Moderation and mediation

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Statistics lecture 5

Outline

1. Moderation

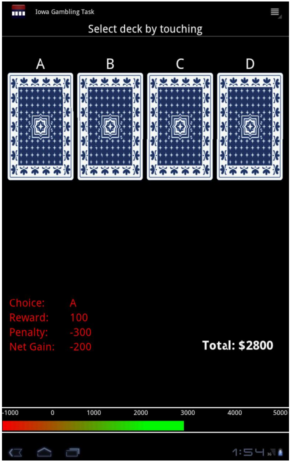
2. Mediation

Somatic markers and interoception in decision making

According to the Somatic Marker Hypothesis, decision-making is guided by emotional signals arising in the body. Dunn *et al.* (2010) conducted a study with an artificial gambling task and measured anticipatory reactions (somatic markers) to decisions.

Dunn *et. al.* (2010). Listening to your heart: How interoception shapes emotion experience and intuitive decision making. *Psychological Science*, *21*, 1835–1844.

Somatic markers and interoception in decision making



Iowa Gambling Task:

- Two decks are profitable in the long run, two decks unprofitable
- If somatic markers offer good guidance, arousal when choosing a profitable deck will be lower than when choosing an unprofitable deck.
- But any effect should depend on the extent to which somatic markers are actually perceived

Somatic markers and interoception in decision making

Hypothesis

The effect of somatic markers is moderated by "interoceptive accuracy": when somatic markers offer good guidance, better interoception should improve decision-making, but when markers offer poor guidance, better interoception should worsen decision-making.

Somatic markers and interoception in decision making

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Each of $n = 92$ participants performed 100 trials of the gambling task. The following measures were obtained:

- Decision quality (`Dqual`): number of profitable deck choices - number of unprofitable deck choices (between -100 and 100)
- Somatic marker (`SoMa`): mean arousal to unprofitable decks - mean arousal to profitable decks.
- Interoceptive accuracy (`Iacc`): performance in the Schandry heartbeat-perception task.

Somatic markers and interoception in decision making

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Relation between `SoMa` and `Dqual` should be *positive* when `Iacc` is high, but 0 when `Iacc` is 0.

Somatic markers and interoception in decision making

I have simulated data in accordance with Dunn et al's results. A linear model

$$\texttt{Dqual}_i = \beta_0 + \beta_1 \times \texttt{SoMa}_i + \beta_2 \times \texttt{Iacc}_i + \epsilon_i \qquad \epsilon_i \sim N(0,\sigma)$$

gave the following results

	b	SS	df	F	p
(intercept)	4.018	6.57	1	.01	.92
SoMa	3.43	8446.1	1	10.11	.002
Iacc	16.818	3.87	1	0.00	0.95
error		74364.94	89		

Somatic markers and interoception in decision making

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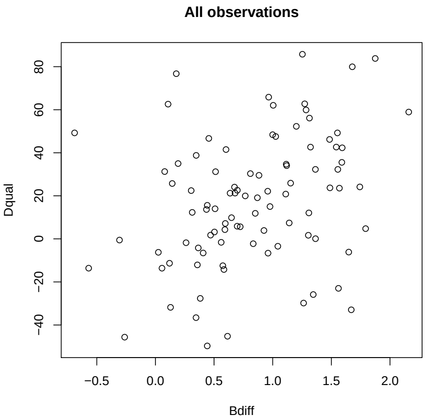
Does interoceptive accuracy have no effect on decision making?

Relation between `Dqual` and `SoMa` by `Iacc`



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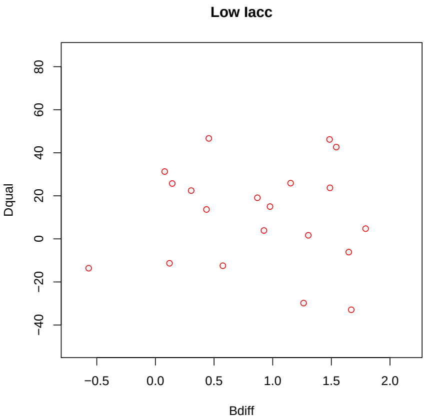

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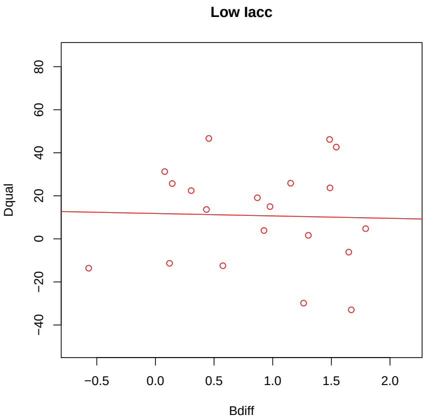

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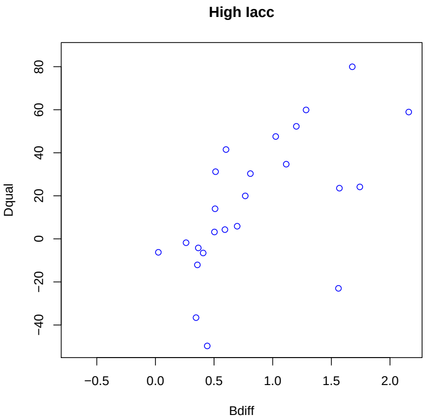

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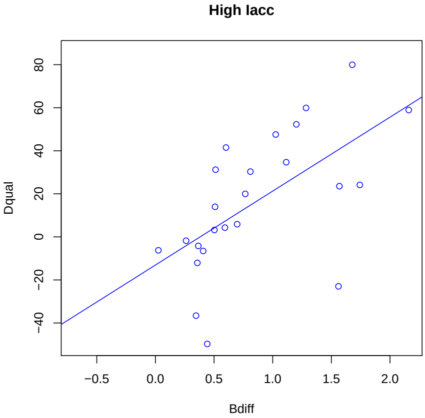
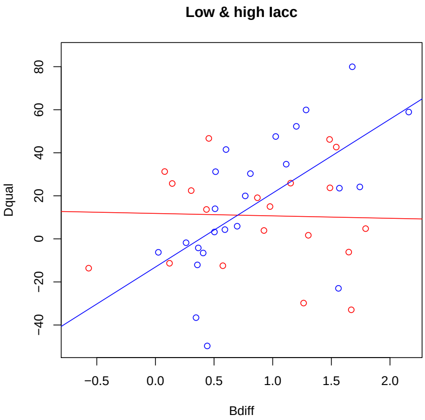


Relation between `Dqual` and `SoMa` by `Iacc`



Relation between `Dqual` and `SoMa` by `Iacc`

Does the slope of `SoMa` increase with the level of `Iacc`?

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$$\texttt{Dqual}_i = \beta_0 + \beta_i \times \texttt{SoMa}_i + \epsilon_i \qquad \epsilon_i \sim N(0, \sigma)$$

Relation between `Dqual` and `SoMa` by `Iacc`

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Idea: model the slope of `SoMa` with a linear model:

$$\beta_i = \beta_1 + \beta_2 \times \texttt{Iacc}_i$$

Relation between Dqual and SoMa by Iacc

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$$\beta_i = \beta_1 + \beta_2 \times \texttt{Iacc}_i$$

$$\begin{aligned} \texttt{Dqual}_i &= \beta_0 + (\beta_1 + \beta_2 \times \texttt{Iacc}_i) \times \texttt{SoMa}_i + \epsilon_i \\ &= \beta_0 + \beta_1 \times \texttt{SoMa}_i + \beta_2 \times (\texttt{Iacc}_i \times \texttt{SoMa}_i) + \epsilon_i \end{aligned}$$

Relation between Dqual and SoMa by Iacc

Does the slope of SoMa increase with the level of Iacc?

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β_1 is the slope of SoMa when $\texttt{Iacc} = 0$

Relation between `Dqual` and `SoMa` by `Iacc`

Does the slope of `SoMa` increase with the level of `Iacc`?

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Idea: model the slope of `SoMa` with a linear model:

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β_1 is the slope of `SoMa` when $\texttt{Iacc} = 0$
β_2 is the *increase* in the slope of `SoMa` for each 1-unit increase in `Iacc`

Modelling the dependence of the slope of `Iacc`

$$\texttt{Dqual}_i = \beta_0 + \beta_1 \times \texttt{SoMa}_i + \beta_2 \times (\texttt{Iacc}_i \times \texttt{SoMa}_i) + \epsilon_i$$

	b	SS	df	F	p
(intercept)	3.010	264.16	1	0.33	0.57
`SoMa`	-54.01	1254	1	1.55	0.22
`SoMa`×`Iacc`	80.13	2188.1	1	2.70	0.10
error		72181	89		

Modelling the dependence of the slope of `Iacc`

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That's a shame. . . or . . .

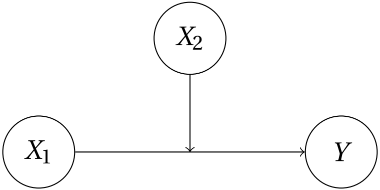
Modelling the dependence of the slope of `Iacc`

We should also include `Iacc` itself. Although it was not significant before, that was in a different model. . .

$$\texttt{Dqual}_i = \beta_0 + \beta_1 \times \texttt{SoMa}_i + \beta_2 \times \texttt{Iacc}_i + \beta_3 \times (\texttt{Iacc}_i \times \texttt{SoMa}_i) + \epsilon_i$$

	b	SS	df	F	p
(intercept)	167.54	3956.2	1	5.09	.026
`SoMa`	-184.91	5031.5	1	6.48	.012
`Iacc`	-185.37	3833.9	1	4.94	.029
`SoMa`×`Iacc`	228.21	6018.1	1	7.75	.007
error		68347	88		

Interactions: moderating relations

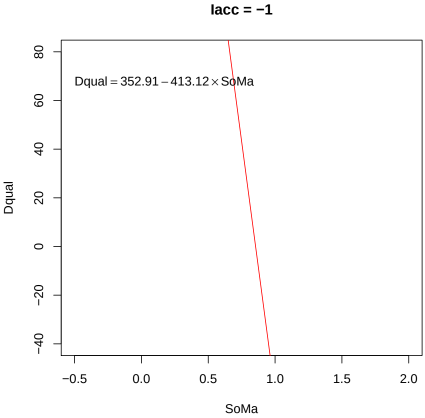


Two predictors interact when their slope depends on one another

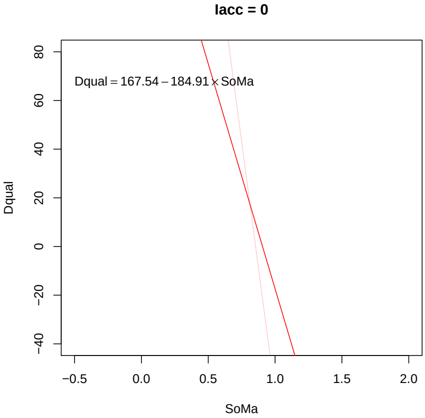
$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_1 \times X_2)_i + \epsilon_i$$

- β_0 is the (predicted) value of Y when $X_1 = 0$ and $X_2 = 0$
- β_1 is the slope for X_1 when $X_2 = 0$
- β_2 is the slope for X_2 when $X_1 = 0$
- β_3 is the increase in β_1 for each 1-unit increase in X_2, and simultaneously the increase in β_2 for each unit increase in X_1

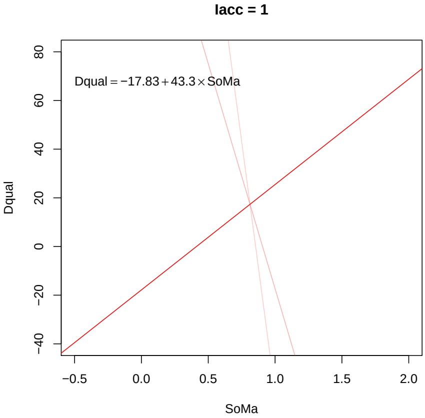
Relation between `Dqual` and `SoMa` by `Iacc`



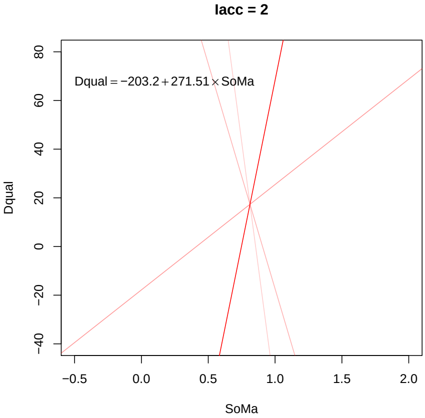
Relation between `Dqual` and `SoMa` by `Iacc`



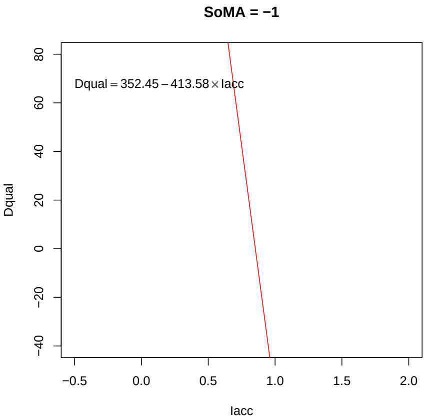
Relation between `Dqual` and `SoMa` by `Iacc`



Relation between Dqual and SoMa by Iacc



Relation between `Dqual` and `Iacc` by `SoMa`



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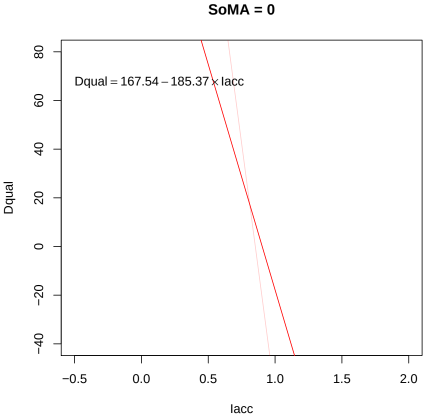
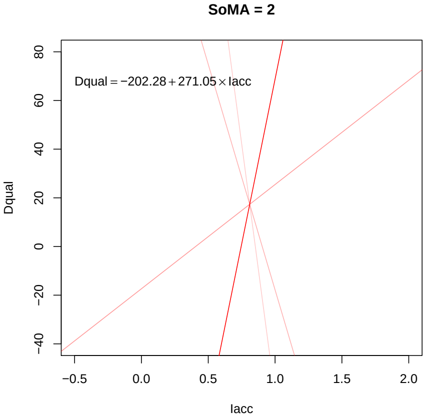


Relation between Dqual and Iacc by SoMa

Relation between `Dqual` and `Iacc` by `SoMa`



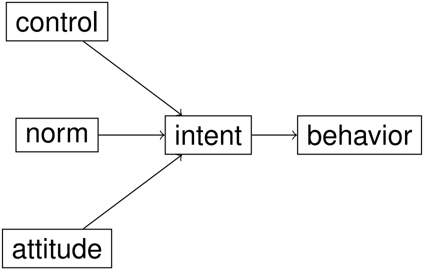
Centering predictors

$$Y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 (X_1 X_2)_i + \epsilon_i$$

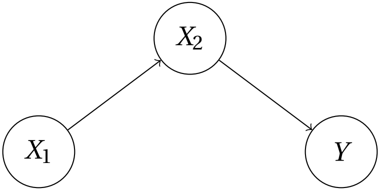
- "Simple slope" β_1 is effect of X_1 when $X_2 = 0$
- Test $H_0 : \beta_1 = 0$ tests whether slope is 0 when $X_2 = 0$. Might be more interesting to test H_0 at *average* levels of other predictors.
- This can be achieved by centering predictors (subtracting their mean). E.g. $X_1' = X_1 - \overline{X}_1$
- Centering affects intercept and simple slopes:

	b	SS	df	F	p
(intercept)	17.73	28912	1	37.23	$< .001$
`SoMa_c`	19.24	10747	1	13.84	$< .001$
`Iacc_c`	4.51	6.70	1	0.01	0.93
`SoMa_c`×`Iacc_c`	228.21	6018.1	1	7.75	.007
error		68347	88		

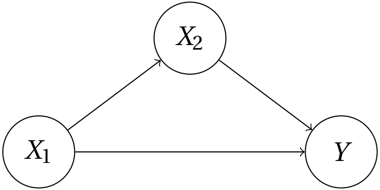
Theory of Planned Behaviour (Ajzen & Fishbein)



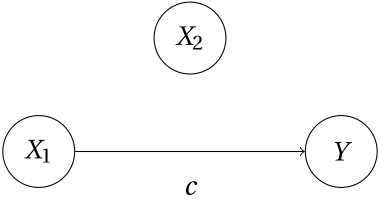
Mediation



Causal steps (Baron & Kenny, 1986)

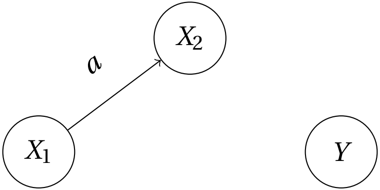


Causal steps (Baron & Kenny, 1986)



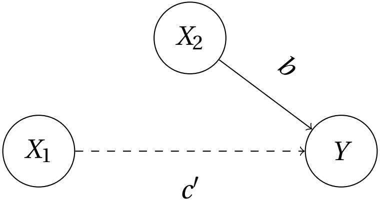
Model 1: $Y_i = \beta_{0(1)} + cX_{1i} + \epsilon_i$ $\quad c \neq 0 \quad$ ($X_1 \rightarrow Y$ signif.)

Causal steps (Baron & Kenny, 1986)



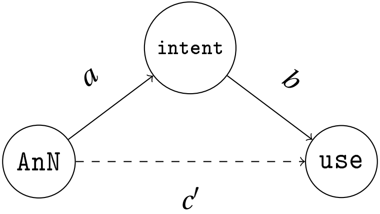
Model 1:	$Y_i = \beta_{0(1)} + cX_{1i} + \epsilon_i$	$c \neq 0$	($X_1 \rightarrow Y$ signif.)
Model 2:	$X_{2i} = \beta_{0(2)} + aX_{1i} + \epsilon_i$	$a \neq 0$	($X_1 \rightarrow X_2$ signif.)

Causal steps (Baron & Kenny, 1986)



Model 1:	$Y_i = \beta_{0(1)} + cX_{1i} + \epsilon_i$	$c \neq 0$	($X_1 \rightarrow Y$ signif.)
Model 2:	$X_{2i} = \beta_{0(2)} + aX_{1i} + \epsilon_i$	$a \neq 0$	($X_1 \rightarrow X_2$ signif.)
Model 3:	$Y_i = \beta_{0(3)} + c'X_{1i} + bX_{2i} + \epsilon_i$	$b \neq 0$	($X_2 \rightarrow Y$ signif.)
		$c' = 0$	($X_1 \rightarrow Y$ not signif.)

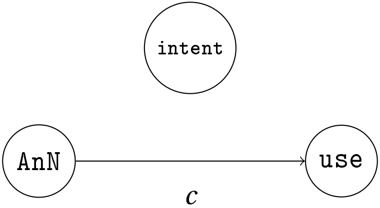
TPB: causal steps



Contraception use by 16-24 year male adolescents

- `use` (Y): usage frequency ("How often do you...")
- `intent` (X_2): intent ("In general do you intend to...")
- `AnN` (X_1): Attitude & norms, an aggregate of
  - 3 attitude measures
  - 3 social norm measures

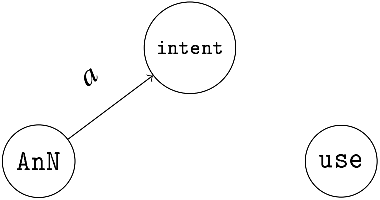
TPB: causal steps



Model 1: $\texttt{use}_i = \beta_0 + c \times \texttt{AnN}_i + \epsilon_i$

effect	b	SS	df	F	p
intercept	-0.08	0.06	1	0.07	0.79
AnN	1.02	90.55	1	100.57	<.001
error		223.30	248		

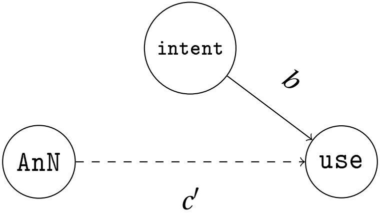
TPB: causal steps



Model 2: $\texttt{intent}_i = \beta_0 + a \times \texttt{AnN}_i + \epsilon_i$

effect	b	SS	df	F	p
intercept	-0.76	5.41	1	8.37	.004
`AnN`	1.24	133.01	1	205.89	<.001
error		160.21	248		

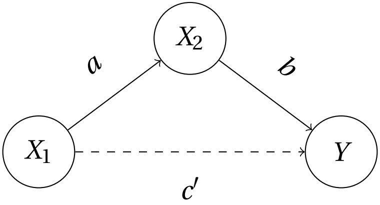
TPB: causal steps



Model 3: $\texttt{use}_i = \beta_0 + c' \times \texttt{AnN}_i + b \times \texttt{intent}_i + \epsilon_i$

effect	b	SS	df	F	p
intercept	0.44	1.72	1	2.84	.093
`AnN`	0.18	1.53	1	2.54	.112
`intent`	0.68	74.05	1	122.54	<.001
error		149.25	247		

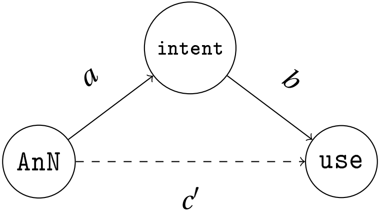
Sobel-Aroian test



- Tests the significance of the indirect effect of X_1 on Y
- Product of regression coefficients, $a \times b$
- Divide by standard error of $a \times b$ and compare to standard Normal (Z) distribution

$$Z = \frac{ab}{\sqrt{a^2\sigma_b^2 + b^2\sigma_a^2 + \sigma_a^2\sigma_b^2}}$$

TPB: Sobel test



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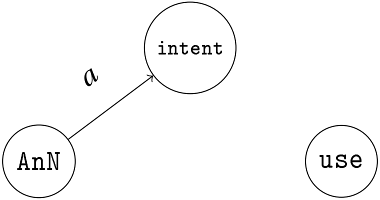
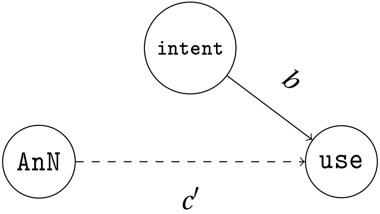


Model 2: $\texttt{intent}_i = \beta_0 + a \times \texttt{AnN}_i + \epsilon_i$

effect	b	SE(b)	t	p
intercept	-0.76	0.263	-2.89	.004
AnN	1.24	0.086	1.59	<.001

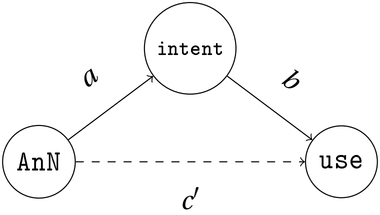
TPB: Sobel test



Model 3: $\texttt{use}_i = \beta_0 + c' \times \texttt{AnN}_i + b \times \texttt{intent}_i + \epsilon_i$

effect	b	SE(b)	t	p
intercept	0.44	0.259	1.69	.093
AnN	0.18	0.113	2.54	.112
intent	0.68	0.061	11.07	<.001

TPB: Sobel-Aroian test



$$Z = \frac{ab}{\sqrt{a^2\sigma_b^2 + b^2\sigma_a^2 + \sigma_a^2\sigma_b^2}} = \frac{1.24 \times 0.68}{\sqrt{1.53 \times 0.061^2 + 0.46 \times 0.086^2 + 0.007 \times 0.004}}$$
$$= 8.75$$

$$P(|Z| \geq 8.75) < .001$$

Problems in mediation analysis

- Causal steps procedure often low power
  - Requirement of significant effect of X_1 on Y problematic
  - Simultaneous test of a and b might be better
- Using standard normal distribution in Sobel test often bad approximation
  - True distribution can be approximated by simulation
- More complex mediation models can be estimated and tested using Structural Equation Modelling (SEM)

Summary of key points

- Moderation means that the effect of a predictor depends on the value of another
  - Include the product of two predictors as a new predictor
  - Changes interpretation of “simple slopes”
  - Centering can be sensible
  - Moderation is “symmetric”
- Mediation means that a predictor affects the dependent variable only “through” another predictor
  - A “causal chain” model
  - Two ways of assessing mediation:
    - The causal steps procedure involves testing slopes in three regression models
    - The Sobel-Aroian assesses tests the indirect effect directly

Further reading

This week:

- Judd, McClelland & Ryan (2009), chapter 7
- MacKinnon, D.P., Fairchild, A.J. & Fritz, M.S. (2007). Mediation analysis. *Annual Review of Psychology*, *58*, 593–614. (you can skip "Extensions of the single-mediator model")

Further reading:

- Baron, R.M. & Kenny, D.A. (1986) The moderator-mediator variable distinction in social psychological research: Conceptual. strategic, and statistical considerations. *Journal of Personality and Social Psychology*, *51*, 1173–1182.

For next week:

- Judd, McClelland & Ryan (2009), chapters 8 and 9