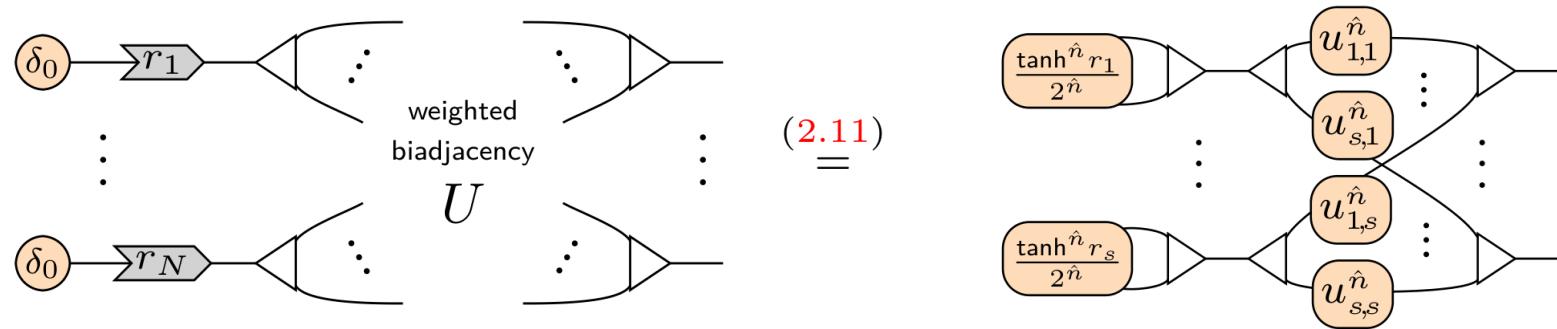
Interferometer normal form

(de Felice et. al 2022), (Bonchi et. al. 2014)

Normal form for Gaussian boson sampling

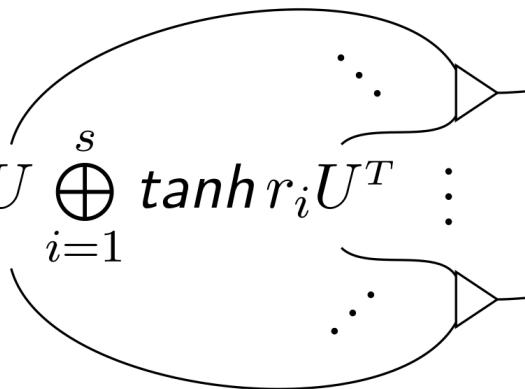


Normal form for Gaussian boson sampling



Normal form for Gaussian boson sampling

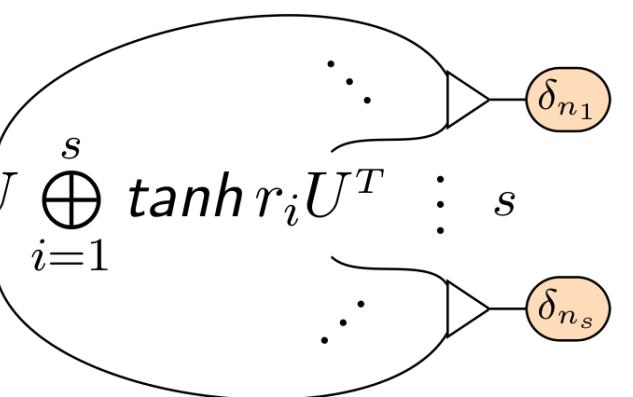
Theorem 6.6. *The circuit of Gaussian boson sampling can be reduced to the following normal form:*

$$\left(\prod_{i=1}^s \frac{1}{\sqrt{\cosh r_i}} \right) U \bigoplus_{i=1}^s \tanh r_i U^T$$


where U is the matrix of the interferometer, r_i represents the amount of squeezing

Sampling amplitude

Theorem 6.12. *The amplitude of observing $n_1, \dots, n_s \in \mathbb{N}$ photons in the Gaussian boson sampling circuit is*

$$\left(\prod_{i=1}^s \frac{1}{\sqrt{\cosh r_i}} \right) U \left(\bigoplus_{i=1}^s \tanh r_i U^T \right)^s = \prod_{i=1}^s \frac{1}{\sqrt{n_i \cosh r_i}} \text{haf} \left[U \left(\bigoplus_{i=1}^s \tanh r_i U^T \right) \right]_{\text{sub}}$$


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Conclusion

Conclusion

Generalized ZX calculus to infinite dimensions

- With continuous and discrete generators
- Complete for the Gaussian fragment

Conclusion

Generalized ZX calculus to infinite dimensions

- With continuous and discrete generators
- Complete for the Gaussian fragment

Graphical analysis of

- GKP quantum error correction
- Gaussian Boson sampling

Future work

Future work

ZX-based algorithms for

- Compiling and circuit optimization
- Classical simulation
- MBQC with hybrid-CV cluster states

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Completeness

- Gaussian completeness for the Fock-W fragment
- Completeness for the approximately universal fragment

Future work

ZX-based algorithms for

- Compiling and circuit optimization
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Completeness

- Gaussian completeness for the Fock-W fragment
- Completeness for the approximately universal fragment

Quantum error correction

- Pauli-webs and floquetification for GKP
- Multi-mode GKP code and concatenated GKP code
- Cat codes and binomial codes