GLM assumptions and diagnostics

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Statistics lecture 4

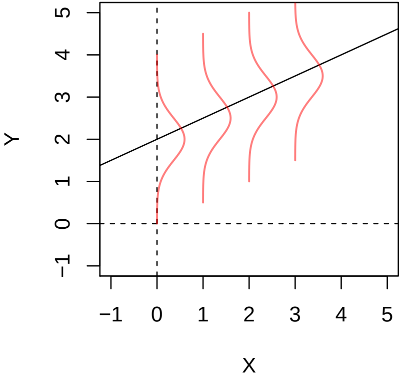
Outline

1. GLM assumptions
   - Unbiasedness
   - Normality
   - Homoscedasticity
   - Independence
2. Transformations
3. Practical problems
   - Outliers
   - Multicollinearity
4. Polynomial regression

General Linear Model assumptions

$$Y_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_{p-1} X_{p-1,i} + \epsilon_i \qquad \epsilon_i \sim N(0, \sigma)$$

- Normality: ϵ_i is Normally distributed
- Unbiasedness: the mean of ϵ_i is 0 (the model predictions are unbiased)
- Homoscedasticity: ϵ_i has constant variance σ^2
- Independence: ϵ_i is independent of ϵ_j (for all i,j)



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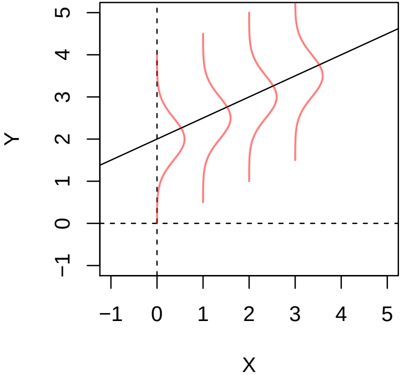

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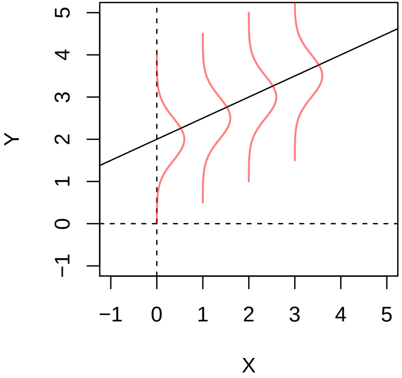

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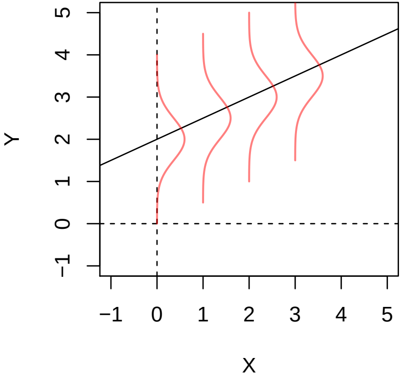

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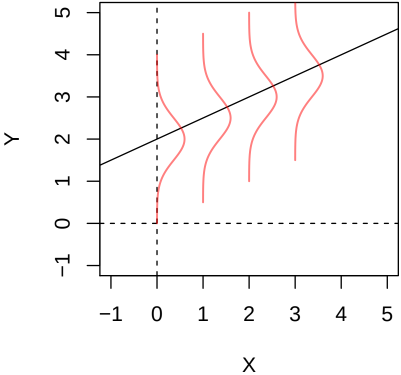
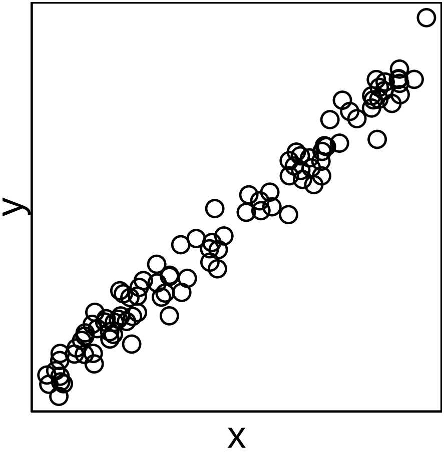


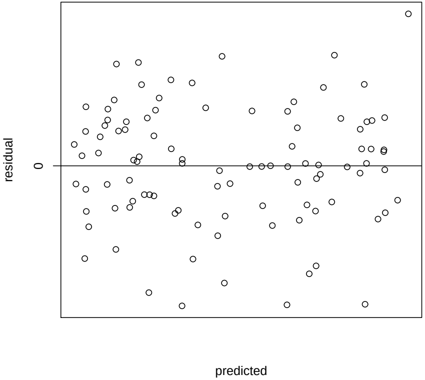
Useful graphs: Scatterplot

- Useful to assess bivariate linearity (unbiased model predictions)
- Can be misleading when there are multiple predictors in the model



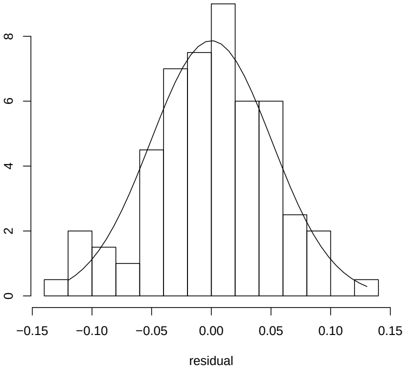
Useful graphs: Predicted vs residual

- There should be no relation between predictions and error (residuals)
- Useful to assess unbiasedness and homoscedasticity



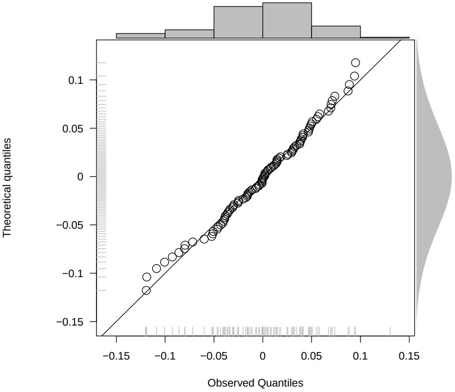
Useful graphs: Histogram of residuals

- Should look like a normal distribution



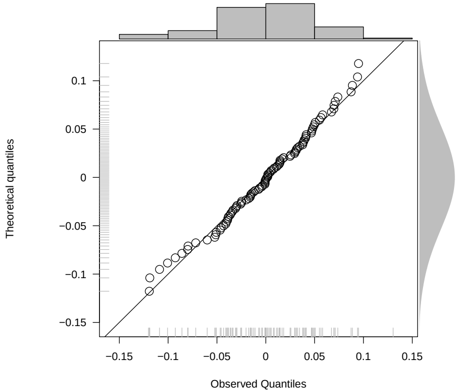
Useful graphs: Quantile-Quantile (QQ)

- A quantile is the value of a variable such that a certain percentage of the distribution has values equal to or smaller than it
  - e.g., the 25% quantile is a value y such that $p(Y \leq y) = .25$



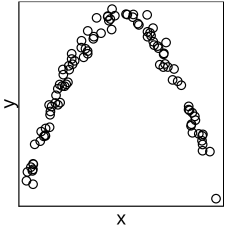
Useful graphs: Quantile-Quantile (QQ)

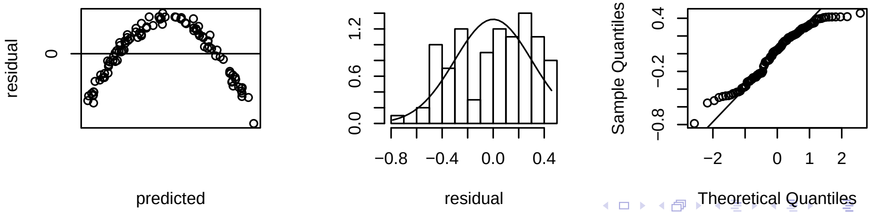
- A Q-Q plot compares *observed* quantiles to *theoretical* quantiles
  - For each Y_i, estimate $\hat{p}_i \approx p(Y \leq Y_i)$ as the proportion of values that are equal to or smaller than Y_i
  - Use a standard Normal distribution to determine Q_i such that $p(Y \leq Q_i) = \hat{p}_i$
  - Plot Y_i against Q_i. If the distribution is Normal, they should roughly lie on straight line.



Unbiasedness

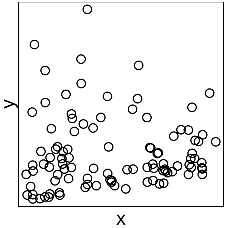
- Assess
  - Predicted-residual plot (and scatterplots)
- If violated
  - Biased predictions
- Remedies
  - Transform predictors
  - Polynomial regression
  - Use alternative model (e.g., nonlinear or nonparametric regression)

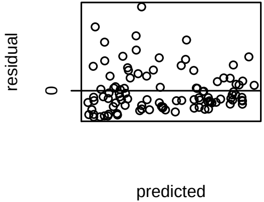


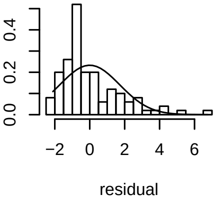


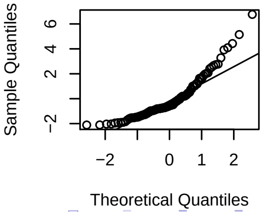
Normality

- Assess
  - Q-Q plot, histogram
  - Tests (Shapiro-Wilk, Kolmogorov-Smirnov)
- If violated
  - Biased test results
- Remedies
  - Transform dependent variable



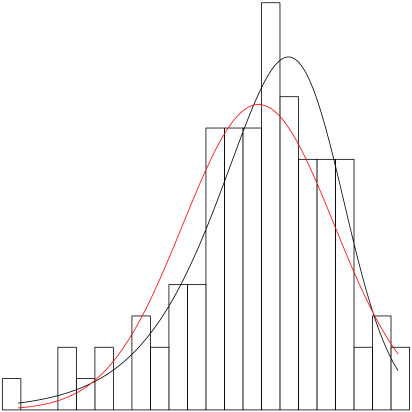


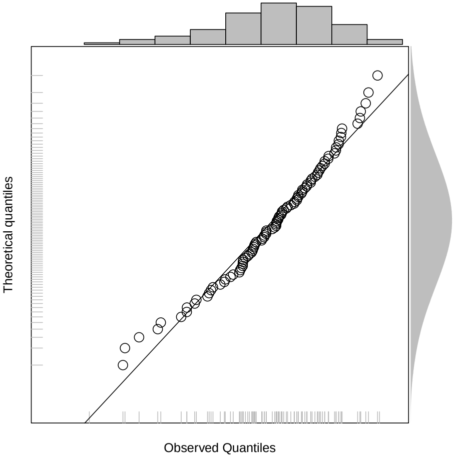




Examples of non-normal distributions

Negatively (left) skewed distribution





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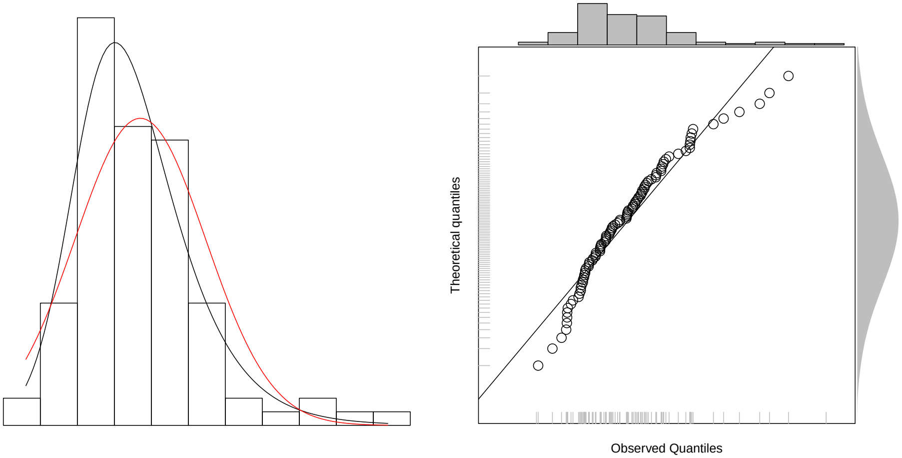
Positively (right) skewed distribution



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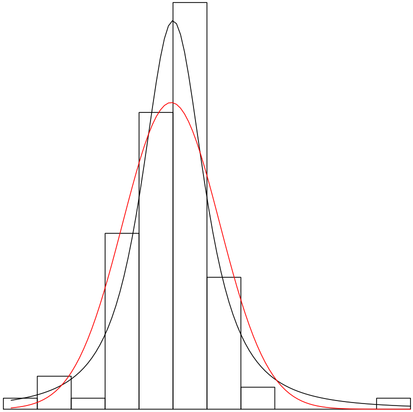
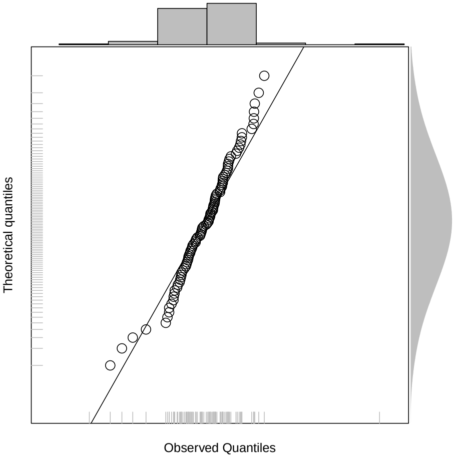
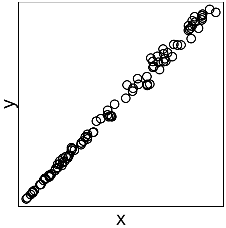
Heavy-tailed distribution

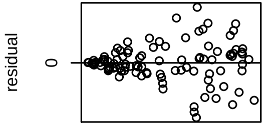


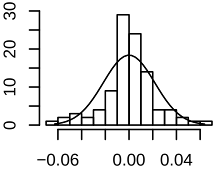


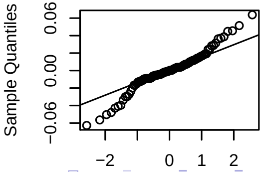
Homoscedasticity



- Assess
  - Predicted-residual plot
  - Breusch-Pagan or Koenker test
  - Levene test (for grouped data)
- Violation (heteroscedasticity)
  - unbiased parameter estimates
  - biased test results
- Remedies
  - Weighted least squares estimation
  - Transform dependent variable

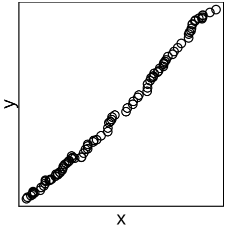


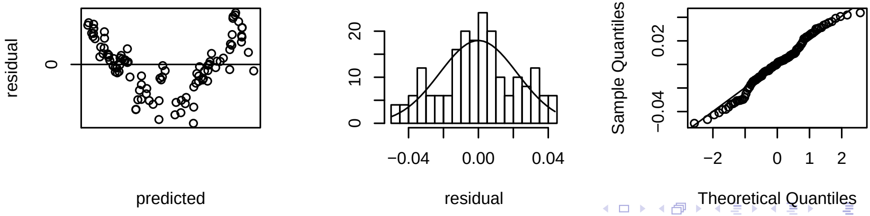




Independence

- Assess
  - A priori (by design)
  - Sequential dependence (for ordered data):
    - Predicted-residual plot
    - Durbin-Watson test (sequential dependence)
- Violation (dependent errors)
  - model mis-specification
- Remedies
  - Repeated measures/multilevel analysis





Transformations

- Why transform?
  - achieve unbiasedness (linearity)
  - achieve homoscedasticity
  - achieve normality (or symmetry about the regression line)
- Can transform both dependent variable and predictors
  - start with dependent to achieve homoscedasticity/normality
  - transform predictors to achieve unbiasedness (linearity)
- Transforming dependent variable changes the distribution of the errors!

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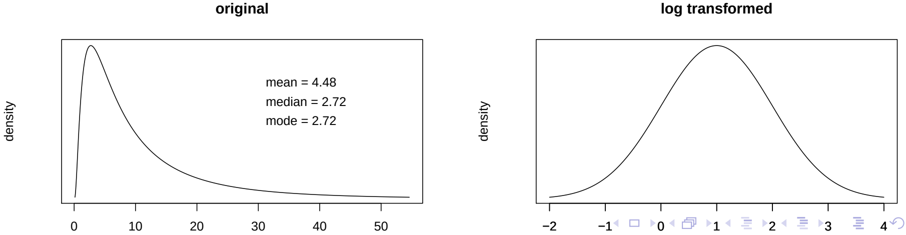

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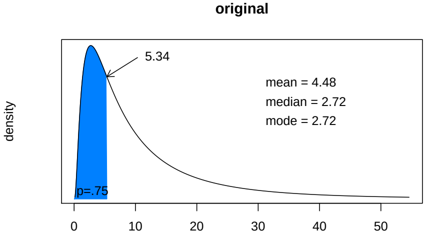
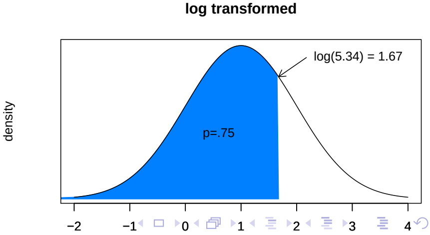
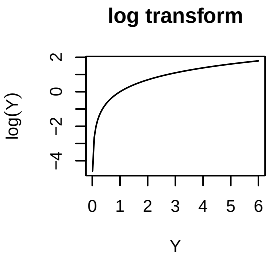
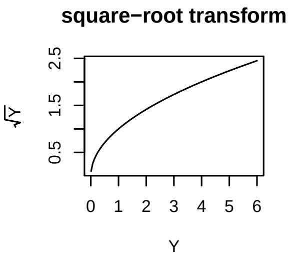




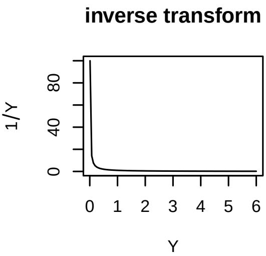
Some common transformations



$$Y_i' = \log(Y_i)$$

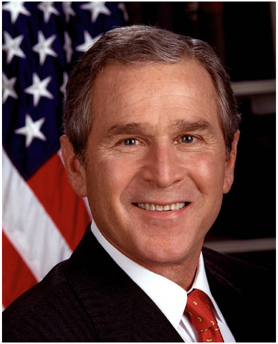


$$Y_i' = \sqrt{Y_i}$$



$$Y_i' = \frac{1}{Y_i}$$

The 2000 US elections



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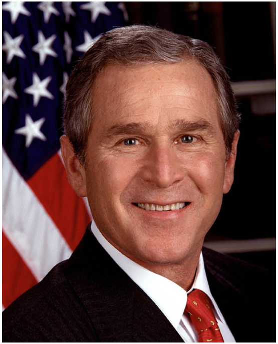
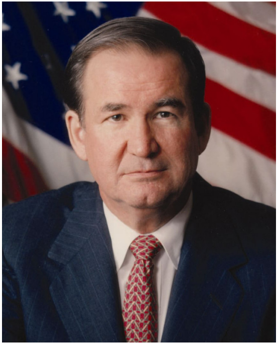
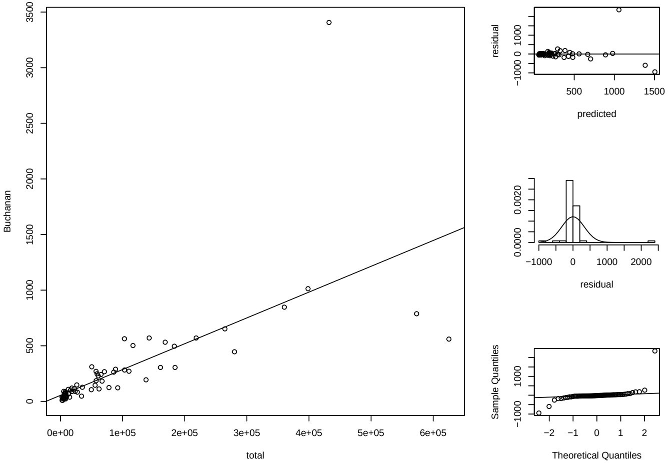




Buchanan votes in Florida

$$\texttt{Buchanan}_i = \beta_0 + \beta_1 \texttt{total}_i + \epsilon_i$$



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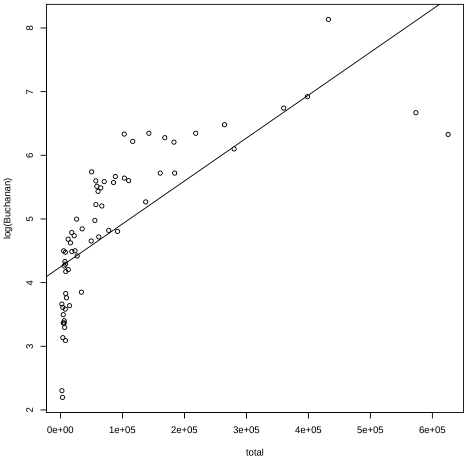
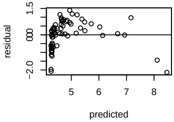
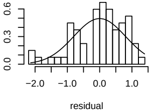
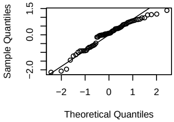
$$\log(\texttt{Buchanan})_i = \beta_0 + \beta_1 \texttt{total}_i + \epsilon_i$$









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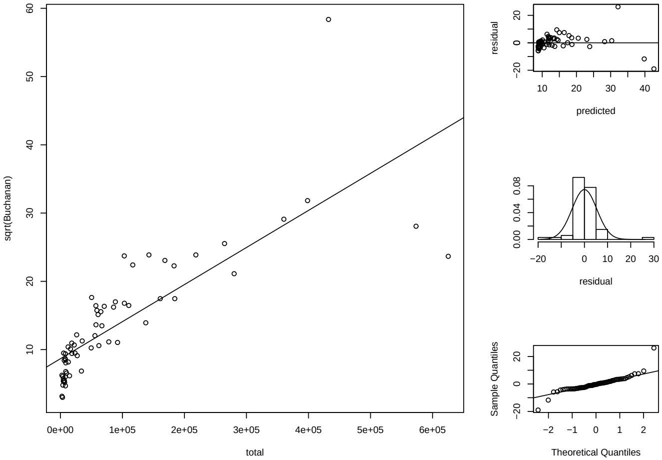

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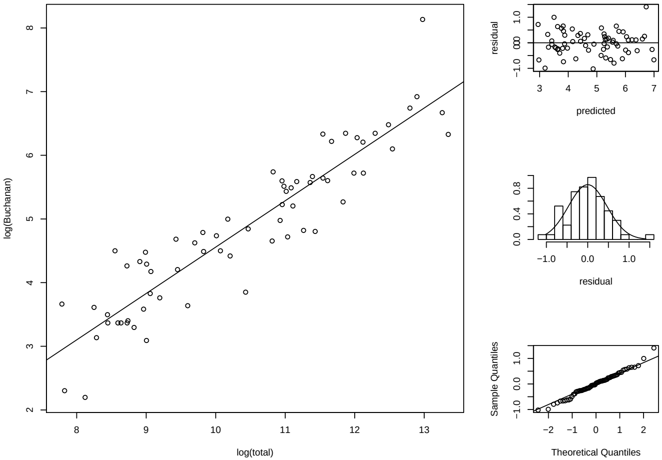

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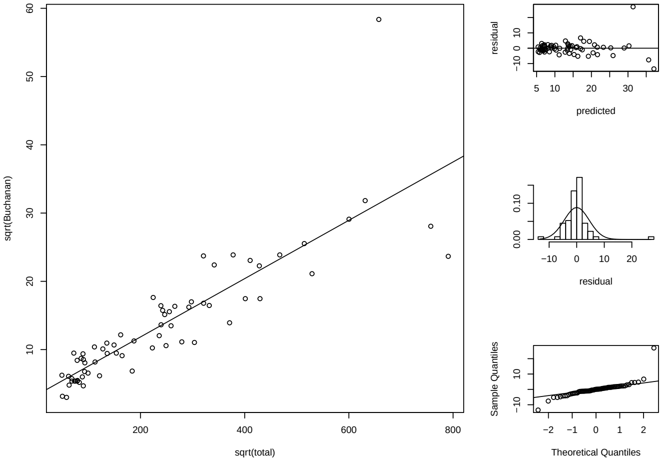
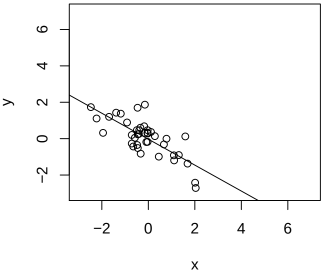


Outliers

- What are they?
  - "Unusual" data points
  - Far removed from other data points
- Consequences
  - Can have severe effect on estimates
- Detection
  - Residuals
  - Mahalanobis distance, Leverage, studentized deleted residual, Cook's distance
- Remedies
  - Remove them ...but carefully



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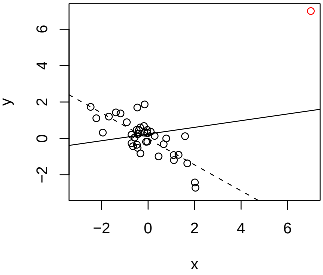

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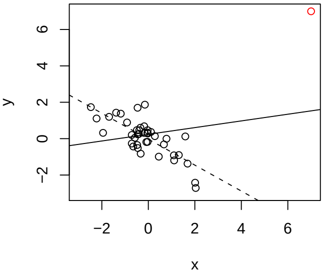

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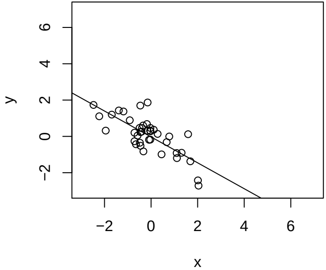

Measures for outlier detection

- Mahalanobis distance
  - Distance of a (multivariate) data point from the center (means)
  - Follows χ^2-distribution with $p-1$ degrees of freedom (for $p-1$ predictors)
- Leverage (lever)
  - Weight of data point in parameter estimates
  - Average leverage is $\bar{h} = \frac{p}{n}$, where p=number of parameters
  - High values (e.g., $> \frac{2p}{n}$) indicate possible problems
- Studentized deleted residual
  - Does a data point require its “own intercept”?
  - Follows t distribution with $n-p-1$ degrees of freedom
- Cook’s distance
  - Does omission of a data point change model predictions?
  - Combination of leverage and studentized deleted residual
  - Values larger than 1 (or 2) indicate possible problems

Multiple tests and Type I error

- When using outlier tests (e.g., studentized deleted residual), effectively performing n tests. Each test has

$$p(\text{type I error}) = p(\text{reject } H_0 | H_0 \text{ true}) = \alpha$$

- When performing multiple tests, probability of making *at least one* type I error is (much) larger than α! For n independent tests:

$$p(\text{at least 1 type I error}) = \alpha_{\text{FW}} = 1 - (1 - \alpha)^n$$

  e.g., with $n = 100$, $p(\text{at least 1 type I error}) = .994$
- To keep family-wise significance level α_{FW} under control, need to adjust α for each individual test. The Bonferroni correction is

$$\alpha = \frac{\alpha_{\text{FW}}}{n}$$

  which is easy to use but rather conservative

Multiple tests and Type I error

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$$p(\text{type I error}) = p(\text{reject } H_0 | H_0 \text{ true}) = \alpha$$

- When performing multiple tests, probability of making *at least one* type I error is (much) larger than α! For n independent tests:

$$p(\text{at least 1 type I error}) = \alpha_{\text{FW}} = 1 - (1 - \alpha)^n$$

e.g., with $n = 100$, $p(\text{at least 1 type I error}) = .994$

- To keep family-wise significance level α_{FW} under control, need to adjust α for each individual test. The Bonferroni correction is

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which is easy to use but rather conservative

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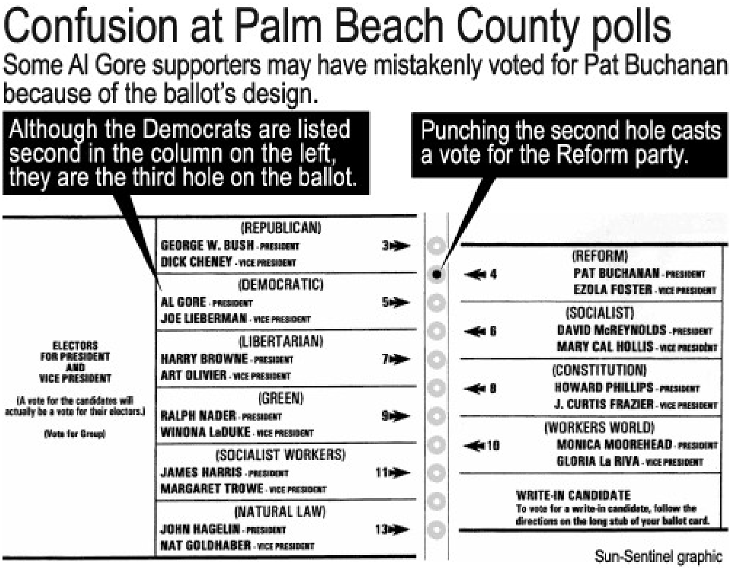
The 2000 US elections: The Palm Beach ballot



The 2000 US elections: The Palm Beach ballot

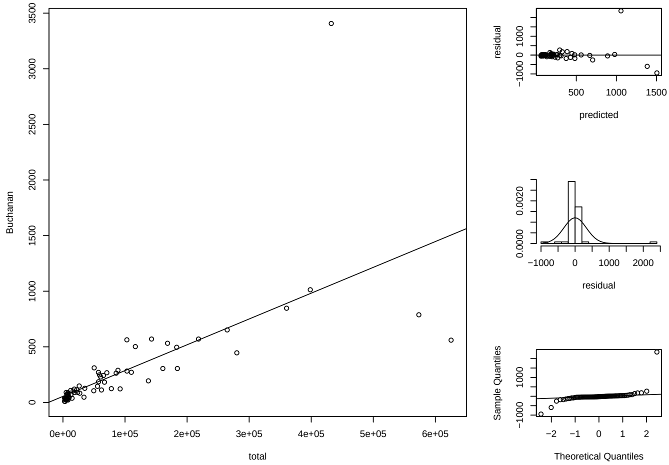
"Palm Beach County is a Pat Buchanan stronghold and that's why Pat Buchanan received 3,407 votes there."

Ari Fleischer, Spokesman for George W. Bush

"That's nonsense. [...] the number of Buchanan activists in the county [is] between 300 and 500 – nowhere near the 3,407 who voted for him."

Jim Cunningham, Palm Beach County's Reform Party

Is Palm Beach an outlier?



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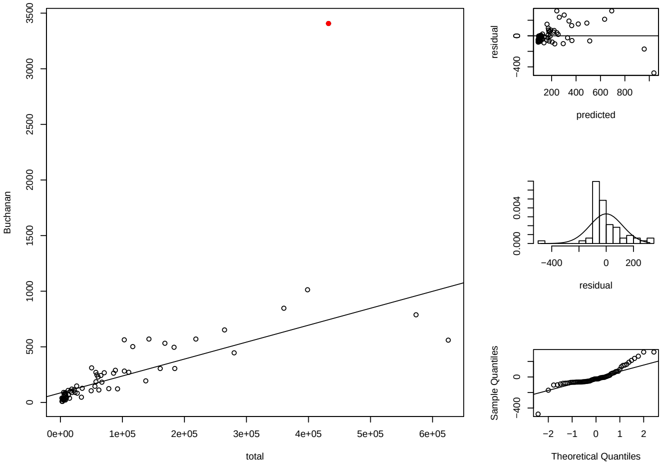
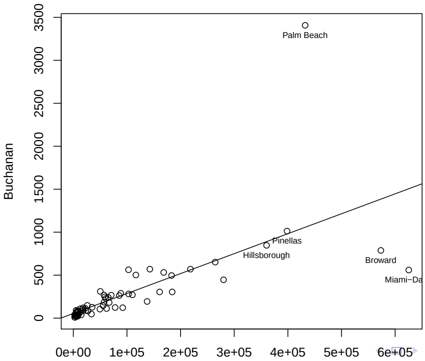
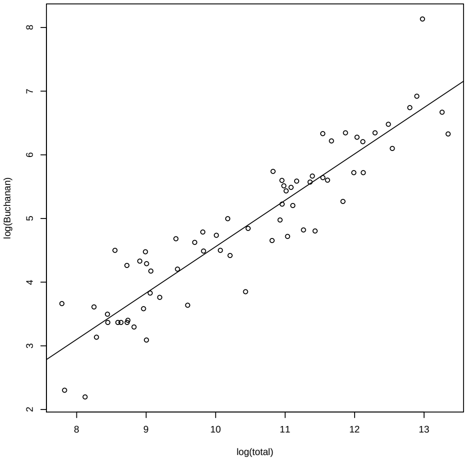


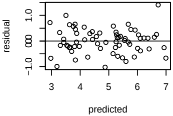
Is Palm Beach an outlier?

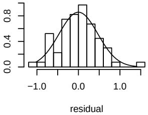
county	Malahanobis	Leverage	Studentized DR	Cook's D
Palm Beach	6.788	0.118*	20.735*	3.776*
Miami-Dade	16.568*	0.266*	-3.612*	1.994*
Pinellas	5.517	0.099*	0.107	0.001
Hillsborough	4.24	0.079*	-0.135	0.001
Broward	13.513*	0.220*	-2.083	0.581

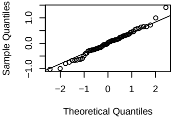


Is Palm Beach an outlier? (on log scale)









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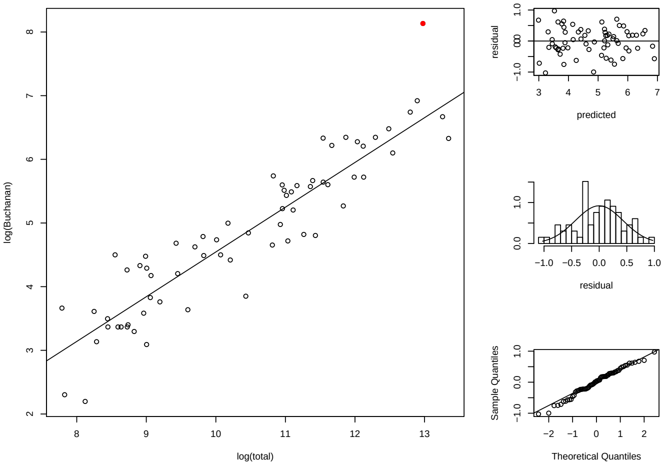
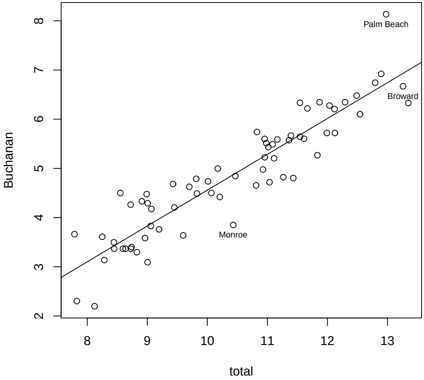


Is Palm Beach an outlier? (on log scale)

county	Malahanobis	Leverage	Studentized DR	Cook's D
Palm Beach	-33.997*	0.059*	3.327%	0.299
Monroe	-36.543*	0.015	-2.263	0.036
Broward	-33.715*	0.069*	-0.576	0.012



Multicollinearity

- What is it?
  - High correlation between predictor variables
  - Predictors account for same variation of Y
- Consequences
  - Estimation of parameters unreliable
  - Significance tests biased
- Detection
  - Tolerance $(1 - R_j^2)$, VIF $(\frac{1}{1-R_j^2})$
  - Correlation matrix
- Remedies
  - Remove collinear/correlated predictors
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Confidence interval slope:

$$b_j \pm \sqrt{\frac{F_{1,n-p;\alpha}\text{MSE}}{(n-1)S_{X_j}^2(1 - R_j^2)}}$$

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Partitioning SSE

MODEL C: $Y_i = b_0 + e_i$

MODEL A1: $Y_i = b_0 + b_1 X_{1i} + e_i$

MODEL A2: $Y_i = b_0 + b_2 X_{2i} + e_i$

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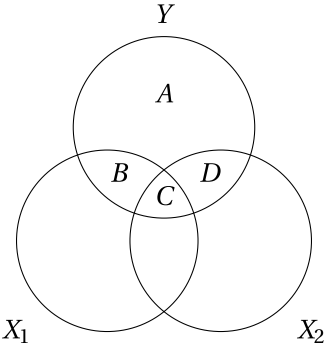

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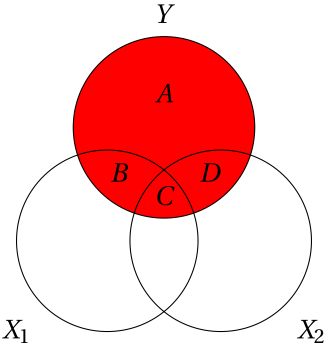
$A + B + C + D =$ SSE(C)



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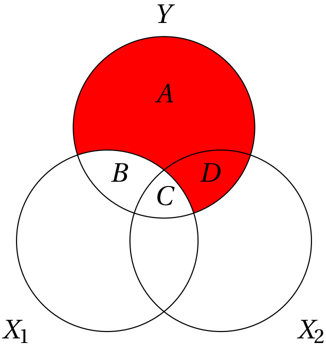

$A + B + C + D = \text{SSE(C)}$
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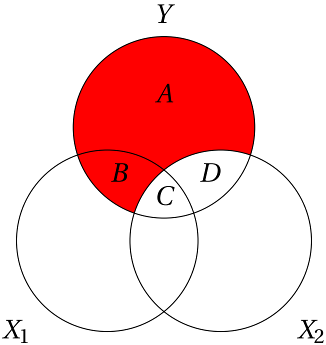

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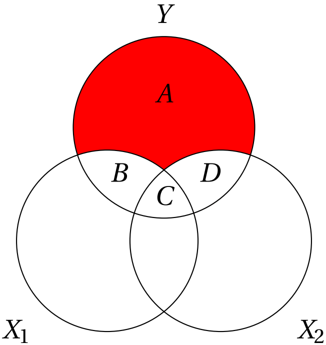

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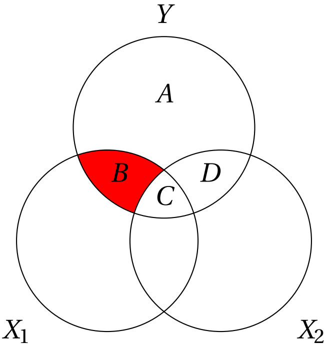

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Comparing A3 to A2:
$B = \text{SSR}(X_1)$

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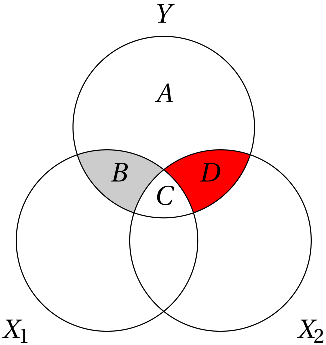

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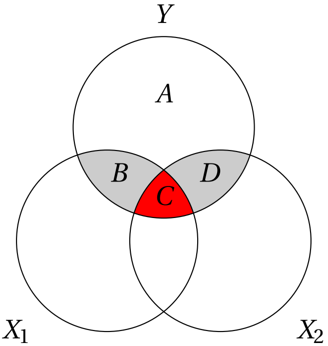

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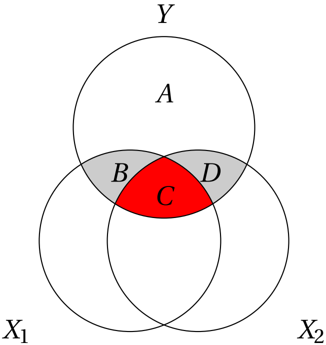

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Comparing A3 to A2:
$B = \text{SSR}(X_1)$
Comparing A3 to A1:
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$C = ?$

Polynomial regression

- Attempts to capture non-linear effects by also including powers of predictor

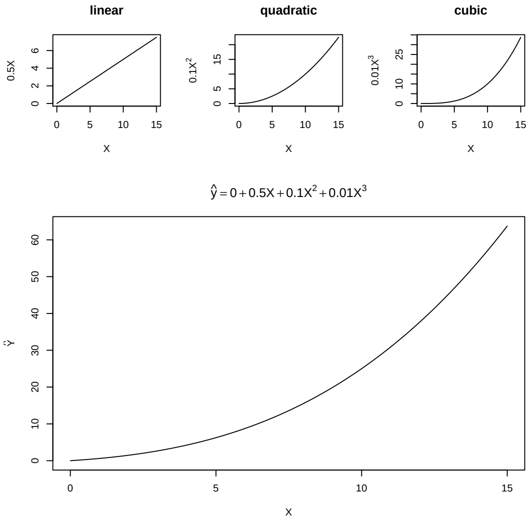
$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \epsilon_i$$

- Can include higher-order terms as well

$$Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \beta_3 X_i^3 \ldots + \beta_{p-1} X_i^{(p-1)} + \epsilon_i$$

(maximum $p = n$ parameters with n observations: perfect fit)

Polynomial regression is flexible



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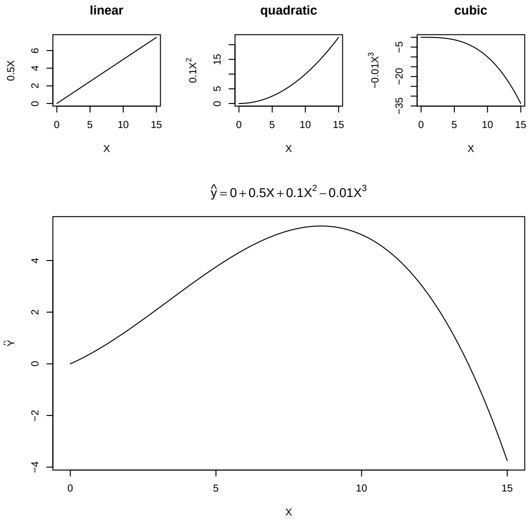

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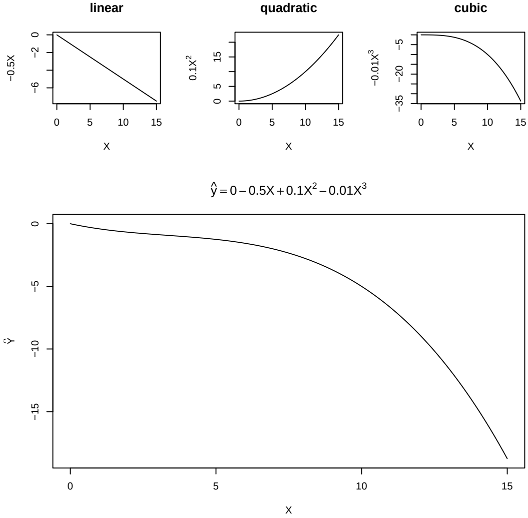

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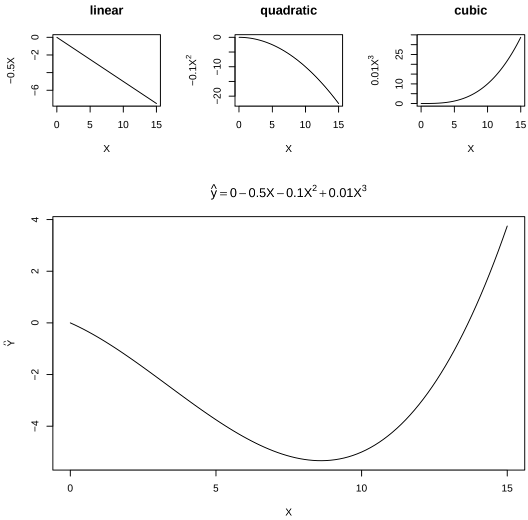
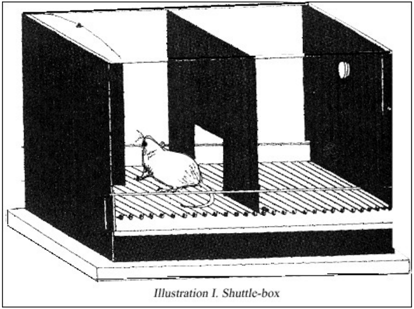
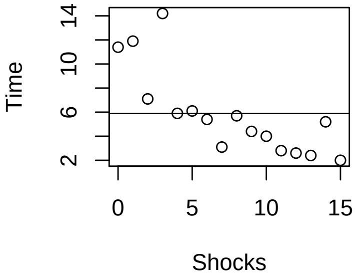


Example: avoidance learning in rats

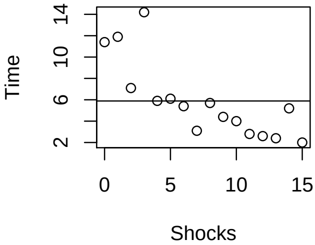




Example: avoidance learning in rats

Y = time to escape $\qquad$ X = number of shocks received

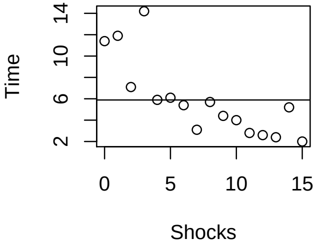
MODEL C: $Y_i = \beta_0 + \epsilon_i$



Example: avoidance learning in rats

Y = time to escape $\quad\quad$ X = number of shocks received

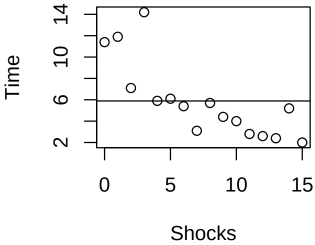
MODEL C: $Y_i = b_0 + e_i$



Example: avoidance learning in rats

Y = time to escape X = number of shocks received

MODEL C: $Y_i = 5.88 + e_i$



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Y = time to escape $\qquad$ X = number of shocks received

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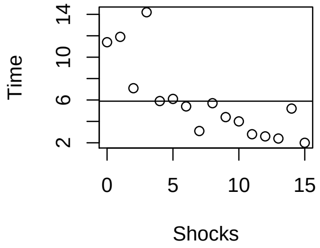
SSE(C) $= 199.05$



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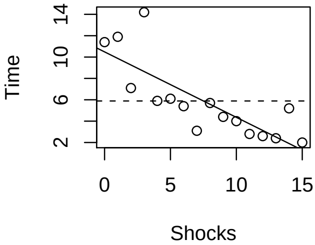

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MODEL C: $Y_i = 5.88 + e_i$
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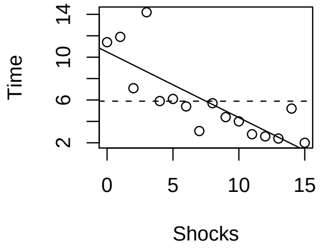

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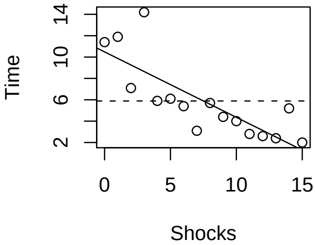
$F_{1,14} = 25.07, p < .001$, PRE $= 0.642$



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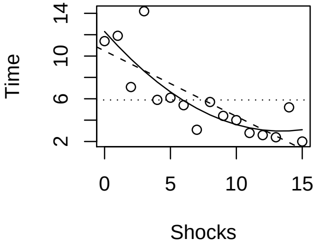
MODEL A2: $Y_i = \beta_0 + \beta_1 X_i + \beta_2 X_i^2 + \epsilon_i$



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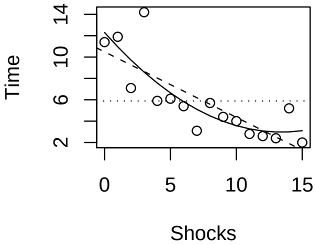
MODEL A1: $Y_i = 10.48 - 0.61X_i + e_i$
SSE(A1) $= 71.32$, $R^2 = 0.642$
$F_{1,14} = 25.07, p < .001$, PRE $= 0.642$

MODEL A2: $Y_i = 12.30 - 1.39X_i + 0.05X_i^2 + e_i$



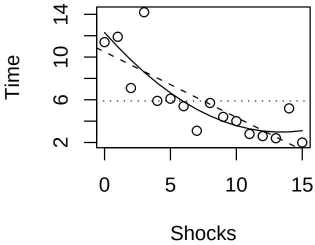
Example: avoidance learning in rats

Y = time to escape X = number of shocks received

MODEL C: $Y_i = 5.88 + e_i$
SSE(C) $= 199.05$

MODEL A1: $Y_i = 10.48 - 0.61X_i + e_i$
SSE(A1) $= 71.32$, $R^2 = 0.642$
$F_{1,14} = 25.07, p < .001$, PRE $= 0.642$

MODEL A2: $Y_i = 12.30 - 1.39X_i + 0.05X_i^2 + e_i$
SSE(A2) $= 55.98$, $R^2 = 0.719$
$F_{1,13} = 3.56, p = .08$, PRE $= 0.216$



Further reading

Judd, McClelland & Ryan:

- Chapter 13 for outliers and violation of model assumptions
- Part of Chapter 7 for polynomial regression

For next week:

- Remainder of Chapter 7 in Judd, McClelland & Ryan (2009)
- MacKinnon, Fairchild & Fritz (2007). Mediation analysis. *Annual Review of Psychology* (on Moodle, you can skip “Extensions of the single-mediator model”)